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Application of the Scattering Matrix Method to Evaluate Acoustic Mufflers Properties

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Abstract The aim of the paper is to analyse acoustic reflective muffler applying the scattering matrix method. In general, the method is based on dividing the muffler/system into separate subsystems and apply the acoustic multi-ports theory to calculate the scattering matrix of each element to finally combine the results and obtain the scattering matrix of the entire system/muffler. The multi-port procedure is derived from the theory of electric networks and allows to analyse acoustic devises of complex geometry with prescribed accuracy. Based on the scattering matrix, the transmission loss was determined.

Keywords: cylindrical duct, reflective muffler, scattering matrix, multi-port method.

1. Introduction

Waveguides are often straight pipes with a circular or rectangular cross-section inside which an acoustic wave propagates. Waveguides with mufflers attached to them are part of the heating, ventilation or air conditioning systems (HVAC). Mufflers are an important element of such systems because they reduce noise propagating inside the waveguide and thus reduce the sound pressure level emitted by the outlet into the rooms or environment. The methods of calculating and simulating mufflers (acoustics or electric) are constantly developed and newer and newer methods are used both for calculating the acoustics parameters of mufflers or the two-port network in electrical systems [1, 2]. The construction of acoustic mufflers may be more or less varied both in terms of geometry and the materials used (microperforated materials, metamaterials). The construction of some mufflers is so complicated that determining their transmission loss is only possible by means of an experiment or FEM, BEM method.

This work focuses on reflective mufflers, the operation of which is based on the principle of interference of the incident and reflected waves. The reflected wave arises as a result of impedance change, which in reflective mufflers is caused by a change in the channel cross-section. In the calculations of acoustic mufflers, the propagation of higher modes is often not taken into account (plane wave approximation), i.e. analysis is carried out below the cut-off frequency of the higher modes, which corresponds to the reduced frequency $ka = 1.84$ for first radial mode [3]. The use of the plane wave approximation in the calculation of the transition loss (TL) may lead to errors in the estimation of the sound pressure level, especially for higher frequencies, because this approach does not take into account all the occurring phenomena (such as mode transformation) which ensure the fulfillment of the boundary condition at junctions of pipes with different cross-sections [4]. The analysis of the multimode acoustic wave was carried out by many authors, who used the concept of calling an acoustic multi-port. In theoretical calculations of the attenuation efficiency of acoustic multiports based on the scattering matrix, the multimode wave propagation through the system approach is justified [6]. Experimental studies related to side orifices with flow were also carried out [7]. Some authors define the order of acoustic multi-port depending on the number of modes which can propagate in waveguide [1, 2, 5, 8].

In this paper, the multiport is defined by the number of ducts at the junction. Note that in each duct many modes can propagate so it constitutes a multimode port. The analysis is concerns to the numerical calculations of the transmission loss of a simple reflective muffler taking into account the propagation of the multimode wave. The obtained results were compared with the published results of experimental studies for the analogous case of a muffler with a simple expansion chamber [9]. A comparison of the method of determining the transmission loss parameter (TL) of a reflective muffler on the basis of the scattering matrix ($\boldsymbol{S}$) using computational and measurements methods is briefly discussed. Analysis of the results of a more complicated geometry with additional bypass is also presented.

2. Theoretical basics

According to the assumptions of linear theory, a complex system can be divided into separate subsystems that will be analysed separately. The use of the scattering matrix formalism allows to determine the transmission loss parameter (TL) of a system composed of several subsystems connected by ducts create a complex structure. The scattering matrix of a system determines the relationship between two state variables, which are $\boldsymbol{P^{in}}$ sound acoustic pressure of the ingoing wave to the system and $\boldsymbol{P^{out}}$ sound acoustic pressure of the outgoing wave from the system.

$$\boldsymbol{P^{out}} = \boldsymbol{S} \times \boldsymbol{P^{in}} \tag{1}$$

where $\boldsymbol{P^{in}}$ – one column matrix of complex amplitudes of acoustic pressure of all mode of ingoing wave to the system from each multimode port, $\boldsymbol{P^{out}}$ – as previously, but outgoing wave from the system.

Depending on the size of the ducts attached to the muffler chamber, the acoustic wave may consist a different number of modes that can propagate. For numerical calculations modes are numbered sequentially by single-number index according to increasing cut-off frequency while in theory the modes are denoted by two indices (circumferential m and radial l). In the case a two port system, the ingoing and outgoing wave matrix can be written as (see Figure 1 – port 1)

$$\boldsymbol{P^{in}} = \begin{bmatrix} \boldsymbol{P^{in,A}} \\ \boldsymbol{P^{in,B}} \end{bmatrix}; \quad \boldsymbol{P^{out}} = \begin{bmatrix} \boldsymbol{P^{out,A}} \\ \boldsymbol{P^{out,B}} \end{bmatrix} \tag{2}$$

$$\boldsymbol{P^{in,A}} = \begin{bmatrix} P_1^{in,A} \\ \vdots \\ P_N^{in,A} \end{bmatrix}; \quad \boldsymbol{P^{out,A}} = \begin{bmatrix} P_1^{out,A} \\ \vdots \\ P_N^{out,A} \end{bmatrix}; \quad \boldsymbol{P^{out,B}} = \begin{bmatrix} P_1^{out,B} \\ \vdots \\ P_N^{out,B} \end{bmatrix}; \quad \boldsymbol{P^{in,B}} = \begin{bmatrix} P_1^{in,B} \\ \vdots \\ P_N^{in,B} \end{bmatrix} \tag{3}$$

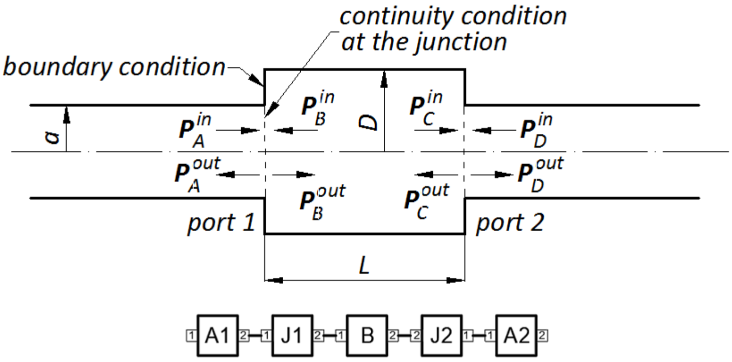

Figure 1. Geometry of the analysed muffler with simple expansion chamber with topological representation, where: A1 – inlet pipe, J1 – junction between pipe A1 and chamber, B – chamber, J2 – junction between chamber and outlet pipe, A2 – outlet pipe.

In cylindrical coordinates (ρ, φ, z), for harmonic time excitation using the $e^{-i\omega t}$ convention acoustic pressure of single mode ingoing and outgoing waves can be written in the form

$$p_{ml}^{in}(\rho, \varphi, z) = P_{ml}^{in} \psi_{ml}(\rho, \varphi) e^{ik_{z,ml}z} \tag{4}$$

$$p_{ml}^{out}(\rho, \varphi, z) = P_{ml}^{out} \psi_{ml}(\rho, \varphi) e^{-ik_{z,ml}z} \tag{5}$$

where $P_{ml}^{in/out}$ – mode amplitude, $k_{z,ml}$ – axial wave number, ψ_{ml} – mode shape function. Mode shape function is expressed as:

$$\psi_{ml}(\rho, \varphi) = \Lambda_{ml} e^{im\varphi} J_m \left(\mu_{ml} \frac{\rho}{a} \right) \tag{6}$$

where Λ_{ml} – modal normalisation constant, $J_m(x)$ – Bessel function, $\frac{\mu_{ml}}{a}$ – radial wave number.

In the presented method, the subsystems are analysed separately to satisfy the continuity condition at each junction. The conditions of acoustic pressure continuity at the junction takes the form

$$p^A(\rho, \varphi) = p^B(\rho, \varphi), \quad |\rho| \le a_A \tag{7}$$

Taking into account multimode excitation

$$\sum_k P_k^A \psi_k^A(\rho,\varphi) = \sum_j P_j^B \psi_j^B(\rho,\varphi), \quad |\rho| \le a_A \tag{8}$$

The axial acoustic velocity

$$v^A(\rho,\varphi) = v^B(\rho,\varphi), \quad |\rho| \le a_A \tag{9}$$

Boundary condition of normal component acoustic velocity on the hard wall

$$v^B(\rho,\varphi) = 0, \quad a_A \le |\rho| \le a_B \tag{10}$$

Modal representation takes the form

$$\sum_k V_k^A \psi_k^A(\rho,\varphi) = \sum_j V_j^B \psi_j^B(\rho,\varphi), \quad |\rho| \le a_A \tag{11}$$

$$\sum_j V_j^B \psi_j^B(\rho,\varphi) = 0, \quad a_A \le |\rho| \le a_B \tag{12}$$

Applying the orthonormality conditions to the acoustic pressure continuity equations and the boundary conditions, we obtain

$$\sum_K Q_{nk}^{AA} P_k^A = \sum_j Q_{nj}^{BA} P_j^B, \quad n = 1, \dots \tag{13}$$

where Q_{kj}^{AA} and Q_{kj}^{BA} are diagonal matrices

$$Q_{kj}^{AA} = \int_{S_A} \psi_k^A(\rho,\varphi) \left(\psi_j^A(\rho,\varphi)\right)^* dS = S_A \delta_{jk} \tag{14}$$

$$Q_{kj}^{BA} = \int_{S_A} \psi_k^B(\rho,\varphi) \left(\psi_j^A(\rho,\varphi)\right)^* dS \tag{15}$$

As a result of applying the mode matching method between the acoustic pressure amplitudes at the junction between the inlet duct and the muffler chamber, we obtain

$$\boldsymbol{P^B} = (\boldsymbol{Q^{BA}})^{-1} \times \boldsymbol{Q^{AA}} \times \boldsymbol{P^A} \tag{16}$$

3. Analyzed cases and results of numerical calculations

The muffler can be thought of as a "black box" to which more than two ducts are connected. Such a system is called an acoustic multiport. The occurring phenomena can be analysed assuming their linearity. Joints are the places of connections of individual multiport with a constant cross-section with the analysed acoustic system. In the joint the acoustic wave can be decomposed into waves ingoing and outgoing the analysed system and the near field effect close to the discontinuity is neglect (the joints are long enough). The total sound pressure field is a superposition of the modes that creates the acoustic wave [10]. Using the scattering matrix formalism in a selected port, an acoustic wave can be described by means of a single-column matrix containing complex amplitudes of acoustic pressure wave ingoing and outgoing to the muffler. The most common approach is that non-propagating modes are not taken into account, although their impact may be significant especially if their cutoff frequencies are close to but above the excitation reduced frequency ka (k is the wave number, a is the waveguide radius).

The analysis concerned two cases of reflective mufflers differing in the design of the expansion chamber. The first case refers to a simple expansion chamber, while in the second case the expansion chamber has a more complicated structure. Numerical calculations of the transmission loss (TL) of the reflective mufflers were carried out in order to compare them with the value obtained on the basis of the previously published experimental results [9]. Numerical calculations were performed using scripts in Matlab software written by J. Jurkiewicz. The scheme of a reflective muffler with a simple expansion chamber is shown in Figure 1.

The geometric dimensions of the analysed muffler are as follows: radius of the inlet (outlet) pipe a = 103.2 mm, radius of the expansion chamber D = 183.2 mm and length of the expansion chamber L = 444 mm. The calculations do not take into account the length of the channels on both sides of the muffler. In this case, for comparative purposes, the analysis was performed to the value of ka = 4. Only axisymmetric modes were taken into account in the calculations.

The sketch and topological scheme of a muffler with a more complicated geometry with additional bypass is presented schematically in Figure 2.

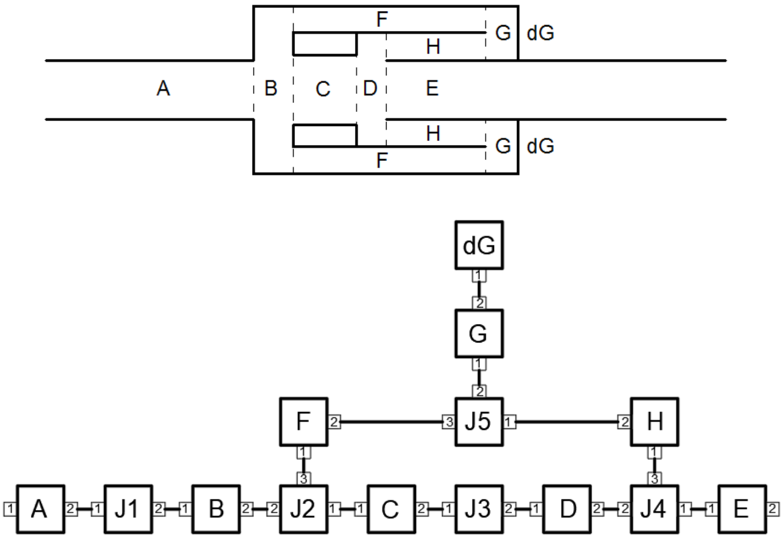

Figure 2. Geometry scheme of asymmetrical muffler with additional bypass with topological representation, where: A, B, C, D, E, F, G, dG, H – multi-ports, J1, J3 – junction between 2-ports, J2, J4, J5 – junction between 3-ports.

The muffler presented in Fig. 2, contains the bypass in the form of an air gap between pipes C and F which can generate phase difference between the waves propagating in the main tract. It differs from the previous muffler in that it is asymmetrical with respect to the left and right side. The calculations have been carried out for the following dimensions of the elements: ducts A, C and E are of unit radius and furthermore duct C length is 3 units, ducts B is of radius 1.88 and unit length, ducts D is of radius 1.49 and also unit length, annular ducts F have the inner and outer radiuses 1.59 and 1.88 respectively and is of 7 units length. Annular duct G is of one unit length, but its inner and outer radiuses are 1.1 and 1.88. The dimensions of the annular duct H are: inner radius – 1.59, outer – 1.88 and the length 3 units. Here the dimensions are presented in any units, because due to the ka parameter the scaling of phenomena occurring in waveguides with different radius values is possible. In this case, the transmission loss parameter was determined only on the basis of numerical calculations.

Figure 3 shows a chart of the transmission loss parameter (TL) for a muffler in the form of a simple expansion chamber. In the mode matching method at the junction of the inlet / outlet ducts with the muffler chamber, in case of insufficient number of modes, errors arise due to neglecting the near-field influence. For this reason, it should be taken from a few to several dozen excited modes on a junction [11]. Thus more accurate results can be obtained by considering a larger number of modes in the calculations (also those axially symmetric modes that cannot propagate) and then finally reduce the size of the scattering matrix taking into account the exponential attenuation of the modes above their cut-off frequency.

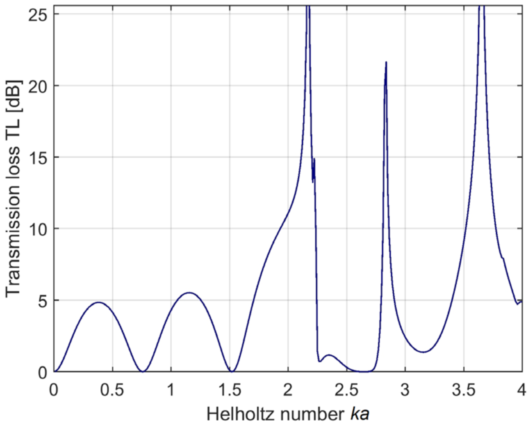

Figure 3. Results of numerical calculations transmission loss of the muffler with simple expansion chamber.

In the case of numerical calculations, the transmission loss parameter (TL) defined as the ratio of the acoustic power of the plane wave incident and transmitted in a logarithmic scale. The single-number value of the transmission loss parameter of the simple reflective muffler (Figure 3) for $ka = 4$ is $TL = 4.8$ dB. Two axisymmetric modes (0.0) and (0.1) were taken into account in the final calculations scattering matrix $\boldsymbol{S}$. In relation to the experimental results, the transmission loss value for single-mode plane wave excitation (mode (0.0)) with a small contribution from mode (0.1) was $TL = 10.6$ dB. However, in the case of excitation with a single mode (mode (0.1)) with a small contribution from the mode (0.0), it was $TL = 0.6$ dB [9]. The discrepancy between the numerical calculations and the experiment results is due the different number of modes taking into account in the calculations parameter TL. Numerical calculations take into account only the plane wave (mode (0,0)) and the experiment two allowed due to the parameter ka, axially symmetric modes (modes (0,0) and (0,1)). In the case of an experiment there is a difficulties in generation of the independent acoustic pressure fields in the form of a single mode, which in turn greatly simplifies the method of determining the scattering matrix $\boldsymbol{S}$. To generate a single mode in a waveguide the measurement setup containing a rotating matrix of sound sources and the measurement in four cross sections in the duct, i.e. two before and two after the muffler is required [12, 13]. Additionally, the determination of complex amplitudes of sound sources in order to generate a single mode on the basis of theory may result in the appearance of other modes which, due to the ka parameter, can propagate. Besides an acoustic wave of excitation should be generated in the form of single mode for the entire range of the analysed frequencies. The construction of the anechoic termination may affects on the waves propagating in the duct on the right side of the muffler. In numerical calculations, the duct on the right (left) side of the muffler was not considered, hence phase differences may appear, especially as individual modes propagate at different velocity. The experimental TL values are presented only as a comparison the effectiveness of the same muffler for different number of modes included in the final calculation of TL parameter. There are significant differences from the well-known plots of the TL parameter curve, taking into account only the propagation of a plane wave in the ducts of such an muffler.

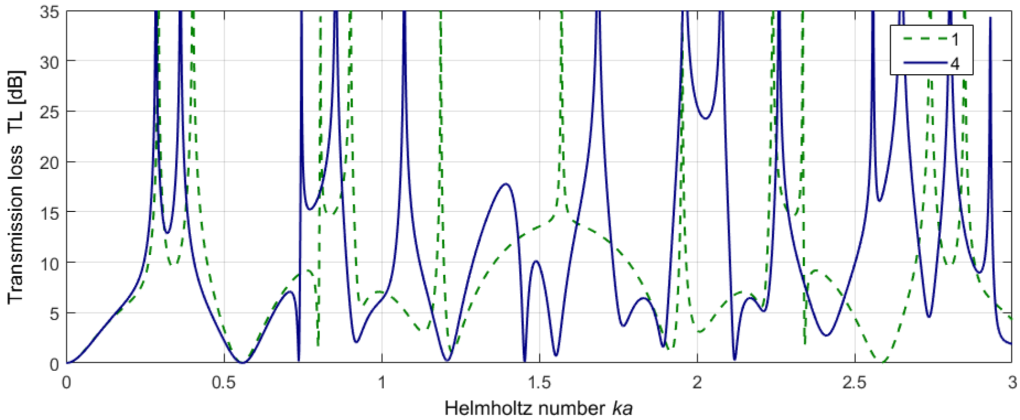

Figure 4. Result of numerical calculations transmission loss of the asymmetrical muffler with additional bypass; dashed line – calculations taking into account only one axisymmetric mode (0,0), continuous line – taking into account four consecutive axially symmetric modes in the calculations of the matrix $\boldsymbol{S}$.

Figure 4 shows transmission loss (TL) value for a reflective muffler with additional bypass. The curves in the figure show the effect of the number of modes involved in the computation of the scattering matrix on the junction. The result of the calculations (Figure 4) is a comparison of the case when higher modes are not taken into account in the calculation of the scattering matrix (only a plane wave can transform) with the case of taking into account higher order modes – four axially symmetric modes. Further increasing the number of modes does not significantly affect the obtained results. There are clear differences in the values of the TL parameter depending on the reduced frequency ka and the number of modes taken into account in the calculations. In this case similar to the previous one, in order to approximate the continuity conditions at the junction with sufficient accuracy, the dimensions of the $\boldsymbol{S}$ matrix should exceed the number of cutoff modes. The dimensions of these matrices depend on the nature and parameters of the analysed joint. After calculating the matrix $\boldsymbol{S}$, its dimension can be reduced to represent only the allowed modes and possibly some weakly attenuated modes propagated in ducts connected to the each junction, as presented in the paper by Snakowska and Jurkiewicz [4]. Figure 4 shows plane wave attenuation because the maximum value of the reduced frequency for which the calculations were performed is $ka = 3$.

4. Conclusions

The paper describes the effectiveness of transmission loss parameter (TL) of two types of reflective mufflers – with simple expansion chamber and more complicated geometry of chamber with additional bypass. The results of numerical calculations and the experimental research for the first type of muffler are presented. The attenuate efficiency of the second type of muffler was determined on the basis of numerical calculations. Experimental results were only provided for the purpose of comparing the effect of the different number of modes taken into account in the final calculation of the transmission loss parameter (TL). In addition, the experimental results may differ from the numerical calculations due to different measurement conditions, such as imperfect anechoic termination and excitation in the form of a single mode which accompanied by other modes with smaller amplitude. Determination of the scattering matrix of reflective mufflers provides all possible information on the modification of the acoustic pressure field caused by the muffler. From the scattering matrix, the transmission loss parameter (TL) of the reflective mufflers can be calculated. The increase in accuracy can be obtained by taking into account in the calculations of $\boldsymbol{S}$ matrix a greater number of modes than those that can propagate without attenuation. The dimension of the scattering matrix $\boldsymbol{S}$ should be reduced after its determination.

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