Journal of Machine Engineering, Vol. 15, No. 4, 2015

Received*: 09 July 2015* / Accepted: *15 October 2015* / Published online: *10 November 2015*

in-plane generating trajectories,
CATIA,
rolling generating tools

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THE METHOD OF "IN-PLANE GENERATING TRAJECTORIES" FOR TOOLS WHICH GENERATE BY ENVELOPING - APPLICATION IN CATIA

The method of "in-plane generating trajectories" is a method designed to study the enveloping profiles associated with a pair of rolling centrodes. This method assumes knowledge the analytical form of the in-planes trajectories described by the points onto the profiles to be generated. These trajectories are known in the reference system of the generating tool, which may be a rack-gear, a gear-shaped or a rotary cutter tool. The envelope of these trajectories represents the profile of the generating tool. An original method for determining the enveloping condition is presented in this paper. Application for ordered curling of non-involute profiles were developed based on the specific enveloping condition. A new solution is proposed. This solution uses the capabilities of the CATIA graphical environment. Graphical solutions are proposed versus analytical solutions in order to validate the proposed method.

1. INTRODUCTION

The issue of rack-gear tool profiling is solved based the Olivier theorem I [4] and, also, based on the Gohman theorem [4],[5],[8].

$$\vec{N}_{\Sigma} \cdot \vec{v} = 0 \tag{1}$$

In equation (1), $\vec{N}_{\Sigma}$ is the normal direction to the profile to be generated, in the surface's own reference system and $\vec{v}$ is a vector with the same direction as the velocity in the relative motion between the rack-gear tool and the surface to be generated.

The analytical solution of this problem is universal and leads to rigorous results. Complementary theorems were elaborated concerning this issue [5]: the "method of the family of the substitution circles" and the "method of the minimum distance". These

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methods, developed at "Dunarea de Jos" University of Galaţi, expressed the enveloping condition in specific forms. These forms may represent intuitive images of the enveloping process.

The "in-plane trajectories method" [8] was developed at "Dunarea de Jos" University of Galaţi, by a research team directed by professor Nicolae Oancea and associate professor Virgil Teodor and was presented as part of doctoral thesis of Virgil Teodor. This method is a specific solution for which the author present applications based on a specific enveloping condition.

The issue of generation with tools associated with a pair of rolling centrodes is a permanent concerning as is presented in specialized literature.

An original enveloping condition form is proposed in this paper. The enveloping condition is determined starting from the process of slotting with the rack-gear tool.

2. GENERATION BY ROLLING WITH RACK-GEAR TOOL

The rolling centrodes, the reference systems and the generated profile are presented in Fig. 1.

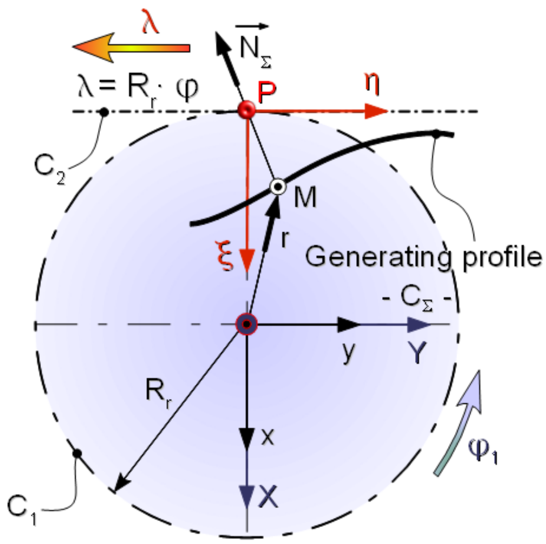

Fig. 1. Generation with rack-gear tool; C_1 and C_2 pair of rolling centrodes

The reference systems are defined:

xy is the global reference system;

XY – relative reference system, joined with the generated profile and with the C_1 centrode;

$\xi\eta$ – relative reference system joined with the C_2 centrode.

The generating process kinematics includes the absolute movements of the *XY* relative reference system,

$$x=\omega_3^T(\varphi)\cdot X \tag{2}$$

and of the $\xi\eta$ relative reference system,

$$x=\xi+A,\ A=\begin{pmatrix}-R_r\\-R_r\cdot\varphi\end{pmatrix}. \tag{3}$$

The relative motion is determines from the absolute motions (2) and (3). The relative motion describes the movement of a point belongs to the *XY* space regarding the $\xi\eta$ space.

The coordinates of point from the C_Σ profile in the *XY* reference system are

$$C_\Sigma\left|\begin{aligned}X&=X(u);\\Y&=Y(u),\end{aligned}\right. \tag{4}$$

with *u* variable parameter.

If the director parameters of the normal direction to the C_Σ profile are defined as

$$\vec{N}_{C_\Sigma}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \dot{X}_u & \dot{Y}_u & 0\\ 0 & 0 & 1\end{vmatrix}, \tag{5}$$

or

$$\vec{N}_{C_\Sigma}=\vec{i}\cdot\dot{Y}_u-\vec{j}\cdot\dot{X}_u, \tag{6}$$

then, the perpendicular drawn from the current point to the C_Σ profile, has the equations:

$$\left(\vec{N}_{C_\Sigma}\right)_\delta\left|\begin{aligned}X&=X(u)+\delta\cdot\dot{Y}_u;\\Y&=Y(u)-\delta\cdot\dot{X}_u,\end{aligned}\right. \tag{7}$$

with δ variable parameter.

The family of normals is determined from (4) and (7) when the *XY* reference system is moved regarding the $\xi\eta$ reference system

$$\left(\vec{N}_{C_\Sigma}\right)_{\delta,\varphi}\left|\begin{aligned}\xi&=\left[X(u)+\delta\cdot\dot{Y}_u\right]\cdot\cos\varphi-\left[Y(u)-\delta\cdot\dot{X}_u\right]\cdot\sin\varphi+R_r;\\ \eta&=\left[X(u)+\delta\cdot\dot{Y}_u\right]\cdot\sin\varphi+\left[Y(u)-\delta\cdot\dot{X}_u\right]\cdot\cos\varphi+R_r\cdot\varphi.\end{aligned}\right. \tag{8}$$

For δ = 0, the equations (8) represents the trajectories family of points from the C_Σ profile, regarding the $\xi\eta$ reference system, namely the in-plane generating trajectories.

According to the Willis theorem [4], the necessary and sufficient condition for the profile C_Σ admits an enveloping curve is that the perpendiculars to the C_Σ profile pass through the gearing pole. The gearing pole is the tangency point between the two centrodes C_1 and C_2, see Fig. 1.

The coordinates of gearing pole in the $\xi\eta$ reference system are, see Fig. 2:

$$P\begin{vmatrix}\xi_P = 0; \\ \eta_P = R_r \cdot \varphi.\end{vmatrix} \tag{9}$$

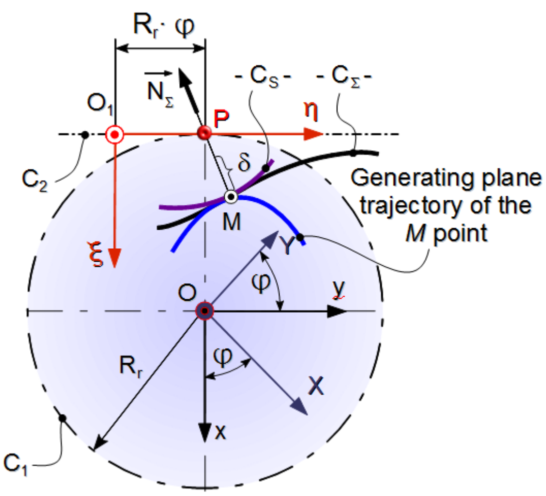

Fig. 2. The gearing pole and the $\overset{u}{N}_{C_\Sigma}$ normal direction to the in-plane trajectory C_Σ

The condition that the family of normals to the C_Σ profile pass through the gearing pole is give by the equations assembly, see (8) and (9):

$$\begin{aligned}&\left[X(u)+\delta\cdot\dot{Y}_u\right]\cdot cos\,\varphi-\left[Y(u)-\delta\cdot\dot{X}_u\right]\cdot sin\,\varphi+R_r=0;\\&\left[X(u)+\delta\cdot\dot{Y}_u\right]\cdot sin\,\varphi+\left[Y(u)-\delta\cdot\dot{X}_u\right]\cdot cos\,\varphi+R_r\cdot\varphi=R_r\cdot\varphi,\end{aligned} \tag{10}$$

The equations system (10), allows to establish a dependency between the u and φ variables parameters:

$$u=u(\varphi). \tag{11}$$

The dependency (11) represents the enveloping condition between the C_Σ trajectories and the C_S profile of the rack-gear tool. Also, the equation (11) allows determining the δ scalar value which represents the distance from the current point of the C_Σ trajectory to the gearing pole.

In the rolling process of the two centrodes, at a certain moment, which means a defined value for the φ parameter, the profiles: C_Σ associated with the C_1 centrode; the family of in-plane trajectories generated by the point from the C_Σ curve and the profile of the future rack-gear tool, admit a common normal which pass through the gearing pole, P.

The specific enveloping condition is determines from the equations assembly (10), removing the parameter δ:

$$\dot{X}_u \cdot cos\,\varphi - \dot{Y}_u \cdot sin\,\varphi = -\frac{X(u)\cdot \dot{X}_u + Y(u)\cdot \dot{Y}_u}{R_r}, \tag{12}$$

The rack-gear tool's profile is defined as give by the assembly of equations (8) and (12).

The rack-gear tool's profile may be defined as the enveloping of the assembly of curves which represents the in-plane generating trajectories of the points from the generated profile. These trajectories are described in the rack-gear tool's own reference system, $\xi\eta$:

$$T(u)_\varphi \left\| \begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} cos\,\varphi & -sin\,\varphi \\ sin\,\varphi & cos\,\varphi \end{pmatrix} \cdot \begin{pmatrix} X(u) \\ Y(u) \end{pmatrix} + \begin{pmatrix} R_r \\ R_r\cdot\varphi \end{pmatrix}. \right. \tag{13}$$

The distances from the $T(u)_\varphi$ trajectories to the gearing pole should have minimum values, for various rolling positions.

Indeed, the δ scalar value is measured onto the normal direction to the generating trajectory of a point from the profile C_Σ. This normal direction passes through the gearing pole and, as a consequence, the value of the δ parameter represents the minimum distance from the trajectory to this pole.

3. APLICATIONS

In the following, applications of the in-plane generating trajectories method are presented for the determination of the rack-gear profile. A graphical methodology is developed in connection with these applications.

3.1. RACK-GEAR TOOL FOR GENERATION OF A SHAFT WITH HEXAGONAL FRONTAL PROFILE

The profile of the hexagonal shaft, the rolling centrodes assembly and the reference systems are presented in Fig. 3 (*xy* is the world reference system; *XY* - reference system joined with the Σ surface and C_1 centrode; ξη - reference system joined with the rack-gear and the C_2 centrode).

The absolute movements of the reference systems joined with the C_1 and C_2 centrodes are give by equations (2) and (3).

The relative motion is defined by:

$$\xi = \omega_3^T(\varphi)\cdot X - A. \tag{14}$$

The equation (14) represents the relative motion of the *XY* space regarding the $\xi\eta$ space.

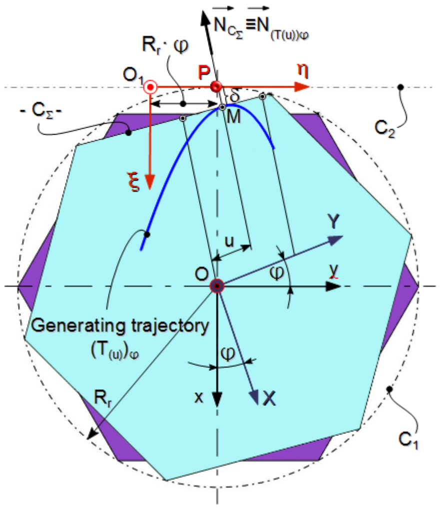

Fig. 3. Hexagonal shaft; reference systems and the in-plane generating trajectory of the point *M*

The parametrical equations of the C_Σ profile, with u variable parameter, are:

$$C_\Sigma \begin{vmatrix} X = -b; \\ Y = u. \end{vmatrix} \tag{15}$$

Thereby, the normal vector to C_Σ curve has form:

$$\vec{N}_{C_\Sigma} = 1 \cdot \vec{i} - 0 \cdot \vec{j}\,. \tag{16}$$

The normal in the current point to the C_Σ curve has equations, see (7):

$$\left(\vec{N}_{C_\Sigma}\right)_\delta \begin{vmatrix} X = -b + 1 \cdot \delta = -b + \delta; \\ Y = u - 0 \cdot \delta = u. \end{vmatrix} \tag{17}$$

The family of normals, in relative motion regarding the $\xi\eta$ space, has the equations:

$$\left(\vec{N}_{C_\Sigma}\right)_{\delta,\varphi} \begin{vmatrix} \xi = \left(-b + \delta\right) \cdot \cos\varphi - u \cdot \sin\varphi + R_r; \\ \eta = \left(-b + \delta\right) \cdot \sin\varphi + u \cdot \cos\varphi + R_r \cdot \varphi. \end{vmatrix} \tag{18}$$

If, a member of family (18) is constraint to pass through the gearing pole is obtained the form:

$$\begin{aligned} &\left(-b + \delta\right) \cdot \cos\varphi - u \cdot \sin\varphi = -R_r; \\ &\left(-b + \delta\right) \cdot \sin\varphi + u \cdot \cos\varphi = 0. \end{aligned} \tag{19}$$

The specific enveloping condition is determined by removing the δ parameter:

$$u = R_r \cdot \sin\varphi\,. \tag{20}$$

In the same time, the scalar value δ is determined:

$$\delta = \sqrt{R_r^2 - u^2} = R_r \cdot \cos\varphi + b\,. \tag{21}$$

The δ scalar represents the distance measured along $\vec{N}_{C_\Sigma}$ from the generating trajectory to the gearing pole, P. For δ=0, the (18) equations assembly represents the in-plane generating trajectories family, is space $\xi\eta$:

$$\left(T(u)\right)_\varphi \left| \begin{array}{l} \xi = \left(-b+\delta\right)\cdot \cos\varphi - u\cdot \sin\varphi + R_r\,; \\ \eta = \left(-b+\delta\right)\cdot \sin\varphi + u\cdot \cos\varphi + R_r\cdot\varphi. \end{array} \right. \tag{22}$$

The enveloping of the family (22) is the profile of the rack-gear which generates the hexagonal shaft.

The hexagonal shaft, the generating trajectories and the rack-gear tool's profile are presented in Fig. 4 and Fig. 5. The rolling radius is 32 mm.

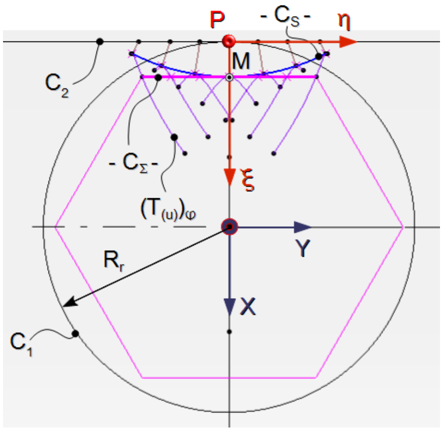

Fig. 4. Generating trajectories; rack-gear tool's profile and hexagonal shaft profile

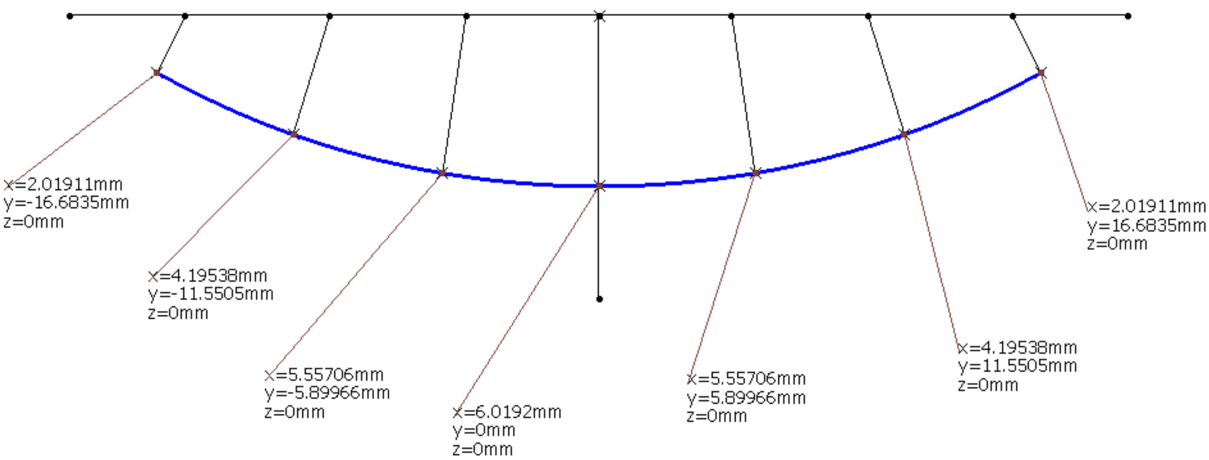

Fig. 5. Coordinates of points from the rack-gear tool's profile

The 3D model of the hexagonal shaft is presented in Fig. 6 (R_r=32 mm). A mechanism is constructed according to the kinematics of generating process.

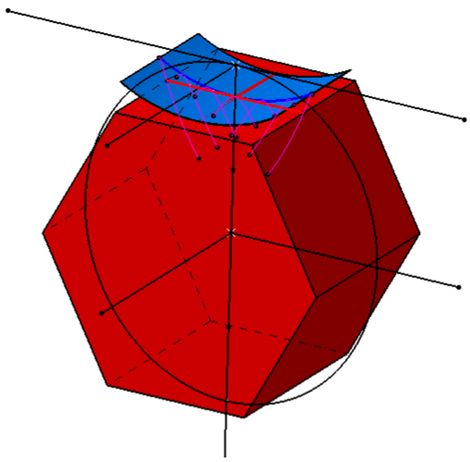

Fig. 6. The 3D model of the hexagonal shaft

The differences between the coordinates of points obtained by this method and the coordinates obtained by an analytical method are presented in Table 1.

Table 1. Error of graphical method

Crt. no.	Analytical method		Graphical method		Error [mm]
	ξ [mm]	η [mm]	ξ [mm]	η [mm]	
1	2.01911	16.6835	2.01911	16.6835	0
2	4.19538	11.5505	4.19538	11.5505	0
3	5.55706	5.89966	5.55706	5.89966	0
4	6.0192	0	6.0192	0	0
5	5.55706	-5.89966	5.55706	-5.8997	0
6	4.19538	-11.5505	4.19538	-11.551	0
7	2.01911	-16.6835	2.01911	-16.683	0

It is obviously that the two methods lead to identical profiles.

3.2. RACK-GEAR FOR A CIRCULAR PROFILE

It is considered the profile presented in Fig. 7. The parametrical equations of the profile are:

$$C_{\Sigma}\begin{vmatrix} X = -R_0 + r \cdot cos\,\theta; \\ Y = r \cdot sin\,\theta. \end{vmatrix} \tag{23}$$

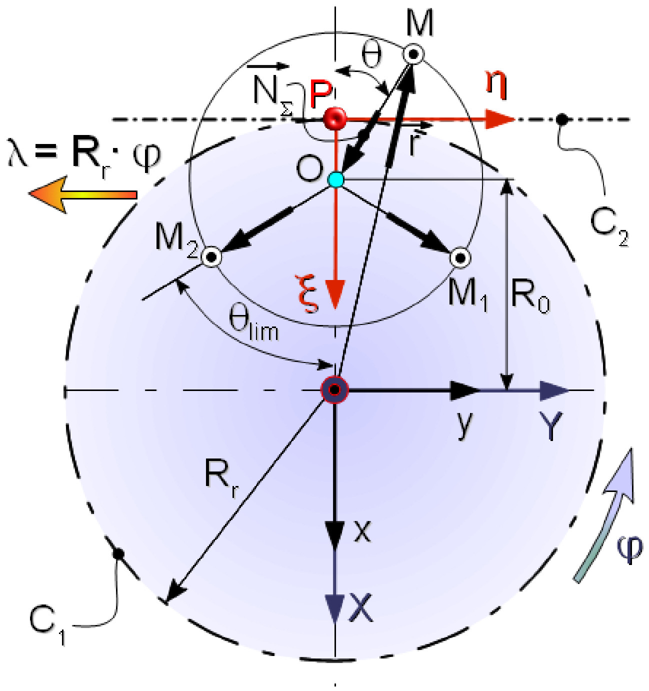

Fig. 7. Circular profile; C_1 and C_2 centrodes

The family of normals to the C_Σ profile has equations:

$$\left(\vec{N}_{C_\Sigma}\right)_{\delta,\varphi}\left|\begin{aligned}&\xi=\left[-R_0+(1-\delta)\cdot r\cdot cos\,\theta\right]\cdot cos\,\varphi-(1-\delta)\cdot r\cdot sin\,\theta\cdot sin\,\varphi+R_r;\\&\eta=\left[-R_0+(1-\delta)\cdot r\cdot cos\,\theta\right]\cdot sin\,\varphi+(1-\delta)\cdot r\cdot sin\,\theta\cdot cos\,\varphi+R_r\cdot\varphi.\end{aligned}\right.\tag{24}$$

The constraint that the normal pass through the gearing pole leads to the specific enveloping condition:

$$\varphi=-\theta+arcsin\left[\frac{R_0\cdot sin\,\theta}{(1-\delta)\cdot R_r}\right].\tag{25}$$

For δ=0, from (24), the generating trajectories family is obtained:

$$\left(T_\theta\right)_\varphi\left|\begin{aligned}&\xi=-R_0\cdot cos\,\varphi+r\cdot cos(\theta+\varphi)+R_r;\\&\eta=-R_0\cdot sin\,\varphi+r\cdot sin(\theta+\varphi)+R_r\cdot\varphi.\end{aligned}\right.\tag{26}$$

The enveloping of the generating trajectories family $\left(T_\theta\right)_\varphi$ represents the rack-gear tool's profile.

For the M_1M_2 arc, the constant values are: R_r = 50 mm; R_0 = 48 mm; r = 6.5 mm and θ_{lim} = 60°. The form and coordinates of the rack-gears profile are given in Fig. 8, Fig. 9 and Table 2.

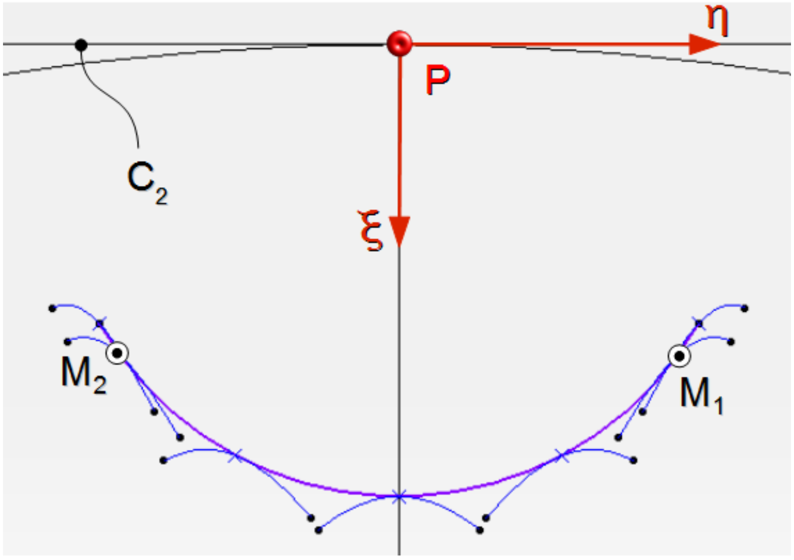

Fig. 8. Rack-gear’s profile for the arc M_1M_2

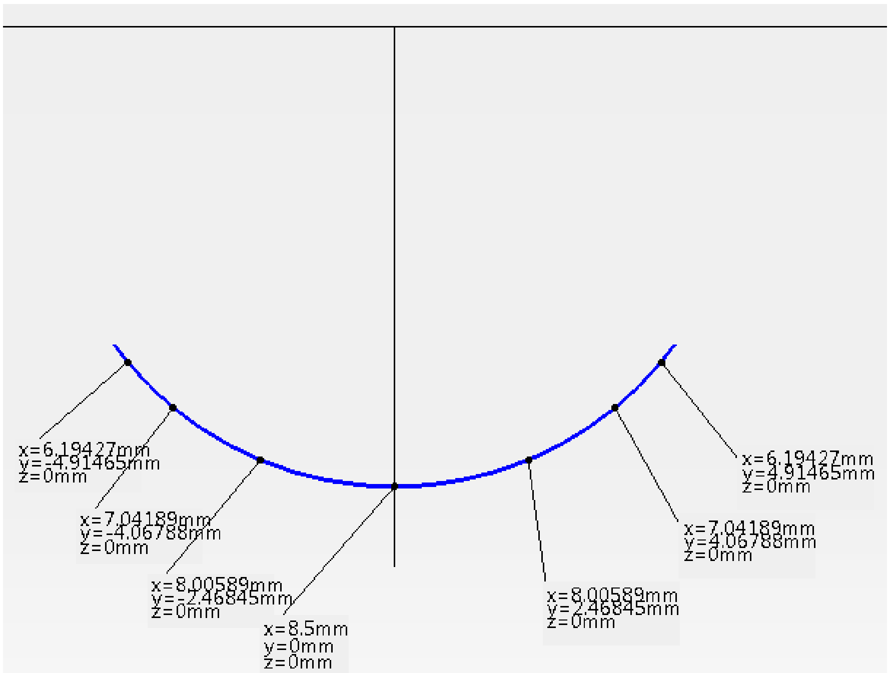

Fig. 9. Coordinates of points from the rack-gear tool’s profile

Table 2. Rack-gear’s profile

Crt. no.	Analytical method		Graphical method		Error [mm]
	ξ [mm]	η [mm]	ξ [mm]	η [mm]	
1	6.19427	-4.91465	6.19427	-4.91465	0
2	7.04189	-4.06788	7.04189	-4.06788	0
3	8.00589	-2.46845	8.00589	-2.46845	0

4	8.50000	0.00000	8.50000	0.00000	0
5	8.00589	2.46845	8.00589	2.46845	0
6	7.04189	4.06788	7.04189	4.06788	0
7	6.19427	4.91465	6.19427	4.91465	0

3.3. GRAPHICAL APPLICATION

In order to solve the generating issue, a mechanism was constructed, reproducing the generating movements when machining with rack-gear tool. The mechanism is composed from three elements: tool, which is defined as fixed element; piece, which has the relative motion regarding the tool and base which assure the relative position of the two previously presented elements. The joint between tool and piece is on type "rack", composed from a "prismatic" joint (between tool and base) and a "revolute" joint (between base and piece).

The mechanism simulation was done in the DMU Kinematics module of the CATIA software, using the "simulation" command. Consequently, this simulation was compiled ("compile simulation" command) and was saved for further replay. The trajectories of points from the piece's profile were drawn using the "trace" command. The points which are nearest to the gearing pole and belong to each trajectory were identified in the Generative Shape Design module, using the "extremum polar" command. These points are tangency points between the curves family and its enveloping. The tool's profile was obtained as a spline curve which admits as control points the previously determined points. The identity between these points and those obtained by analytical way is obviously.

4. CONCLUSION

The method of generating trajectories is based on the gearing basic theorem – the Willis theorem.

The family of trajectories is generated by the points of the generating profile, in the relative motion regarding the rack-gear. The tool's profile results as enveloping of a generating trajectories family.

A graphical method was presented. This method is developed in the CATIA design environment. It is simple and easy to apply due to the capabilities of the CATIA software.

The graphical method is based on the capability of this software to draw trajectories of points which belongs to specific mechanism.

ACKNOWLEDGMENTS

The work has been funded by the Sectoral Operational Program Human Resources Development 2007-2013 of the Ministry of European Funds through the Financial Agreement POSDRU/159/1.5/S/132397.

REFERENCES

[1] BAICU I., 2002, *Graphic algorithm to profile generating tools by the rolling method of compound profiles,* “Ghe. Asachi” Technical University, Iaşi, Tome LVIII (LII), 301-305.

[2] BERBINSCHI S., TEODOR V., FRUMUŞANU G., OANCEA N., 2014, *Graphical method for profiling the tools wich generate internal surface by rolling*, Academic Journal of Manufacturing Engineering - AJME, 12, 12-17.

[3] FRUMUŞANU G., TEODOR V.G., OANCEA N., *Graphical Method in CATIA to Profile the HOB Mill for Generating the Slitting Saw Cutter Teeth*, Applied Mechanics and Materials, 811, 85-91, Trans Tech Publications, ICMERA 2015, doi:10.4028/www.scientific.net/AMM.811.85.

[4] LITVIN F.L., 1984, *Theory of gearing*, Reference Publication 1212, NASA, Scientific and Technical Information Division, Washington D.C.

[5] OANCEA N., 2004, *Surfaces generation by enwrapping*, Vol. I, II, Ed. Fundaţiei Universitare „Dunarea de Jos” - ISBN 973-627-170-6, Galaţi.

[6] POPA, C.L., TEODOR V.G., 2014, *Graphical method in AutoCAD for profiling tools for generation of helical surfaces*, The Annals of “Dunarea de Jos” University of Galati, Fascicle V, ISSN 1221-4558, 9-12.

[7] RADZEVICH S.P., 2008, *Kinematic geometry of surface machining*, CRC Press, Boca Raton, London, New York.

[8] TEODOR V.G., 2010, *Contribution to elaboration method for profiling tools which generate by enveloping*, ISBN 978-3-9433-8261-8, Lambert Academic Publishing.

[9] TEODOR V.G, PAUNOIU V., BERBINSCHI S., OANCEA N., 2013, *The profiling of rack-gear tool for generation of the helical surfaces*, Proceedings of the International Conference on Advanced Manufacturing Engineering and Technologies NewTech 2013, Sweden, ISBN 978-91-7501-892-8, 63-72.

[10] TEODOR V.G., BERBINSCHI S., BAICU I., BAROIU N., OANCEA N., 2014, *Graphical complementary theorems of enveloping surfaces*, The Annals of “Dunarea de Jos” University of Galati, Fascicle V, ISSN 1221-4558, 75-80.

[11] LI G., SUN J., LI J., 2014, *Process modeling of end mill groove machining based on Boolean method*, International Journal of Advanced Manufacturing Technology, doi: 10.1007/s00170-014-6187-7, 75/58, 959-966.

[12] HAN Q., LIU R., 2014, *Theoretical modeling and error analysis for CNC whirling of the helical surfaces of custom screws using common inserts*, Proceedings of the Institution of Mechanical Engineers Part C-Journal of Mechanical Engineering Science, doi: 10.1177/0954406213513573, 228/11, 1948-1957.

[13] LIU R., ZHU H., 2014, *On Helical Projection and Its Application in Screw Modeling*, Advances in Mechanical Engineering, doi: 10.1155/2014/901047.

[14] LU H., LIU Z., WANG S., 2014, *Digitization modeling and CNC machining for enveloping surface parts*, International Journal of Advanced Manufacturing Technology, 73/1-4, 20 -227, doi: 10.1007/s00170-014-5777-8, 2014.