# 3D HYDRODYNAMIC PRESSURE IN GAP HEIGHT DIRECTION FOR CYLINDRICAL BEARING VISCOELASTIC LUBRICATION

**Krzysztof Wierzcholski**

*Technical University of Koszalin, Institute of Technology and Education*
*Śniadeckich Street 2, 75-453 Koszalin, Poland*
*tel.:+48 94 3478344, fax: +48 94 3426753*
*e-mail: krzysztof.wierzcholski@wp.pl*

***Abstract***

*In this paper we show the new method of solving the lubrication problem for visco-elastic oils occurring in cylindrical micro-bearing gap. In this method are taking into account all viscoelastic terms without simplifications. In the method presented in this paper for lubricants with non-Newtonian visco-elastic properties are questioning the hitherto prevailing assumptions using in hydrodynamic theory of lubrication such as for example constant value of the hydrodynamic pressure in gap height direction. The objective of the research under the paper topic is an analytical, unified formulation and a new view of general a non-classical solution of hydrodynamic problem of microbearing lubrication using algorithm to determine oil viscoelastic properties of the lubricant and the 3D hydrodynamic pressure distribution taking into account pressure changes in gap height direction.Up to now the complete influences of non-Newtonian visco-elastic oil features on the 3D hydrodynamic pressure distribution in cylindrical micro-bearing gap were not considered in analytical way.*

*We assume viscous and complete visco-elastic non-Newtonian lubricant directly near the cooperating surfaces. Viscoelastic oil properties are described by means of total form of Rivlin-Ericksen constitutive relations without simplifications.During the solution process the oil velocity components and hydro dynamic pressure are derived. The equation of motion are considered in cylindrical coordinates. Present paper elaborates the analytical method of oil velocity, pressure and friction forces determination for cylindrical slide micro-bearing. In this paper are derived the analytical formulae for velocity components, pressure distributions, in cylindrical micro-bearings.*

***Keywords:*** *pressure changes in gap height direction, HDD micro-bearings, viscoelastic lubrication*

## 1. Introduction

In this paper are assumed a hypothetical assumption that investigations of the hydrodynamic lubrication of slide micro-bearings starting from the foundations of the problem shall result in questioning in some areas of solutions the basic simplifications that have been in use so far, e.g. the constant hydrodynamic pressure in gap height direction. The theory of hydrodynamic lubrication that has been valid so far is based on the abovementioned simplification assumptions and it leads to classical form of Reynolds equations that are more or less modified and that determine the distributions of the values of the hydrodynamic pressure in two dimensional form [1, 3]. The research practice that has been accepted so far by many authors for the formulation of various problems in the area of hydrodynamics comes down to modifications of the Reynolds equation that was defined 100 years ago without any thorough derivations. Present paper shows that Reynolds equation is the result of an imposing concrete boundary conditions that are different in almost each problem on the components of the distribution of the velocities of the lubricating liquid in the bearing gap.Thus abovementioned Reynolds equation can not be the same for each case. The measurements performed by AFM in micro- bearing gap demonstrate that particularly in ultra thin microbearing gap i.e. in thin lubricant layer the hydrodynamic pressure varies considerably in gap height direction. In this paper are derived abovementioned changes [5].

## 2. Formulation of the problem

HDD micro-bearing geometry described in cylindrical coordinates (r,φ,z) is presented in Fig. 1.

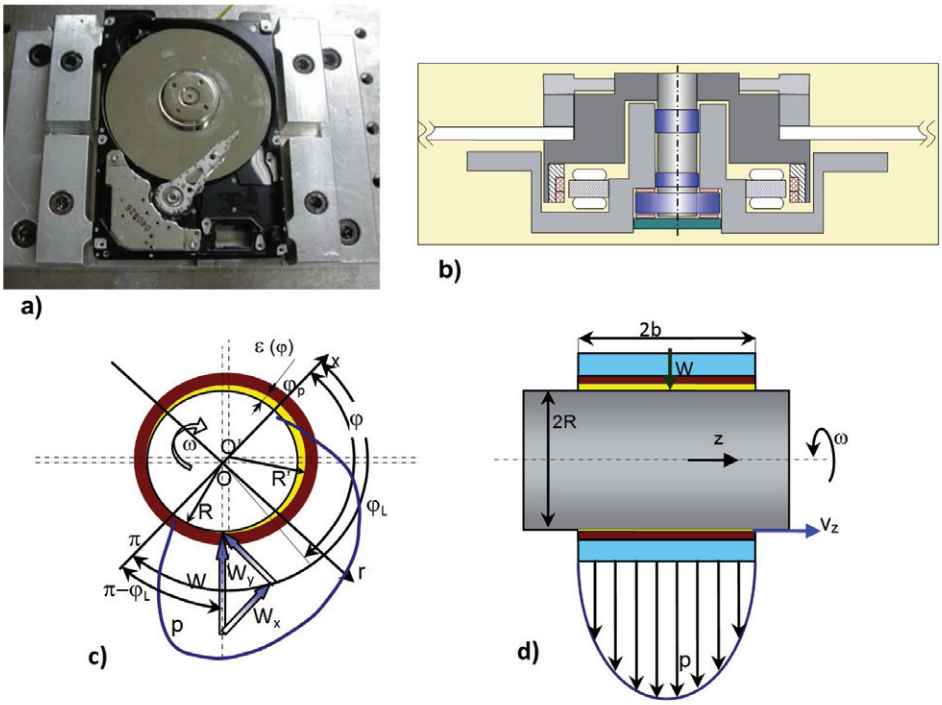


*Fig. 1. The geometry of HDD microbearing: a) View of HDD, b) micobearing construction, c) classical pressure distribution in cross section, d) classical pressure distribution in length direction*

In this micro-bearing lubrication problem for the visco-elastic properties of the oil the presented solutions are described by means of Rivlin-Ericksen constitutive relations with the two pseudo-viscosity constant coefficients α and β in $Pas^2$ [2,4]. Here is considered the unsteady liquid flow with full viscoelastic properties for enough large Deborah numbers inside thin boundary layer. Viscous fluid forces are much greater than density forces. Liquid inertia forces are neglected. After boundary layer simplifications, i.e. when the terms of the order $\psi=\varepsilon_\Sigma/R\approx 0.001$ for radial clearance $\varepsilon_\Sigma$ and radius of the micro-bearing journal R are neglected, then for the oil flow in the thin layer the equations of motion and the continuity equation have the following dimensional form:

$$0=-\frac{1}{R}\frac{\partial p}{\partial\varphi}+\frac{\partial}{\partial r}\left(\eta\frac{\partial v_\varphi}{\partial r}\right)+\alpha O_{\alpha\varphi}(\varphi,r,z,t)+\beta O_{\beta\varphi}(\varphi,r,z,t), \tag{1}$$

$$\frac{\partial p}{\partial r}=\frac{(\alpha+2\beta)}{\varepsilon_\Sigma}\int_0^{\varepsilon_\Sigma}\frac{\partial}{\partial r}\left[\left(\frac{\partial v_\varphi}{\partial r}\right)^2+\left(\frac{\partial v_z}{\partial r}\right)^2\right]dr_1, \tag{2}$$

$$0=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial r}\left(\eta\frac{\partial v_z}{\partial r}\right)+\alpha O_{\alpha z}(\varphi,r,z,t)+\beta O_{\beta z}(\varphi,r,z,t), \tag{3}$$

$$\frac{1}{R}\frac{\partial v_\phi}{\partial\varphi}+\frac{\partial v_r}{\partial r}+\frac{\partial v_z}{\partial z}=0, \tag{4}$$

where $0<r<\varepsilon_\Sigma$, $0<\varphi<2\pi$, $-b<z<+b$ and t denotes time. Symbol 2b describes bearing length.

System of equations (1)-(4) determines unknown oil velocity components $v_\varphi$, $v_r$, $v_z$, in (φ, r, z) i.e. circumferential, radial, longitudinal directions and unknown hydrodynamic pressure function p. Equation (2) indicates that hydrodynamic pressure p changes in gap height direction.

Micro-bearing sleeve is motionless in circumferential φ and longitudinal z direction. Hence on the sleeve surface for $r=\varepsilon_\Sigma$ all oil velocity components are equal to zero. Oil flow in micro-bearing gap is generated by the rotation of the journal with angular velocity ω and rotational speed 20 000 rpm. Thus on the journal surface for r=0 the velocity component in circumferential φ direction attains value Rω. On the ground of abovementioned remarks, we assume the following boundary conditions for the oil velocity components [5]:

$$v_\varphi(\varphi,r=0,z)=\omega R,\ v_\varphi(\varphi,r=\varepsilon_\Sigma,z)=0,\ v_r(\varphi,r=\varepsilon_\Sigma,z)=0,\ v_r(\varphi,r=0,z)=0,\ v_z(\varphi,r=0,z)=0,\ v_z(\varphi,r=\varepsilon_\Sigma,z)=0. \quad (6)$$

On the hydrodynamic pressure function we impose the following boundary conditions:

$$p=p_A\approx 0 \text{ for } \varphi=\varphi_p=0 \text{ and } p=p_A\approx 0 \text{ for } \varphi=\varphi_k,\ p_A - \text{atmosferic pressure}, \quad (7a)$$

where origin angle $\varphi_p$ we know and unknown end coordinate $\varphi_k$ we determine from the condition:

$$\frac{\partial p}{\partial\varphi}(\varphi=\varphi_k)=0, \quad \text{for } \varphi=\varphi_k,\ p=p_A\approx 0 \text{ for } z=-b,\ z=+b \text{ and } \frac{\partial p}{\partial z}=0 \quad \text{for } z=0. \quad (7b)$$

The changes of viscoelastic oil properties in gap height direction are small hence terms describing mentioned features are described by the following averaged form in gap height direction [5]:

$$O_{\alpha\varphi}(v_\varphi,v_z,t)\equiv\frac{1}{\varepsilon_\Sigma}\int_0^{\varepsilon_\Sigma}\left\{\frac{1}{R}\frac{\partial}{\partial\phi}\left[\left(\frac{\partial v_\varphi}{\partial r}\right)^2\right]+\frac{\partial}{\partial r}\left[\frac{\partial v_z}{\partial r}\left(\frac{1}{R}\frac{\partial v_z}{\partial\varphi}+\frac{\partial v_\varphi}{\partial z}\right)-2\frac{\partial v_\phi}{\partial r}\frac{\partial v_z}{\partial z}\right]+\frac{\partial}{\partial z}\left(\frac{\partial v_\varphi}{\partial r}\frac{\partial v_z}{\partial r}\right)\right\}dr, \quad (8)$$

$$O_{\beta\varphi}(v_\varphi,v_z,t)\equiv\frac{1}{\varepsilon_\Sigma}\int_0^{\varepsilon_\Sigma}\left\{\frac{\partial^2}{\partial r^2}\left(\frac{\partial v_\varphi}{\partial t}+v_\varphi\frac{1}{R}\frac{\partial v_\varphi}{\partial\varphi}+v_r\frac{\partial v_\varphi}{\partial r}+v_z\frac{\partial v_\varphi}{\partial z}\right)+2\frac{\partial}{\partial r}\left(\frac{1}{R}\frac{\partial v_\varphi}{\partial\varphi}\frac{\partial v_\varphi}{\partial r}+\frac{1}{R}\frac{\partial v_z}{\partial\varphi}\frac{\partial v_z}{\partial r}\right)\right\}dr, \quad (9)$$

$$O_{\alpha z}(v_\varphi,v_z,t)\equiv\frac{1}{\varepsilon_\Sigma}\int_0^{\varepsilon_\Sigma}\left\{\frac{\partial}{\partial z}\left(\frac{\partial v_z}{\partial r}\right)^2+\frac{\partial}{\partial r}\left[\frac{\partial v_\varphi}{\partial r}\frac{\partial v_\varphi}{\partial z}-\frac{2}{R}\frac{\partial v_\phi}{\partial\phi}\frac{\partial v_z}{\partial r}+\frac{1}{R}\frac{\partial v_\varphi}{\partial r}\frac{\partial v_z}{\partial\varphi}\right]+\frac{1}{R}\frac{\partial}{\partial\varphi}\left(\frac{\partial v_\varphi}{\partial r}\frac{\partial v_z}{\partial r}\right)\right\}dr, \quad (10)$$

$$O_{\beta z}(v_\varphi,v_z,t)\equiv\frac{1}{\varepsilon_\Sigma}\int_0^{\varepsilon_\Sigma}\left\{\left[\frac{\partial^2}{\partial r^2}\left(\frac{\partial v_z}{\partial t}+v_\varphi\frac{1}{R}\frac{\partial v_z}{\partial\varphi}+v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right)+2\frac{\partial}{\partial r}\left(\frac{\partial v_\varphi}{\partial z}\frac{\partial v_\varphi}{\partial r}+\frac{\partial v_z}{\partial z}\frac{\partial v_z}{\partial r}\right)\right]\right\}dr, \quad (11)$$

where $0<r<\varepsilon_\Sigma$, $0<\varphi<2\pi$, $-b<z<+b$.

## 3. Initial basic solutions

Neglecting the visco-elastic terms for α=0 and β=0 in equations (1)-(4) we obtain by virtue of Eq.(2) the classical constant value of hydrodynamic pressure in gap height direction. In this case the initial basic solutions of the system of partial differentia equations (1)-(4) under the boundary conditions (6), (7ab) lead to the following classical particular solutions:

$$v_\varphi^{(0)}(r,\varphi,z)=\omega R\left(1-\frac{r}{\varepsilon_\Sigma}\right)+\frac{1}{2\eta}\frac{1}{R}\frac{\partial p^{(0)}}{\partial\varphi}\left(r^2-r\varepsilon_\Sigma\right), \quad (12)$$

$$v_z^{(0)}(r,\varphi,z)=\frac{1}{2\eta}\frac{\partial p^{(0)}}{\partial z}\left(r^2-r\varepsilon_\Sigma\right), \quad (13)$$

$$v_r^{(0)}(r,\varphi,z)=\frac{1}{6}\left(\varepsilon_\Sigma r^2-r^3\right)\left[\frac{1}{R^2}\frac{\partial}{\partial\varphi}\left(\frac{1}{\eta}\frac{\partial p^{(0)}}{\partial\varphi}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\eta}\frac{\partial p^{(0)}}{\partial z}\right)\right], \quad (14)$$

$$\frac{1}{R^2}\frac{\partial}{\partial\varphi}\left(\frac{\varepsilon_\Sigma^3}{\eta}\frac{\partial p^{(0)}}{\partial\varphi}\right)+\frac{\partial}{\partial z}\left(\frac{\varepsilon_\Sigma^3}{\eta}\frac{\partial p^{(0)}}{\partial z}\right)=6\omega\frac{\partial\varepsilon_\Sigma}{\partial\varphi}, \quad (15)$$

where $0<r<\varepsilon_\Sigma$, $0<\varphi<2\pi$, $-b<z<+b$.

### 3. First step of approximation

Now we are going to the first step approximation. At first we determine the pressure changes in gap height direction. Therefore we use the averaged terms describing visco-elastic oil properties in equation (2). Into abovementioned terms we put initial basic solutions (12) and (13), thus after integration respect to the variable r, we have following expression for the pressure approximation:

$$\frac{\partial p_{\Sigma}^{(1)}}{\partial r}=(\alpha+2\beta)\frac{1}{\varepsilon_{\Sigma}}\int_{0}^{\varepsilon_{\Sigma}}\frac{\partial}{\partial r}\left[\left(\frac{\partial v_{\varphi}^{(0)}}{\partial r}\right)^{2}+\left(\frac{\partial v_{z}^{(0)}}{\partial r}\right)^{2}\right]dr=(\alpha+2\beta)\frac{1}{\varepsilon_{\Sigma}}\left[\left(\frac{\partial v_{\varphi}^{(0)}}{\partial r}\right)^{2}+\left(\frac{\partial v_{z}^{(0)}}{\partial r}\right)^{2}\right]_{r=0}^{r=\varepsilon_{\Sigma}}=$$

$$=(\alpha+2\beta)\frac{1}{\varepsilon_{\Sigma}}\left\{\left[\frac{\omega R}{\varepsilon_{\Sigma}}-\frac{1}{2\eta R}\frac{\partial p^{(0)}}{\partial\varphi}(2r-\varepsilon_{\Sigma})\right]^{2}+\left[\frac{1}{2\eta}\frac{\partial p^{(0)}}{\partial z}(2r-\varepsilon_{\Sigma})\right]^{2}\right\}_{r=0}^{r=\varepsilon_{\Sigma}}. \tag{16}$$

After terms reduction formula (16) leads finally to the form:

$$\frac{\partial p_{\Sigma}^{(1)}}{\partial r}=-2(\alpha+2\beta)\frac{\omega}{\eta\varepsilon_{\Sigma}}\frac{\partial p^{(0)}}{\partial\varphi}. \tag{17}$$

Integration of equation (17) with respect to the variable r gives the first pressure approximation in following general form:

$$p_{\Sigma}^{(1)}(r,\varphi,z,t)=-2(\alpha+2\beta)\frac{\omega r}{\eta\varepsilon_{\Sigma}}\frac{\partial p^{(0)}}{\partial\varphi}+C_1,\ \text{where}\quad p_{\Sigma}^{(1)}(r=0,\varphi,z,t)=p^{(1)}(\varphi,z,t), \tag{18}$$

where $C_1$ denotes integration constant. It is easy to see, that the total pressure in first approximation $p_{\Sigma}^{(1)}(r,\varphi,z,t)$ varies in gap height direction r and on the journal surface in place r=0 equals unknown function $p^{(1)}(\varphi,z,t)$. On the ground of above mentioned assumption we obtain following form of particular solution for pressure in first approximation step:

$$\begin{gathered}p_{\Sigma}^{(1)}(r,\varphi,z,t)=-2(\alpha+2\beta)\frac{\omega r}{\eta\varepsilon_{\Sigma}}\frac{\partial p^{(0)}}{\partial\varphi}+p^{(1)}(\varphi,z,t)=\alpha rA^{(0)}+\beta rB^{(0)}+p^{(1)}(\varphi,z,t),\\ A^{(0)}\equiv-2\frac{\omega}{\eta\varepsilon_{\Sigma}}\frac{\partial p^{(0)}}{\partial\varphi},\quad B^{(0)}=2A^{(0)},\quad 0\le r\le\varepsilon_{\Sigma},\,0\le\varphi<2\pi,\,-b\le z\le+b.\end{gathered} \tag{19}$$

In system of equations (1), (3), (4) we replace functions $v_{\varphi}$, $v_r$, $v_z$ by the unknown velocity components $v_{\varphi}^{(1)},v_{r}^{(1)},v_{z}^{(1)}$ in first step of approximation and we replace pressure p by the function $p_{\Sigma}^{(1)}$ determined in equation (19), where we average in gap height direction the terms describing viscoelastic properties. Into the terms $O_{\alpha\varphi}$, $O_{\alpha z}$, $O_{\beta\varphi}$, $O_{\beta z}$ we put velocity component (12), (13) obtained in the basic step approximation. We obtain following system of equations, where all terms describing visco-elastic oil properties are averaged in gap height direction:

$$0=-\frac{1}{R}\frac{\partial p^{(1)}}{\partial\varphi}+\frac{\partial}{\partial r}\left(\eta\frac{\partial v_{\varphi}^{(1)}}{\partial r}\right)+\alpha O_{\alpha\varphi}^{(0)}+\beta O_{\beta\varphi}^{(0)}, \tag{20}$$

$$0=-\frac{\partial p^{(1)}}{\partial z}+\frac{\partial}{\partial r}\left(\eta\frac{\partial v_{z}^{(1)}}{\partial r}\right)+\alpha O_{\alpha z}^{(0)}+\beta O_{\beta z}^{(0)}, \tag{21}$$

$$\frac{1}{R}\frac{\partial v_{\varphi}^{(1)}}{\partial\varphi}+\frac{\partial v_{r}^{(1)}}{\partial r}+\frac{\partial v_{z}^{(1)}}{\partial z}=0, \tag{22}$$

where:

$$O_{\alpha\varphi}^{(0)} \equiv O_{\alpha\varphi}\left(v_\varphi^{(0)}, v_z^{(0)}, t\right) - \frac{\varepsilon_\Sigma}{2R}\frac{\partial A^{(0)}}{\partial\varphi}, \quad O_{\beta\varphi}^{(0)} \equiv O_{\beta\varphi}\left(v_\varphi^{(0)}, v_z^{(0)}, t\right) - \frac{\varepsilon_\Sigma}{R}\frac{\partial A^{(0)}}{\partial\varphi}, \tag{23}$$

$$O_{\alpha z}^{(0)} \equiv O_{\alpha z}\left(v_\varphi^{(0)}, v_z^{(0)}, t\right) - \frac{\varepsilon_\Sigma}{2}\frac{\partial A^{(0)}}{\partial z}, \quad O_{\beta z}^{(0)} \equiv O_{\beta z}\left(v_\varphi^{(0)}, v_z^{(0)}, t\right) - \varepsilon_\Sigma\frac{\partial A^{(0)}}{\partial z}, \tag{24}$$

whereas $0<r<\varepsilon_\Sigma$, $0<\varphi<2\pi$, $-b< z<+b$.

In this case the first approximation solutions of the system of partial differential equations (20)-(22) under the boundary conditions (6), (7ab) has the following particular solutions:

$$v_\varphi^{(1)}(r,\varphi,z,t) = \omega R\left(1-\frac{r}{\varepsilon_\Sigma}\right) + \frac{1}{2\eta}\frac{1}{R}\frac{\partial p^{(1)}}{\partial\varphi}\left(r^2 - r\varepsilon_\Sigma\right) - \frac{1}{2\eta}\left(r^2 - r\varepsilon_\Sigma\right)\left(\alpha O_{\alpha\varphi}^{(0)} + \beta O_{\beta\varphi}^{(0)}\right), \tag{25}$$

$$v_z^{(1)}(r,\varphi,z,t) = \frac{1}{2\eta}\frac{\partial p^{(1)}}{\partial z}\left(r^2 - r\varepsilon_\Sigma\right) - \frac{1}{2\eta}\left(r^2 - r\varepsilon_\Sigma\right)\left(\alpha O_{\alpha z}^{(0)} + \beta O_{\beta z}^{(0)}\right), \tag{26}$$

where $0<r<\varepsilon_\Sigma$, $0<\varphi<2\pi$, $-b< z<+b$.

To determine radial oil velocity component $v_r^{(1)}$ and unknown pressure $p^{(1)}$ in first approximation step, we integrate continuous equation (22) with respect to the variable r in following form:

$$v_r^{(1)}(r,\varphi,z,t) = -\int_0^r \frac{1}{R}\frac{\partial v_\varphi^{(1)}}{\partial\varphi}dr - \int_0^r \frac{\partial v_z^{(1)}}{\partial z}dr + C. \tag{27}$$

Symbol C denotes the integration constants. On the equation (27) we impose following boundary condition: $v_r^{(1)} = 0$ for r=0 and for $r=\varepsilon_\Sigma$. Hence integration constant C=0 and we obtain:

$$\int_0^{\varepsilon_\Sigma} \frac{1}{R}\frac{\partial v_\varphi^{(1)}}{\partial\varphi}dr + \int_0^{\varepsilon_\Sigma} \frac{\partial v_z^{(1)}}{\partial z}dr = 0. \tag{28}$$

In further transformation of equation (28) we use following gut known integral identities:

$$\frac{\partial}{R\partial\varphi}\int_0^{\varepsilon_\Sigma(\varphi,z)} v_\varphi^{(1)}(r,\varphi,z)dr = \frac{1}{R}\int_0^{\varepsilon_\Sigma(\varphi,z)} \frac{v_\varphi^{(1)}(r,\varphi,z)}{\partial\varphi}dr + \frac{\partial\varepsilon_\Sigma(\varphi,z)}{R\partial\varphi}v_\varphi^{(1)}(r=\varepsilon_\Sigma,\varphi,z), \tag{29}$$

Previously satisfied conditions:

$$v_\varphi^{(1)}(\varphi, r=\varepsilon_\Sigma, z, t) = 0, \quad v_z^{(1)}(\varphi, r=\varepsilon_\Sigma, z, t) = 0, \quad v_r^{(1)}(\varphi, r=\varepsilon_\Sigma, z, t) = 0. \tag{30}$$

we put into expressions (29) and such obtained results we substitute into (28). Hence we have:

$$\frac{1}{R}\frac{\partial}{\partial\varphi}\int_0^{\varepsilon_\Sigma} v_\varphi^{(1)}dr + \frac{\partial}{\partial z}\int_0^{\varepsilon_\Sigma} v_z^{(1)}dr = 0. \tag{31}$$

Oil velocity components $v_\varphi^{(1)}, v_z^{(1)}$ in first approximation step (25), (26) we put into equation (31). After calculations and term ordering we obtain following equation for pressure determination $p^{(1)}(\varphi,z)$ in first approximation step:

$$\frac{1}{R^2}\frac{\partial}{\partial\varphi}\left(\frac{\varepsilon_\Sigma^3}{\eta}\frac{\partial p^{(1)}(\varphi,z,t)}{\partial\varphi}\right) + \frac{\partial}{\partial z}\left(\frac{\varepsilon_\Sigma^3}{\eta}\frac{\partial p^{(1)}(\varphi,z,t)}{\partial z}\right) = \Pi\left(O_1 = O_{\alpha\varphi}^{(0)}, O_2 = O_{\beta\varphi}^{(0)}, O_3 = O_{\alpha z}^{(0)}, O_4 = O_{\beta z}^{(0)}\right), \tag{32}$$

$$\Pi(O_1,O_2,O_3,O_4) \equiv 6\omega\frac{\partial\varepsilon_\Sigma}{\partial\varphi} + \frac{1}{R}\frac{\partial}{\partial\varphi}\left[\frac{\varepsilon_\Sigma^3}{\eta}(\alpha O_1 + \beta O_2)\right] + \frac{\partial}{\partial z}\left[\frac{\varepsilon_\Sigma^3}{\eta}(\alpha O_3 + \beta O_4)\right]. \tag{33}$$

Pressure function $p^{(1)}$ is determined from (32) using boundary conditions (7ab) i.e.:

$$p^{(1)} = p_A \approx 0 \ \text{ for } \ \varphi = \varphi_p, \varphi = \varphi_k, \frac{\partial p^{(1)}}{\partial\varphi}(\varphi = \varphi_k) = 0, \text{ and } \ \text{ for } z = \pm b. \tag{34}$$

We put into equation (27), oil velocity components $v_\varphi^{(1)}, v_z^{(1)}$ in first approximation step (25), (26). After calculations and term ordering we obtain radial component of oil velocity in first approximation step for $0 \le r \le \varepsilon_\Sigma$, $0 \le \varphi < 2\pi$, $-b \le z \le b$ [5]:

$$v_r^{(1)}(r,\varphi,z,t) = v_r\left(r, p^{(1)}, O_{\alpha\varphi}^{(0)}, O_{\beta\varphi}^{(0)}, O_{\alpha z}^{(0)}, O_{\beta z}^{(0)}\right) \equiv \frac{1}{6}\left(\varepsilon_\Sigma r^2 - r^3\right)\left[\frac{1}{R^2}\frac{\partial}{\partial\varphi}\left(\frac{1}{\eta}\frac{\partial p^{(1)}}{\partial\varphi}\right) + \frac{\partial}{\partial z}\left(\frac{1}{\eta}\frac{\partial p^{(1)}}{\partial z}\right)\right] +$$

$$+\frac{1}{6}\left(\varepsilon_\Sigma r^2 - r^3\right)\left\{\frac{1}{R}\frac{\partial}{\partial\varphi}\left[\frac{1}{\eta}\left(\alpha O_{\alpha\varphi}^{(0)} + \beta O_{\beta\varphi}^{(0)}\right)\right] + \frac{\partial}{\partial z}\left[\frac{1}{\eta}\left(\alpha O_{\alpha z}^{(0)} + \beta O_{\beta z}^{(0)}\right)\right]\right\}, 0 \le r \le \varepsilon_\Sigma, 0 \le \varphi < 2\pi, -b \le z \le +b. \quad (35)$$

## 4. Scetch of n-th approximation step

Now we are going to the n-th (2,3,…) step approximation of solutions. At first we determine the pressure changes in gap height direction. Therefore we use the averaged terms describing visco-elastic oil properties in equation (2). Into abovementioned terms we put n−1 step approximation i.e. solutions $v_\varphi^{(n-1)}, v_z^{(n-1)}$, for n=2,3,…, thus after integration respect to the variable r, we obtain following expression for the first derivative of the n-th pressure approximation step:

$$\frac{\partial p_\Sigma^{(n)}}{\partial r} = (\alpha + 2\beta)\frac{1}{\varepsilon_\Sigma}\int_0^{\varepsilon_\Sigma}\frac{\partial}{\partial r}\left[\left(\frac{\partial v_\varphi^{(n-1)}}{\partial r}\right)^2 + \left(\frac{\partial v_z^{(n-1)}}{\partial r}\right)^2\right]dr =$$

$$= (\alpha + 2\beta)\frac{1}{\varepsilon_\Sigma}\left\{\left[\frac{\omega R}{\varepsilon_\Sigma} - \frac{1}{2\eta}\left(\frac{\partial p^{(n-1)}}{R\partial\varphi} - \alpha O_{\alpha\varphi}^{(n-2)} - \beta O_{\beta\varphi}^{(n-2)}\right)(2r - \varepsilon_\Sigma)\right]^2 +\right.$$

$$\left. + \left[\frac{1}{2\eta}\left(\frac{\partial p^{(n-1)}}{\partial z} - \alpha O_{\alpha z}^{(n-2)} - \beta O_{\beta z}^{(n-2)}\right)(2r - \varepsilon_\Sigma)\right]^2\right\}_{r=0}^{r=\varepsilon_\Sigma}, \text{ for } n=2,3,4,\dots . \quad (36)$$

After calculations, terms ordering and reduction and neglecting the terms of order $\alpha^2$, $\alpha\beta$, $\beta^2$ formula (36) leads finally to the form:

$$\frac{\partial p_\Sigma^{(n)}}{\partial r} = -2(\alpha + 2\beta)\frac{\omega}{\eta\varepsilon_\Sigma}\frac{\partial p^{(n-1)}}{\partial\varphi} + O\left(\alpha^2, \alpha\beta, \beta^2\right). \quad (37)$$

Integration of equation (37) with respect to the variable r gives the n-th pressure approximation in following general form:

$$p_\Sigma^{(n)}(r,\varphi,z,t) = -2(\alpha + 2\beta)\frac{\omega r}{\eta\varepsilon_\Sigma}\frac{\partial p^{(n-1)}}{\partial\varphi} + C_n, \text{ where } \quad p_\Sigma^{(n)}(r = 0,\varphi,z) = p^{(n)}(\varphi,z) \quad (38)$$

and $C_2$ denotes integration constant. It is easy to see, that the total pressure in n-th approximation $p_\Sigma^{(n)}(r,\varphi,z)$ varies in gap height direction r and on the journal surface in place r=0 attains unknown function $p^{(n)}(\varphi,z)$. On the ground of above mentioned assumption we obtain following form of particular solution for pressure in second approximation step:

$$p_\Sigma^{(n)}(r,\varphi,z,t) = -2(\alpha + 2\beta)\frac{\omega r}{\eta\varepsilon_\Sigma}\frac{\partial p^{(n-1)}(\varphi,z,t)}{\partial\varphi} + p^{(n)}(\varphi,z,t) =$$

$$= \alpha r A^{(n-1)} + \beta r B^{(n-1)} + O\left(\alpha^2, \alpha\beta, \beta^2\right) + p^{(n)}(\varphi,z,t), \quad (39)$$

$$A^{(n-1)} \equiv -2\frac{\omega}{\eta\varepsilon_\Sigma}\frac{\partial p^{(n-1)}}{\partial\varphi}, \quad B^{(n-1)} = 2A^{(n-1)}, \text{ where } 0 \le r \le \varepsilon_\Sigma, 0 \le \varphi < 2\pi, -b \le z \le +b.$$

In system of equations (1), (3), (4) we replace functions $v_\varphi$, $v_r$, $v_z$ by the unknown velocity components $v_\varphi^{(n)}, v_r^{(n)}, v_z^{(n)}$ in n-th step of approximation and we replace pressure p by the function $p_\Sigma^{(n)}$ determined in equation (39). We average in gap height direction the terms describing viscoelastic properties. Into the terms $O_{\alpha\varphi}$, $O_{\alpha z}$, $O_{\beta\varphi}$, $O_{\beta z}$ we put velocity component (25), (26) obtained in the n-th step approximation. Hence we have $O_{\alpha\varphi}^{(n-1)}, O_{\alpha z}^{(n-1)}, \ldots$. Finally we obtain following system of equations, where all terms describing visco-elastic oil properties are averaged in gap height direction:

$$0 = -\frac{1}{R}\frac{\partial p^{(n)}}{\partial\varphi} + \frac{\partial}{\partial r}\left(\eta\frac{\partial v_\varphi^{(n)}}{\partial r}\right) + \alpha O_{\alpha\varphi}^{(n-1)} + \beta O_{\beta\varphi}^{(n-1)},\quad 0 = -\frac{\partial p^{(n)}}{\partial z} + \frac{\partial}{\partial r}\left(\eta\frac{\partial v_z^{(n)}}{\partial r}\right) + \alpha O_{\alpha z}^{(n-1)} + \beta O_{\beta z}^{(n-1)},\tag{40}$$

$$\frac{1}{R}\frac{\partial v_\varphi^{(n)}}{\partial\varphi} + \frac{\partial v_r^{(n)}}{\partial r} + \frac{\partial v_z^{(n)}}{\partial z} = 0,\tag{41}$$

where

$$O_{\alpha\varphi}^{(n-1)} \equiv O_{\alpha\varphi}\left(v_\varphi^{(n-1)}, v_z^{(n-1)}, t\right) - \frac{\varepsilon_\Sigma}{2R}\frac{\partial A^{(n-1)}}{\partial\varphi},\quad O_{\beta\varphi}^{(n-1)} \equiv O_{\beta\varphi}\left(v_\varphi^{(n-1)}, v_z^{(n-1)}, t\right) - \frac{\varepsilon_\Sigma}{R}\frac{\partial A^{(n-1)}}{\partial\varphi},\tag{42}$$

$$O_{\alpha z}^{(n-1)} \equiv O_{\alpha z}\left(v_\varphi^{(n-1)}, v_z^{(n-1)}, t\right) - \frac{\varepsilon_\Sigma}{2}\frac{\partial A^{(n-1)}}{\partial z},\quad O_{\beta z}^{(n-1)} \equiv O_{\beta z}\left(v_\varphi^{(n-1)}, v_z^{(n-1)}, t\right) - \varepsilon_\Sigma\frac{\partial A^{(n-1)}}{\partial z},\tag{43}$$

whereas $0<r<\varepsilon_\Sigma$ , $0<\varphi<2\pi$, $-b< z<+b$, $n=2,3,4,\ldots$ .

We take into account the same procedure of solutions as in n-1 approximation step from equation (25) to (35). Hence the system of partial differential equations (40)-(41) under the boundary conditions (6), (7ab) has the following particular solutions in n-th approximation step:

$$\begin{gathered}
v_\varphi^{(n)}(r,\varphi,z,t) = \omega R\left(1-\frac{r}{\varepsilon_\Sigma}\right) + \frac{1}{2\eta}\frac{1}{R}\frac{\partial p^{(n)}(\varphi,z,t)}{\partial\varphi}\left(r^2 - r\varepsilon_\Sigma\right) - \frac{1}{2\eta}\left(r^2 - r\varepsilon_\Sigma\right)\left(\alpha O_{\alpha\varphi}^{(n-1)} + \beta O_{\beta\varphi}^{(n-1)}\right),\\
v_z^{(n)}(r,\varphi,z,t) = \frac{1}{2\eta}\frac{\partial p^{(n)}(\varphi,z,t)}{\partial z}\left(r^2 - r\varepsilon_\Sigma\right) - \frac{1}{2\eta}\left(r^2 - r\varepsilon_\Sigma\right)\left(\alpha O_{\alpha z}^{(n-1)} + \beta O_{\beta z}^{(n-1)}\right),\\
v_r^{(n)}(r,\varphi,z,t) = v_r\left(r, p^{(n)}, O_{\alpha\varphi}^{(n-1)}, O_{\beta\varphi}^{(n-1)}, O_{\alpha z}^{(n-1)}, O_{\beta z}^{(n-1)}\right)
\end{gathered}\tag{44}$$

for $0\le \varphi\le \pi$, $-b< z<+b$, $0 \le r \le \varepsilon_\Sigma$.

Hydrodynamic pressure in n-th approximation step determines the following partial differential equation:

$$\frac{1}{R^2}\frac{\partial}{\partial\varphi}\left(\frac{\varepsilon_\Sigma^3}{\eta}\frac{\partial p^{(n)}(\varphi,z,t)}{\partial\varphi}\right) + \frac{\partial}{\partial z}\left(\frac{\varepsilon_\Sigma^3}{\eta}\frac{\partial p^{(n)}(\varphi,z,t)}{\partial z}\right) = \\ = \Pi\left(O_1 = O_{\alpha\varphi}^{(n-1)}, O_2 = O_{\beta\varphi}^{(n-1)}, O_3 = O_{\alpha z}^{(n-1)}, O_4 = O_{\beta z}^{(n-1)}\right).\tag{45}$$

## 5. Conclusions

This paper presents the method of non-classic solution of the oil velocity components, and hydrodynamic pressure distribution in slide cylindrical micro-bearing gap taking into account the full terms of viscoelastic Rivlin-Ericksen constitutive relations. The main achievements are referring to the calculation algorithm elaboration for determination of 3D pressure distribution in journal bearing gap where hydrodynamic pressure changes in gap height direction.

## References


[1] Asada, T. et al, *Hydrodynamic bearings and applied technologies*, Matsushita Tech., 46 (1), pp. 54-76, 2000.

[2] Böhme, G., *Strömungsmechanik nicht-Newtonscher Fluide*, Teubner Studienbücher Mechanik, Stuttgart 1981.

[3] Jang, G. H., Park, S. J., Kim, C. S., Han, J. H., *Investigation of the electromechanical variables of the spindle motor and the actuator of HDD due to positioning and the free fall*, Microsystem Technologies, 13, pp. 797-809, 2007.

[4] Teipel, I., *Die räumliche Staupunktströmung für ein viscoelastisches Fluid*, Reologica Acta, 25, pp.75-79, 1986.

[5] Wierzcholski, K., *The Viscoelastic Lubrication Problem of Micro-Bearing*, Tribologia, 3 (231), pp. 231-240, 2010.