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The Hilbert Transform Adaptation for Measuring Amplitude and Phase Low-Frequency Disturbances in Power System Voltage

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***Summary*: Phase disturbances in electric power signals are neglected and only amplitude (envelope) distortions are discussed in a literature. Amplitude disturbances are interpreted as Amplitude Modulation (AM) of main 50 Hz frequency. Detailed analysis of all disturbances and their sources are presented in the paper. It is shown that AM and narrow band PM (phase modulation) take place in power network simultaneously and those two modulations interact with each other. Voltage variation is proposed as disturbances measurement. Theoretical results are verified by field experiments and their results are presented.**

Key words:
flickermeter,
analytical signal,
fundamental component compensation,
function variation

1. INTRODUCTION

Low-frequency disturbances in electrical power systems are caused mainly by a slow, with regard to the 50 Hz main component frequency, variation of load impedances, particularly those of large power. The mechanism of these disturbances consists in transferring the variation of load impedance to the variation of load voltage supply. Voltage supply fluctuations are caused by time-varying load currents and internal impedances of supply. Thus the system voltage modulation by a slowly-changing, low-frequency signal results in voltage drops across the impedances of supply transformers and on power system conductors. As shown in [1, 3], this modulation is nonlinear with respect to the modulating factor. Under normal operating conditions it is contained within frequency band from 15 Hz to 85 Hz in the neighbourhood of 50 Hz (in Europe).

The low frequency disturbances deteriorate the quality of electric power delivered by suppliers, causing, among the other things, flickering of light sources. For this reason (in terms of international standardization) a standardized method for determining the measurements of such disturbances has been proposed. It takes into account physiological properties of human eye and brain that are suffering fatigue from light unsteadiness [5÷8]. Their definitions are rather complex, they have no simple physical interpretation and comprise heuristically chosen coefficients [7, 8].

The method, described above, defines the state of the art recommendations for designing flickermeters. Though these instruments employ the most recent system solutions and digital measurement techniques they have several drawbacks being a consequence of the adopted measuring method. Their most significant disadvantages are:

1) experimentally proved discrepancy of measurements results [5],
2) the lack of additivity with regard to disturbance level, and complicated calibration method [7].

It should be mentioned that low-frequency disturbances, apart of causing light flicker, can result also in an irregular operation of AC equipment.

The concept of a new method of measuring low-frequency disturbances in a power system voltage is presented in the paper. It challenges the existing flickermeter method and moves away from flicker-meter structure. A low-frequency disturbance signal is determined in it by means of high-resolution measurement of the network, voltage bandpass filtering and amplitude and phase demodulation using the Hilbert transform and analytical signal concept. Its variation stands for a measure of the low-frequency disturbances. Basic properties of the method and preliminary experimental results of its application are presented in the paper.

It has been verified, both analytically and experimentally, that low-frequency disturbances have the nature of complex, simultaneous amplitude and phase modulation. This modulation is different from the one assumed in the documents defining principles of operation and calibration of flickermeters [7]. This fact explains the observed discrepancy of low-frequency disturbance measurements obtained from using different flicker-meters [5].

2. THE GENERATION MODEL OF LOW-FREQUENCY DISTURBANCES

The model of low-frequency disturbances we adopt is presented below. For simplicity the model is based on a typical radial network configuration, which consists of a high voltage (HV) transformer (110 kV) a system of medium voltage MV (15 kV) and low voltage (LV) (400/230 V) transformers and distribution lines. We also assume that loads are only on the low-voltage side. The single-line diagram of the network and its simplified, equivalent circuit are shown in Figure 1.

The following simplifying assumptions have been made:

1. The network is symmetrical both with respect to the supply and load, thus the three-phase system can be converted to a single-phase one.
2. Each transformer is simplified to its equivalent impedance, reflected by the transformation ratio to the low-voltage side; the parameters of this impedance can be calculated from the short-circuit voltage, load losses

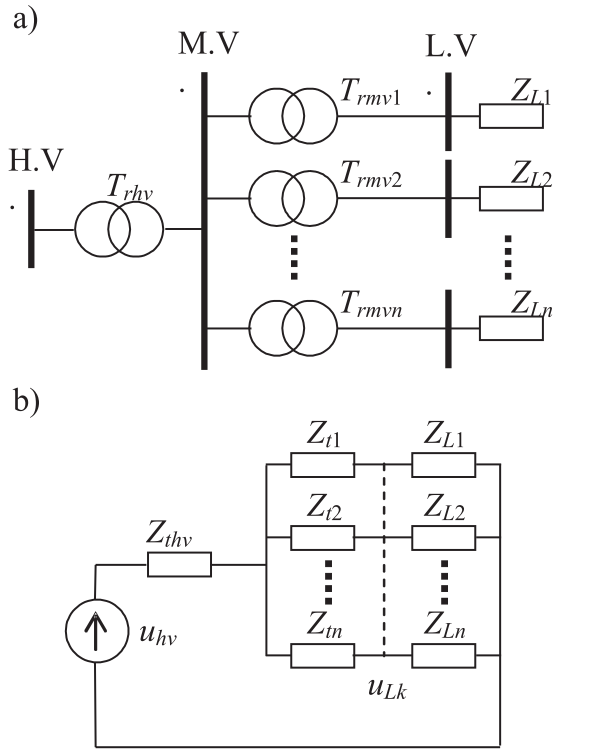


Fig. 1. The diagram of the analysed network configuration model, a) schematic diagram, b) simplified equivalent circuit, T_{rhv}, T_{rmv} – HV and MV transformers, Z_{thv}, Z_{t1}... Z_{tn} equivalent impedances of transformers, converted to the LV side, Z_{L1}... Z_{Ln} – load impedances, u_{hv} – HV supply voltage converted to the LV side, u_{Lk} – the voltage at the k-th load.

as well as apparent power and transformers' nominal voltages; the impedances of network conductors are neglected.

3. The high voltage (HV) side is load invariant; therefore only the high-voltage transformer T_{rhv} impedance Z_{thv}, reflected to the low-voltage side, is taken into account.
4. For each k-th medium-voltage (MV) transformer, $k = 1, 2, 3,...n$ all loads at the low-voltage (LV) side are substituted by one equivalent load Z_{Lk}; it is assumed that only the first load impedance Z_{L1} is varying and our goal is to determine the influence of this variability on the supply voltage u_{Lk} of an arbitrary other load.
5. Slow (with respect to the network voltage) variability with time of the impedance Z_{L1} is assumed while the other load impedances Z_{L2}, Z_{L3},...Z_{Ln} are fixed.
6. The load and transformers' impedances do not depend on the current (the equivalent network system is nonstationary).

Taking the initial phase angle φ_{hv} equal zero, the voltage u_{hv} will be represented in a complex form as:

$u_{hv} = u_{hvm} e^{j\varphi_{hv}} e^{j\omega t} = u_{hvm} e^{j\omega t}$. The term $e^{j\omega t}$ for $\omega = 2\pi 50$ [rad/s], represents a cyclic variation in time of phasors u_{hv} and u_{Lk} in the model (Fig. 1).

For the above simplifying assumptions the impedance ratios can be expressed as dimensionless complex numbers:

$$a_k = \frac{Z_{Lk}}{Z_{tk} + Z_{Lk}} = a_{km} e^{j\varphi_k}, \quad k = 1,2,...n$$

$$a(t) = \frac{1}{Z_{thv} \sum_{i=1}^{n} \frac{1}{Z_i} + 1} = a_m(t) e^{j\varphi_m(t)} \tag{1}$$

Dynamic $a_m(t)$ is smaller than fundamental frequency. Thus, taking time-dependence into considerationtakes the form:

$$\begin{aligned} u_{Lk} &= u_{Lkm}(t) e^{j\varphi_{Lk}(t)} e^{j\omega t} = u_{hvm} a_k a(t) e^{j\omega t} = \\ &= u_{hvm} a_{km} a_m(t) e^{j\varphi_k} e^{j\varphi_m(t)} e^{j\omega t} \end{aligned} \tag{2}$$

Expression (2) has the following interpretation:

1. The terms a_k and $a(t)$, influencing the u_{hv}, represent amplitude and phase change of this voltage.
2. The first term $a_k = a_{km} e^{j\varphi_k}$ describes the model of dividing the converted voltage u_{hv} between the impedances of the k-th load and its supply source. It takes into account voltage drops across the impedances of the HV transformer and the k-th MV transformer, reflected to the k-th low voltage. a_k and φ_k are assumed constant in time.
3. The disturbance signal in expression (2) takes a rational form. It is convenient to interpret it by a means of an analytical signal concept, i.e. as a rotating phasor with modulated amplitude and phase.

The aim of the postulated method is measurement of the signal $a_m(t) e^{j\varphi_m(t)}$ and treating is as a disturbance. The result of measurement is therefore a vector [$a_m(t)$, $\varphi_m(t)$] (or, respectively $\left[Re\left(a_m(t)e^{j\varphi_m(t)}\right) Im\left(a_m(t)e^{j\varphi_m(t)}\right)\right]$). This influences the way the measure is defined. According to [5, 6], autor assume the following range of modulus $a_m(t)$ variation: $a_{m,max} = 1.05$; $a_{m,min} = 0.95$. The range of angle $\varphi_m(t)$ variation is not specified in [7, 8], since the angle is not taken into account there.

3. THE METHOD OF MEASUREMENT OF A SIGNAL DISTURBANCE

Since the modulating factor $a_m(t) e^{j\varphi_m}$ in (1) differs only slightly from unity, the variation in amplitude of the modulated network voltage signal u_{Lk} is small. This causes a problem of measuring it with sufficient resolution.

In presented method the measured voltage signal $u_{Lk}(t)$ was sampled with frequency f_s=2 kHz over 10-minute intervals and stored in a bulk memory. Further processing was performed offline using Matlab environment. Calculation of signals $a_m(t)$ and $\varphi_m(t)$ was addressed.

The signal $u_{Lk}(t)$ was converted by means of the Hilbert transform to the analytical signal form:

$$u_{Lka}(t) = u_{Lk}(t) + jH\{u_{Lk}(t)\} = u_{Lkm}(t) e^{j\varphi_{Lk}(t)} \tag{3}$$

The modulus of $u_{Lka}(t)$ (its envelope) and phase $\varphi_{Lk}(t)$ (between fundamental 50 Hz and $u_{Lka}(t)$) have been computed from (4) as follows:

$$u_{Lkm}(t)=\sqrt{u_{Lk}^2(t)+\left[H(u_{Lk}(t))\right]^2} \quad (4)$$

$$\varphi_{Lk}(t)=tan^{-1}\frac{H(u_{Lk}(t))}{u_{Lk}(t)} \quad (5)$$

The signals $u_{Lkm}(t)$ and $\varphi_{Lk}(t)$ represent instantaneous amplitude and phase of the signal $u_{Lka}(t)$. The process of their reconstruction can be interpreted as complex demodulation.

The modulus $a_m(t)$ of the modulating factor and its phase angle $\varphi_m(t)$ was determined from signals $u_{Lkm}(t)$ i $\varphi_{Lk}(t)$ using (3):

$$a_m(t)=\frac{u_{Lkm}(t)}{u_{hvm}a_{km}} \quad (6)$$

$$\varphi_m(t)=\varphi_{Lk}(t)-\omega t-\varphi_k \quad (7)$$

$a_m(t)$ in (6) represents a ratio of the time-varying instantaneous amplitude, determined from expression (4), and constant amplitude value of the k-th load voltage (according to the assumption in Section 2, $u_{hvm}= const,\ a_{km}= const,\ \varphi_{km}= const$).

In turn, $\varphi_m(t)$ in (7) is a difference of the instantaneous phase, determined from (5), and two terms: a linear one of a generalized phase, proportional to angular power frequency, and a fixed one derived from impedances of the k-th load and k-th transformer. If φ_k is not determined then $\varphi_m(t)+\varphi_k$ should be taken as the modulating factor phase.

The final block diagram of the algorithm for processing the voltage $u_{Lk}(t)$ (2), is shown in Fig. 2.

Digital FIR filter with cut-off frequency of 85Hz is applied. After it decimation by factor of 8 is performed and the sampling frequency f_s is reduced to 250 Hz. Next, the analytical signal is built for the $a_m(t)$ and $\varphi_m(t)$ demodulation. Finally, measures of variation of these functions are computed.

The analytical signal is obtained using the digital Hilbert filter having the ideal impulse response:

$$h_H[n]=\begin{cases}\frac{2}{\pi}\frac{sin^2(\pi n/2)}{n}, & n\neq 0\\ 0, & n=0\end{cases} \quad (8)$$

The applied Hilbert filter was designed using the time-window method. Error of its amplitude characteristic do not exceed 1% over the range 0.75 Hz to 124.25 Hz.

Figure 3 presents the modulus u_{Lk} and phase φ_{Lk} of the actual voltage signal, measured described methods. Assessing the quality of these method the following conclusions can be formulated:

- The measurement resolution was obtained;
- After employing efficient protection tools against the disturbances the influence of disturbances in the presented method within the observed band of (0÷35) Hz was negligible;
- The preliminary system version for the presented method and its realization was characterized by relatively large phase errors due to the low-pass filter and phase corrector (shifter) properties. The maximum value of the zero — level error was estimated at the level of 3 rad, and the rate-of-rise error at 0.02 rad/s.

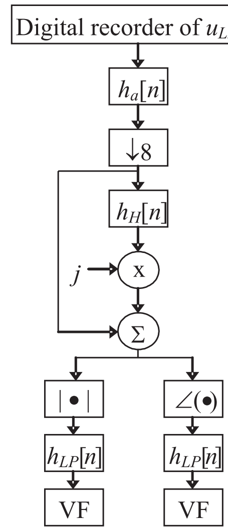


Fig. 2. Block diagram of voltage $u_{Lkm}(t)$ (3) processing. Denotations: $h_a[n]$ – antialiasing FIR filter with cut-off frequency 85 Hz, ↓8 – decimation algorithm 1:8, $h_H[n]$ – Hilbert filter, $h_{LP}[n]$ – low-pass FIR filter with cut-off frequency 35 Hz, |•| – modulus of a complex number, ∠(•) – argument of a complex number, VF – algorithm computing the variation of a function.

4. THE NEW MEASURE OF LOW-FREQUENCY DISTURBANCES

In this paper, the norm of a function variation has been applied to the signals $a_m(t)$ and $\varphi_m(t)$ for determining the measure of low-frequency disturbances. First application of the definition of this norm valid for an arbitrary, continuous and bounded function $f(t)$ was presented in [2]. Variation of the function $f(t)$ in the [0, T] interval is defined as a functional with non-negative rational values:

$$\underset{[0,T]}{Var}\left[f(t)\right]=\underset{[0,T]}{sup}\sum_{i=0}^{n}\left|f(t_{i+1})-f(t_i)\right| \quad (9)$$

for such partitioning of the interval [$0,\ T$] by points t_i: $0 = t_0 < t_1 < t_2 < ... < t_n = T$, $i = 0, 1, 2, ..., n$, that functional reaches its least upper bound. The measures of disturbances based on equation (10) take the form:

$$\underset{[0,T]}{V}(a_m(t))=\frac{1}{T}\underset{[0,T]}{Var}\left[a_m(t)\right]-a_m(0)+a_m(T) \quad (10)$$

$$\underset{[0,T]}{V}(\varphi_m(t))=\frac{1}{T}\underset{[0,T]}{Var}\left[\varphi_m(t)\right]-\varphi_m(0)+\varphi_m(T) \quad (11)$$

Variations $\underset{[0,T]}{V}\left(a_m(t)\right)$ and $\underset{[0,T]}{V}\left(\varphi_m(t)\right)$ of the modulating signals $a_m(t)$ (7) and $\varphi_m(t)$ (7) have been determined over intervals $[0, T]$ with 10-second duration.

5. VERIFICATION OF THE PROPOSED METHOD IN AN ACTIVE EXPERIMENT

In order to verify the effectiveness of the proposed method for determining the measure of low-frequency disturbances an active experiment was carried out on a real power system. It's equivalent circuit is shown in Figure 1 and mathematically modelled by (1,2). Voltage fluctuation on the load Z_{L1} was produced deliberately. The experiment consisted in cyclic switching of the Z_{L1} load impedance, and measuring effects of this variability, i.e. amplitude and phase modulation of voltages $u_{L1}(t)$ and $u_{L2}(t)$, measured at the T_{rmv1} and T_{rmv2} transformers, respectively (see Fig.1). The modulating factors, determined from these voltages, were treated as low-frequency disturbances and their variations were calculated. The maximum amplitude of Z_{L1} variation was 3.3 Ω with frequency of 3 Hz.

Obtained results are presented in Figure 5. The first column consists measurements and calculations for the $u_{L2}(t)$ voltage, while the second column – for the $u_{L1}(t)$ voltage.

Both 10-minute recordings of $u_{L1}(t)$ and $u_{L2}(t)$ were performed simultaneously (maximum shift between them was 2 s). The Z_{L1} load was increased at time instants t = 80, 700, 1370, 1950 s and switched off at t = 2600 s.

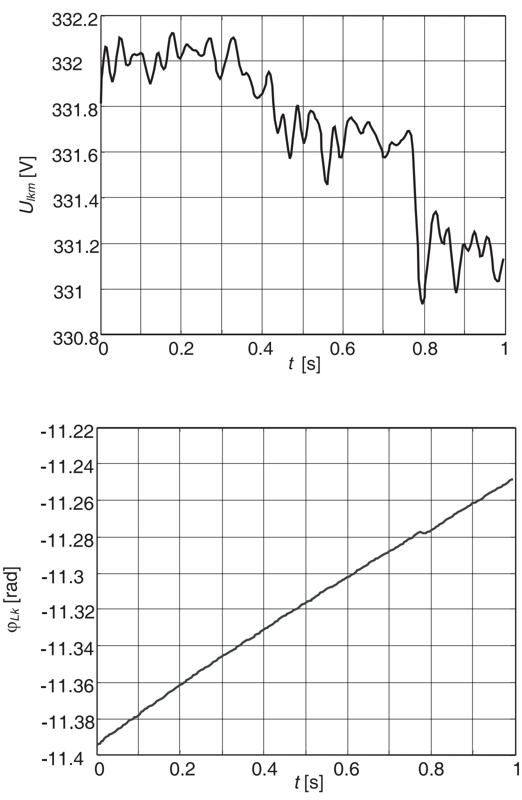


Fig. 3. The results of measurement of the modulus u_{Lk} and phase φ_{Lk} of the actual voltage signal.

The effect of disturbance dispersion is very well seen in Figure 4. Voltage fluctuations can be observed at the terminals of the unloaded transformer T_{rmv2}, being away from the disturbance source. Since the experiment was carried out during the system normal operation, an uncontrolled variations of other system loads are superimposed on the obtained results.

The conducted experimental research and obtained results can be summarized as follows:

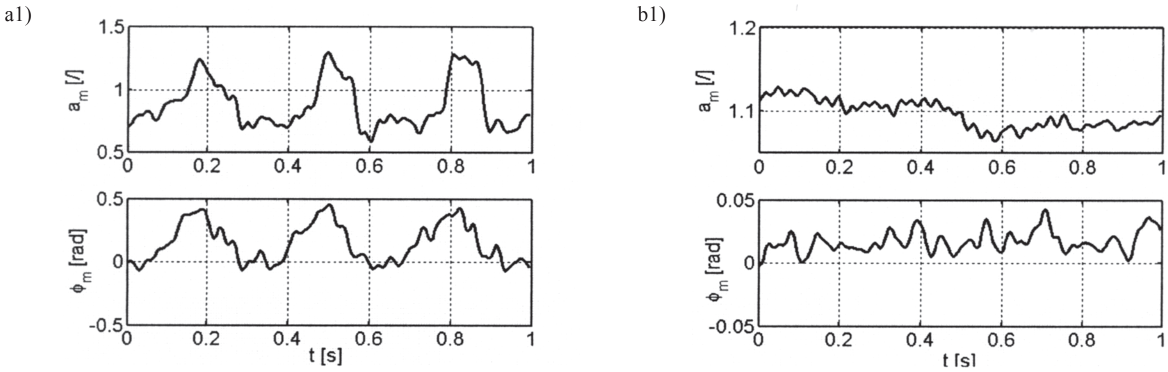
1. The distance from a disturbance source has a significant influence on the amplitude of the periodic component. The first case shows a bigger influence ,when the voltage is measured at the terminals of T_{rmv1} transformer loaded with varying impedance.
2. The thesis about the complex modulation has been experimentally verified since in the a) case both modulus and phase of the modulating factor contain the varying component.
3. The applied measurement method is sensitive. In the second case, for the $u_{L2}(t)$ voltage, a disturbance in the modulus of the modulating factor is visible at the level of 0.1% of the nominal value (0.1% of the voltage amplitude at the terminals of T_{rmv2} transformer, not loaded with variable impedance). The method therefore allows observation of small-magnitude disturbances propagation in a power system.
4. The applied definition of function variation based disturbances measure,is effective even for small-magnitude disturbances. This feature has been demonstrated by a noticeable decrease in the modulus and phase variation at the time instant t = 200 s when the disturbance was switched off (see Fig. 4: a4, b4).

6. CONCLUSIONS

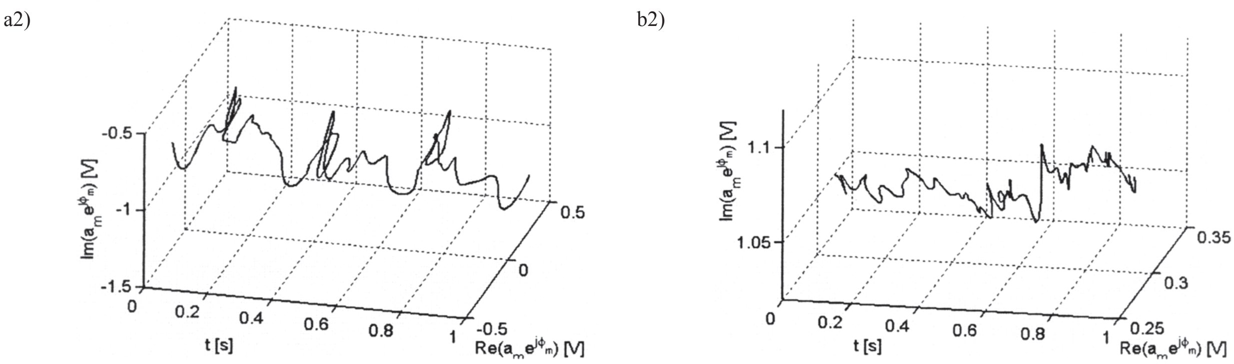
The author would like recapitulate and stress that their work:

1. Presents the new model of low-frequency disturbances generation based on a network model and defines the influence of a load impedance variation on voltages supplying this load, including also mutual dependence between load components.
2. Postulates and proves the thesis about the complex modulation of network voltages, different from that currently adopted in standardization documents concerning flickermeter [6, 7, 8]. The complex modulation is expressed by a variation of the modulating factor modulus and phase angle, computed with applying the Hilbert transform. It explains the origin of observed discrepancy of flickermeter measurements.
3. Proposes the new definition of the low-frequency disturbances measure, based on the signal variation, and – in the authors' opinion – more adequately explains both: the properties and the way of generation of disturbances.
4. Presents the metrological analysis, which lead to the system, hardware and programme solutions that allow to obtain appropriate measurement method resolution and sensitivity.

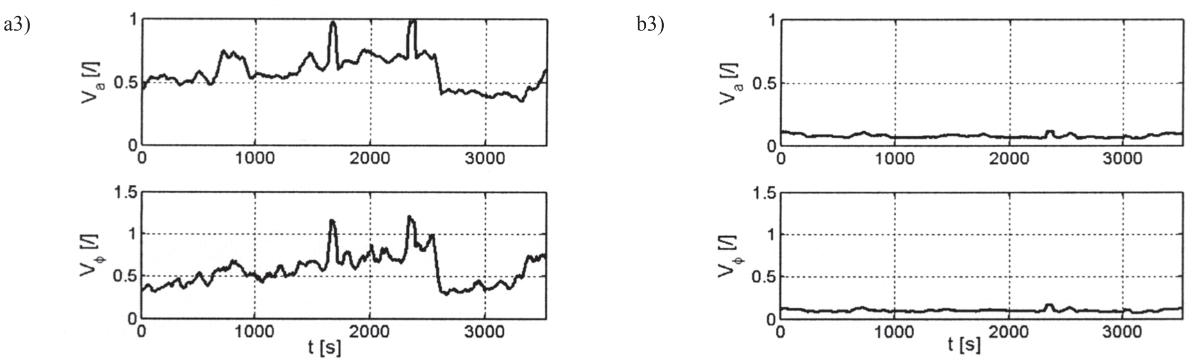
Normalized modulus and phase of the analytical signal for $u_{L1}(t)$ (left) and $u_{L2}(t)$ (right)



Complex variation of the analytical signal after subtracting mean values for $u_{L1}(t)$ (left) and $u_{L2}(t)$ (right)



The amplitude and phase variations during the experiment (averaged in 1-second windows over the period of 60 s) for $u_{L1}(t)$ (left) and $u_{L2}(t)$ (right)



The amplitude and phase variations after filtering the signals from figures a3) and b3) by the low-pass FIR filter with cut-off frequency 5 Hz for $u_{L1}(t)$ (left) and $u_{L2}(t)$ (right)

Fig. 4 The results of measurements in the active experiment (figure continues on the next page)

The experiments proved the effectiveness of the proposed method of measuring disturbances in an electric power system. The method enables quantitative assessment of disturbances and analysis of their propagation on a network. Opposed to standard documents the proposed method allows a phase variation analysis. Phase variation analysis is, in author's opinion, a good tool for power delivery defects investigation.

The described method also provides a possibility of determining a long-term load variation based on the analytical signal phase angle variation.

The method description is brief for obvious reasons. The authors is aware of the necessity of explaining many of the method's properties, not yet investigated, as well as the necessity of multiple verification of its effectiveness in various network configurations.

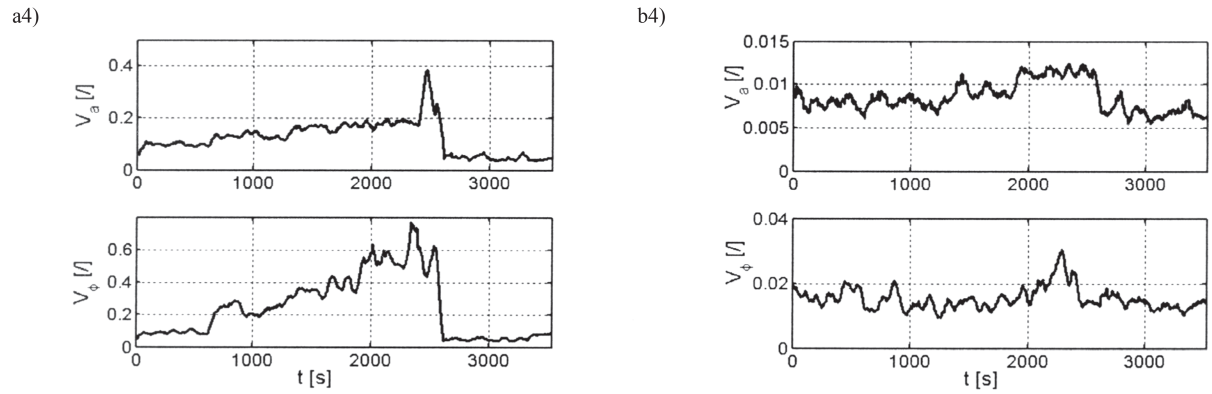


Fig. 5. The results of measurements in the active experiment

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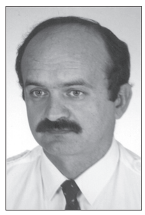

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