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Improvement of accuracy of simple methods for design and analysis of a blazed phase grating microstructure

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In order to precisely analyze and design the transmittance characteristics of a blazed grating, the validity of both the scalar diffraction theory and the effective medium theory is quantitatively demonstrated. By making a comparison of diffraction efficiencies calculated by the two simplified methods and Fourier modal method, the accuracy can be obtained. It is found that when the normalized period is more than three wavelengths of the incident light, the scalar diffraction theory is useful to calculate the transmittance of the blazed grating within the error of less than 3%. The validity of the scalar diffraction theory increases when the normalized period increases. Importantly, by considering the Fresnel reflection effect, the validity of scalar diffraction theory can be significantly enhanced. Furthermore, when no higher-order diffraction waves appear and only zeroth order diffraction wave propagates, the effective medium theory is accurate to compute the diffraction efficiency within the difference of less than 1% between the zeroth order effective medium theory and Fourier modal method. The polarization characteristics of the validity of effective medium theory are also quantitatively demonstrated. The validity of the two simplified theories is dependent on not only the normalized period of surface microstructure but also the normalized groove depth.

Keywords: grating, effective medium theory, scalar diffraction theory.

1. Introduction

The blazed grating is widely applied in diffractive optical elements (DOEs) [1, 2]. In general, the scalar diffraction theory (SDT) can be frequently used in the design and analysis of these diffractive optical elements when the normalized period of the grating is large compared with incident wavelength [2]. With the development of microfabrication technologies, the feature size of the blazed grating can be manufactured in the subwavelength region in which the SDT is inapplicable to design and analyze the diffraction efficiency of the blazed grating. In this domain of feature size of a grating with the period comparable to the incident wavelength, only the rigorous vector theories can be applied to yield accurate diffraction characteristics, such as the Fourier model method (FMM) [3–5], the rigorous coupled wave analysis (RCWA) [6], and the finite - difference time-domain (FDTD) method [7]. However, these rigorous vector methods are difficult to be used for computationally intensive studies. In this paper, we present the accuracy and validity of the scalar diffraction theory for simply designing and analyzing the blazed grating by a comparison of the transmittances calculated by the SDT and FMM. Moreover, the accuracy of the SDT can be clearly enhanced by considering the Fresnel reflection effect in this work.

Furthermore, when the normalized period of the grating profile is much smaller than the wavelength of the incident illumination, it is well known that the effective medium theory (EMT) can be used to compute the diffraction efficiency of the grating. Generally, it is recognized that the EMT is inaccurate for analyzing periodic surface microstructure with the normalized period more than tenth of the incident wavelength [8]. However, the results in this paper verified that the validity of the EMT can be extended as only the zero order waves are to propagate. Therefore, based on the determination of the accuracy of the SDT and EMT, we can intuitively understand and easily analyze the optical characteristics of a sawtooth surface microstructure grating. The SDT and EMT are more intuitive and simple than rigorous treatments.

In this study, the transmittances of the blazed grating with respect to the normalized period and the normalized groove depth at normal incidence are investigated. Through the comparison of the diffraction efficiencies predicted by the simplified methods with those calculated by the FMM, the validity of the SDT and EMT is fully determined quantitatively. By considering the Fresnel reflection effect, the accuracy of SDT in design and analysis of a blazed grating can be significantly improved.

2. Theory

2.1. Fourier modal method

In Figure 1, Λ and d represent the period and groove depth, respectively, and n_0 and n_g are the refractive index of the incident medium and grating, respectively. In this paper, we choose $n_0 = 1.00$ and $n_g = 1.50$. Meanwhile, we assume $n_s = n_g$. Additionally, the absorption loss of the medium and the dispersion effect are ignored in our study for simplification.

According to the rigorous vector method of the FMM [5, 9], a sawtooth grating depicted in Fig. 1 can be approximated by a multilayer lamellar grating along the vertical direction. The electromagnetic field can be gained by solving the Maxwell's equations, which reduces to the solution of an algebraic eigenvalue problem in discrete Fourier space for each of the lamellar grating layers. Afterwards, the reflection and transmission coefficient matrix (RTCM) propagation algorithm [10], a numerically more valid variant of *S*-matrix algorithm than any of known forms [11], was used to calculate the amplitude coefficient matrix of modal fields with the boundary condi-

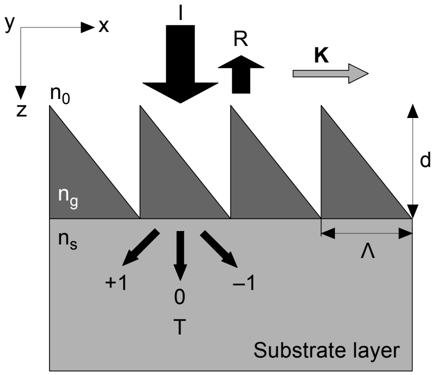

Fig. 1. The schematic diagram of a blazed surface microstructure grating with depth d, period Λ and grating vector **K**. Parameter n_0 denotes the refractive index of incident medium, n_g – the refractive index of grating, and n_s – the refractive index of substrate layer. A plane wave is incident at normal.

tions. Therefore, the reflectivity and transmittance of this kind of diffraction optical component can be derived.

Furthermore, as is known, the accuracy of FMM is dependent on the number of spatial harmonics. The results calculated by FMM can converge to the exact solution with the increase in the number of spatial harmonics. Hence, in order to ensure the accuracy of the calculated diffraction efficiency, the convergence of FMM as a function of Fourier order with the surface microstructure parameter of $d/\lambda = 0.5$ and $\Lambda/\lambda = 0.5$ is given in Fig. 2. In our calculation, the TE (TM) polarization is defined by the electric field vector parallel (perpendicular) to the grating groove. It can be seen in Fig. 2 that the convergence for the TE polarization is more effective than that for TM polarization, which also exists in other rigorous vector methods such as RCWA [12]. So in our calculation the sufficient Fourier orders, 21 and 33 for TE and TM polarizations, respectively, were chosen to insure all convergent results. The adequate divided lamellar grating layers of 20 were set for giving a sufficient approximation to this sawtooth grating.

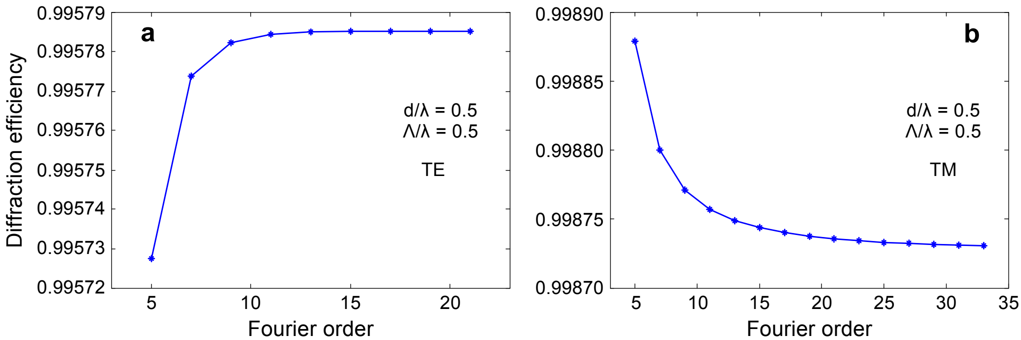

Fig. 2. The convergence of FMM for TE and TM polarizations with adequately divided multilayer lamellar grating for the sawtooth surface profile at normal incidence for TE (**a**) and TM (**b**) polarization with the normalized period of 0.5 and the normalized groove depth of 0.5.

2.2. Scalar diffraction theory

2.2.1. Without the Fresnel reflection effect

Utilizing the scalar Kirchhoff diffraction theory, the diffraction efficiency of a grating can be calculated. It neglects the vectorial polarized nature of light but gives reasonably precise results when the periodicity of surface profile is much larger than the wavelength of incident light. In scalar approximation, the general equation of the diffraction efficiency η_m is represented by [13]

$$\eta_m(\lambda) = \left| \frac{1}{\Lambda} \int_0^{\Lambda} t(x) \exp\left(-\frac{2\pi i m x}{\Lambda}\right) \mathrm{d}x \right|^2 \tag{1}$$

where $t(x)$ is a function defined as the ratio of transmitted (or reflected) and incident wave amplitudes at location x, Λ is the period, and m is the diffraction order. Then the m order transmittance efficiency of a sawtooth surface structure at normal incidence can be derived as [13]

$$\eta_m = \left| \frac{\sin\left[\left(m - \frac{n_\mathrm{g} - 1}{\lambda} d\right)\pi\right]}{\left(m - \frac{n_\mathrm{g} - 1}{\lambda} d\right)\pi} \right|^2 \tag{2}$$

According to this formula, we can obtain the diffraction efficiencies of the grating related to the depth of grating structure and the wavelength of incidence light, but they are independent of the period of surface profiles.

2.2.2. With the Fresnel reflection effect

To more accurately calculate the transmittance efficiency, we take the Fresnel reflection factor into account. Then, the m order transmittance efficiency of the phase blazed grating can be expressed as

$$\eta_m = \frac{n_\mathrm{g} \cos(\theta_\mathrm{g})}{n_0 \cos(\theta_0)} \tau^2(\theta_0) \left| \frac{\sin\left[\left(m - \frac{n_\mathrm{g} - 1}{\lambda} d\right)\pi\right]}{\left(m - \frac{n_\mathrm{g} - 1}{\lambda} d\right)\pi} \right|^2 \tag{3}$$

where $\tau(\theta_0)$ is the Fresnel transmission coefficient. The factor $n_\mathrm{g}\cos(\theta_\mathrm{g})/n_0\cos(\theta_0)$ is also considered because the transmitted light and the incident light are in different mediums. Specifically, according to the Fresnel formulae, the Fresnel transmission factor in Eq. (3) can be computed to be 0.96 at the normal incidence with $n_0 = 1.00$ and $n_\mathrm{g} = 1.50$. The quantitative analysis and comparison between two cases, with and without the Fresnel reflection effect, respectively, will be shown in next sections.

2.3. Effective medium theory

When the period Λ of the grating is much smaller than the incident wavelength, the EMT is useful to evaluate the transmittance characteristics exactly. In the quasi-static limit (the period-to-wavelength ratio $\Lambda/\lambda \to 0$), the zero order EMT can be utilized to describe the effective refractive index. But with the increase of the period of the grating which is only several times smaller than the wavelength of an incident wave, the use of higher order EMT is indispensable. It is well known that RYTOV deduced the second order EMT which made no static-field approximations and expanded the expressions on the term of Λ/λ [14]. As Λ/λ becomes larger, higher order terms from the series expansions of transcendental equations must be analyzed. Consequently, the zeroth and the second orders EMT are chosen to calculate the transmittances in our study. The estimated transmittivities are compared with the results calculated by FMM, to evaluate the valid one of the EMT. Besides, one should be noted that it is inappropriate to use EMT when higher order diffraction waves rather than zeroth order begin to propagate because EMT is based on the premise of only the zeroth diffraction order propagating.

For the second-order EMT on the Λ/λ, the effective indices of the refraction (assuming that permeabilities of both the incident and substrate medium are equal to free space's permeability) are represented by [15, 16]

$$n_{\mathrm{TE}}^{(2)} = \left[\left(n_{\mathrm{TE}}^{(0)}\right)^2 + \frac{1}{3}\left(\frac{\Lambda}{\lambda}\right)^2 \pi^2 f^2 (1-f)^2 \left(n_{\mathrm{g}}^2 - n_0^2\right)^2 \right]^{1/2} \tag{4}$$

when **E** is perpendicular to grating vector **K** (TE polarization) and by

$$n_{\mathrm{TM}}^{(2)} = \left[\left(n_{\mathrm{TM}}^{(0)}\right)^2 + \frac{1}{3}\left(\frac{\Lambda}{\lambda}\right)^2 \pi^2 f^2 (1-f)^2 \left(\frac{1}{n_{\mathrm{g}}^2} - \frac{1}{n_0^2}\right)^2 \left(n_{\mathrm{TM}}^{(0)}\right)^6 \left(n_{\mathrm{TE}}^{(0)}\right)^2 \right]^{1/2} \tag{5}$$

when **E** is parallel to grating vector **K** (TM polarization). In Eqs. (4) and (5), f is the duty cycle of lamellar structure, and $n_{\mathrm{TE}}^{(0)}$ and $n_{\mathrm{TM}}^{(0)}$ represent the effective indices of refraction for the zero order EMT approximations, respectively, as

$$n_{\mathrm{TE}}^{(0)} = \left[(1-f) n_0^2 + f n_{\mathrm{g}}^2 \right]^{1/2} \tag{6}$$

$$n_{\mathrm{TM}}^{(0)} = \left(\frac{1-f}{n_0^2} + \frac{f}{n_{\mathrm{g}}^2} \right)^{1/2} \tag{7}$$

For a sawtooth surface grating profile in Fig. 1, it can be approximated as a large number of lamellar grating layers in which they have different duty cycles f in depth direction as shown in Fig. 3. As the number of the divided layers is increased, the consistency to the shape of the blazed grating is enhanced. Hence the region of grating surface can be considered as a stack of effective homogeneous thin films. The effective

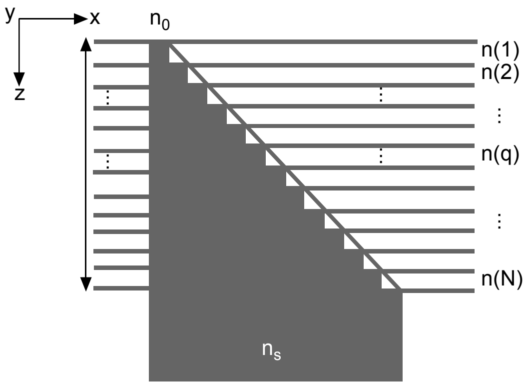

Fig. 3. The effective film stack of an approximated N-level for a period of surface microstructure. The $n(q)$ is the effective index of refraction for each layer, and the thickness of each layer is d/N.

refractive index changes from incident medium to substrate. Using the optical film theory with the effective refractive index in each divided layer, the transmittance of this layered structure can be calculated.

In Fig. 3, a single period of a sawtooth structure profile with the total thickness d is divided into N layers. Thus, the thickness of each layer is d/N, and the filling factor for the q-th layer is

$$f_q = q/N \tag{8}$$

According to the matrix method of the film theory [5], the characteristic matrix of a gather of N layers is

$$\begin{bmatrix} B \\ C \end{bmatrix} = \left\{ \prod_{q=1}^{N} \begin{bmatrix} \cos(\delta_q) & i\sin(\delta_q)/\eta_q \\ i\eta_q \sin(\delta_q) & \cos(\delta_q) \end{bmatrix} \right\} \begin{bmatrix} 1 \\ \eta_s \end{bmatrix} \tag{9}$$

where $\delta_q = 2\pi d n(q)\cos(\theta_q)/N\lambda$ and $\eta_q = \eta_0 n(q)_{TE}\cos(\theta_q)$ for TE polarization, $\eta_q = \eta_0 n(q)_{TM}/\cos(\theta_q)$ for TM polarization, where δ_q is the phase for q-th layer, $n(q)$ is the effective refractive index of q-th layer, η_0 is the optical admittance in free space ($\eta_0 = (\varepsilon_0/\mu_0)^{1/2} = 2.6544 \times 10^{-3}$ S) and η_s is the optical admittance of substrate. We can calculate the value of θ_q according to Snell's law if θ_0, the incident angle, is known,

$$n_0\sin(\theta_0) = n_s\sin(\theta_s) = n_q\sin(\theta_q) \tag{10}$$

Let Y be C/B, the reflectance of the blazed grating is

$$R = \left(\frac{\eta_0 - Y}{\eta_0 + Y}\right)\left(\frac{\eta_0 - Y}{\eta_0 + Y}\right)^* \tag{11}$$

then regardless of the loss of the grating material, the transmittance is

$$T = 1 - R \tag{12}$$

3. Calculated results and discussion

3.1. Diffraction efficiency of rigorous vector method of FMM

Figure 4 shows the transmittances calculated by FMM as a function of the normalized period at normal incidence for a sawtooth phase grating. The transmission property with two groove depths, 0.5λ and 1.0λ, is estimated for TE and TM polarizations, respectively.

From Fig. 4, we can see that the transmittances of both 0 and ±1 orders trend to a constant when the normalized period increases to a few wavelengths of the incident light. It is clear that the transmittance of +1 order is different from that of –1 order because of the asymmetric structure of the grating. Furthermore, when the period of the surface structure is much smaller or larger than the wavelength, the diffraction efficiency as a function of the normalized period can be approximately represented by a straight line. So, it is expected that the simplified methods of the EMT and SDT can be accurately applied to analyze the performance of the grating for two ranges of the

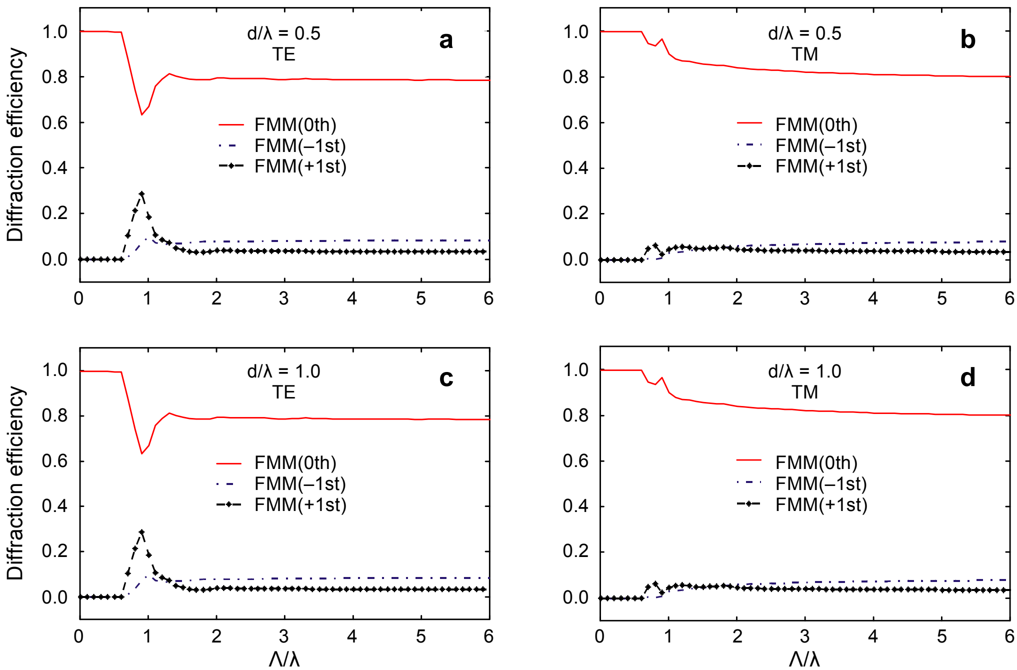

Fig. 4. Transmittances for the refractive index $n_g = 1.5$ *versus* the normalized period at normal incidence. TE (**a**) and TM (**b**) polarization with the normalized groove depth $d/\lambda = 0.5$; TE (**c**) and TM (**d**) polarization with the normalized groove depth $d/\lambda = 1.0$.

period-to-wavelength ratio. However, when the period approaches to the wavelength of incident light, the characteristics of transmittance do change significantly, and the methods of the EMT and SDT are inapplicable. In this case, a rigorous vector theory must be applied to calculate the diffraction characteristics. The quantitative analysis and comparison between the rigorous vector method of FMM and both simplified methods of SDT and EMT will be implemented in next sections.

3.2. Validity of scalar diffraction theory

To quantitatively analyze how normalized period, normalized depth, and polarization of incident light influence the accuracy of the SDT, we present plots of 0 and 1 orders

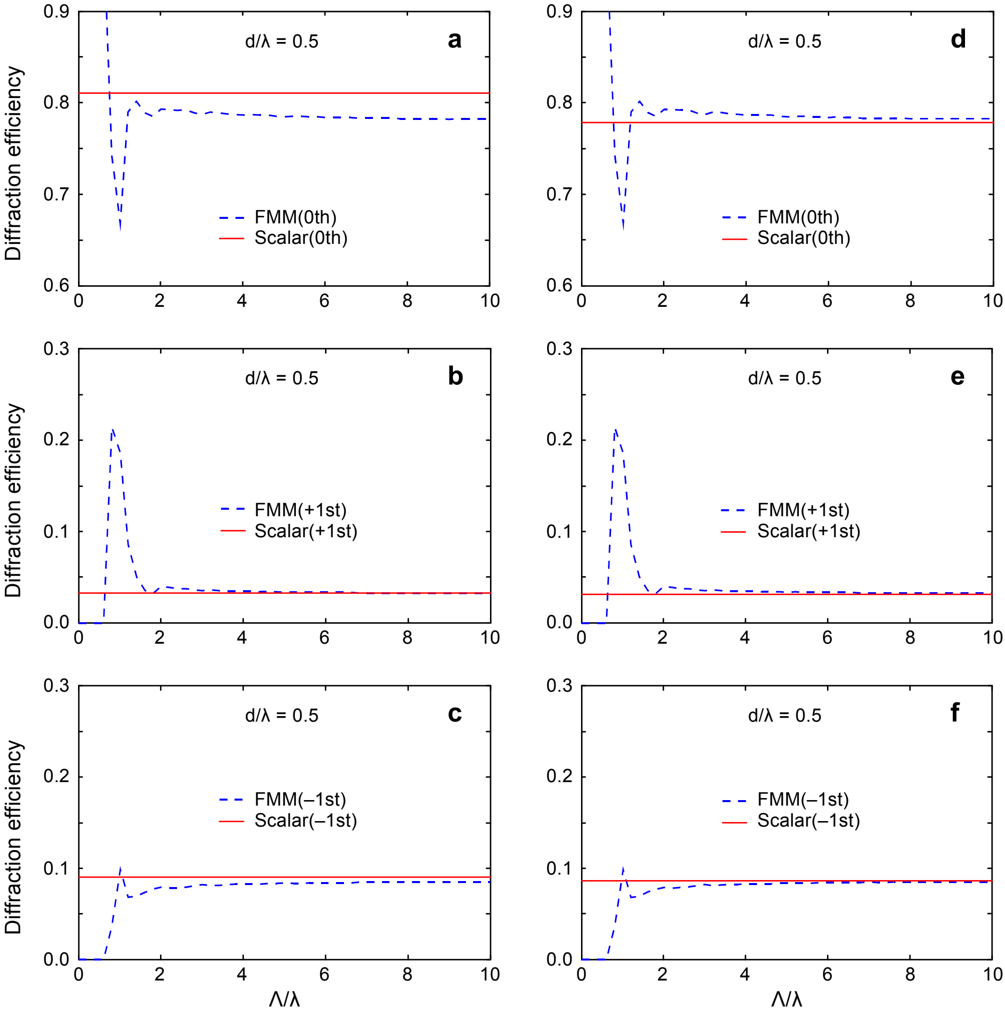

Fig. 5. The comparison of diffraction efficiencies between the SDT and FMM at $d/\lambda = 0.5$ as a function of normalized period: without (**a**–**c**) and with (**d**–**f**) the Fresnel reflection effect.

transmission efficiencies *versus* period/wavelength and *versus* depth/wavelength at normal incidence. In this part, we also demonstrate a significant enhancement of the accuracy of SDT by considering Fresnel reflection effect.

Figure 5 shows the comparison of the transmittances calculated by the scalar diffraction theory and FMM *versus* the normalized period for TE polarization at $d/\lambda = 0.5$. In Figs. 5**a**–5**c**, the diffraction efficiencies were calculated without the Fresnel reflection effect. This effect is considered in Figs. 5**d**–5**f**. From Eq. (2) we can obtain that the transmittance is independent of the period of a surface profile so that the result predicted by the SDT is shown by a straight line. Additionally, it is clear that the transmittance from the scalar diffraction is well in agreement with that estimated by FMM

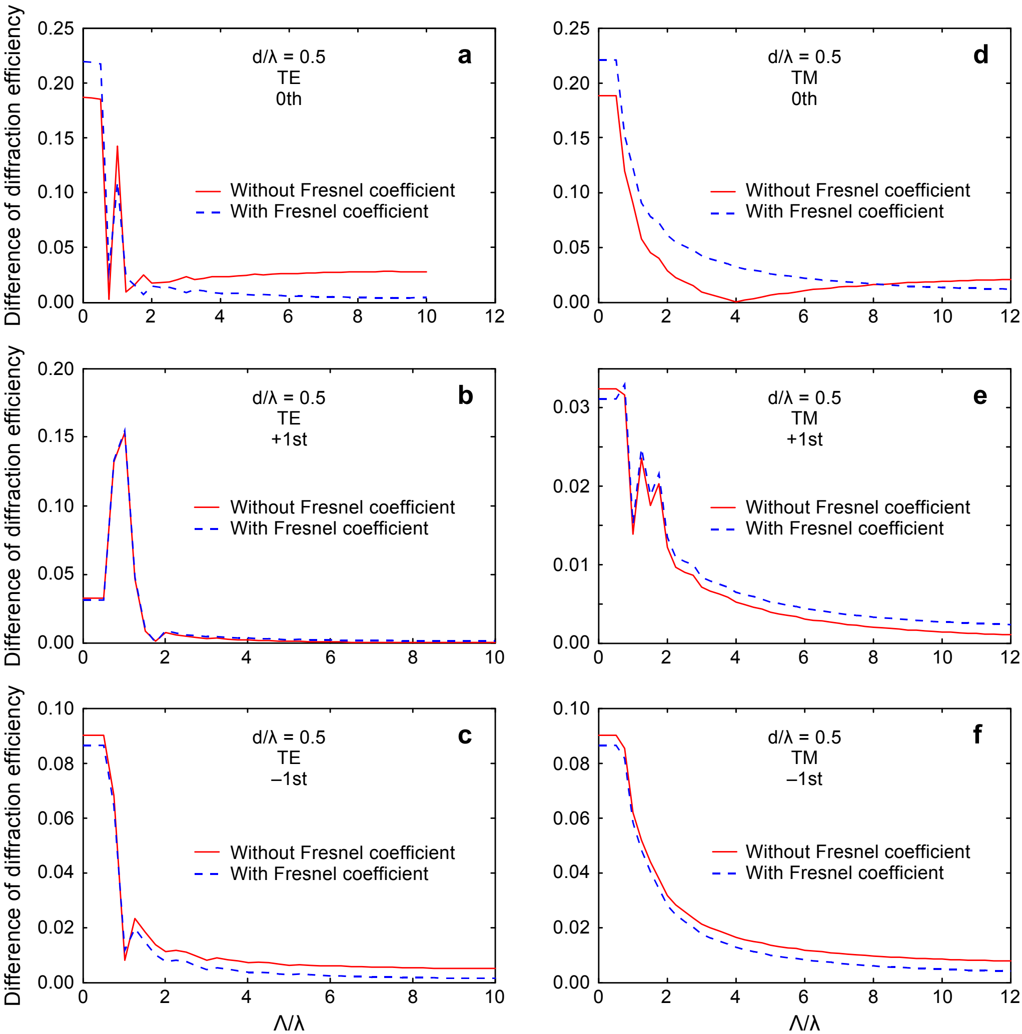

Fig. 6. The difference of diffraction efficiencies *versus* the normalized period between two cases with and without the Fresnel reflection effect at $d/\lambda = 0.5$: TE (**a**–**c**), and TM (**d**–**f**) polarization.

as $\Lambda/\lambda > 3$ regardless of diffractive order. In Fig. 5**a**, it can be seen that the discrepancy between the outcome of the scalar diffraction theory and that of FMM tends to a constant. The difference is less than about 3% when the value of the normalized period Λ/λ is more than three wavelengths of incident light. In Figs. 5**b** and 5**c**, the results of the SDT are consistent with that of the FMM. Figures 5**d**–5**f** show the transmittances calculated by the scalar diffraction theory with the Fresnel reflection effect. Although the accuracy of SDT with the Fresnel reflection effect for the diffraction efficiencies of ±1 order is slightly influenced, the validity of SDT is indeed enhanced. Importantly, it can be seen in Fig. 5 that the validity of SDT for calculated diffraction performance is significantly improved by considering the Fresnel reflection effect, especially for zeroth diffraction order.

In order to intuitively analyze the difference of calculated results between FMM and SDT, we demonstrate quantitatively the error of diffraction efficiencies between these two methods with respect to the normalized period, see Fig. 6.

In Figure 6 it is clear that the error with the Fresnel reflection effect is smaller than that without the Fresnel reflection effect as the normalized period increases. Also, the Fresnel reflection effect to enhance the accuracy of SDT for TE polarization is more effective than that for TM polarization. Particularly, for the +1 order diffraction efficiency as shown in Figs. 6**b** and 6**e**, the error between FMM and SDT with the Fresnel reflection effect is slightly larger than that without this effect, regardless of the polarization of incident light. The reason causing this phenomenon is in research. Meanwhile, it is worth mentioning that the diffraction efficiency of +1 order from SDT is in agreement with that from FMM with the divergence less than 2% for TE wave. In Fig. 6**d**, in the region of $0 \le \Lambda/\lambda \le 8.3$, the transmittivity of zero order without the Fresnel reflection effect calculated by the SDT is closer to the consequence by FMM, but when $\Lambda/\lambda > 8.3$ the result is opposite.

To display the accuracy of the scalar method with respect to the normalized groove depth in detail, the comparison of diffraction efficiencies calculated by the FMM and SDT for TE polarization with discrete values of Λ/λ is shown in Fig. 7. Here, the diffraction efficiencies were calculated by SDT without the consideration of the Fresnel reflection effect.

Figure 7**a** indicates that the SDT seems to be valid for zeroth order diffraction efficiency when the normalized groove depth is less than 1.0 with the error of less than 5% for $\Lambda/\lambda = 2.0$, but for ±1 orders the validity is good only when the normalized depth is less than 0.5. When $\Lambda/\lambda = 4.0$, the zeroth order diffraction efficiency calculated by the SDT is well in agreement with the result estimated by FMM for the range of $d/\lambda \le 2.0$, but the diffraction efficiencies of ±1 orders from the scalar method are coincident with that computed by FMM for $d/\lambda < 1.0$ shown in Fig. 7**b**. Moreover, when $\Lambda/\lambda = 6.0$ in Fig. 7**c**, the validity of the SDT begins to become better, especially for 0th and +1st orders. At $\Lambda/\lambda = 8.0$ in Fig. 7**d**, the results of diffraction efficiencies calculated by the scalar theory are well in agreement with that of FMM for both 0th and +1st orders in the region of $0 \le d/\lambda \le 5.0$, but for –1 order the accuracy of the scalar theory is lower at $1.5 \le d/\lambda \le 2.5$. Therefore, it can be concluded that the accuracy of

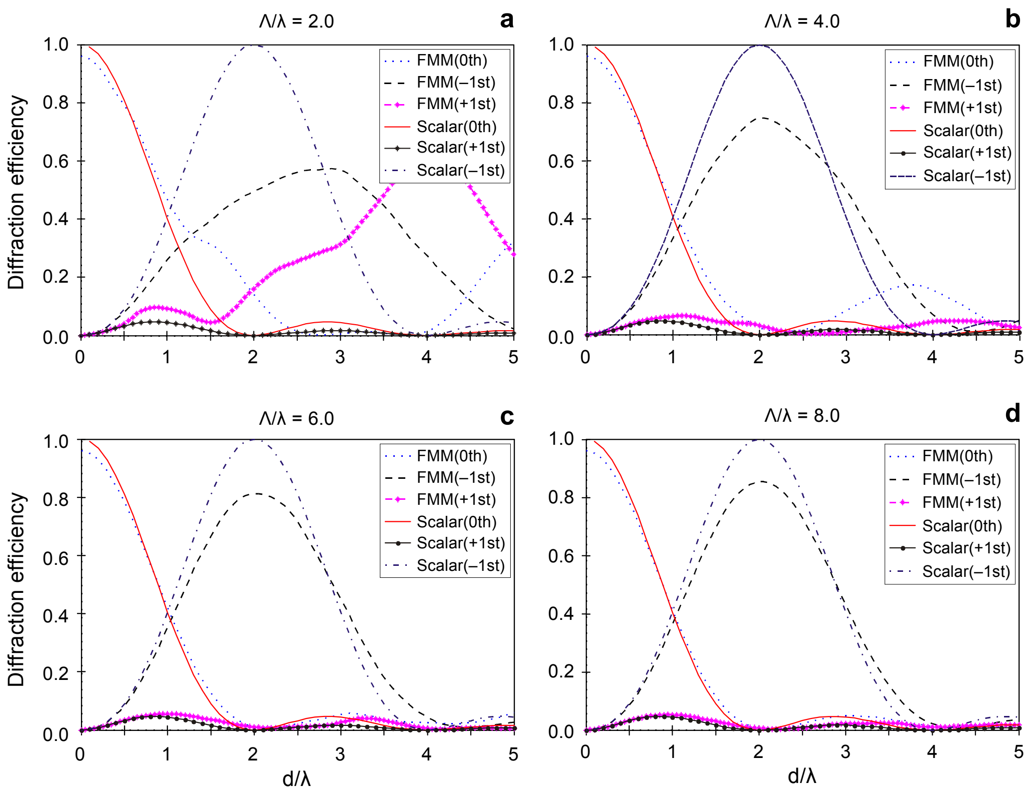

Fig. 7. The comparison of transmittance characteristics between the scalar method neglecting the Fresnel reflection effect and the FMM for the 0th and ±1st orders, respectively, with respect to normalized groove depth. The outcomes are computed at normal incidence for $n_g = 1.5$, and for the normalized period of 2.0 (**a**), 4.0 (**b**), 6.0 (**c**) and 8.0 (**d**).

the scalar theory with respect to the normalized groove depth increases as the normalized period of the surface microstructure increases. From the scalar diffraction theory it can be concluded that the greater is the normalized period the better is the transmittance which agrees with the calculations done with the rigorous vector method.

Furthermore, we also quantitatively compared the diffraction efficiencies from FMM and those of SDT with the consideration of the Fresnel reflection effect as shown in Fig. 8. It is obvious that the results considering the Fresnel reflection effect are more precise than that without this effect, especially at lower groove depth. In Fig. 8, at $0 \le d/\lambda \le 0.5$ for the comparison of zeroth order diffraction efficiency, the results estimated by FMM and SDT are almost the same. However, a larger error between FMM and SDT is found in this range of groove depth in Fig. 7. Hence, the validity of SDT with the Fresnel reflection is significantly enhanced.

Additionally, the comparison of diffraction efficiencies predicted by the two methods for TM polarization incidence shows that the accuracy of the scalar theory is comparable with that of the TE case. Thus we can come to a conclusion that the results

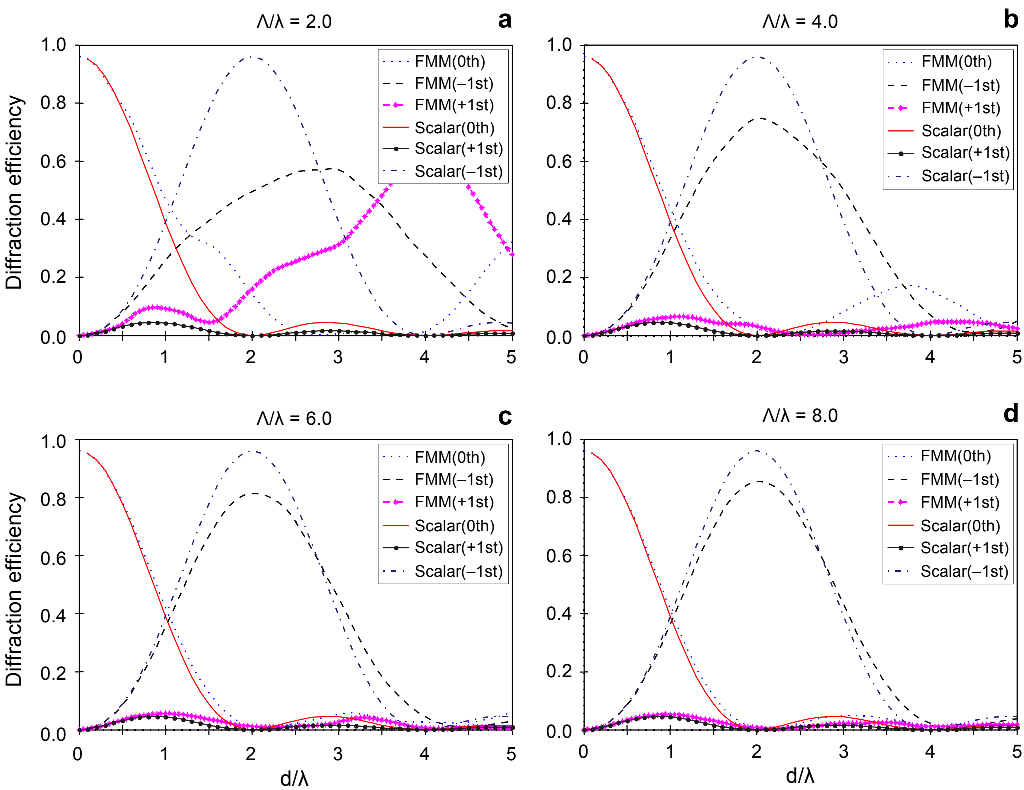

Fig. 8. The comparison of transmittance characteristics between the scalar method considering the Fresnel reflection effect and the FMM for the 0th and ±1st orders, respectively, with respect to normalized groove depth. The outcomes are computed at normal incidence for $n_g = 1.5$, and for the normalized period of 2.0 (**a**), 4.0 (**b**), 6.0 (**c**) and 8.0 (**d**).

calculated by the scalar theory considering the Fresnel reflection effect are better than those without the Fresnel reflection effect especially for 0th and +1st orders.

3.3. Accuracy of effective medium theory

When the period of a surface microstructure is much smaller than the wavelength of the incident light, in general, EMT can be used to estimate its optical property. In order to apply EMT effectively for designing and analyzing a sawtooth blazed phase grating, we quantitatively evaluate the validity of the zero and second order EMT at normal incidence.

Figure 9 shows the transmittances calculated by the rigorous vector theory and effective medium theory of both zeroth and second orders at the fixed groove normalized depth, 0.5 and 1.0, *versus* the normalized period for TE and TM polarization, respectively. In Fig. 9**a**, it is intuitively clear that the transmittances for zeroth and second

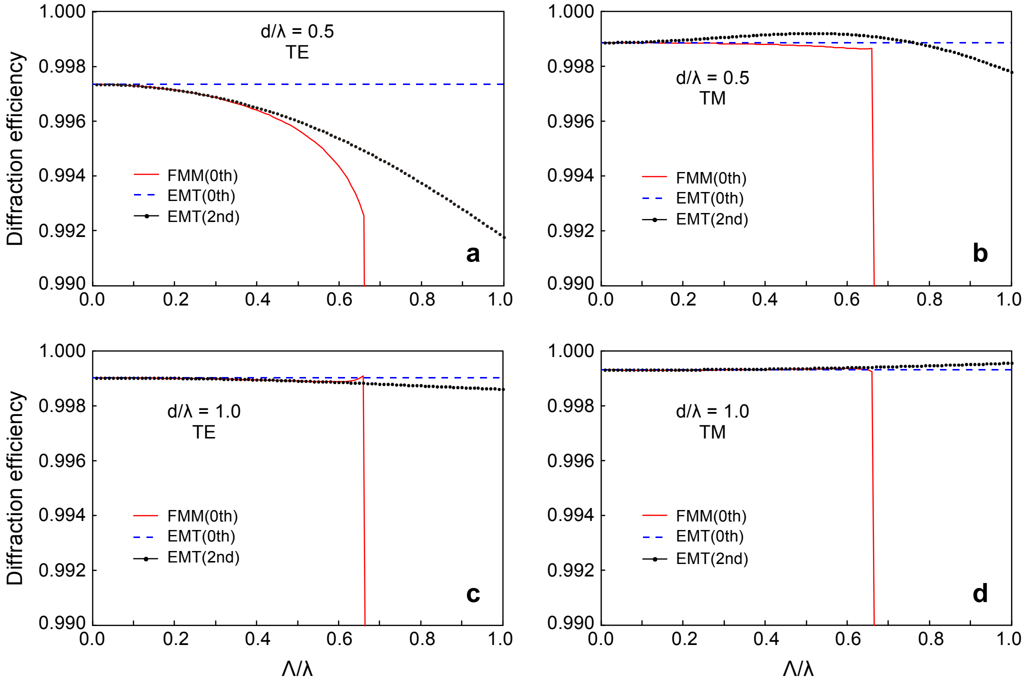

Fig. 9. The comparison of transmittance characteristics for FMM, zeroth and second orders EMT as a function of normalized period at normal incidence: for TE (**a**) and TM (**b**) polarization with $d/\lambda = 0.5$, and for TE (**c**) and TM (**d**) polarization with $d/\lambda = 1.0$.

orders EMT are extremely approximate to the results derived by FMM as $\Lambda/\lambda \le 0.1$ which is a quasi-static limit. When the normalized period exceeds this range, the difference of the results from zero order EMT to FMM increases and the results from zero order EMT are greater than those of FMM. However, the calculated transmittivity from second EMT is well in agreement with that estimated by FMM at $\Lambda/\lambda \le 0.4$. Thus, it is clear that the accuracy of second order EMT to calculate transmittance of phase gratings is higher than that of zero order one. Generally, it is believed that the effective medium theory is invalid as the feature size of surface structure larger than 1/10 of incident wavelength. From our conclusions, it can be seen that the validity of EMT is immensely extended in an applicable range.

For TM polarization in Fig. 9**b**, the results are in a good agreement with those calculated by both EMT and FMM when $0 \le \Lambda/\lambda \le 0.667$ in which the maximum difference is less than 0.1%. However, beyond this range, the diffraction efficiency calculated by FMM rapidly declines because the higher-order diffraction waves begin to propagate. Hence it is inappropriate to estimate the performance of optical elements using the EMT. Furthermore, it is clear that for the refractive index of 1.5 and the fixed normalized groove depth of 0.5λ, the accuracy of EMT for TM wave *versus* the nor-

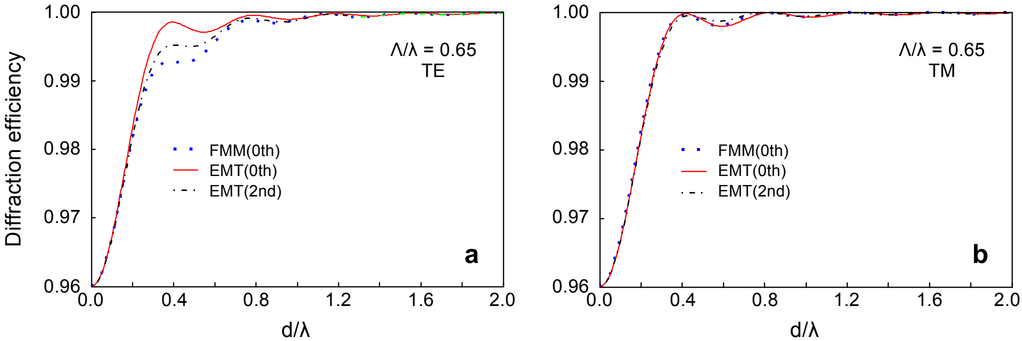

Fig. 10. The transmittances for rigorous vector theory, zeroth and second order EMT *versus* normalized groove depth; TE (**a**) and TM (**b**) polarization with the normalized period $\Lambda/\lambda = 0.65$.

malized period is higher than that for TE wave at $0 \le \Lambda/\lambda \le 0.667$. Figures 9**c** and 9**d** show that for both TE and TM polarizations the diffraction efficiencies calculated by both zeroth and second order EMT are extremely approximate to that derived by the FMM when higher diffraction orders rather than zeroth order are not propagating in the range of $0 \le \Lambda/\lambda \le 0.667$. It seems to be concluded that the larger the normalized depth is, the better the results from EMT agree with those of FMM.

In order to further research the influence of groove depth for the limitation of EMT, Figure 10 shows the transmittance as a function of the normalized groove depth for TE and TM polarizations, respectively. And the period of the grating is fixed to be 0.65λ according to the above discussion. For TE polarization shown in Fig. 10**a**, it is clear that when $0.25 \le \Lambda/\lambda \le 0.6$, the error is larger, reaching to about 0.5%, but beyond this range the results calculated by the EMT are in agreement with those by the FMM. For the TM polarization shown in Fig. 10**b**, it is obvious that the transmittances of EMT agree well with those from FMM in our calculated domain, and the maximum divergence between the zeroth order EMT and FMM is about less than 0.1%. Then, it is concluded that the accuracy of EMT for TM polarization is higher than that of TE mode. Also, the validity of EMT is primarily dependent on the normalized period for a sawtooth blazed surface microstructure, and the effect caused by groove depth is slight.

4. Conclusions

The validity of both the SDT and the EMT is quantitatively demonstrated for precisely analyzing and designing a sawtooth phase grating with the refractive index $n_g = 1.5$ at the normal incidence. It is useful for the SDT without the Fresnel reflection effect that when the normalized period is more than three wavelengths of the incident light within the error of less than 3% regardless of the polarization and diffraction order. And the validity of the SDT increases when the normalized period increases. Besides, the results calculated by the SDT with the Fresnel reflection effect are more precise than that from the SDT without the Fresnel reflection effect. Thus, by considering the effect of Fresnel reflection, the accuracy of SDT can be significantly enhanced. Furthermore, as $0 \leq \Lambda \leq 0.667$ in which only zeroth order diffraction wave is to propagate, the EMT is accurate to compute the diffraction efficiency within the difference of less than 1%. For TE polarization, the diffraction efficiencies of both zeroth and second orders EMT are well in agreement with that of FMM except for $0.25 \leq d/\lambda \leq 0.6$. However, for TM polarization, the results from EMT are more exact than those of TE polarization as a function of the normalized groove depth.

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