WIDEBAND BEAMFORMER IN THE FREQUENCY DOMAIN

ANDRZEJ ELMINOWICZ

Marine Technology Centre,
Dickmana 62, 81-109 Gdynia, POLAND
andrzeje@ctm.gdynia.pl

In modern high resolution wideband sonars using linear array, beamforming constitutes one of the most difficult problems. This paper presents a new efficient method of beamforming in the frequency domain. Applying fast Fourier transforms: to transfer into the frequency domain – temporal FFT (TFFT); to make spatial filtration – spatial FFT (SFFT) and to transfer from the frequency domain – inverse FFT (IFFT), as well as fast convolution in the frequency domain correlating the conjugated and corrected replicas for each beam with picked up samples of SFFT for each frequency and each beam we obtain a processing that is less calculations burdened than other known processing algorithms.

INTRODUCTION

Beamforming, in the time domain, is realized by equalization of wave front impinging on the multielements array. The wave front equalization for a beam with steering angle Θ is accomplished by inserting time delays, determined from formula (1), into the received signals.

$$\Delta\tau(\Theta) = (d/c)\sin(\Theta) \tag{1}$$

where:

$\Delta\tau$ – time delay of signals between adjacent elements,
Θ – beam steering angle,
d – interelements spacing,
c – acoustic velocity in water.

Delay in the time domain corresponds to phase shift in the frequency domain. The basic operation of beamforming in the frequency domain is thus the phasing and summing of signals from particular array elements. The input signals are recorded in time and therefore, to make beamformer in the frequency domain, a prior signal transformation from time into frequency domain is necessary. A series of complex signals in the time domain, obtained for each element of array, is segmented into blocks (segments) with M length, synchronous for each channel. To ensure an undistorted realization of beamformer and correlation processing (eliminating the edge effect due to segmentation), the length of the segments should be doubled and the segments should overlap in 50%. Using FFT for each segment, we obtain factors $X_n(k)$ for f_k frequency, determined from the formula:

$$f_k = f_c - \frac{BW}{2} + \frac{k \cdot f_s}{M} \tag{2}$$

where:

M – the length of the segment,
f_s – sampling frequency,
f_c – central frequency,
BW – beam width,
k – frequency factor.

The transform enabling conversion from time into frequency domain, called TFFT (temporal FFT), has thereby been made. Complex coefficients $X_n(k)$ for each frequency f_k are obtained. Beams for each f_k are calculated from:

$$|B(k,\Theta,\Theta_m| = \left|\sum_{n=0}^{N-1} X_n(k,\Theta)\exp\left[-j2\pi f_k \cdot \frac{d}{c}\sin(\Theta_m)\right]\right| \tag{3}$$

where:

Θ_m - beam direction

For simplification reasons, weighting and focusing are ignored in the above formulae. A shortcoming of beamforming in the frequency domain is beams directions dependence on frequency. The beam direction is calculated from the formula:

$$\Theta_m = \sin^{-1}(cm/f_k N' d) \tag{4}$$

where:

m – beam index,
c – acoustic velocity,
N' – number of array elements,
d – interelements spacing.

The above dependence means that we have to make separate FFT for each direction, what requires unprofitable large number of complex multiplication. Since beams computed for different frequencies, at desired direction Θ_m, may be calculated with a small error, approximate methods of correcting beams directions, for different frequencies, may be applied.

Paper [1] presents two approximate methods of correcting the beams directions. In the first method, the so–called zero–padded FFT, the number of beams is increased by extending SFFT with a large number of zeroes; SFFT size may be extended 8 or 16 times. In the second method (De Muth's method), short FIR filter is utilized for interpolating data obtained by SFFT. Paper [1] presents also 4 exact methods of beamforming, i.e.: direct method (dot product), Horner's rule, Goertsel's algorithm and chirp Z transform (CZT). All these methods require a greater number of computations than that required by *N*–point SFFT for each frequency. Papers [2] and [3] present the methods of wideband processing by 2DFFT. This method, widely used in English sonars, is an exact procedure, but it also requires a greater number of calculations than it is in the case of *N*–point SFFT for each frequency.

1. TFFT and SFFT BEAMFORMER

To realize FFT beamformer, separately for each frequency *k*, TFFT is to be performed on segments of signals, obtained from *N*–element array and then *N*–point SFFT, for each frequency, is to be carried out. As a result of these operations, we get a spatial spectrum for frequency *k*, marked with "•", as shown in Fig. 1. The spectrum samples for B_{-4} synchronous beam, given in Fig. 1, are marked with "*".

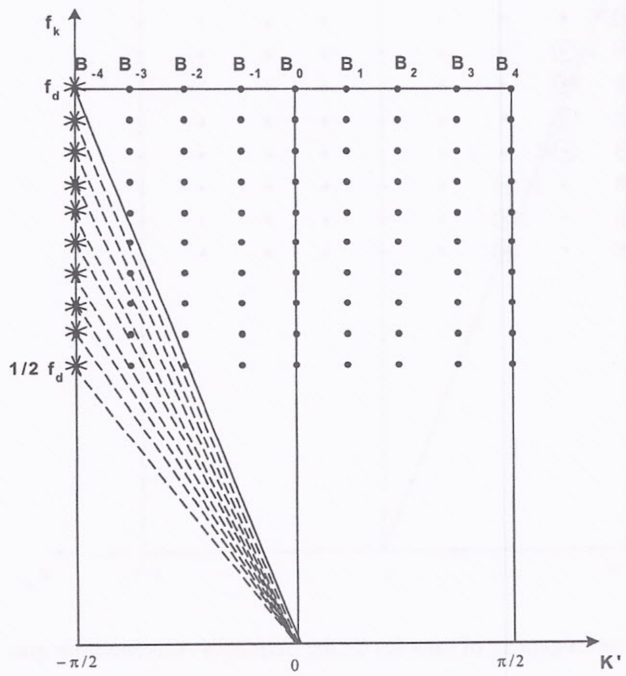

Fig. 1 The arrangement of different frequencies samples and the corresponding beams directions for B_{-4} synchronous beam.

The samples correspond to beams, marked with ---· in Fig. 1, which are formed in different directions calculated from formula (4), creating thereby the total broadening beam for *k* frequencies. Fig. 2 presents the arrangement of spectrum samples on the frequency–wavenumber grid, where the exact samples for B_{-4} direction are marked with "*".

The SFFT spectrum samples decrease for frequencies receding from frequency f_d due to the fact that beams directions, for those frequencies, recede from B_{-4} direction in accordance with formula (4).

It should be noted that for the remaining B_0, B_{-1}, B_{-2} and B_{-3} directions, we obtained the spectrum samples marked with "⊙" , located close to B_{-4} direction, determined by samples marked with "x", formed exactly in B_{-4} synchronous direction. Picking–up the spectrum samples, marked with "⊙", located close to B_{-4} direction (Fig. 2), yields appreciably lower broadening of the total beam for all frequencies. For beams, marked with "⊙" , corresponding to particular frequencies, we receive signals which are lower than B_{-4} synchronous beam signals what results in the distorted amplitudes of particular spectrum samples. Correcting the amplitudes of signals for particular frequencies k and Θ_m directions, we can realize the exact beamformer in frequency domain.

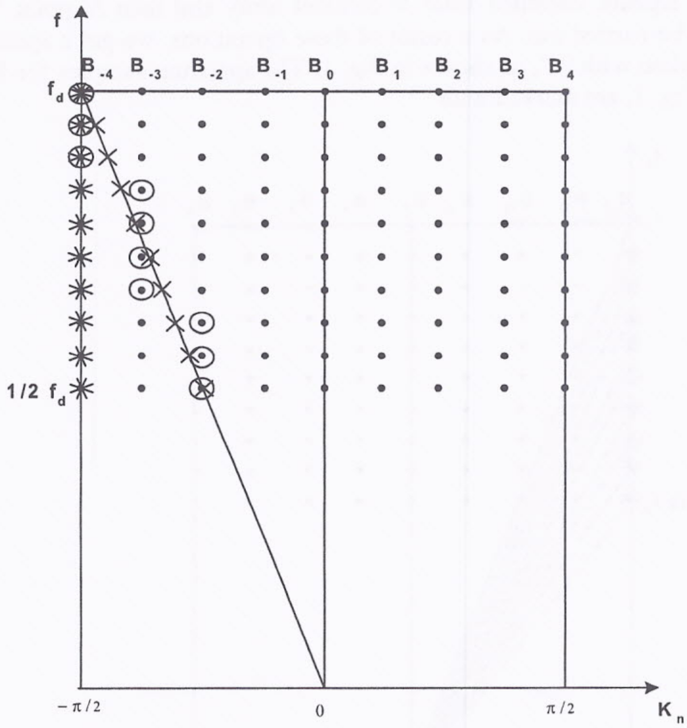

Fig. 2 The arrangement of samples on the frequency–wavenumber grid

- • – samples from SFFT – all beams,
- X – samples for synchronous beam, obtained from the beamformer in time domain,
- ✳ – samples for B_{-4} beam from SFFT beamformer
- ⊙ – picked–up samples for B_{-4} beam from SFFT beamformer (samples placed close directions of B_{-4} beam).

Comparing the distorted (after SFFT) and the exact spectra of T_{xNm} replica, for the same *m* direction, we can determine frequency–amplitude coefficients which, multiplied by particular samples of the distorted spectrum, yield the exact spectrum of T_{xNm} replica for *m* direction. Similarly, we can determine frequency–amplitude coefficients for all directions of beams coming from different ranges and directions. Comparing the spectra, marked with "⊙", from SFFT beamformer with the spectra for a beam from Θ_m direction, we get:

$$X_n(k)\exp\left(-j2\pi nf_k \frac{d}{c}\cdot\sin\Theta_m\right) = X'_n(k)\exp\left(-j\frac{2\pi}{N}mn\right)\cdot h_{k,m} \qquad (5)$$

where:

$h_{k,m}$ – coefficients of amplitude correction for the given *k* and *m* values.

Applying $h_{k,m}$ coefficients, the data obtained from SFFT may be approximated to beams exact values, calculated by the delay–sum beamformer. It should be noted that, for wideband signals, the change of beam directions requires not only the correction of phase $2\pi\frac{m}{N}$ for *m* directions, but also the change of *m* in such a way that the point, obtained from SFFT, will be located closer to the real Θ_m direction. This case, for greater frequencies, is illustrated in Fig.2 where points marked with "*" are replaced by points marked with "⊙", located closer to Θ_m direction, determined by B_{-4} line. The above is called collection of points from SFFT.

Instead of direct multiplication of SFFT points by $h_{k,m}$ coefficients, we can use replica of transmitting signal $T'_{xm}(k)$ for all *m* directions, the replica being subsequently multiplied by $h_{k,m}$. The replica of transmitting signal, in frequency domain, for *m* directions, may be written as:

$$T'_{xm}(k) = \sum_{n=0}^{N-1} T_{xm}(k)\exp\left(-j2\pi nf_k \frac{d}{c}\sin\Theta_m\right) \qquad (6)$$

Multiplying the replica of transmitting signal from *m* directions by $h_{k,m}$ coefficients, we obtain the corrected replica of transmitting signal for *m* direction.

In correlation process – in frequency domain – $X'_n(k)$ is multiplied by $h_{k,m}$ for all *k*:

$$\Phi_{k,m}(f_k) = X'_n(k)\exp\left(-j\frac{2\pi}{N}mn\right)\cdot T_{xm}(k)\cdot h_{km}\exp\left(+j2\pi nf_k \frac{d}{c}\sin\Theta_m\right) \qquad (7)$$

In the same way we can correct unfocussed signals when sonar operates in a near field.

In lieu of phase shifts, which depend on frequency, we can perform, for each frequency after SFFT execution, frequency–amplitude correction in the way analogous to the correction method for the far field.

Correlation process results in frequency–amplitude correction which yields the focussing of signals separately for each m direction. Fig. 3 presents the diagram of signal receiving processing, realised according to the above described method.

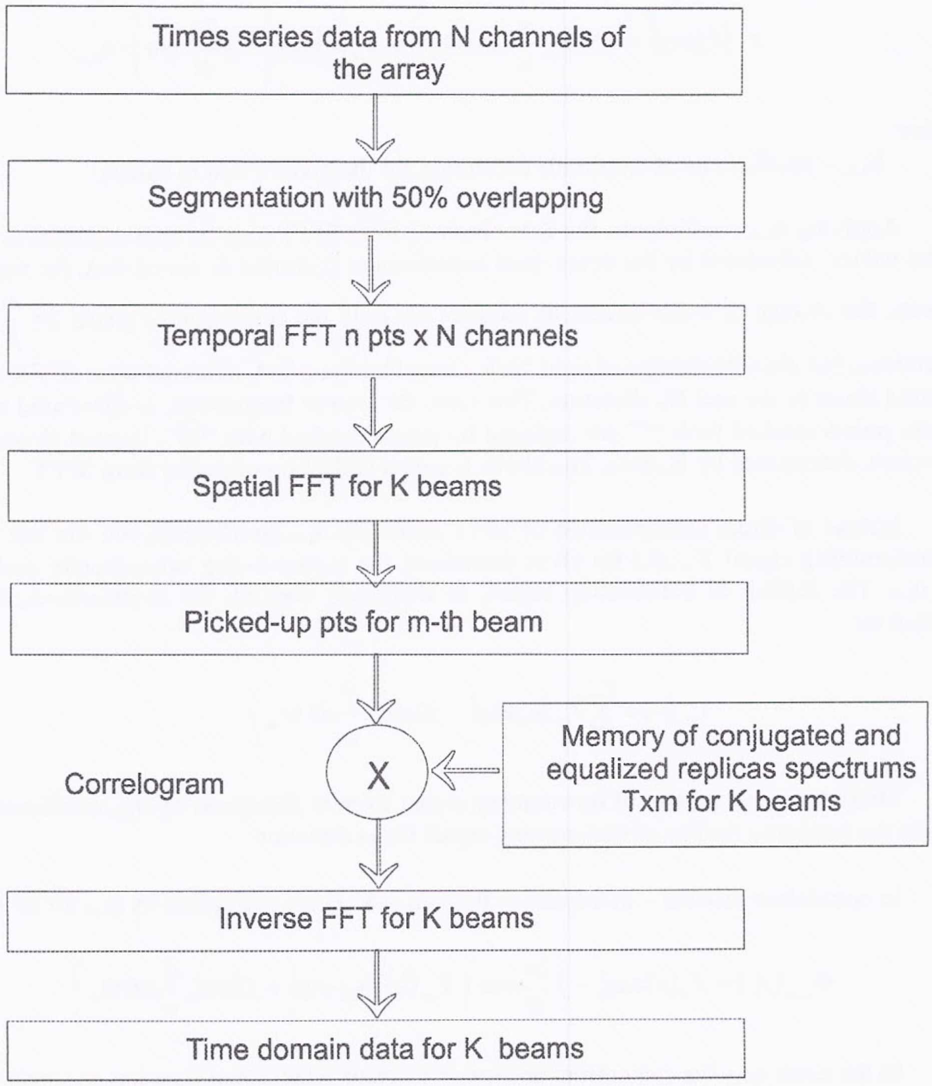

Fig. 3 Efficient frequency domain beam

Table 1 presents number of complex multiplications in particular algorithm steps for processing realization using TFFT and 2DFFT method, as well as TFFT and SFFT method. When we realise beamformer using TFFT and SFFT method, we perform by approx. 40% less complex multiplications than it is required in the case of TFFT and 2DFFT method.

Table 1 Number of mathematical operation for the processing method TFFT and 2DFFT or TFFT and SFFT

Number of channels	N=512	N=512	equation
Processing method	TFFT and 2DFFT	TFFT and SFFT	
	Number of complex multiplecation		
TFFT(t)	2906112	2906112	$N' \cdot n/2 \cdot \log_2(n)$
H^*W1	528384		$N \cdot n$
SFFT(x)	4718592	4718592	$n \cdot N' \cdot n/2 \cdot \log_2 (N)$
W2	10488576		$n \cdot N'$
IFFT(x)	4718592		$n \cdot N' \, n/2 \cdot \log_2 (N)$
W3 (correlogram)	292864	292864	$n \cdot K$
IFFT(t)	1610752	1610752	$K \cdot n/2 \cdot \log_2 (n)$
Number of operation for segment	15823872	9528320	Σ for segment
Number of operation for sample	15903	9576	

where:

N – number of channels with zero-padded FFT,
N' – number of real channels (array elements),
n – number of samples,
K – number of beams.

2. CONCLUSIONS

1. A wideband beamformer in the frequency domain, intended for a linear array, presented in this paper, is a very efficient method of processing, since *N* n–points TFFT, *n* N–points SFFT, *K*· *n* multiplication in the frequency domain and *K* n–points IFFT are required. We obtain processing that is less calculations about 40% than known 2DFFT processing. This has been achieved using the corrected replicas of signals T_{xm} for beams coming from particular directions.

2. Applying the corrected replicas spectrums T_{xm} for beams coming from particular directions, we can made additional correction of the replica so as to include the weighting of array aperture and focusing of the beams in near field, made separately for each direction.

3. Array aperture weighting and beams focusing require no additional mathematical operations in the processing channel, because all calculations of the corrected replicas T_{xm} are made only once at a design stage of the processing channel and what's more, the corrected replicas are stored in memory.

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