COMPARATIVE ANALYSIS OF RESULTS OF APPLICATION OF MARKOV AND SEMI-MARKOV PROCESSES TO RELIABILITY MODELS OF MULTI-STATE TECHNICAL OBJECTS

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Abstract

During rational operation of technical objects and systems various operational decisions are made and decision-making process itself should be consisted in selecting that considered most favourable out of all possible to be taken. Choice of such decision is possible after taking into account many different information items but it never be completely correct without accounting for data and indices dealing with reliability.

In the case when probabilistic principle of failure occurrence is determined , values of reliability indices can be estimated by using reliability mathematical models. During operation of many technical devices and systems (e.g. ship main propulsion systems) was observed many times the fact that their correct operation time is not a unique measure of their wear. Therefore for description of their serviceability and reliability can be used models of change of their reliability states in the form of semi-Markov processes based on the assumption of multi-state character of technical objects.

This paper presents a comparative analysis of results of simulation reliability tests of a hypothetical technical object , obtained with the use of reliability models in the form of Markov processes and semi-Markov ones, corresponding to them, called also "half-Markov" processes.

Keywords: reliability, Markov processes, semi-Markov processes, multi-state technical objects

1. Introduction

During operation of many mechanical devices (including ship power plant devices) it was many times stated that their correct operation time is not a unique measure of their wear [6], [7], [9], [10].

Therefore the following hypothesis can be considered correct : „ *Operational process of a given technical object , which takes place in a rational system of operation is that whose state considered in the arbitrary instant t_n (n = 0, 1, ..., m; $t_0 < t_1 < ... < t_m$) depends on the state directly preceding it and does not depend stochastically on the earlier occurred states and their duration intervals.*"

The hypothesis explains the fact observed in practice , which consists in the predicting of correct operation time of devices, sufficiently good for practice , by knowing only their current technical state and conditions of fulfilling the tasks , as well as energy and material resources.

The operational decision- making is hence carried out in the situation in which it is necessary to introduce ,in the random variable form, at least one of the parameters of decision-making model. It more completely reflects operational reality although it leads exclusively to more or less probable conclusions , that results from the fact that not only one value of criterion function but many values which occur with different probabilities, are assigned to particular values of decision variables.

Among the probabilistic decision models usually applied in practice , can be found those in which an expected value of decision-making consequences is principally taken into account in selecting an optimum value of decision variable [3, 8].

In this situation , from the formal point of view it is more favourable to present decision-making procedure in one of the most often used structural form : i.e. the decision tree or decision table.

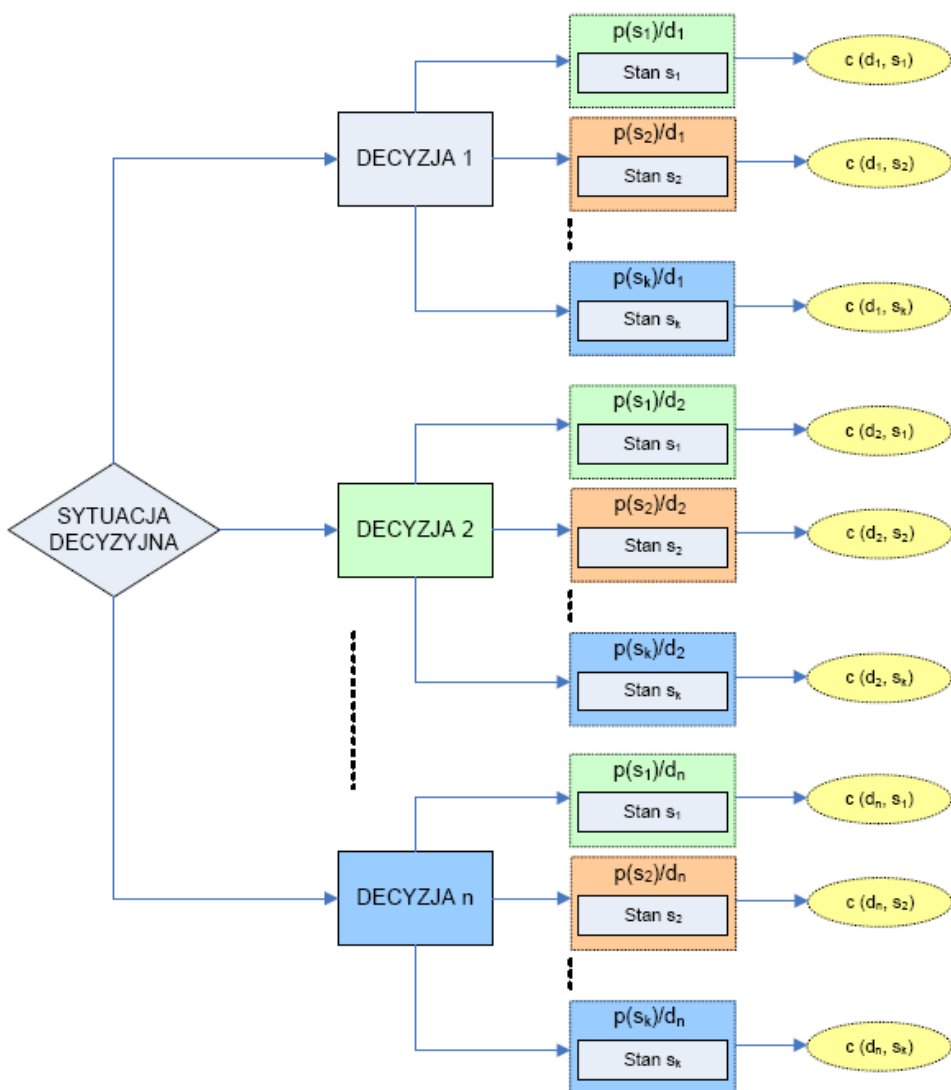A decision tree is generally presented as shown in Fig. 1.

Fig. 1 General case of the decision tree. n – number of considered decisions, $p(s_i)/d_j$ – relative occurrence probability of the state s_i in case of making the decision d_j, $c(d_j, s_i)$ – consequences of occurrence of the state $_si$ in case of making the decision d_j

For the presented tree the criterion function is to maximize the expected value of consequences $c(d_j, s_i)$, which , for particular tree nodes symbolizing event of making the decision d_j , can be determined as follows [1]:

$$E(c/d_j) = \sum_{i=1}^{k} \left[p(s_i)/d_j \cdot c(d_j, s_i) \right] \quad i = 1, 2, \dots, k \quad j = 1, 2, \dots, n \quad (1)$$

And, it should be observed that , in spite of inclusion of occurrence probabilities of the state s_i (i = 1,2, …, k) under condition of making the decision d_j (i = 1,2, …, n) , the situation is deterministic as it consists in choosing only one decision out of *n* possible to be done.

Application of the decision procedure presented in Fig. 1, to operational process requires, apart from determination of a set of decisions possible to be taken , also to have knowledge which make it possible to do :

a) specification of distinguished states (state classes) of the process ,
b) determination of values of the relative probabilities $p(s_i)/d_j$,
c) estimation of the consequences $c(d_j, s_i)$.

Even if to omit the points a) and c) in this work , which are separate and broad problems , the making of the simplest operational decision requires to determine occurrence probabilities of concrete, distinguished classes of operational process states. Taking into account the earlier mentioned hypothesis, for determination of their values one is able to use reliability models in the form of Markov and semi-Markov processes.

2. Reliability – functional models of technical objects

The using of the theory of stochastic processes makes it possible to resign from the assumption on two-state process of technical states changes and the splitting of the space S of possible states into a countable and limited number of subspaces differently distant from extreme set of states of a new object [39]. Therefore the multi-state character of the object constitutes a crucial assumption for forming reliability –functional models. It results in possible selection , within the distinguished set , series of states of different degrees of serviceability , as well as series of non-serviceability states , moreover (in contrast to majority of classical models) it makes it possible to include renewability-the important feature which characterizes machines and mechanical devices - into the model. This is especially important in case of complex technical objects which , being objects of a high-degree complexity, may suffer failures of various modes with different probabilities and consequences for fulfilling their tasks.

In functional approach, operational process is that of simultaneous changes in technical and operational states , which - being mutually depending - simultaneously occur in phase of operation [6].

In this case the process model of changes in the technical state significant from the reliability and durability point of view, belongs to stochastic processes of discrete set of distinguished states and continuous time of their duration. Elements of the set of distinguished technical states, S={s_i; i=1, 2, 3, ..., I} , are values of the process {W(t): t > 0} which is consisted of the states $s_i \in S$, one-by-one following and mutually depending on each other [6, 7].

In investigations of complex technical objects has been first of all used so far the theory of semi-Markov stochastic processes as well as ,in special cases, that of Markov processes.

3. Assumptions for the performed analysis

To perform the comparative analysis in question the following assumptions were taken:
- Simulation investigations will be aimed at a hypothetical object simple in the sense of reliability.
- Realizations of random variables which describe duration time of staying in distinguished states, will be generated by using a pseudo-random numbers generator , and point estimation of theoretical parameters of the considered random variables will be then made.
- Permissibility of application of selected theoretical distributions will be verified by using

the procedure of testing non-parametric hypotheses , employing the test χ^2.

- The reliability model produced with the use of semi-Markov processes (the process $\{W(t): t \geq 0\}$) , can be presented by means of the graph of states and transitions , shown in Fig. 2.
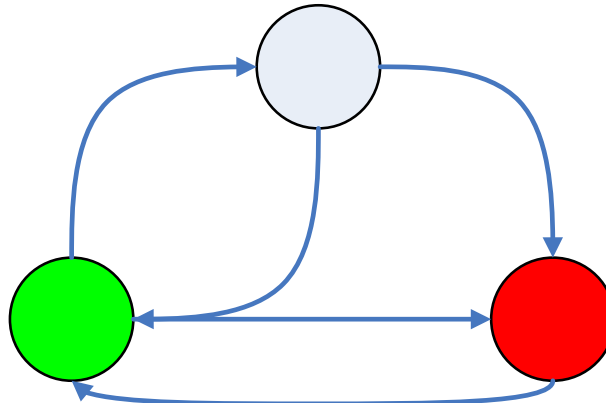

Fig. 2 Graph of changes in states of the process $\{W(t): t \geq 0\}$: u_1 – full serviceability state; u_2 – task non-serviceability state; u_3 – full non-serviceability state; T_{ij} – random variable which describes duration time of the state under condition of transition of the process to the state u_j (i, j=1, 2, 3, i≠j); p_{ij} – probability of transition of the process from the state u_i to the state u_j (i, j=1, 2, 3, i≠j); $E(T_i)$ – expected value of duration time of the state u_i, irrespective of to which state the process transition takes place.

The functional matrix of the considered process takes hence the following form (see the graph) [7]:

$$Q(t) = \begin{bmatrix} 0 & \int_0^t [1 - F_{13}(x)]F_{12}(x)dx & \int_0^t [1 - F_{12}(x)]F_{13}(x)dx \\ \int_0^t [1 - F_{23}(x)]F_{21}(x)dx & 0 & \int_0^t [1 - F_{21}(x)]F_{23}(x)dx \\ \int_0^t dF_{31}(x) & 0 & 0 \end{bmatrix} \quad (2)$$

By using the relation :

$$Q_{ij}(t) = p_{ij} \cdot F_{ij}(t) \quad (3)$$

where:

p_{ij} – probability of transition from the state s_i to the state s_j of the Markov chain inserted into the semi-Markov process, (i, j = 1, 2, 3 i≠j),

$F_{ij}(t)$ – cumulative distribution function of the random variable T_{ij} which describes the duration time u_i of the process state, under condition that the following will be the state u_j ,

it is possible to determine functional forms of particular elements of the process core.

- The object reliability model produced with the use of the Markov processes (the process $\{W'(t): t \geq 0\}$) , can be presented by means of the graph of states and transitions , shown in Fig. 3.
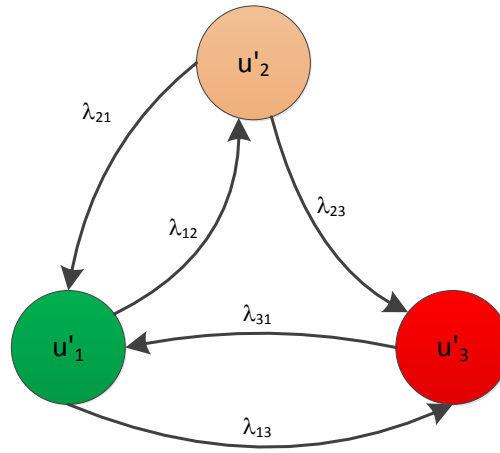
Fig. 3 Graph of states and transitions of the process {W'(t): t ∈ T}. u'₁– full serviceability state; u'₂– task non-serviceability state; u'₃– full non-serviceability state; λ_{ik} – intensity of transition from the state u_i under condition that the process transits to the state u_k (i, k=0, 1, 2, 3 i≠k);

Determination of probabilities of object staying in particular states requires to form a set of Kolmogorov equations for the assumed model of state changes. To this end, instead of the transition probabilities p_{ij} (Fig. 3), are applied the transition intensities λ_{ik} (i, k = 1, 2, 3; i ≠ k) of the following interpretation [10, 11] :

$$\lambda_{ik}(t) = \lim_{\tau \to 0} \frac{P\left(W(t+\tau) = u_j / W(t) = u_i\right)}{\tau} \tag{4}$$

In practice, a favourable and credible estimation of the above given quantity may be determined on the basis of experiments or technical documentation and producer's recommendations, as follows :

$$\lambda_{ik} = \frac{1}{E(T_{ik})} \cong \frac{1}{\overline{T}_{ik}}, \qquad (i,k = 1,2,3; \ i \neq k) \tag{5}$$

where:
$E(T_{ik})$ – expected value of random variable which describes duration time of the state u'ᵢ under condition that the next will be the state u'ₖ,

$\overline{T}_{ik}$ - average time of staying in the state u'ᵢ under condition that the next will be the state u'ₖ - on the basis of experiments or/and technical documentation and producer's recommendations.

- For carrying out the comparative analysis in question, the following reliability indices calculated on the basis of the considered models , will be taken into account :
  o The instantaneous distribution of the process $P_j(t)$, which stands for probability of finding itself the process in the state s_j, in the instant *t*.
  o Limit distribution of the process $p_k = \lim_{t \to \infty} P\{W(t) = u_k\}$

4. Calculation example
4.1. Semi- Markov reliability model

In line with the made assumptions, in 1ˢᵗ phase   1000 realizations of random variables which describe time of staying the object in the distinguished states : u_1, u_2, u_3 , were generated with the use of the generator of pseudo-random numbers.

Next, also in a random way , subsequent changes of states of the process *{W(t): t ≥ 0}* were simulated , and, as a result its subsequent realizations were reached. Graphical illustration of a part of process run is presented in Fig. 4.
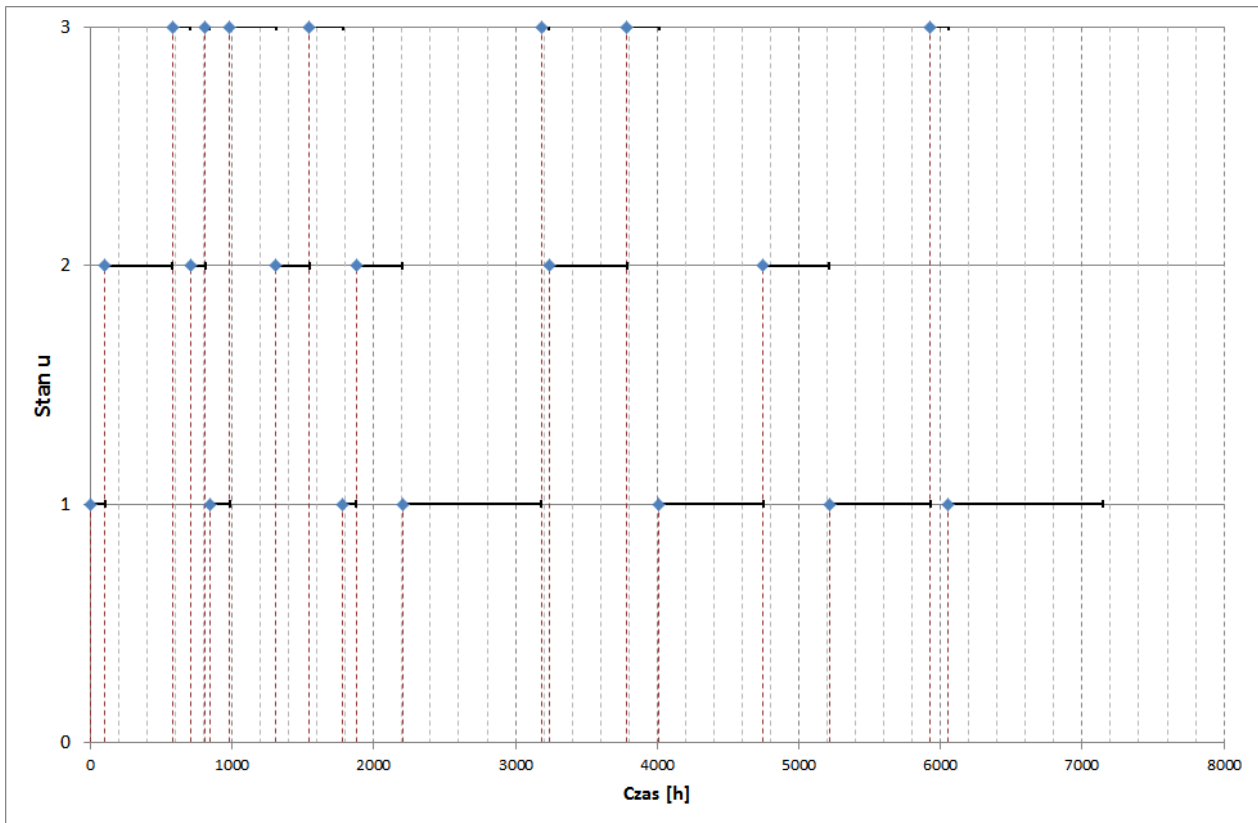
Fig. 4 The states-transitions graph of the of the process {W'(t): t $\in$ T}. u'1 – full serviceability state.

In the considered reliability – functional model it was next assumed that the process of changes in reliability states of objects of the investigated type is the semi-Markov process {W(t): t$\geq$ 0} of the set of states S = {u_1, u_2, u_3}, whose graphical form (graph of transitions) was presented in Fig. 2. From the graph form results the form of the process core determined by Eq. 2 , and , the initial distribution of the process {W(t): t$\geq$ 0} can be presented as follows :

$$p_1 = P\{W(0) = s_1\} = 1, p_i = P\{W(0) = s_i\} = 0 \ \text{dla} \ i = 2, 3. \tag{6}$$

Eq.(3) was used to determine functional forms of particular elements of the process core. In order to make use of Eq. (2) and (3) , realizations of the particular random variables T_{ij} were determined on the basis of an analysis of the generated data set , and, an analysis of the considered process of technical state changes was performed to determine sequence and number of the transitions of the process from the state u_i to the state u_j (i, j $\in$ S, i $\neq$ j) , n_{ij}. Tab.1 shows results of the analysis.

Tab. 1 Number of the transitions n_{ij} from the state u_i to the state u_j of the process of changes in technical state of the considered engines

n_{12}	355	$\Sigma \ n_{1j}$	474
n_{13}	119		
n_{21}	303	$\Sigma \ n_{2j}$	355
n_{23}	52		
n_{31}	171	$\Sigma \ n_{3j}$	171

In the next phase , the estimation of parameters of cumulative distribution functions of the random variables T_{ij} was performed and then verification of the hypothesis on conformity of the assumed

theoretical distributions with empirical results, was done. The verification was carried out with the use of the test of goodness of fit,$\chi 2$, and , in order to select out the true hypothesis H_0, i.e. that which is taken true as there is no basis to reject it , the following principle of accepting the hypotheses was used :

- if $g_0 \geq g_\alpha$ for $\alpha = 0,05$ then H_0 should be rejected;
- if $g_0 < g_\alpha$ for $\alpha = 0,10$ then H_0 should be accepted;
- if $g_0 < g_\alpha$ for $\alpha = 0,05$ and for $\alpha = 0,10$ $g_0 \geq g_\alpha$ then correctness of H_0 is dubious and tests should be continued ;

where :
- g_0 – test characteristic value determined on the basis of experimental results;
- g_α - limit value for the significance level α, i.e. α-order quantile value of selected statistics .

The applied test made it possible to state that there is no basis for rejection of subsequent hypotheses H_0 on conformity between the considered variables T_{ik} and gamma - distribution as well as exponential one - in the case of the random variable T_{31} ; this way they were accepted to be true.

Tab. 2 Values of distribution parameters of the random variables T_{ij}.

Random variable	Distribution type	Distribution parameters	
T_{12}	gamma	g = 1,29	b = 0,010 [h⁻¹]
T_{13}	gamma	g = 0,74	b = 0,007 [h⁻¹]
T_{21}	gamma	g = 1,29	b = 0,027 [h⁻¹]
T_{23}	gamma	g = 1,32	b = 0,023 [h⁻¹]
T_{31}	exponential	λ = 0,17 [h⁻¹]	

Therefore :

$$F_{12}(t) = \int_0^t \frac{0,010^{1,29}}{\Gamma(1,29)} \cdot t^{0,29} \cdot \exp(-0,010 \cdot t) \tag{7}$$

$$F_{13}(t) = \int_0^t \frac{0,007^{0,74}}{\Gamma(0,74)} \cdot t^{-0,26} \cdot \exp(-0,007 \cdot t) \tag{8}$$

$$F_{21}(t) = \int_0^t \frac{0,027^{1,29}}{\Gamma(1,29)} \cdot t^{0,29} \cdot \exp(-0,027 \cdot t) \tag{9}$$

$$F_{23}(t) = \int_0^t \frac{0,023^{1,32}}{\Gamma(1,32)} \cdot t^{0,32} \cdot \exp(-0,023 \cdot t) \tag{10}$$

$$F_{31}(t) = 1 - \exp(-0,17 \cdot t) \tag{11}$$

To estimate the particular probabilities p_{ij} the following quantity was used [?]:

$$p_{ij}^* = \frac{n_{ij}}{\sum_j n_{ij}} \tag{12}$$

where :

n_{ij} – number of transitions of the process from the state u_i to the state u_j (Tab. 2).
Values of the statistics described by Eq. (12) are as follows :

$$p_{12}^{*} = \frac{n_{12}}{n_{12} + n_{13}} = \frac{150}{150 + 50} = 0,750 \tag{13}$$

$$p_{13}^{*} = \frac{n_{13}}{n_{12} + n_{13}} = \frac{50}{150 + 50} = 0,250 \tag{14}$$

$$p_{21}^{*} = \frac{n_{21}}{n_{21} + n_{23}} = \frac{128}{128 + 22} = 0,853 \tag{15}$$

$$p_{23}^{*} = \frac{n_{23}}{n_{21} + n_{23}} = \frac{22}{128 + 22} = 0,147 \tag{16}$$

$$p_{31}^{*} = \frac{n_{31}}{n_{31}} = \frac{72}{72} = 1,000 \tag{17}$$

hence :

$$Q(t) = \begin{bmatrix} 0 & p_{12} \cdot F_{12}(t) & p_{13} \cdot F_{13}(t) \\ p_{21} \cdot F_{21}(t) & 0 & p_{23} \cdot F_{23}(t) \\ p_{31} \cdot F_{31}(t) & 0 & 0 \end{bmatrix} \tag{18}$$

This way the considered semi-Markov process of changes in technical states of the considered type of engines was determined. The produced model makes it possible to determine all reliability characteristics necessary to further analysis.

a. The relative probabilities $P_{ij}(t)$ – transition probabilities .

Basing on [7] one is able to obtain unknown transforms of the function $P_{ij}(t)$ by solving the set of equations:

$$\widetilde{P}(s) = \frac{1}{s} \cdot (I - \widetilde{g}(s)) + \widetilde{q}(s) \cdot \widetilde{P}(s) \tag{19}$$

where :

$$\widetilde{P}(s) = \begin{bmatrix} \widetilde{p}_{11}(s) & \widetilde{p}_{12}(s) & \widetilde{p}_{13}(s) \\ \widetilde{p}_{21}(s) & \widetilde{p}_{22}(s) & \widetilde{p}_{23}(s) \\ \widetilde{p}_{31}(s) & \widetilde{p}_{32}(s) & \widetilde{p}_{33}(s) \end{bmatrix}, \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{20}$$

$$\widetilde{q}(s) = L \left\{ \begin{bmatrix} 0 & p_{12} \cdot \frac{dF_{12}(t)}{dt} & p_{13} \cdot \frac{dF_{13}(t)}{dt} \\ p_{21} \cdot \frac{dF_{21}(t)}{dt} & 0 & p_{23} \cdot \frac{dF_{23}(t)}{dt} \\ p_{31} \cdot \frac{dF_{31}(t)}{dt} & 0 & 0 \end{bmatrix} \right\}, \tag{21}$$

$$\widetilde{g}(s) = L \left\{ \begin{bmatrix} \frac{d\left(p_{12} \cdot F_{12}(t) + p_{13} \cdot F_{13}(t)\right)}{dt} & 0 & 0 \\ 0 & \frac{d\left(p_{21} \cdot F_{21}(t) + p_{23} \cdot F_{23}(t)\right)}{dt} & 0 \\ 0 & 0 & \frac{d\left(p_{31} \cdot F_{31}(t)\right)}{dt} \end{bmatrix} \right\}, \tag{22}$$

The relative probabilities $P_{ij}(t)$ make it possible to determine the distribution $P_j(t)$:

$$P_j(t) = P\{W(t) = s_j\}, j \in S \tag{23}$$

On the basis of the expression for total probability [43] , as well as for the assumed initial distribution of the process {W(t): t≥ 0}, the distribution in question is as follows :

$$P_j(t) = P_{1j}(t) \qquad (24)$$

The probabilities P_j (t) constitute elements of the first row of the matrix :

$$P(t) = [P_{ij}(t)] \qquad (25)$$

The probabilities are presented in Fig. 5.

     The assumed initial distribution of the process {W(t): t≥ 0} is justified when the engine, before starting to execute its task, is in the state s_1. In this case it is important to analyze the probabilities: P_{11} (t) = P_1 (t), P_{12} (t) = P_2 (t) and P_{13} (t) = P_3 (t), which for t→∞ may be interpreted as follows : P_{11} (t) = P_1 (t), P_{12} (t) = P_2 (t) i P_{13} (t) = P_3 (t), (they stand for the probabilities of finding itself the object in the states : u_1, u_2, and u_3, respectively, under condition that the state u_1 was initial.).
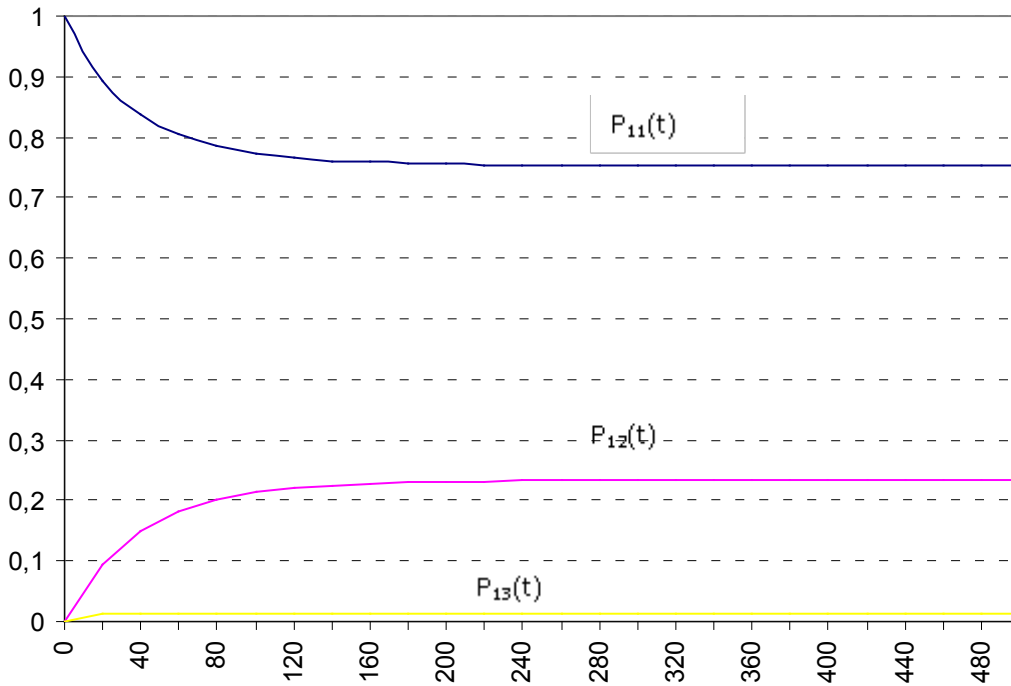

Fig. 5 The relative probabilities $P_{11}(t)$, $P_{12}(t)$, $P_{13}(t)$

4.2. Markov reliability model

    In the case of application of Markov processes , to determine probabilities of staying the system in particular states is possible by solving the set of Kolmogorov equations.

    By taking into account the graph of changes in states of the process {W'(t) : t ≥ 0} and its initial distribution for the case in question the set of equations is formed as follows :

$$\left. \begin{aligned} \frac{dP_1(t)}{dt} &= -(\lambda_{12} + \lambda_{13}) \cdot P_1(t) + \lambda_{21} \cdot P_2(t) + \lambda_{31} \cdot P_3(t) \\ \frac{dP_2(t)}{dt} &= -(\lambda_{21} + \lambda_{23}) \cdot P_2(t) + \lambda_{12} \cdot P_1(t) \\ \frac{dP_3(t)}{dt} &= -\lambda_{31} \cdot P_3(t) + \lambda_{13} \cdot P_1(t) + \lambda_{23} \cdot P_2(t) \end{aligned} \right\} \qquad (26)$$

    Empirical distributions of the random variables T_{ik} were used to verify statistical hypotheses on their conformity with exponential distribution , and, it was assumed that the values $\hat{\lambda}_{ik}$ determined in compliance with Eq. (5) may serve as estimators of parameters of these distributions, λ_{ik} . The values are given in Tab. 3.

Random variable	$\hat{\lambda}$ $[\text{h}^{-1}]$
T_{12}	0,0078
T_{21}	0,0210
T_{23}	0,0170
T_{13}	0,0094
T_{31}	0,1700

The verification was performed with the use of the test of goodness of fit, $\chi 2$, with applying the earlier described inference rules. The applied test made it possible to state that there is no basis for rejecting the subsequent hypotheses H_0 on conformity of the considered variables T_{ik} with exponential distribution; therefore they were taken to be true.

By making use of Laplace transform [2], initial distribution of the process $\{W(t) : t \geq 0\}$ and the values $\hat{\lambda}_{ik}$, the following set of linear equations in the domain of transforms was obtained for solving the set of equation (26):

$$\left. \begin{aligned} s \cdot P_1^*(s) - 1 &= -(0,0078 + 0,0094) \cdot P_1^*(s) + 0,021 \cdot P_2^*(s) + 0,17 \cdot P_3^*(s) \\ s \cdot P_2^*(s) &= 0,0078 \cdot P_1^*(s) - (0,021 + 0,017) \cdot P_2^*(s) \\ s \cdot P_3^*(s) &= 0,0094 \cdot P_1^*(s) + 0,017 \cdot P_2^*(s) - 0,17 \cdot P_3^*(s) \end{aligned} \right\} \quad (27)$$

The solution of the presented set of equations and execution of Laplace converse transform made it possible to find the instantaneous distribution of the process $p_j(t) = P\{W'(t) = u'_j\}$. Fig. 6 shows the solution in the graphical form.
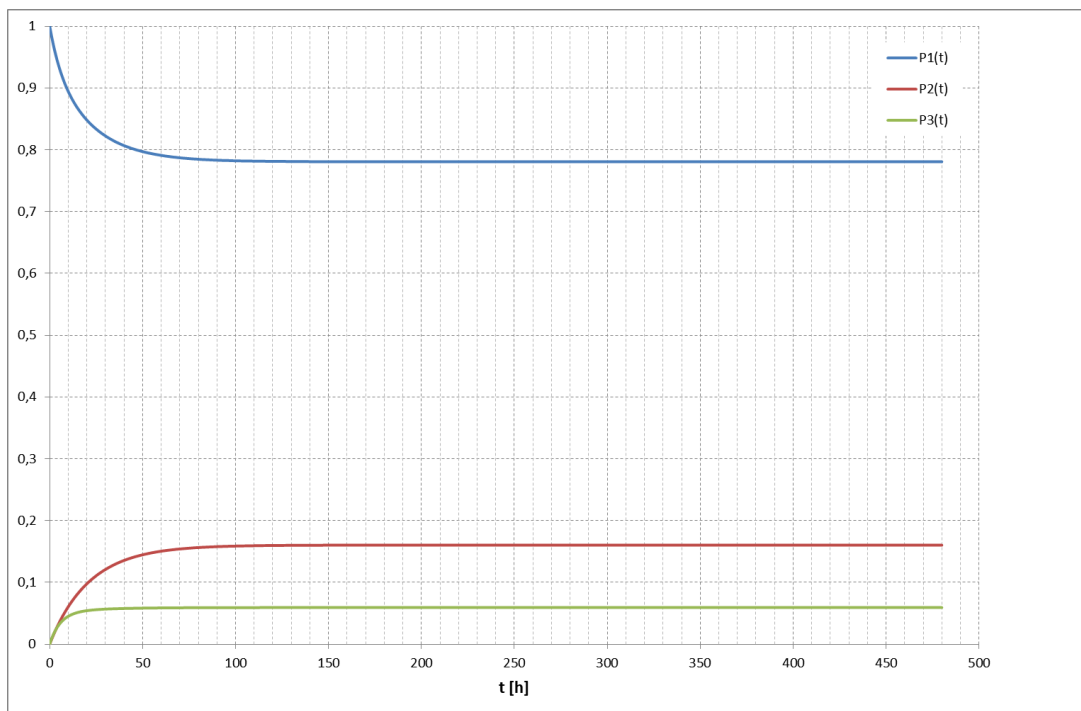

Fig. 6. Results of calculation of the instantaneous distribution for the initial distribution $p_1 = P\{W'(0) = u'_1\} = 1$, $p_i = P\{W(0) = u'_i\} = 0$ for i = 2, 3;

5. Summary

Fig. 7 shows the results obtained with the use of the presented models for one set of the data which represent simulated results of empirical tests , and Fig. 8 presents graphical illustration of relative error (under assumption that the semi-Markov model serves as basic one).
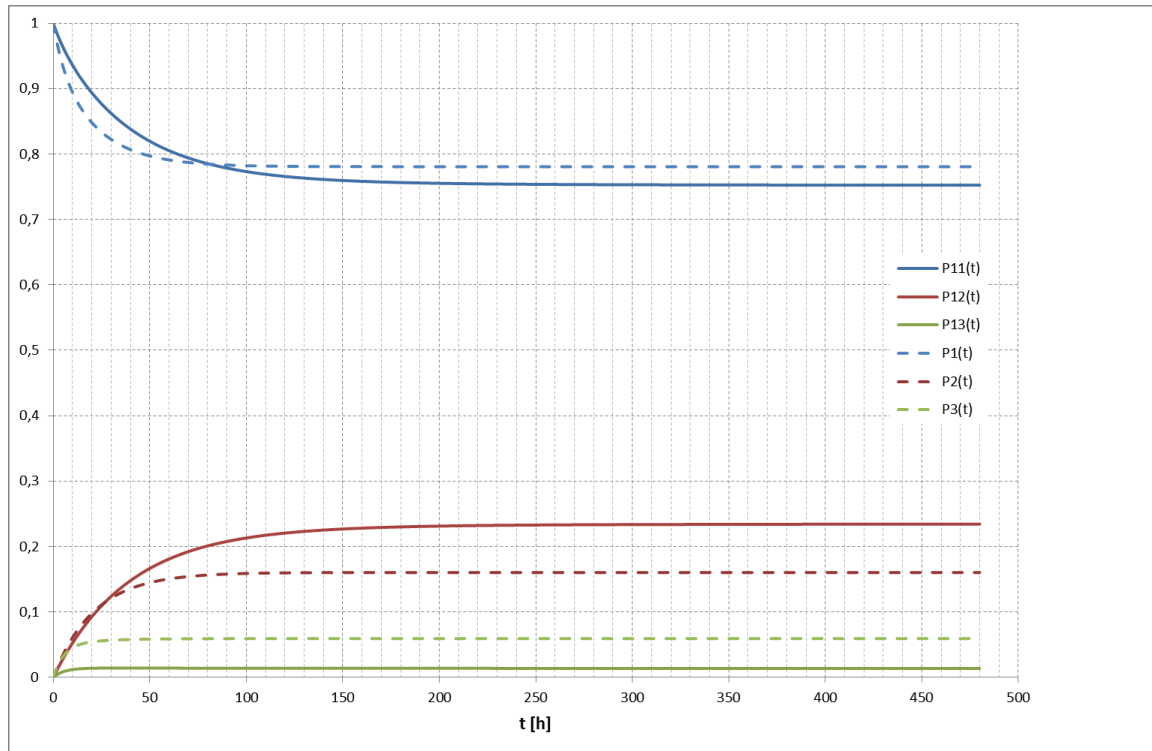

Fig. 7. Comparison between results of calculation with the use of semi-Markov model and those obtained with the Markov process model (the used notation is the same as in the text)
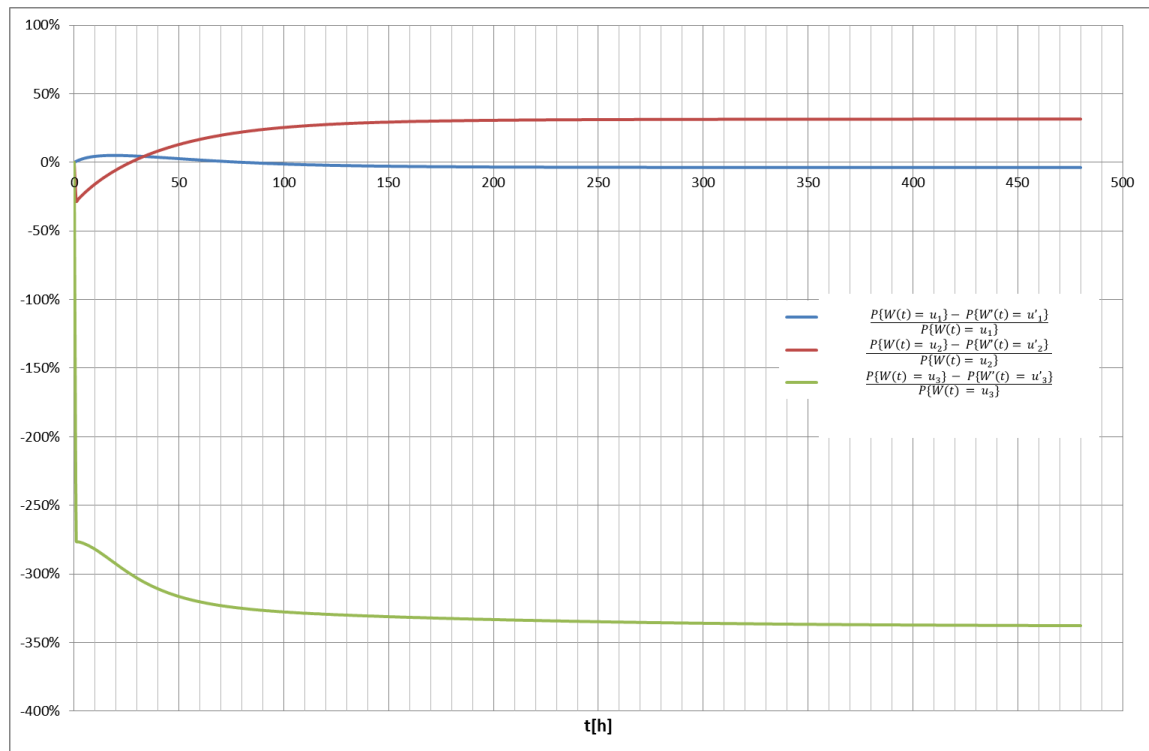

Fig. 8. Relative errors in application of Markov model

The analysis of Fig. 7 and 8 leads to the following conclusions:

- Application of the semi-Markov model results in obtaining lower values of probability of staying the object in full serviceability state , that leads in practice to making more cautious decisions;
- Relative error in estimating the probability of staying the object in full serviceability state is negligible and , in the considered case , it amounted to about 4% ;
- Therefore it should be considered if the complex mathematical tool (Eq. 19 through 22) is worth to be applied in concrete situations as it is known that one of the conditions of practical usability of the models based on semi-Markov processes is its relatively not complicated mathematically, functional matrix Q(t)., apart from its moderate complexity in the sense of the lowest necessary number of distinguished state classes. The condition is crucial in the case of calculation of the instantaneous distribution of process states $p_k(t)$. The distribution can be calculated [7] if only initial distribution of the process and the function $p_{ij}(t)$ is known. Calculating the probability $p_{ij}(t)$ consists in solving the set of Volterra equations of 2nd type , in which the functions $Q_{ij}(t)$ - the elements of the process functional matrix Q(t) - are known quantities. In case when number of process states is low and the process functional matrix not complicated , the set in question can be solved by means of operators – applying Laplace transforms. On the other hand , when number of process states is high or if its functional matrix (process core) is very complex, then only an approximate solution of the set of equations may be achieved. The solution (numerical one) does not provide possibility of determining values of occurrence probabilities of particular process states when t is of large value (theoretically $t \to \infty$). Such numerical solution does not give any answer for the question which is very important for operational practice : how are probabilities of states of semi-Markov process changing when t is large ? As results from the theory of semi-Markov processes the probabilities, in case of ergodic semi-Markov processes, tend along with time to strictly determined , constant numbers [7, 13]. The numbers are called the limit probabilities of states, and , sequence of the numbers produce limit distribution of the process. The distribution can be calculated much more easily than instantaneous distribution, moreover it makes it possible to define the availability coefficient for an engine, as well as profit or cost per unit of time of operation. The quantities may serve as criterion functions in solving optimization issues of engine operational processes.
- In case of a rather complex graph of states and transitions an additional possibility of estimating values of the probability appears namely by producing a model of the process {W(t) : t ≥ 0} in the form of Markov process {W'(t) : t ≥ 0}. Such models constitute a simplification of semi-Markov models. Markov process differs from semi-Markov one by that unconditional duration time intervals of staying the process in particular states as well as duration time intervals of a given state of the process , under condition that the next will be one of the remaining states in which it may find itself , are random variables of exponential distribution [4, 5]. For this reason, application of such processes is restricted, however in the case of the above described difficulties associated with producing the semi-Markov model or also when random variables under consideration have unknown distributions,   the obtained results may be assumed to be the first approximation. Use of exponential distributions makes it possible to achieve very simple relationships which constitute distribution of the process in question.

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