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The effect of reduced heat transfer in a micropolar fluid in natural convection

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Abstract This paper presents the numerical solution to the unsteady natural convection problem in micropolar fluid in the vicinity of a vertical plate, heat flux of which rises suddenly at a given moment. In order to solve this problem the method of finite differences was applied. The numerical results have been presented for a range of values of the dimensionless material properties and fluid Prandtl number. The analysis of the results shows that the intensity of the heat transfer in micropolar fluid is lower compared to the Newtonian fluid.

Keywords: Micro polar fluid; Microrocation; Boundary layer theory; Local Nusselt number

Nomenclature

a	–	fluid thermal diffusivity , m^2/s
Gr_x	–	local Grashof number
g	–	gravitational acceleration, m/s^2
j	–	microinertia density, m^2
M	–	total number of spatial steps in x directions
N	–	microrotation component normal to (x,y)-plane, 1/s; total number of spatial steps in y directions
n	–	microrotation parameter
Pr_∞	–	Prandtl number

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t	–	temperature of the fluid, K
T	–	dimensionless temperature
u, v	–	the x- and y-components of the velocity field, m/s

Greek symbols

β	–	volumetric coefficient of thermal expansion, 1/K
δ	–	boundary layer thickness, m
κ	–	rotational viscosity coefficient Pa s
λ	–	thermal conductivity, W/(mK)
ρ	–	density, m^3/kg
γ	–	spin gradient viscosity, Ns
μ	–	dynamic viscosity, Pa s
τ	–	time, s
τ_w	–	shear stress of a vertical surface, Pa

Subscripts

i, j	–	grid locations in x, y directions
n	–	number of time steps
w	–	refers to conditions at the wall
∞	–	refers to conditions far away from the wall

1 Introduction

The concept of micropolar and thermomicropolar fluids introduced by Eringen [1] deals with the class of fluids which exhibit certain microscopic effects arising from the local structure and micromotions of the fluid elements. Eringen's theory may form suitable non-Newtonian fluid models that can be used to analyse the behaviour of exotic lubricant, colloidal suspensions or polymeric fluids, liquid crystals and animal blood [2–4].

The process of natural convection in micropolar fluid located in the vicinity of a vertical plate heated by the constant heat flux has been considered by Chang [5]. Studies of the convection heat transfer in micropolar fluids have been focused on vertical and horizontal plates. El-Hakiem [3] presented a similar solution for the steady laminar natural convection along an isothermal vertical plate in a micropolar fluid with internal heat generation. Gorla [2] considered the unsteady mixed convection flow of a laminar micropolar boundary layer over a vertical plate. Mohammadein and Gorla [6] presented an analysis to study the heat transfer characteristics of a steady laminar micropolar fluid over a linearly stretching, continuous surface. They have taken into consideration the surface with prescribed uniform surface temperature and the surface with prescribed uniform wall heat flux.

The present paper deals with numerical solution to the unsteady natural convection problem in micropolar fluid of variable material coefficients κ and j. The fluid under consideration is placed in the vicinity of a vertical surface, heat flux of which rises suddenly at a given moment. The numerical results have been presented for a range of values of the dimensionless material properties and fluid Prandtl number.

2 Problem formulation

Let us consider the unsteady natural convection flow of a micropolar fluid along a vertical plate. The heat flux of the plate rises suddenly at the initial time $\tau = 0$.

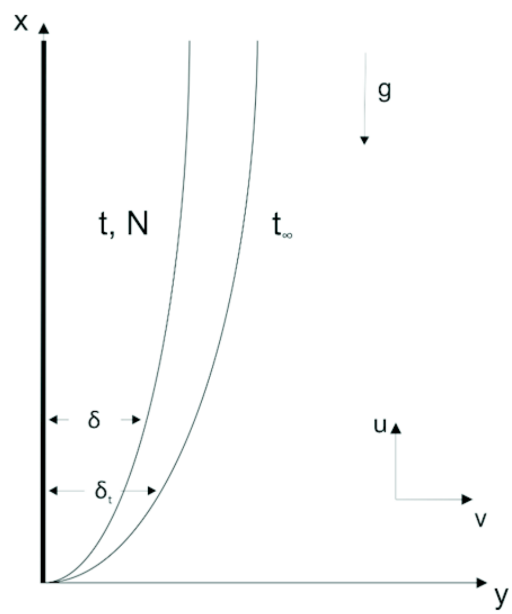

Figure 1. Considered fluid schema.

Due to a degree of complexity of general differential equations resulting from a balance of mass, momentum, angular momentum and heat, we are going to introduce the following simplifying assumptions:

- The analysed flows geometry justifies the use of the boundary layer theory.

- Oberbeck-Boussinesq approximation is assumed.
- The viscous dissipation motion, pressure and volumetric energy source are neglected.
- Eringen's theory of thermomicrofluids is assumed.

Taking into account the simplification resulting from the boundary layer theory and fluid density changes according to the Oberbeck-Boussinesq approximation, the following system of equations can be obtained:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\ , \tag{1}$$

$$\frac{\partial u}{\partial \tau}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\left(\mu+\kappa\right)\frac{\partial^2 u}{\partial y^2}+\frac{\kappa}{\rho}\frac{\partial N}{\partial y}+g\beta\left(t-t_\infty\right)\ , \tag{2}$$

$$\frac{\partial N}{\partial \tau}+u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{\kappa}{\rho j}\left(2N+\frac{\partial u}{\partial y}\right)\ , \tag{3}$$

$$\frac{\partial t}{\partial \tau}+u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}=a\frac{\partial^2 t}{\partial y^2}\ . \tag{4}$$

The above system of partial differential equations together with the following boundary conditions:

$$\tau<0,\quad u=v=0,\quad t=t_\infty \tag{5}$$

$$\tau\geq 0,\quad x=0,\quad u=v=0,\quad t=t_\infty\ , \tag{6}$$

$$y=0,\ \ u=v=0,\ \ \frac{\partial t}{\partial y}=-\frac{q_0}{\lambda},\ \ N=-n\frac{\partial u}{\partial y}\ , \tag{7}$$

$$y\to\infty,\ \ u=0,\ N=0,\ \ t=t_\infty\ , \tag{8}$$

formulates the mathematical description of momentum, angular momentum and heat transfer driven by the unsteady convection in micropolar fluid. In Eqs. (1)–(4), x and y are the coordinates measured along and perpendicular to the plate, u and v being the velocity components in the x and y directions, N is the microrotation component in the xy-plane, τ is the time, ρ is the density, μ and κ are the dynamic and rotational viscosity coefficient, respectively , γ is the spin-gradient viscosity, j is the microinertia density, α is the thermal diffusivity, β is the coefficient of volumetric expansion, t is the fluid temperature, and q_0 denotes the surface heat flux. In the present analysis, the spin gradient viscosity is assumed to be [2–4]:

$$\gamma = \left(\mu + \frac{\kappa}{2}\right) j \, . \tag{9}$$

In the last condition listed in (7) we have assumed that the microcirculation on the boundary layer is equal to the angular velocity, namely, $N(x, 0, \tau) = -n\frac{\partial u}{\partial y}$. As the suspended particle cannot get closer than its radius to the wall, the microstructure effect must be negligible on the boundary. Therefore, in the vicinity of the boundary, the rotation is due to fluid shear and thus the microrotation must be equal to the angular velocity of the boundary.

The parameter n is a number between 0 and 1 and that relates microgyration vector to the shear stress. The value $n = 0$ corresponds to the case of the high density of liquid microparticles that prevents them from performing rotational movements in the vicinity of the wall. The value $n = 0.5$ is indicative of weak concentrations, at $n = 1$ flows are believed to represent turbulent boundary layers [2,6]. Taking into account the constitutive equations of the micropolar fluid [1,2], the shear stress of a considered vartical surface was determined:

$$\tau_w = \left[(\mu + \kappa)\frac{\partial u}{\partial y} + \kappa N\right]_{|y=0} \, . \tag{10}$$

The fluid differential equations are recast in a dimensionless form by introducing:

$$T = \frac{t - t_\infty}{\left[\nu_\infty^2 \left(\frac{q_0}{\lambda}\right)^3 \frac{1}{g\beta}\right]^{\frac{1}{4}}} \, , \qquad \overline{\tau} = \frac{\tau}{\left[\left(\frac{\lambda}{q_0}\right)\frac{1}{g\beta}\right]^{\frac{1}{2}}} \tag{11}$$

$$U = \frac{u}{\left(\nu_\infty^2 \frac{q_0}{\lambda} g\beta\right)^{\frac{1}{4}}} \, , \qquad V = \frac{v}{\left(\nu_\infty^2 \frac{q_0}{\lambda} g\beta\right)^{\frac{1}{4}}} \tag{12}$$

$$X = \frac{x}{\left(\nu_\infty^2 \frac{\lambda}{q_0}\frac{1}{g\beta}\right)^{\frac{1}{4}}} \qquad Y = \frac{y}{\left(\nu_\infty^2 \frac{\lambda}{q_0}\frac{1}{g\beta}\right)^{\frac{1}{4}}} = \frac{y}{x}\left(\mathrm{Gr}_x\right)^{\frac{1}{4}} \tag{13}$$

$$\mathrm{Gr}_x = \frac{g\beta}{\nu_\infty^2}\frac{q_0}{\lambda}x^4 \, , \qquad \overline{N} = N\left(g\beta\frac{q_0}{\lambda}\right)^{-\frac{1}{2}} \tag{14}$$

$$\Delta = \frac{\kappa}{\nu_\infty \rho} \, , \qquad P = \frac{\nu_\infty}{j}\frac{1}{\left(\frac{q_0}{\lambda}g\beta\right)^{\frac{1}{2}}} = \frac{x^2}{j}\left(\mathrm{Gr}_x\right)^{-\frac{1}{2}} \tag{15}$$

Substituting Eqs. (11)–(15) into the governing Eqs. (1)–(4), respectively, leads to

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0\ , \tag{16}$$

$$\frac{\partial U}{\partial \overline{\tau}}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=(1+\Delta)\,\frac{\partial^2 U}{\partial Y^2}+\Delta\frac{\partial \overline{N}}{\partial Y}+T\ , \tag{17}$$

$$\frac{\partial \overline{N}}{\partial \overline{\tau}}+U\frac{\partial \overline{N}}{\partial X}+V\frac{\partial \overline{N}}{\partial Y}=\left(1+\frac{\Delta}{2}\right)\frac{\partial^2 \overline{N}}{\partial Y^2}-\Delta\left(2\overline{N}+\frac{\partial U}{\partial Y}\right)P\ , \tag{18}$$

$$\frac{\partial T}{\partial \overline{\tau}}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{\mathrm{Pr}_\infty}\frac{\partial^2 T}{\partial Y^2}\ . \tag{19}$$

The boundary conditions (5)–(8) are then given by the following dimensionless form:

$$\overline{\tau}<0,\quad U=V=T=0\ , \tag{20}$$

$$\overline{\tau}\geq 0,\quad X=0,\quad U=V=T=0\ , \tag{21}$$

$$Y=0,\ \ U=V=0,\ \ -\frac{\partial T}{\partial Y}=1,\ \ \overline{N}=-n\frac{\partial U}{\partial Y}\ , \tag{22}$$

$$Y\to\infty,\ \ U=V=T=\overline{N}=0\ . \tag{23}$$

Substituting into Eq. (10) dimensionless expressions (11)–(14) we obtain

$$\overline{\tau}_w=\frac{\tau_w}{\frac{\rho_\infty \nu_\infty^2}{j\,P}\,(1+\Delta-n\Delta)}=\frac{\partial U}{\partial Y}_{|y=0}\ . \tag{24}$$

The system of differential Eqs. (15)–(18) together with the initial and boundary conditions (19)–(22) has been solved using the method of finite differences [7,8].

3 Dimensionless form of balance equations and their numerical solution

The numerical scheme used for solving the natural convection problem was also an explicit finite difference scheme. According to the method applied, differential equations resulting from the balance of mass, momentum, angular momentum and energy have been replaced by corresponding difference equations. Spatial distribution grid contains $M\times N$ points in the X and Y direction respectively, $\Delta\overline{\tau}$ is the time step. Due to the intensive heat, momentum, angular momentum and mass transfer, only in the direct vicinity of the considered vertical surface, the maximum values of dimensionless coordinates $X = 100$ and $Y = 30$ were assumed [7,8]. A characteristic feature of the difference equations was to determine the temperature field, the velocity field components and the microrotation component $\overline{N}$ at a time $\overline{\tau}_{n+1}$ ($\overline{\tau}_{n+1} = \overline{\tau}_n + n\Delta\overline{\tau},\ n = 1, 2..$) depending on the certain parameters, but determined at a time τ_n. Convection terms of balance equations comprising time $\overline{\tau}$ and spatial Y coordinate derivatives were approximated by "forward" formulas whereas spatial X coordinate derivatives were approximated by "backward" formulas. Diffusion terms were approximated by central differences. Derivatives appearing in the boundary conditions (22) were approximated by higher order difference formulas taken in the form [7]

$$\frac{\partial T}{\partial Y_{|ij}} = \frac{1}{2\Delta Y}\left(-3T_{ij} + 4T_{i,j+1} - T_{i,j+2}\right) + O\left(\Delta Y\right)^2 \ , \tag{25}$$

$$\begin{aligned} -\frac{1}{n}\overline{N}_{|i,j} = \frac{\partial U}{\partial Y_{|ij}} = \frac{1}{12\Delta Y}&\left(-25U_{i,j} + 48U_{i,j+1} - \right. \\ &\left. -36U_{i,j+2} + 16U_{i,j+3} - 3U_{i,j+4}\right) + O\left(\Delta Y\right)^4 \ . \end{aligned} \tag{26}$$

These difference formulas are statically stable and exhibit characteristics of conservation [7].

Before performing the basic calculations for established, nonzero values of parameters Δ and P describing the properties of micropolar fluid, calculation tests were done. In these computational tests, the influence of the spatial and time steps length on the result accuracy was investigated. In the process of natural convection in a Newtonian fluid exact analytical solutions are known [9], which were compared to the corresponding calculation results. These results and the relevant results of exact solutions [9] are compared in Tab. 1.

The comparison presented in Tab. 1 leads to the finding that the ratio of Nusselt and Grashof number, $\mathrm{Nu}_x/(\mathrm{Gr}_x)^{\frac{1}{5}}$, calculated for the relatively sparse spatial area division and considerable size of time step has a high degree of accuracy with respect to the corresponding exact solution [9].

In the case of the numerical results of dimensionless shear stress $\overline{\tau}_w$ (dimensionless velocity component gradient) considerably smaller time and spatial steps are required in order to achieve satisfactory accuracy in conformity to exact solutions. On the basis of trial calculations further ones, taking into account the non-zero values of Δ and P parameters, were performed with the following spatial area division: $M \times N = 250 \times 450$, the set size of time step $\Delta\overline{\tau} = 0.001$.

Table 1. Spatial and time steps size impact on calculation precision.

Pr	M	N	$\Delta\overline{\tau}$	$\frac{\mathrm{Nu}}{\mathrm{Gr}_x^{\frac{1}{5}}}$	$\overline{\tau}_w$
0.72	190	250	0.0120	0.4898179	0.784502
	190	390	0.0100	0.4898179	0.793826
	290	290	0.0050	0.4898179	0.789164
	290	292	0.0020	0.4898179	0.794909
	250	450	0.0010	0.4898179	0.796920
				0.489802 *	**0.814100 ***

* Exact analytical solution by [9]

4 Results and discussion

The system of Eqs. (16)–(19) together with the initial (20) and boundary conditions (21), (22) and (23) has been integrated for the selected parameter values $\mathrm{Pr}_\infty, P, \Delta$, and n. To implement the numerical integration of the equations an algorithm in FORTRAN was developed.

Figure 2 presents the U velocity component in the X-axis direction of the fluid described by Prandtl number $\mathrm{Pr}_\infty = 0.72$ at fixed times of the process $\overline{\tau} = 10, 15$, and 40. In order to simplify the analysis of the rheological properties of the considered micropolar fluid impact in Fig. 2 three different cases are shown. Line marked with a circle represents the results obtained for the case of Newtonian fluid, that does not demonstrate micropolar effects. Lines marked with a triangle and a diamond depict the profiles of velocity U component in micropolar fluids characterized by different values of Δ and P parameters respectively. Additionally, to obtain unambiguous description, U component of velocity profiles obtained for the process at $\overline{\tau} = 15$ is marked with black symbols. According to the velocity profiles U component of the considered fluid described by Prandtl number $\mathrm{Pr}_\infty = 0.72$ reaches a steady state in a relatively short time ($\overline{\tau} = 40$).
It is also worth noting that the maximum U velocity component in each of the listed times, $\overline{\tau}$, is located at a greater distance from the vertical surface,

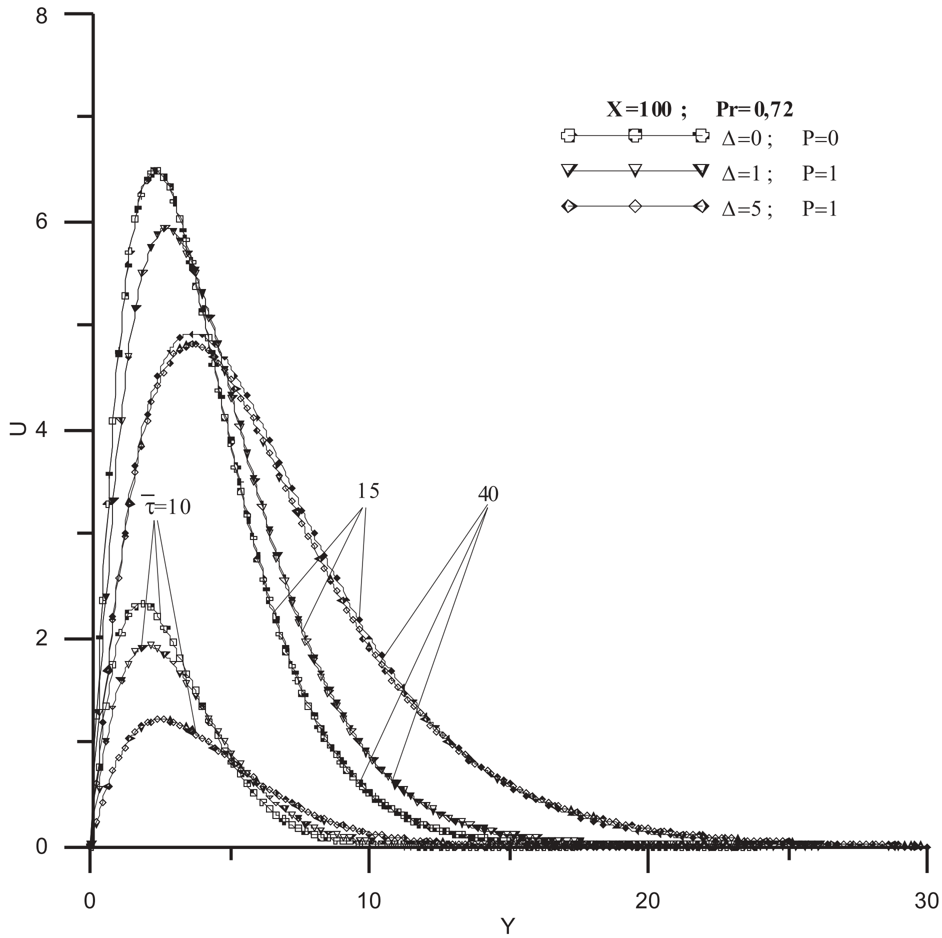

Figure 2. Profiles of the fluid velocity component, U, at selected moments of the process (profiles obtained at $\overline{\tau} = 15$ are marked with solid symbols).

compared to the case of a Newtonian fluid. Moreover, in the initial stage of the heating process the corresponding velocities maxima for considered micropolar fluids are smaller than the maximum velocity of a Newtonian fluid.

Figure 3 shows the temperature profiles in the considered liquid ($\mathrm{Pr}_\infty = 0.72$) at certain moments of the process $\overline{\tau} = 5, 15$, and 40. Similarly as for the U velocity component, proper values of parameters describing the physical properties of the fluid were assumed.

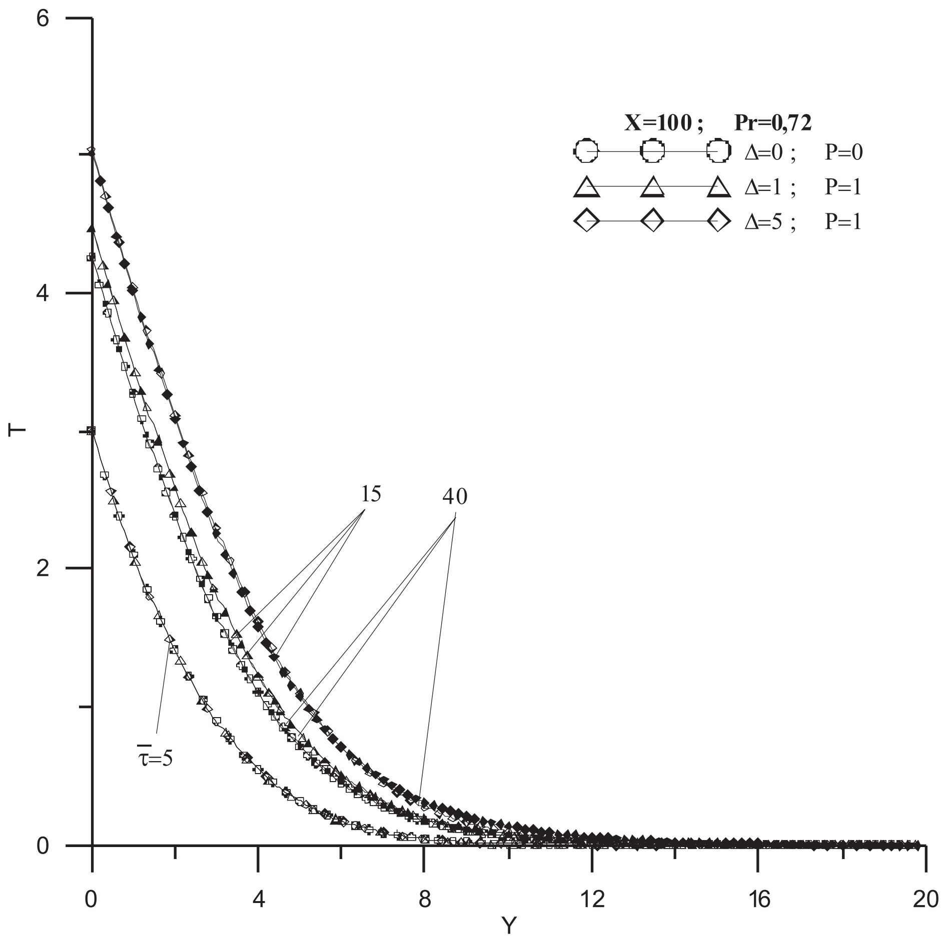

Figure 3. Profiles of the fluid temperature changes (profiles obtained at $\overline{\tau} = 15$ are marked with solid symbols).

In contrast to the changes in the U component of velocity shown in Fig. 2, the temperature profiles in the initial phase of the process, $\overline{\tau} \leq 5$, presented in Fig. 3 demonstrate the same course for Newtonian fluid and micropolar fluid characterized by the corresponding values of Δ and P parameters. In the final stage of the process (steady state) vertical surface temperature increases significantly with increasing values of Δ and P parameters.

Based on a numerically calculated transient temperature field in the fluid, local Nusselt number changes in time were determined on the suddenly heated vertical surface. The local Nusselt number is defined as follows:

$$\mathrm{Nu}_x = \frac{q_o}{t_w - t_\infty} \frac{x}{\lambda} \,. \tag{27}$$

Substituting in (27) dimensionless expressions (11), (13) and (14) we obtain after transformation

$$\frac{\mathrm{Nu}_x}{\mathrm{Gr}_x^{\frac{1}{5}}} = X^{\frac{1}{5}} \frac{1}{T_w} \,, \tag{28}$$

where

$$T_w = \frac{t_w - t_\infty}{\left[\nu_\infty^2 \left(\frac{q_0}{\lambda}\right)^3 \frac{1}{g\beta}\right]^{\frac{1}{4}}} \,. \tag{29}$$

The relationship (28) is shown graphically in Fig. 4. For the sake of comparison, Fig. 4 comprises the corresponding curve obtained for Newtonian fluid. It is worth noting that the dimensionless value coordinate $X = 100$ corresponds, according to the Eqs. (13) and (14), to the value of Grashof number $\mathrm{Gr}_x = X^4 = 10^6$. According to the analysis of the curves in Fig. 4, representing the change in Nusselt number $[\mathrm{Nu}_x/(\mathrm{Gr}_x)]^{\frac{1}{5}}$, the heat transfer intensity in micropolar fluids is much lower than in the corresponding Newtonian fluids.

Based on a numerically calculated velocity field, the shear stress of a considered vertical surface was determined.

Table 2. A comparison of obtained results.

Pr = 0.72			
Δ	P	$\frac{\mathrm{Nu}}{\mathrm{Gr}_x^{\frac{1}{5}}}$	$\overline{\tau}_w$
0.0	0.0	0.48980*	0.81410*
0.0	0.0	0.48982	0.79620
1.0	0.1	0.46520	0.61532
1.0	1.0	0.46470	0.62433
1.0	5.0	0.46545	0.63844
2.0	1.0	0.44712	0.52976
5.0	1.0	0.41446	0.38555
5.0	5.0	0.41446	0.38813

* Exact analytical solution by [9]

In order to make a comparative analysis, Tab. 2 summarizes the Nusselt number values according to the (28) formula and the dimensionless shear stress $\overline{\tau}_w$ in accordance with (24) formula, obtained from the numerical calculations performed for the various parameters Δ and P.

Summarized results relate to the steady state with regard to dimensionless coordinate $X = 100$ and fixed value of the parameter $n = 0.5$. The first line of Tab. 2 brings the corresponding exact solutions found in [9].

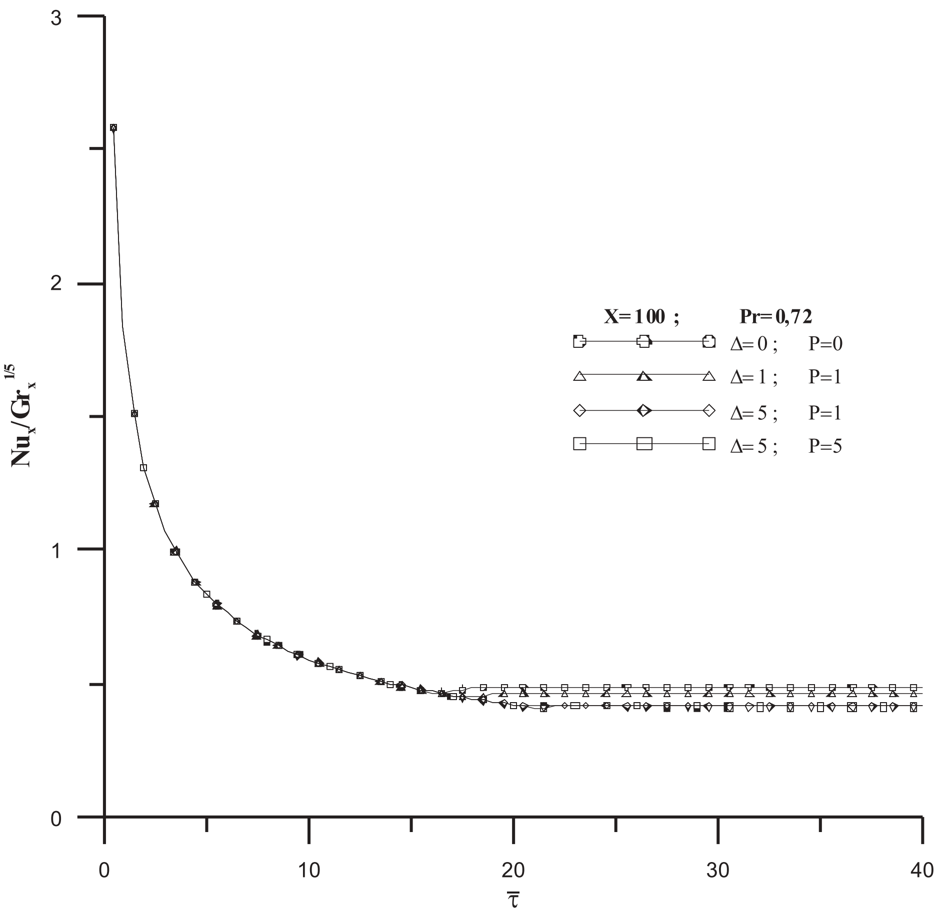

Figure 4. Transient changes of the local Nusselt number.

Figure 5 presents the dimensionless component profiles of microrotation $\overline{N}$ in the selected moments of the process. As previously indicated, the results for the case of fluid parameters $\Delta = P = 0$ and $n = 0.5$ are presented with a line marked with circles. Lines marked with a triangle, star and diamond present microrotation component profiles $\overline{N}$ for selected, nonzero values of Δ and P. Additionally, in order to obtain unambiguous description profiles of microrotation obtained for the time $\overline{\tau}$ are marked with solid symbols.

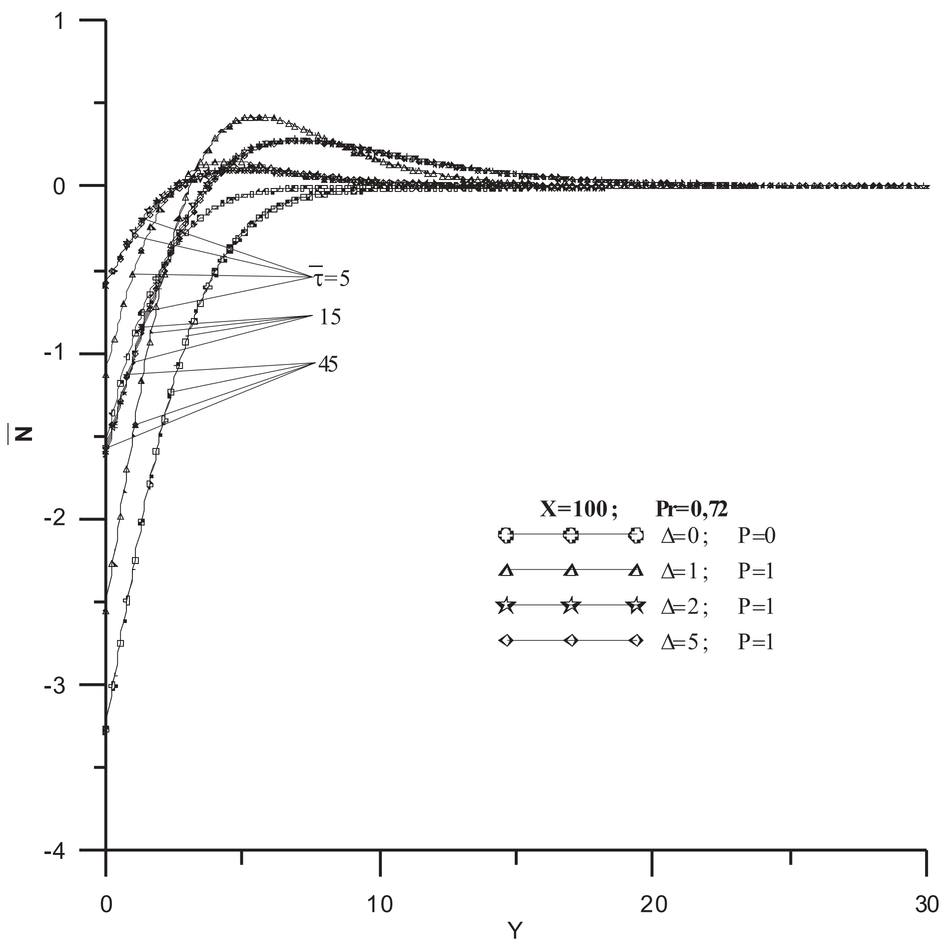

Figure 5. The changes of the profiles of dimensionless microrotation at certain moments of the process.

A comparison of the presented curves in Fig. 5 shows that the dimensionless microrotation component $\overline{N}$ attains the maximum value for the fluid described by $\Delta = P = 1$, namely $\overline{N} = 0.409$ for $\overline{\tau} = 5$.

5 Concluding remarks and conclusions

In the present paper we have investigated the processes of momentum and heat transfer in the range of natural convection in the micropolar fluid. For the description of these processes, the equations of both hydrodynamic and thermal boundary layer were applied. It is worth noting that the coupled system of differential equations describing the analyzed exchange processes also includes, in accordance with the boundary layer theory, simplified equation for the, N, microrotation component, arising from the angular momentum principle.

In order to solve this problem the method of finite differences was applied. That the results obtained are consistent for the entire course of the considered process, and therefore in the full range of the time variable $\overline{\tau}$.

The impact of changes in the Δ and P parameter values on dimensionless shear stress $\overline{\tau}_w$ and the Nu_x number is clearly noticeable for the longer process durations ($\overline{\tau} > 20$), that is in the time variable range where the convection heat transfer between the suddenly heated vertical plate and considered micropolar fluid prevails. Due to the obtained calculation results the U velocity component gradient is much smaller for the micropolar fluid compared to a corresponding Newtonian fluid throughout the entire heating process. The maximum relative change ($\partial U/\partial Y_{Y=0}$) is about 50%.

Significantly higher temperatures of the heated vertical wall at times $\overline{\tau} > 5$ in the vicinity of micropolar fluid indicate lower intensity of heat transfer compared to the Newtonian fluid. The relative decrease in the $\mathrm{Nu}_x/(\mathrm{Gr}_x)^{\frac{1}{5}}$ ratio for the micropolar fluid ($\Delta = 5.0\,; P = 5.0$) with reference to the Newtonian fluid equals 15%. The biggest change in the $\overline{N}$ microrotation component is observed, in turn, for the initial time of the process ($\overline{\tau} < 20$).

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