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Influence of zero and non-zero boresight pointing errors on bit-error rate free-space optic transmission over Málaga atmospheric channel

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In this paper, the influence of a zero and a non-zero boresight pointing errors on the performances of free-space optic transmission over the Málaga atmospheric turbulence channel is considered. Closed form expressions for a zero boresight channel model probability density function, non-zero boresight channel model probability density function, as well as a bit-error rate over a binary phase shift keying modulation transmission are provided. Numerical results for zero and non-zero boresight pointing errors are graphically presented.

Keywords: free-space optic (FSO) system, Málaga atmospheric turbulence channel, zero boresight pointing error, non-zero boresight pointing error, average bit-error-rate (ABER).

1. Introduction

Free-space optics (FSO) transmission has emerged recently as an efficient solution for obtaining secure, high data rate, wide bandwidth communication, due to its lack of licensing requirements, non-susceptibility to interferences and cost-effectiveness [1]. During transmission, the transmitted signal is affected by various effects such as atmospheric turbulence, irradiance, misalignment between the transmitter and the receiver (pointing error) and scintillation index. These effects influence the performances of the transmitted signal.

There are many articles in the open technical literature that are dealing with characterization of FSO transmission properties. A well known turbulence model with general properties that is used for modeling transmission under the influence of atmospheric turbulence is Málaga turbulence model. Málaga (M) distribution, was proposed in [2] to model the irradiance fluctuation of an unbounded optical wave front (plane or spherical waves), propagating through a turbulent medium, under all irradiance conditions in homogeneous, isotropic turbulence [3]. This Málaga distribution unifies most of the proposed statistical models derived until now in the literature in a closed-form expression providing an excellent agreement with published simulation data over a wide range of turbulence conditions (weak to strong) [2].

In addition to performance degradation caused by atmospheric turbulence, pointing errors are also significantly deteriorating the performance of FSO systems. It should be noted that the pointing errors are not only due to misalignments in the installation process, but also to vibrations on the transmitter and receiver platforms. Mechanical vibration of the transmitter beam causes a misalignment between the transmitter and the receiver. For horizontal links, the vibration comes from transceiver stage oscillations and buildings sway caused by wind, while for vertical links – *i.e.* ground-satellite links – satellite wobbling oscillations are the main source of pointing errors. The pointing error consists of two components: a fixed error, called boresight, and a random error, called jitter, superimposed over the fixed boresight error.

Pointing error occurs due to thermal expansion of a laser beam and it represents a fixed shift between the center of an optical beam and the detector at the reception. Several statistical models have been proposed for modeling a pointing error [4–6], depending on its values, and characterization type of precision of a laser beam and jitter. To describe a pointing error, a model proposed by FARID and HRANILOVIC [4] was used, which, unlike the model given in [5], does not ignore the relation between the size of the detector and the width of the optical beam at the reception. It is also assumed that a satisfactory laser precision has been used. Due to boresight and jitter, each received intensity sample can be thought of as a randomly sampled point on a random irradiance profile.

Impacts of zero and non-zero boresight pointing errors on FSO transmission over Málaga modeled channels are presented in [7–9]. The impact of zero boresight pointing error on a bit-error rate (BER) and the outage probability are given in [8]. Also, the impact of a non-zero boresight pointing error on the outage probability are given in [9, 10]. Since being a general turbulence model, Málaga model can be reduced to other turbulence models:

- by setting $\rho = 0$ and $\mathrm{Var}[|UL|] = 0$, Málaga model reduces to Rice–Nakagami model;
- by setting $\rho = 0$ and $\gamma = 0$, it reduces to gamma model;
- by setting $\mathrm{Var}[G] = 0$, $\rho = 0$ and $X = \gamma$, it reduces to HK distribution model;
- by setting $\rho = 1$, $\gamma = 0$ and $\Omega = 1$, it reduces to gamma–gamma distribution;
- by setting $\mathrm{Var}[|X|] = 0$, it reduces to shadowed–Rician distribution;
- by setting $\rho = 0$, $\mathrm{Var}[|UL|] = 0$ and $\gamma \to 0$, it reduces to lognormal model;
- by setting $\Omega = 0$ and $\rho = 0$ or $\beta = 1$, it reduces to K-distribution;
- by setting $\Omega = 0$, $\rho = 0$ and $\alpha \to 0$, it reduces to exponential distribution;
- by setting $\beta \to 0$, it reduces to gamma–Rician distribution.

For these models, the impact of a zero and a non-zero boresight pointing errors are presented in [10–14].

However, up to now, there is no study in literature that provides closed-form expressions for calculating BER, under the influence of the zero and the non-zero boresight pointing errors, when FSO transmission is carried out over Málaga turbulence channels. The contribution of this paper consists in providing such closed form expressions for analyzing the impact of the zero and the non-zero boresight pointing errors on BER for such FSO transmission scenario.

2. Channel and system model

2.1. Málaga atmospheric turbulence model

We consider a FSO system using IM/DD with OOK, which is widely deployed in commercial systems. The laser beams propagate along a horizontal path through a M turbulence channel with additive white Gaussian noise (AWGN) in the presence of pointing errors. The receiver integrates the photocurrent signal which is related to the incident optical power by the detector responsivity for each bit period

$$y = Ix + n \tag{1}$$

where x is the binary transmitted signal, I is the normalized channel fading coefficient considered to be constant over a large number of transmitted bits, and n is AWGN with variance σ_{s}^2. Hence, the atmospheric turbulence and the pointing error are independent. Subsequently, the channel gain can be expressed as $I = I_{\mathrm{l}} I_{\mathrm{a}} I_{\mathrm{p}}$, where I_{l} is the path loss that is a constant in a given weather condition and link distance, I_{a} is a random variable that signifies the atmospheric turbulence loss factor, and I_{p} is another random variable that represents the pointing error loss factor.

Before we start with numerical calculation, it is necessary to define atmospheric turbulence channel, and both pointing error models, including zero and non-zero boresight. The Málaga turbulence model is based on a physical model that involves a line -of-sight (LOS) contribution U_{L}, a component that is quasi-forward scattered by the eddies on the propagation axis and coupled to the LOS contribution $U_{\mathrm{S}}^{\mathrm{C}}$, and another component due to energy that is scattered to the receiver by off-axis eddies $U_{\mathrm{S}}^{\mathrm{G}}$. $U_{\mathrm{S}}^{\mathrm{C}}$ and $U_{\mathrm{S}}^{\mathrm{G}}$ are statistically independent random processes

$$U = (U_{\mathrm{L}} + U_{\mathrm{S}}^{\mathrm{C}} + U_{\mathrm{S}}^{\mathrm{G}})\exp(\chi + jS) \tag{2}$$

with χ and S being real random variables representing the log-amplitude and phase fluctuations of the optical field. LOS components are defined as $U_{\mathrm{L}} = \sqrt{G}\,\sqrt{\Omega}\,\exp(j\varphi_{\mathrm{L}})$, while scattered components are defined as $U_{\mathrm{S}}^{\mathrm{C}} = \sqrt{\rho G 2b_0}\exp(j\varphi_{\mathrm{C}})$ and $U_{\mathrm{S}}^{\mathrm{G}} = \sqrt{1-\rho}\,U_{\mathrm{S}}'$ with the parameter $\Omega = E[|U_{\mathrm{L}}|^2]$, and the total scattered component denoted as $2b_0 = E[|U_{\mathrm{S}}^{\mathrm{C}}|^2 + |U_{\mathrm{S}}^{\mathrm{G}}|^2]$. Parameter ρ, $0 \le \rho \le 1$, denotes the factor expressing the amount of scattering power coupled to the LOS component and depends on the propagation path length, while U'_{S} denotes a circular Gaussian complex variable, and G denotes gamma random process with a unit mean value, respectively. Constants φ_{L} and φ_{C} denote deterministic phases of the LOS and coupled-to-LOS scatter components.

Received irradiance can be expressed as:

$$I = \left|U_{\mathrm{L}} + U_{\mathrm{S}}^{\mathrm{C}} + U_{\mathrm{S}}^{\mathrm{G}}\right| \exp(2\chi) = YX \tag{3}$$

where $X = \exp(2\chi)$ denotes large-scale fluctuations and $Y = \left|U_{\mathrm{L}} + U_{\mathrm{S}}^{\mathrm{C}} + U_{\mathrm{S}}^{\mathrm{G}}\right|$ denotes small-scale fluctuations.

Málaga model is given as:

$$f_{\mathrm{a}}(I_{\mathrm{a}}) = A \sum_{m=1}^{\beta} a_m I_{\mathrm{a}}^{(\alpha+m)/2-1} K_{\mathrm{a}-m}\left(2\sqrt{\frac{\alpha\beta I_{\mathrm{a}}}{\gamma\beta+\Omega'}}\right), \qquad I_{\mathrm{a}} > 0 \tag{4}$$

where

$$A \approx \frac{2\alpha^{\alpha/2}}{\gamma^{1+\alpha/2}\Gamma(\alpha)}\left(\frac{\gamma\beta}{\gamma\beta+\Omega'}\right)^{\beta+\alpha/2}$$

$$a_m \approx \binom{\beta-1}{m-1}\frac{(\gamma\beta+\Omega')^{1-m/2}}{\Gamma(m)}\left(\frac{\Omega'}{\gamma}\right)^{m-1}\left(\frac{\alpha}{\beta}\right)^{m/2}$$

where $K_v(\cdot)$ denotes the modified Bessel function of the second kind [11, 12], $\Gamma(\cdot)$ denotes the gamma function [15] and $\binom{n}{m}$ represents the binomial coefficient. The parameter α represents the effective number of large-scale cells of the scattering process, while the parameter β represents the effective number of small-scale effects, in the same form as was explained.

2.2. Pointing error models

The pointing error effects in FSO systems can also contribute to channel impairments. In order to study the influence of pointing error on system performance, we propose a statistical model for pointing error with zero and non-zero boresight error, which takes into account the laser beam width, detector aperture size, and jitter variance. The fading due to pointing errors I_{p} has been modeled as the result of considering independent identical Gaussian distributions, with variance σ_{s}^2, for the elevation and horizontal displacement (sway) [16]. In the first case, we presented a zero boresight pointing error model [4]:

$$f_{I_{\mathrm{p}}}(I_{\mathrm{p}}) = \frac{g^2}{A_0^{g^2}} I_{\mathrm{p}}^{g^2-1}, \qquad 0 \le I_{\mathrm{p}} \le A_0 \tag{5}$$

where $g = \omega_{zeq}/(2\sigma_{\mathrm{s}})$ is the ratio between the equivalent beam radius at the receiver ω_{zeq} and the pointing error displacement standard deviation at the receiver σ_{s}; $A_0 = [\mathrm{erf}(v)]^2$

is the fraction of the collected power, where $v = (\sqrt{\pi}\, a/\sqrt{2}\omega_z)$ with erf$(\cdot)$ denoting the error function, whereas the square of the equivalent beam width is given by:

$$\omega_{zeq}^2 = \omega_z^2 \frac{\sqrt{\pi}\,\mathrm{erf}(v)}{2v\exp(-v^2)} \tag{6}$$

In the second case, we consider a non-zero boresight pointing error. The non-zero boresight pointing error is given as:

$$f_{I_\mathrm{p}}(I_\mathrm{p}) \approx \frac{g^2 \exp\left(-s^2/(2\sigma_\mathrm{s}^2)\right)}{A_0^{g^2}} I_\mathrm{p}^{g^2-1} I_0\left(\frac{s}{\sigma_\mathrm{s}^2}\sqrt{\frac{-\omega_{zeq}^2 \ln(I_\mathrm{p}/A_0)}{2}}\right), \quad 0 \le I_\mathrm{p} \le A_0 \tag{7}$$

where s is the boresight displacement, σ_s^2 is the jitter variance at the receiver, $I_0(\cdot)$ is the modified Bessel function of the first kind with order zero. The atmospheric attenuation I_l can be described by the exponential Beers–Lambert law as:

$$I_l(z) = \exp(-\sigma z) \tag{8}$$

where z denotes the propagation distance and σ is the attenuation coefficient.

After defining both pointing error models, we can calculate the probability density functions (PDFs). PDF is obtained by calculating the mixture of the two distributions presented above in Eqs. (4) and (5) or Eqs. (4) and (7)

$$f_I(I) = \int_0^\infty f_{I|I_\mathrm{a}}(I|I_\mathrm{a}) f_{I_\mathrm{a}}(I_\mathrm{a}) \mathrm{d}I_\mathrm{a} \tag{9}$$

where $f_{I|I_\mathrm{a}}(I|I_\mathrm{a})$ is the conditional probability given a turbulence state I_a and it is expressed as

$$f_{I|I_\mathrm{a}}(I|I_\mathrm{a}) = \frac{g^2}{A_0^{g^2} I_\mathrm{a}}\left(\frac{I}{I_\mathrm{a}}\right)^{g^2-1}, \quad 0 \le I_\mathrm{p} \le A_0 I_\mathrm{a} \tag{10}$$

for the zero boresight pointing error and

$$f_{I|I_\mathrm{a}}(I|I_\mathrm{a}) = \frac{g^2 \exp\left[-s^2/(2\sigma_\mathrm{s}^2)\right]}{A_0^{g^2} I_\mathrm{a} I_1}\left(\frac{I}{I_\mathrm{a} I_1}\right)^{g^2-1} I_0\left(\frac{s}{\sigma_\mathrm{s}^2}\sqrt{\frac{-\omega_{zeq}^2}{2}\ln\left(\frac{I}{A_0 I_\mathrm{a} I_1}\right)}\right) \tag{11}$$

for the non-zero boresight pointing error.

Substituting expression (10) in (9) we get the expression for PDF for the zero boresight pointing error which is represented as:

$$f_I(I) = \frac{g^2 A}{A_0^{g^2}} I^{g^2-1} \sum_{k=1}^{\beta} a_k \int_{I/A_0}^{\infty} I_\mathrm{a}^{(\alpha+k)/2-1-g^2} K_{\alpha-k}\left(2\sqrt{\frac{\alpha\beta I_\mathrm{a}}{\gamma\beta + \Omega'}}\right) \mathrm{d}I_\mathrm{a} \tag{12}$$

After evaluating the integral in Eq. (12) according to [17], where the modified Bessel function of the second kind $K_v(\cdot)$ can be expressed as a special case of the Meijer G-function, given by the following relationship [15], we obtained the closed form expression for PDF for a zero boresight pointing error

$$f_I(I) = \frac{g^2 A}{2} I^{-1} \sum_{k=1}^{\beta} a_k \left(\frac{\alpha\beta}{\gamma\beta + \Omega'}\right)^{-(\alpha+k)/2} G_{1,3}^{3,0}\left(\frac{\alpha\beta}{\gamma\beta + \Omega'} \frac{I}{A_0} \middle| \begin{matrix} g^2+1 \\ g^2, \alpha, k \end{matrix}\right) \tag{13}$$

Substituting expression (11) in (9), we get the expression for PDF for a non-zero boresight pointing error which is represented as:

$$f_I(I) = \frac{g^2 A \exp\left[-s^2/(2\sigma_s^2)\right]}{(A_0 I_l)^{g^2}} I^{g^2-1} \sum_{k=1}^{\beta} a_k$$
$$\times \int_{I/(A_0 I_l)}^{\infty} I_a^{(\alpha+k)/2-1-g^2} K_{\alpha-k}\left(2\sqrt{\frac{\alpha\beta I_a}{\gamma\beta+\Omega'}}\right) I_0\left(\frac{s}{\sigma_s^2}\sqrt{\frac{-\omega_{zeq}^2}{2}\ln\left(\frac{I}{A_0 I_a I_l}\right)}\right) dI_a \tag{14}$$

After evaluating the integral from Eq. (14), we obtained the closed form expression for PDF for a non-zero boresight pointing error.

$$f_I(I) = \frac{2\pi g^2 A \exp\left[-s^2/(2\sigma_s^2)\right]}{\omega_{zeq}^2} \sum_{k=1}^{\beta} \frac{a_k I^{(\alpha+k)/2-1}}{(A_0 I_l)^{(\alpha+k)/2} \sin\left[\pi(\alpha-k)\right]}$$
$$\times \sum_{p=0}^{P} \left\{ \frac{\left[\frac{\alpha\beta I}{(\gamma\beta+\Omega')A_0 I_l}\right]^{p-(\alpha-k)/2}}{\Gamma\left[p-(\alpha-k)+1\right]p!} \left[\frac{-\omega_{zeq}^2}{4(p+k-g^2)} \exp\left(\frac{-\omega_{zeq}^2 s^2}{8(p+k-g^2)\sigma_s^4}\right)\right] \right.$$
$$\left. - \frac{\left[\frac{\alpha\beta I}{(\gamma\beta+\Omega')A_0 I_l}\right]^{p+(\alpha-k)/2}}{\Gamma\left[p+(\alpha-k)+1\right]p!} \left[\frac{-\omega_{zeq}^2}{4(p+\alpha-g^2)} \exp\left(\frac{-\omega_{zeq}^2 s^2}{8(p+\alpha-g^2)\sigma_s^4}\right)\right] \right\} \tag{15}$$

The study of the average bit-error rate (ABER) of the M probability distribution in the presence of misalignment fading is considered. First of all, we have defined the expression for ABER when transmission is carried out over binary phase shift keying (BPSK) modulation scheme

$$P_\mathrm{e} = \int_0^\infty \frac{1}{2}\mathrm{erfc}\left(\frac{PR}{\sigma_\mathrm{N}}I\right) f_I(I) \tag{16}$$

where erfc(·) is related to the complementary error function. If we represent erfc(·) as a special case of Meijer *G*-function according to [15, 18] and substituting in (16) along with expression (13), the integral from (16) can be solved [17]. A closed form expression for the zero boresight pointing error for ABER is given as

$$P_\mathrm{e} = \frac{2^\alpha g^2 A\left(\dfrac{\gamma\beta+\Omega'}{\alpha\beta}\right)^{\alpha/2}}{32\pi\sqrt{\pi}} \sum_{k=1}^{\beta} 2^k \left(\frac{\gamma\beta+\Omega'}{\alpha\beta}\right)^{k/2} a_k$$

$$\times G_{7,4}^{2,6}\left(\frac{8R^2P^2A_0^2}{\sigma_\mathrm{N}^2}\left(\frac{\gamma\beta+\Omega'}{\alpha\beta}\right)^2 \middle| \begin{matrix} \frac{1-g^2}{2}, \frac{2-g^2}{2}, \frac{1-\alpha}{2}, \frac{2-\alpha}{2}, \frac{1-k}{2}, \frac{2-k}{2}, 1 \\ 0, \frac{1}{2}, \frac{-g^2}{2}, \frac{1-g^2}{2} \end{matrix}\right) \tag{17}$$

ABER for the non-zero boresight pointing error is calculated and represented in a closed form as:

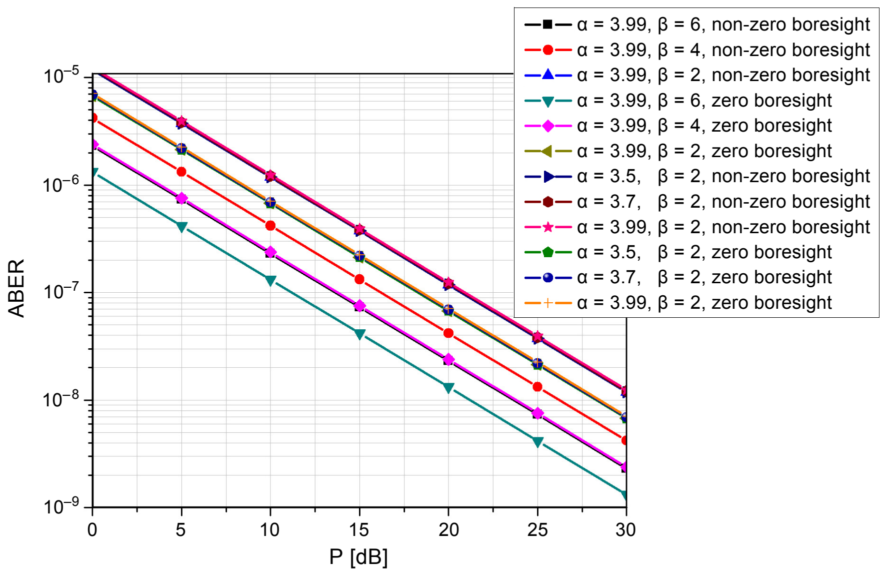

Fig. 1. ABER for non-zero and zero boresight pointing errors when $z = 4$ km, $a = 0.1$ m, $s = 0.3$ m, and $\sigma = 8$ dB/km.

$$P_{\mathrm{e}} = \frac{\sqrt{\pi}\, g^2 A \exp\left[-s^2/(2\sigma_{\mathrm{s}}^2)\right]}{\omega_{zeq}^2} \sum_{k=1}^{\beta} \frac{a_k}{(A_0 I_1)^{(\alpha+k)/2} \sin\left[\pi(\alpha-k)\right]}$$

$$\times \sum_{p=0}^{P} \left\{ \frac{\left[\dfrac{\alpha\beta}{(\gamma\beta+\Omega')A_0 I_1}\right]^{p-(\alpha-k)/2}}{\Gamma\left[p-(\alpha-k)+1\right]p!} \frac{\Gamma(p+k)\Gamma\left(\dfrac{1}{2}+p+k\right)}{\Gamma(1+p+k)} \right.$$

$$\times \frac{-\omega_{zeq}^2}{4p+k-g^2} \exp\left(\frac{-\omega_{zeq}^2 s^2}{8p+k-g^2\sigma_{\mathrm{s}}^4}\right)\left(\frac{R^2P^2}{\sigma_{\mathrm{N}}^2}\right)^{-p-k}$$

$$- \frac{\left[\dfrac{\alpha\beta}{(\gamma\beta+\Omega')A_0 I_1}\right]^{p+(\alpha-k)/2}}{\Gamma\left[p+(\alpha-k)+1\right]p!} \frac{-\omega_{zeq}^2}{4p+\alpha-g^2} \exp\left(\frac{-\omega_{zeq}^2 s^2}{8p+\alpha-g^2\sigma_{\mathrm{s}}^4}\right)$$

$$\left. \times \left(\frac{R^2P^2}{\sigma_{\mathrm{N}}^2}\right)^{-p-\alpha} \frac{\Gamma(p+\alpha)\Gamma\left(\dfrac{1}{2}+p+\alpha\right)}{\Gamma(1+p+\alpha)} \right\} \tag{18}$$

3. Numerical results

Numerical results are obtained for the following values of the parameters: optoelectronic conversion factor $R = 0.5$ A/W, noise standard deviation $\sigma_{\mathrm{N}} = 10^{-7}$ A/Hz, atmospheric attenuation coefficient $\sigma = 8$ dB/km, jitter standard deviation $\sigma_{\mathrm{s}} = 0.2$ m, beam width $\omega_z = 2.5$ m, average optical power of the classic scattering component received by off -axis eddies $\gamma = 0.2$, and average optical power of the coherent contributions $\Omega' = 0.8$.

Figure 1 shows the ABER *versus* the average transmit power for different values of the boresight displacement when $z = 4$ km, and different values of α and β. From Fig. 1 it can be clearly seen that the boresight displacement has an impact on the ABER of the FSO system, since with the increase of the boresight errors, the energy collected at receiver aperture decreases, and ABER increases correspondingly. Also, as expected, for larger values of parameters α and β (less severe fading/scattering conditions), ABER values are smaller and performances improve.

Namely, comparing performances obtained for smaller values of α and β, which are associated with stronger turbulence, *i.e.*, $(\alpha, \beta) = (3.50, 2)$, with performances obtained for higher values of α and β, associated with more moderate turbulence, *i.e.*, $(\alpha, \beta) = (3.99, 4)$, we can acknowledge an evident performance increase (lower values of ABER). Figure 2 shows the influence of a beam width change on ABER for both observed boresight and non-boresight cases. As expected, the growth of a non-zero boresight defining the s parameter beam width change, provides a more significant impact. Figure 3 shows the influence of a propagation link distance change z on ABER for an observed boresight case of $s = 0.3$. One can see how with the propagation the link distance grows from 3 to 4 km and performances deteriorate. Namely, 1 km growth

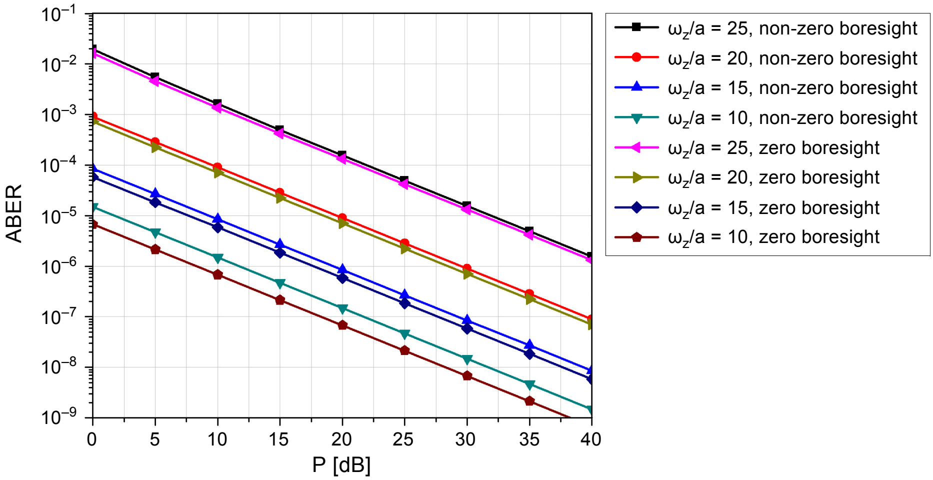

Fig. 2. ABER for fixed values $\alpha = 3.99$, $\beta = 2$, $z = 4$ km, $a = 0.1$ m, $s = 0.3$ m, and different relation ω_z/a.

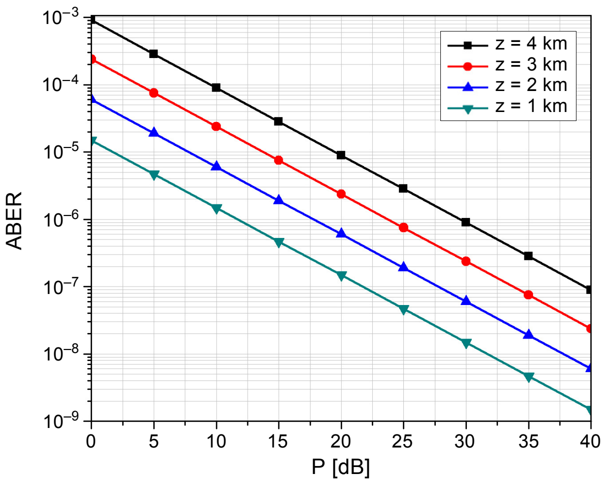

Fig. 3. ABER for fixed values $\alpha = 3.99$, $\beta = 2$, $a = 0.1$ m, $s = 0.3$ m and different distances z.

in the propagation link distance requires about additional 5 dB for achieving the same level of a system ABER quality.

4. Conclusions

In this paper, two analytical closed-form representations are derived for the ABER performance over BPSK of an AOC system operating over Málaga turbulence in the presence of pointing errors. Also, two analytical closed-form representations for the PDF are derived. The impact of pointing errors on FSO system performance has been graphically presented based on obtained numerical results and discussed in the function of system parameters. It has been shown that in the first case, for a zero boresight pointing error, where the displacement is equal to zero, systems have better performance and simpler analytical form. In the second case, for a non-zero boresight pointing error, where the displacement is greater than zero, it has been shown that the impact of a pointing error on the system performance is higher.

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