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# Selected approaches on reliability assessment of complex system with one shot items

**Keywords**

reliability assessment, complex system, one shot item

**Abstract**

This paper deals with the modelling and analysis of the reliability of complex systems that use one-shot items during their operation. It includes an analysis of the impact of the reliability of used one-shot items on the resulting reliability of the system as a whole. Practical application of the theoretical knowledge is demonstrated using a model of reliability of an aircraft gun that was used for optimization of the gun's design during its development and design. Furthermore this paper demonstrates utilization of a proposed model to determine an optimum number of pyrotechnic cartridges that will achieve a required probability of fulfilment of the mission with the given reliability of individual subsystems of the gun, rounds and pyrotechnic cartridges. The proposed procedures of modelling and analysis of the reliability of complex systems with one-shot items were used in practice in designing and development of the PL-20 aircraft gun. Development of the gun was successfully finished in 2005, and the gun was fielded in the armament of the Czech Republic Air Force.

## 1. Introduction

In the paper there is described a procedure for reliability assessment of complex systems using one shot items in its construction. The main principles of the system's function are similar to sequential systems in mathematical terms. Behaviour models of general sequential systems, their function and the assessment of the right and wrong function are described for example in [13], [14], [15], [16], [17], [18], [19], [20], [21], [22]. Some other books deal with very general theoretical suggestions of the test programmes for one shot items [1], or a different specific area of mathematics, e.g. sequential systems impact assessment on mass service process performance ([1], [1], [2], [3]). The books suggesting the methods on how to assess sequential systems reliability offer specific and interesting mathematical applications which can be found in [4], [5], [6], [7], [8], [9], [10], [11], [12]. The articles are mostly very theoretical and do not include in any way the possibility of one shot items/devices practical implementation in the complex system construction. The solution principle is based only on a sequence course of a certain process or a system function. The sequence course is given for example by the main function completion (e.g. pressers, assembly lines, phase course process, shots, etc.).

One shot items/devices implementation in the complex system construction extends the real possibilities of technical systems and improves the level of dependability and safety. This extension could be in both military and civilian systems, or space programs [35].

The basic presumptions of the usage of one shot items, and example applications can be found in [28], [29] and [35]. It has been shown [48] that besides weapon systems the one shot items are also applied to plenty of aircraft applications, e.g. to the aircraft F-16 Fighting Falcon.

The general and basic approach to test the performance and make an assessment in order to obtain and demonstrate the reliability level of one shot items is stated in [10], [32], [33], [34] and [35]. The demonstration of the required reliability level of one shot items/devices in relation to the costs which are spent during the test are described in [23], [24], [26], [27], [29], [31], [32] and [36]. The entire test effectiveness of development test programs for one shot items/devices is discussed, e.g. in [23] or [24]. In the books [24] and [29] there are recommended design changes forms, the usage of highly/ultra reliable materials for one shot items/devices construction (but here they are diverted to reliability growth programs). The method of searching and detection of different failure modes and their impact on development programs of one shot items/devices is described in [25] and [26]. Also in the books [27], [28] and [32] there are recommended forms and methods on how to perform tests, especially acceleration tests with some stress imposed (e.g. thermal or mechanical stress), or highly accelerated tests, HALT, for one shot items/devices. The reliability growth program in relation to the operational state increase of one shot items/devices together with the systems which can use one shot items for their function is very well developed in [23], [25], [26] or [30].

This contribution is aims to contribute to a solution of dependability qualities of the complex (in this case) weapon system as an observed object. Some ways on how to specify a single value of a dependability measure of a set is shown. The aim of the paper is to verify the suggested solution in relation to some functional elements which influence the fulfilment of a required function in a very significant manner ([37] and [38]). The PL-20 aircraft gun is an example of the application of the procedures of a complex system with one shot items was designed for the needs of the Czech Air Force. It was fielded into its armament as an onboard weapon for the L-159 advanced light combat aircraft. It refers to a 20-mm calibre twin gun, the automatic function of which is actuated by powder gases from its barrels. It is a typical example of a complex, sequential, mechatronic, weapon system working with one shot items. It is a barrel shooting gun – fast shooting twin-barrel cannon. Speaking about the system failure it means a failure of the round of the automatic weapons. This will result in discontinuation of firing and a non-fired round remains loaded in a chamber of the gun. An external action is then necessary to eject a failed round from the chamber and to charge a new round in order to continue firing. To this end the aircraft gun is equipped with special pyrotechnic cartridges. When a round fails, a pyrotechnic cartridge is automatically initiated and powder gases generated during its firing provide for an ejection of the failed round and continuation of firing. A probability of accomplishment of a mission is the most important measure for assessment of reliability of a gun as a whole. A mission is considered as successfully accomplished if it was possible to fire all rounds that were charged into the gun ammunition feeding belt prior to the mission.

From the description of the gun, it is evident that the probability of accomplishment of a mission depends both on the reliability of used rounds and pyrotechnic cartridges. Of paramount importance that influences the probability of accomplishment of a mission is the quantity of pyrotechnic cartridges used in the design of the gun. The greater the number of pyrotechnic cartridges that can be applied during an accomplishment of a mission, the higher the probability of fulfilment of a mission. On the other side, a greater number of pyrotechnic cartridges brings about many problems – higher weight of a weapon, more complex design/construction and more complicated fire control system, etc.

A requirement during the development and design of this gun was to determine the optimum number of pyrotechnic cartridges that would ensure fulfilment of a mission. For this purpose a mathematical model of the dependence of the probability of accomplishment of a mission on the number of used rounds and pyrotechnic cartridges and their reliability was created.

Theory of queuing systems is a well-developed mathematical discipline, and in principle relates to this work. Based on it a substantial number of positive results have been generated. The results obtained in studying queuing systems and networks proved to be of significant profundity and importance from mathematical and practical points of view. In fact queuing systems and networks are able to model a broad class of real technical systems, info-telecommunication systems and networks ([2]).

Many scientific works and papers both in journals and conference proceedings have presented the approach to reliability assessment of $k$-out-of-$n$ systems ([1], [2], [39] or [40]). All of them presume that there is a system consisting of $n$ parts from which at maximum $k$ may fail in order to guarantee the system to work. Our assumption is partially similar. The whole system for the main function consists from $n$ queuing elements waiting until they will come into service. The main part of the system (with $n$ elements) is backed up by another system with $m$ elements which are supposed to restore just the $k$ failed elements immediately after a failure occurs. Also the elements from the supporting system may fail which might cause an instant stop of the main function performance with all related consequences (e.g. costs penalties, endanger, etc.).

Some work has even been carried out in the field of testing and costs spent during one-shot items technical life ([41]).

Speaking about the weapon set of the example it consists of mechanical parts, electric, power and manipulation parts, electronic parts and ammunition. For the purpose of use in the paper it will be dealt with as isolated functional blocks and ammunition only. In this case the ammunition is as recommended, standardised rounds and pyrotechnic cartridges.

Single parts of the set can be described with qualitative and most importantly quantitative indices which present their quality. In this paper quality is dealt with in terms of dependability characteristics. Primarily probability values which characterize single indices, and which describe the functional range and required functional abilities of the set, are used. Also focus is given to the part of handling rounds and pyrotechnic cartridges which are crucial for this case. In order to continue the work it is necessary to define some basic terms and specify every function.

We define following basic terms and definitions. The term object is commonly used with reference to reliability analysis. The definition for an object is the same as that used in [42]. Consequently there is a need to describe the measures of the basic object as stated in [42].

A one-shot device/item:

"is an item which is required to perform its function only once during normal use. Such items will usually be destroyed during their normal operation and cannot therefore be fully tested. The reliability required from one-shot devices is normally high" [37].

One-shot items are usually required to perform a function once only since their use is normally accompanied by an irreversible reaction or process, e.g. chemical reaction or physical destruction.

The reliability of a one-shot item could be defined as the ability to perform the required function only once, and only when demanded, under stated conditions and for the specified period of time.

Object's (weapon set) function:

The main function: The main function of the object is putting into effect the fire from a gun using standard ammunition.

The step function: Manipulation with ammunition, its charging, initiation, detection and indication of ammunition failure during initiation, initiation of the backup system used for re-charging of a failed cartridge.

It is expected that the object will be able to work under different operating conditions especially in different temperature spectra, under the influence of varied static, kinetic and dynamic effects, in various zones of atmospheric and weather conditions.

In this case any of the operating conditions mentioned above are not taken into account while performing the assessment. However, their influence might be important while considering successful mission completion.

One of the main terms which is to be developed is:

Mission: It is an ability to complete a regarded mission by an object in specified time, under given conditions and in a required quality.

In the paper it is the case of cannon ability to fire a required amount – a number of shot ammunition at a target in the required time, and under given operating and environmental conditions.

As it follows from the definition of a mission it is the case of a set of various conditions which have to be fulfilled all at once in a way to satisfy the requirements completely. The object is supposed to be able to shoot a required amount of ammunition which has to hit the target with required accuracy (probability). The only focus in this paper is the ability of the object to shoot [42], and there will be no consideration given to the evaluation of the result of shooting, weapon aiming, internal and external ballistics, and weather conditions.

## 2. Reliability of one shot item

It follows from the definition concerning one-shot items that differentiation into two basic types of failures can be made:

- an item does not perform a required function when needed;
- an item performs a required function when not needed.

Reliability of a system where the item is used as well as its ability to complete a required mission is influenced by the first type of failure. System safety in particular is influenced by the latter type of failure because an inadvertent initiation of any one-shot item can lead to the hazard of personnel or equipment. In view of the nature of the problem being solved focus is upon the first type of failures only. They are the kinds of failures which may lead to weapon function disruption.

The reliability of a one-shot item or system with one-shot items should normally be expressed or quantified as a probability of mission success (see equation (1)). The conditions under which the mission is regarded as completed depend on many circumstances – on the nature of a mission, on one-shot items having been used, on the purpose of a system etc. Dealing with automatic weapons the mission is completed only in case when all the rounds placed in a magazine or in ammunition feed belt are able to be shot, and if this happens without any external intervention.

If one-shot item reliabilities are assumed to be statistically independent, the reliability of all one-shot items may be incorporated into the final calculation of the system reliability by multiplying the portion of the model representing the one-shot items by the portion representing other parts of the system [38]:

$$R_s = R_w \prod_{i=1}^{i=n} (1 - p_i) \tag{1}$$

The total reliability $R_w$ of the system is represented by multiplication - $\prod$ - of its partial items reliability – $(1 - p_i)$, where $i = 1, \ldots, n$ , which are not one-shot items. Reliability of these items is time dependent - it means that the probability of failure occurrence of these items rises with operating time. Reliability $R_w$ is used to describe the reliability of a weapon as a whole. On the other hand the reliability of a one-shot-item is regarded as time independent and it means that the probability of a failure is unchanged with operating time [38].

Referring to fire arms the operating time is usually measured by a number of performed shots, and using the number of the shots we quantify relevant reliability performance measures. Mean time between failures for example is expressed as a mean number of shots between failures – *MNSBF* and a failure rate $\lambda$ is related to a single shot. If we take into account an exponential distribution of time between failures, the failure rate can be expressed by equation 1:

$$\lambda = \frac{1}{MNSBF} \tag{2}$$

On the basis of these assumptions the reliability $R_w$ can be described as a function of the number of rounds shot (e.g. [2] and [38]):

$$R_w(n) = \exp(-\lambda n) \tag{3}$$

Let's presume that for an automatic weapon we use rounds with the failure probability $p_r$ and a magazine has a capacity of $n$ rounds. The reliability of a system is described as the probability that all the rounds will be shot without external intervention and it can be expressed in the following formula using (1) and (3):

$$R_s = \exp(-\lambda n)\,(1 - p_r)^n \tag{4}$$

The formula shows that the reliability of a weapon system depends mainly on the failure probability of the rounds $p_r$ and the number of the rounds $n$ with failure rate $\lambda$. The dependence is represented graphically in the diagram in *Figure 1*. The diagram demonstrates that despite relatively high reliability of rounds the reliability of a system decreases rapidly with the growing number of rounds. Regarding each automatic weapon we have to take into account the possibility of a round failure and then to provide recharging of the gun.

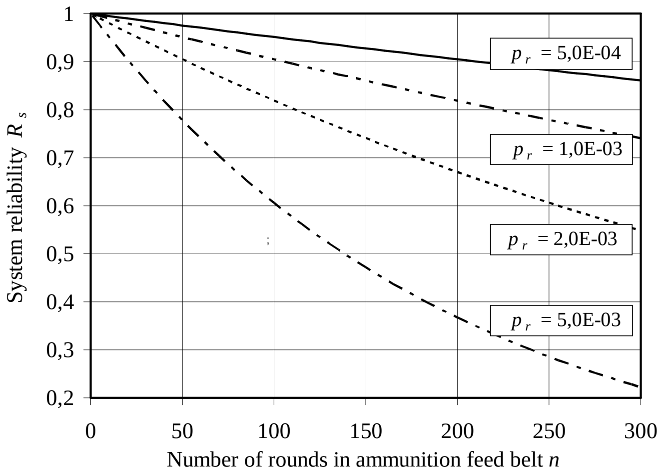


*Figure 1.* Weapon reliability as a function of rounds number

To describe the reliability of a system with one-shot items it is important to be able to specify the occurrence probability of a particular number of faulty items taken from a total number of used items. In most cases the initiation of single one-shot items might be regarded as a succession of mutually independent effects, and that is why we can use the binomial distribution to describe the reliability.

The following formula (5) demonstrates the probability of failure of the $x$ particular items which might occur during operation of a system [44] and [45].

$$\Pr(X = x) = \binom{n}{x} p^x (1 - p)^{n-x} \tag{5}$$

The probability of a failure of at most $x$ items during operation of a system may be specified in a similar way [3] and [4]:

$$\Pr(X \le x) = \sum_{i=0}^{i=x} \binom{n}{i} p^i (1 - p)^{n-i} \tag{6}$$

## 3. Description of a process and the aircraft gun reliability

### 3.1. Direct calculation of aircraft gun reliability

One way of determining the aircraft gun reliability is based on the direct calculation approach. We need to determine the required number of supporting pyrotechnical cartridges which are needed for successful completing of the mission. The next issue is the level at which the probability of mission success is acceptable.

#### 3.1.1. Mathematical model

In general the condition under which the mission is completed may be defined as – during the mission the number of round failures is equal to the number of working pyrotechnic cartridges. This can be expressed as:

$$X \le m - Y \tag{7}$$

It is obvious that during the mission a failure of the aircraft gun itself must not occur either. The probability of completing the mission can be given in the following formula:

$$R_s = R_w(n)\,\Pr(X \le m - Y) \tag{8}$$

A mathematical model of aircraft gun reliability is based on analysis of the possible scenarios of aircraft gun operation which lead to completion of the mission. These particular scenarios are specified below and the probability of their implementation is determined.
A typical feature of the other scenarios leading to completion of the mission is a failure which always occurs to a particular number of rounds and automatic recharging of an aircraft gun is done by pyrotechnic cartridges. These scenarios will be implemented only under the condition put in formula (7) which says that a number of working pyrotechnic cartridges is at least the same as the number of round failures.
The equation (7) will be transformed into the form which enables calculation of the probability of completing the condition using formula (6):

$$Y \le m - X \tag{9}$$

Scenarios of this description will be marked with $S_x$ and the subscript *x* stands for the number of round failures considered in the scenario. Only the scenarios which lead to completing the mission are considered, and that is why the number of considered failures can equal at best the number of used pyrotechnic cartridges $x \le m$. If there is a failure of more rounds than is the number of used pyrotechnic cartridges, the mission will be always uncompleted.
The probability of implementation of the particular scenarios can be specified as a product of the reliability of the gun itself $R_w$ as well as the probability of occurring exactly *x* round failures, and the probability of occurring at most $(m - x)$ failures of the pyrotechnic cartridges. This probability can be put in the following formula:

$$\Pr(S_x) = R_w \Pr(X = x)\Pr(Y \le m - x) \tag{10}$$

Using equations (5) and (6) we can transform the formula (10) into the form as written below:

$$\Pr(S_x) = R_w \binom{n}{x} p_r^x (1 - p_r)^{n-x} \sum_{i=0}^{i=m-x} \binom{m}{i} p_p^i (1 - p_p)^{m-i} \tag{11}$$

The formula mentioned above is universal for all the scenarios where $0 \le x \le m$.
It follows from the analysis that an aircraft gun completes the mission only at that time if some of the scenarios mentioned above $S_x$ are achieved. If the mission completion is marked as an event with S, the conditions under which the mission is completed can be expressed this way [43]:

$$S = S_0 \cup S_1 \cup S_2 \cup \ldots\ldots \cup S_x \cup \ldots\ldots \cup S_m = \bigcup_{x=1}^{x=m} S_x \tag{12}$$

In view of the fact that the particular scenarios present mutually disjunctive events, the probability of event occurrence S in terms of mission completion can be expressed as a sum of probabilities of single scenarios occurrence $S_x$ [37]:

$$R_s = \Pr(S) = \sum_{x=0}^{x=m} \Pr(S_x) \tag{13}$$

Equation (13) allows the reliability of an aircraft gun to be assessed. The next section details how is done.

### 3.2. Calculation based on the possible scenarios

Another approach to determine the probability of mission success as well as the required number of pyrotechnical cartridges supporting the main function is based on the possible scenarios which may occur during the system function.

Let's consider the following blocks:

- manipulation with ammunition, its charging, initiation, failure detection and indication during initiation, initiation of a backup system in order to recharge a failed cartridge, all mechanical parts, all electric and electronic parts, interface elements with a carrying device - Block A;
- ammunition – Block B;
- pyrotechnic cartridges – Block C.

The process as a whole can be described as:

The task is to find out their minimum number which is essential for completing the mission successfully.

### 3.2.1. Mathematical model

To meet the needs of the requirements the successful completion of the mission is to be expressed in a mathematical way. It is known that the number of rounds *n* in an ammunition belt is final. It is also known that an event/failure of a round $\overline{B}$ (ammunition block function – B) can occur with a probability $p_n$. All the requirements and specifications mentioned above will be used in the further steps.

A number of occurrences $X_n$ of an event $\overline{B}$ follows the distribution in Bernoulli's row *n* of independent experiments, and probability of event occurrence $P(\overline{B}) = p_n$. The number $p_n$ is the same in every experiment (e.g. [43] and [44]).

Because there is an occurrence of the number of events in an observed file it has a counting distribution of an observed random variable which is in this case the number of failed rounds. A probability function of a binomial distribution can be expressed as:

$$P(X_n = x) = \binom{n}{x} p_n{}^x (1 - p_n)^{n-x}; \quad x \in \{0,1,2,...,n\} \tag{14}$$

In order to specify the mean number of possible failures in an ammunition belt of a given length it is to quantify the formula (14) and replace *n* by a real number of rounds in an ammunition belt.

On the basis of the facts mentioned above it is obvious that the process of fulfilling the requirements by the back up system of pyrotechnical cartridges follows a geometrical distribution (*Ge*). It means that the process of fulfilling the requirements repeats so often until it meets them in terms of reversion of all the process to an operational state. Pyrotechnic cartridges also have failure rate $p_m$ (failure probability) and there is a limited number of them. It means that a failure can occur up to *m*-times.

It is necessary to assess the succession of independent attempts, and the probability of an observed event occurrence equals the same number $p_m$ in each attempt. The quantity $X_m$ is a serial number of the first success which means that a required event occurs. The event here means a function of a block C, and a probability $p_m$ means an event occurrence $\overline{C}$. Characteristics of the process are as follows. A probability function:

$$P(X_m = x) = p_m{}^{x-1}(1-p_m);\ x \in \{1,2,3,\ldots,m\} \tag{15}$$

It is a special case of a geometrical distribution when a probability of an event occurrence (a pyrotechnic cartridge failure) does not depend on the number of previous unsuccessful attempts of a value "0". Characteristics of geometrical distributions like mean value $E(X_m)$ (a mean number of pyrotechnic cartridges necessary for removing one failed round) and dispersion $D(X_m)$ are obtained by calculation of the formula:

$$E(X_m) = \sum_{x=0}^{\infty} x.P(X_m = x) = \frac{1}{1 - p_m} \tag{16}$$

$$D(X_m) = \frac{1 - p_m}{p_m^2} \tag{17}$$

While completing the mission during either training or real deployment a few scenarios can occur, and the course of them depends on single functional blocks. To complete the mission *Mis* successfully single blocks are expected to be failure free as stated above. The function of the blocks mentioned above are designated as *A, B, C,* the opposite is $\overline{A}; \overline{B}; \overline{C}$. The relation can be expressed by using events this way:

$$Mis = A \cap (B \cup C) \tag{18}$$

Using the probability expression, the probability of mission completion *P*(*Mis*) is given by:

$$P(Mis) = P(A) \,.\, [P(B) + P(C) - P(B \cap C)] \tag{19}$$

### 3.2.2. Description of scenarios

Description of the scenarios which can occur during completion or default of the mission relate only to an ammunition block and to a redundant mechatronics system with pyrotechnic cartridges.

The mission is completed: In the first case there can be a situation when all the ammunition of a certain amount which is placed in an ammunition belt is used up and a round failure occurs or it is used up and a round failure does not occur. In this case a backup system of pyrotechnic cartridges is able to reverse a system into the service. Using up can be single, successive in small bursts with breaks between different bursts, or it might be on mass using one burst. Shooting is failure free or there is a round failure occurrence *n.* In case a round failure occurs, a system which restores the function of pyrotechnic cartridges is initiated. There are two scenarios too – a system restoring a pyrotechnic cartridges function is failure free, or a pyrotechnic cartridge fails. If a function of pyrotechnic cartridges is applied, it can remove a failure *m*-times. So a number of restorations of the function is the same as the number of available pyrotechnic cartridges. Another alternative is the situation that a round fails and in this case a pyrotechnic cartridge fails too. A different pyrotechnic cartridge is to be initiated and it should restore the function. This must satisfy the requirements that an amount of all round failures *n* is lower or at least equal to the number of operational (undamaged) pyrotechnic cartridges *m*. The mission is completed in all the cases mentioned above and when following a required level of readiness of a block A.

The mission is not completed: In the second case the shooting is carried out one at a time, in small bursts or in one burst, and during the shooting there will be *n* round failures. At the time the failure occurs a backup system for restoring the function will be initiated. Unlike the previous situation there will be *m* pyrotechnic cartridges´ failures and a total number of pyrotechnic cartridges´ failures equals at least the number of round failures, and is equal to the number of implemented pyrotechnic cartridges *M* at the most. It might happen in this case that restoring of the function does not take place and the mission is not completed at the same time because there are not enough implemented pyrotechnic cartridges.

Following the mathematical formula (16) it is possible to find out the probability of a number of round failures´ occurrences in an ammunition belt of a length *n*. Following equation (16) it can be specified as an expected mean value of a mean number of round failures in an ammunition belt of a given length.

The mean value result is recommended to be used for the maximum length of an ammunition belt (a maximum number of rounds) which could be implemented into a weapon set concerning construction as well as tactical and technical views. The result informs of a minimum number of pyrotechnic cartridges which are to be applied for a successful completion of the mission.

In this case there is a threat of a pyrotechnic cartridge failure which could cause a system failure (as far as a number of round failures is higher than a number of available pyrotechnic cartridges). In this case it is not possible to complete the mission successfully.

In order to assess dependability of the shooting function it is necessary to know a number of pyrotechnic cartridges and, depending on this, the probability of completing the mission. To fulfil the requirements it is suggested the following three steps are carried out:

1) To determine the required number of pyrotechnic cartridges;
2) To quantify generally probabilities of completing the mission;
3) To quantify exactly probabilities of completing the mission

Following the steps mentioned above it is suggested to follow this method.

Ad 1) To determine the required number of pyrotechnic cartridges

When calculating the mean number of failed rounds $E(X_n)$ which is determined from the maximum number of rounds *n* in a ammunition belt (see above) and the probability of a round failure occurrence $p_n$, see the formula (16), it is to get a minimum recommended number of pyrotechnic cartridges which are supposed to guarantee completing the mission in case a round fails.

The calculation would be successful in case a pyrotechnic cartridge failure does not occur. However, even a system of pyrotechnic cartridges concerning a failure occurrence depends on counting the distribution of a discreet random variable which is specified in this case by a geometrical distribution (because the system is activated so long until the observed and required event occurs – in terms of repairing the failure). It is suggested to calculate the mean number of pyrotechnic cartridges´ failures following formula (20). For the calculation it is required to know only the pyrotechnic cartridge failure probability $p_m$. On the basis of this calculation the average number of pyrotechnic cartridges required to repair a failure of one round can be obtained.

In order to complete the mission a number of available (operational) pyrotechnic cartridges should be at least the same as the number of failed rounds. When the mean values are multiplied then it is possible to obtain the total number of pyrotechnic cartridges $M$ which will guarantee completing the mission (even in the situation when besides failed rounds there are failed pyrotechnic cartridges too).

$$M = E(X_n)\,.\,E(X_m) = \frac{n.p_n}{1-p_m} \tag{20}$$

Logically the number of pyrotechnic cartridges which are essential for completion of the mission successfully is continually proportioned to the number of rounds $n$ and to the probability of their failure $p_n$, and inversely proportioned to the probability of pyrotechnic cartridge "success" $1\text{-}p_m$. The figure 2 shows a typical course of dependability $M$ ($p_n$;$p_m$), it means a invariant $M$ which depends on variables $p_n$ and $p_m$. This way might be the first of the alternatives on how to solve the problem. It suggests the total number of pyrotechnic cartridges which are essential for completing the mission but it does not show the way of how to quantify the probability of mission completion.
While recording the distribution parameters an equivalent $m$ standing for a value $M$ is to be used.

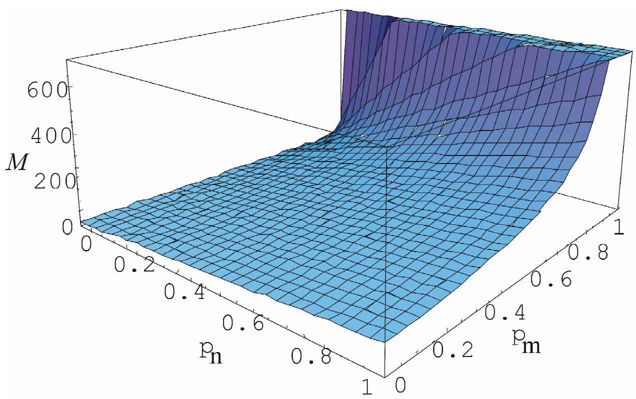


*Figure 2*. Course of dependability of a number of pyrotechnic cartridges M on variables $p_n$ and $p_m$

Ad 2) To quantify generally the probability of completing the mission
In this case it is necessary to follow the solution which has been stated in the part Ad 1. It should be taken into account that there is a number of pyrotechnic cartridges required for completing the mission. So, it has to be determined an $\alpha$ fractile which provides an upper limit of the number of rounds which fail with probability $\alpha$. After this it has to be specified $\beta$ fractile which provides an upper limit of the pyrotechnic cartridges which fail with probability $\beta$.
While working with fractiles this general information needs to be followed. 100% fractile of a random variable $X$ is a number $x_p$, and a probability $p$ where $0<p<1$ is denoted by

$$P(X \le x_p) \ge p \tag{21}$$

And

$$\lim_{x \to x_p^-} P(x) \le p \tag{22}$$

The fractile of an observed random variable which is to work with is expressed by

$$p_n = \sum_{n=0}^{x_\alpha} P(X_\alpha = n) \tag{23}$$

In words this means – the occurrence probability $n$ of a number of events is specified by a sum of the probabilities for the occurrence of all events from *0* to $n$.
In this case it is necessary to take into account that round failures´ distribution is binomial $B_i$=($n$;$p_n$) and a fractile determining an upper limit of a number of rounds which might fail with probability $\alpha$ will be designated as $x_\alpha$. Given as

$$P(X_n \le x_\alpha) = \alpha \tag{24}$$

Given that the general distribution of a pyrotechnic cartridge follows a binomial distribution $Bi$($m$;$p_m$) and a fractile providing an upper limit of a number of pyrotechnic cartridges which fail in probability $\beta$ is denoted by $y_\beta$. Therefore

$$P(Y_m \le y_\beta) = \beta \tag{25}$$

The equation can be expressed as

$$\Pr(m - Y_m \ge m - y_\beta) = \beta \tag{26}$$

The following interpretation of a fractile $y_\beta$ is useful for the other steps – at least $m - y_\beta$ of pyrotechnic cartridges will be available with probability $\beta$.
As it was stated before given the total number of pyrotechnic cartridges $M$ which are essential for completing the mission is known, the requirement is shown in the following equation:

$$(M - y_\beta) \ge x_\alpha \tag{27}$$

The equation shows that the number of available pyrotechnic cartridges (it is obtained when failed pyrotechnic cartridges are subtracted from the total amount of all applied pyrotechnic cartridges) will be at least the same (it would be better to have a higher number) as the number of failed rounds. If this assumption is fulfilled, it can be expected that the mission will be completed with probability $p_{mis}$. Probability of completing the mission can be defined as.

$$p_{mis} = x_{\alpha} \,.\, y_{\beta} \tag{28}$$

The formula can be described like this – the probability of completing the mission equals a multiplication of probabilities $x_{\alpha}$; $y_{\beta} \in (0;1)$ which provide an upper limit of failed rounds and an upper limit of failed pyrotechnic cartridges for required levels of fractiles.
If the level of mission completion probability is known in advance, e.g. it is specified by technical requirements for a set, it is possible to be put into the formula which is based on an assumption that the mission will be completed in case a number of available pyrotechnic cartridges is at least the same as rounds which are supposed to fail.

$$x_{\alpha} \le m - y_{\beta} \tag{29}$$

If it goes this way, the mission will be completed with the probability expressed in the formula (28). If the values $\alpha$, $n$, $\beta$, $p_{mis}$, are known it is possible to find a value $m$ ($M$) using quantitative methods. At the end of this contribution there is an example of this solution.

Ad 3) To quantify exactly the probabilities of completing the mission
In the last step it will be examined how to quantify an exact value of mission completion probability $p_{mis}$. On the basis of the assumption described above it is known that the probability of completing the mission depends on the reliability of two key blocks. It is an ammunition block (B) and a pyrotechnic cartridges´ block (C). Following the last two alternatives it might be specified that both require a total number of pyrotechnic cartridges which is essential to complete the mission (in case all conditions are met), and a general value of mission completion probability in case general conditions are followed. This solution might be satisfied under certain circumstances but it is not always the case. Therefore it is suggested the last way on how to quantify the probability of completing the mission based on a more exact method.
It is necessary to define indices and quantities which effect directly the probability of completing the mission $p_{mis}$. These are the number of rounds $n$, the probability of a round failure occurrence $p_n$, the number of pyrotechnic cartridges $m$, and the probability of a pyrotechnic cartridge failure occurrence $p_m$. A general function of mission completion probability and its variables is given by:

$$p_{mis}(n, p_n, m, p_m) \tag{30}$$

Further steps follow well known assumptions. The function of a rounds´ failure takes the form of a binomial distribution with parameters $n$ and $p_n$ – $Bi(n,p_n)$, and the number of rounds which may fail can be marked with $k$ where $k \in \{0;1;2;\ldots..;n\}$. Moreover, introduced are functions of a pyrotechnic cartridges´ failure $Y_k$ where $k \in \{0;1;2;\ldots..;m\}$. They show the possibility of a pyrotechnic cartridge failure while shooting as soon as it is necessary to remove a failed round. Let's assume that the sum of functions of a pyrotechnic cartridges´ failure will be lower than the number of available pyrotechnic cartridges used for removing a failed round, given by the following formula:

$$\sum_{k=0}^{m} Y_k \le m \approx Y_0 + Y_1 + \ldots\ldots Y_k \le m \tag{31}$$

Following the assumption mentioned above it is considered the case that the first available pyrotechnic cartridge follows a geometrical distribution of a function of its activity $Ge(p_m)$ during the failure of the $k$-th round $Y_k$. It can be described as:

$$Y_k \sim \mathrm{Ge}(p_m) \tag{32}$$

The equation showing the probability of completing the mission is:

$$P(n, p_n, m, p_m) = \sum_{k=0}^{n} P(X = k).P(Y_0 + Y_1 + \ldots. + + Y_k \le m) \tag{33}$$

where in case $k=0$ (it reflects a situation where there is no round failure) a function would be specified additionally provided that $P(Y_1 + \ldots. Y_k \le m) = 1$. And in order to solve the probability value of completing the mission it is possible to use so called completing the formula taking advantage of forming functions. From a mathematical point of view this is much more demanding but it offers a very exact value expressing the probability of completing the mission $p_{mis}$ while using a variation of function factors. On its basis it is easy to prove the dependability of the total number of used pyrotechnic cartridges on a level of mission completion probability $p_{mis}$.

## 4. An example of possible solution

### Ad 3.1

A designed model makes it possible to carry out an analysis which assesses systems´ sensitivity to changes of input parameters of the model. When calculating all the results stated below the reliability of a gun itself $R_w$ = 0,9999 is considered. Final results of mission non-completion probability which are shown in the graphs below were calculated with the formulae (11), (13) and (14).

In the diagram in *Figure 3* there is a dependency of the mission non-completion probability on the number of pyrotechnic cartridges when considering rounds of different reliability. The diagram shows that the bigger number of pyrotechnic cartridges used in the construction of an aircraft gun, the more sensitive reaction of the system is to a change of the rounds´ reliability.

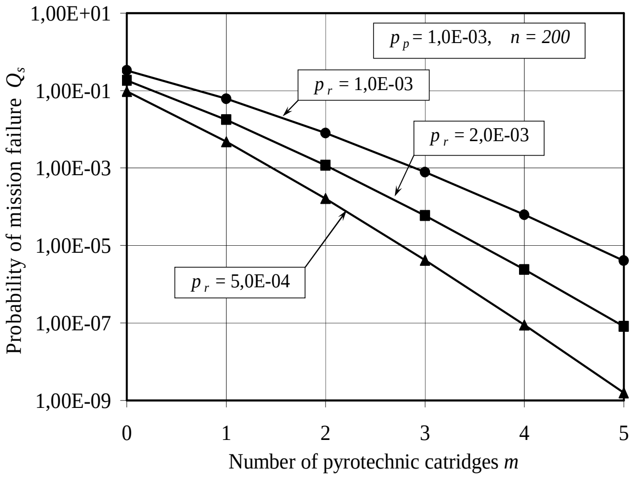


*Figure 3*. Influence of m and $p_r$ on probability of mission failure

In the diagram in *Figure 4* there is a dependency of mission non-completion probability on the number of pyrotechnic cartridges when considering a different number of rounds in an aircraft gun magazine. It follows from a course of dependencies that with a growing number of rounds which are supposed to be used during a mission the final effect of using pyrotechnic cartridges decreases.

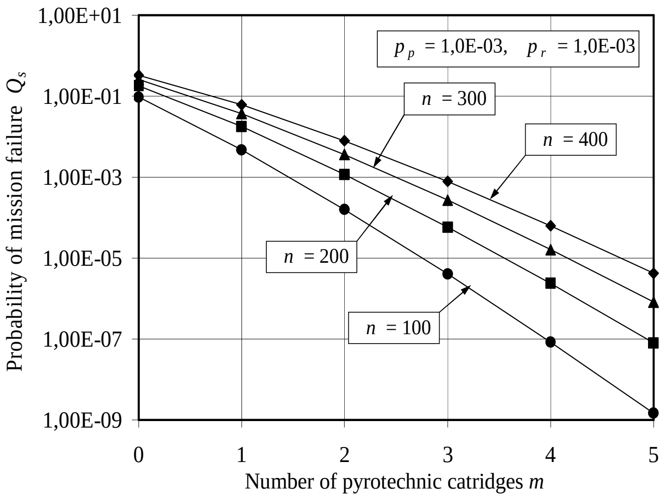


*Figure 4*. Influence of of *n* and *m* on probability of mission failure

In the diagram in *Figure 5* there is a dependency of mission non-completion probability on reliability of pyrotechnic cartridges when considering a different number of pyrotechnic cartridges.

It follows from the diagram that a change of the level of pyrotechnic cartridges reliability influences the final reliability of systems on a very limited scale. It results from this that extremely high requirements for reliability of pyrotechnic cartridges are not legitimate.

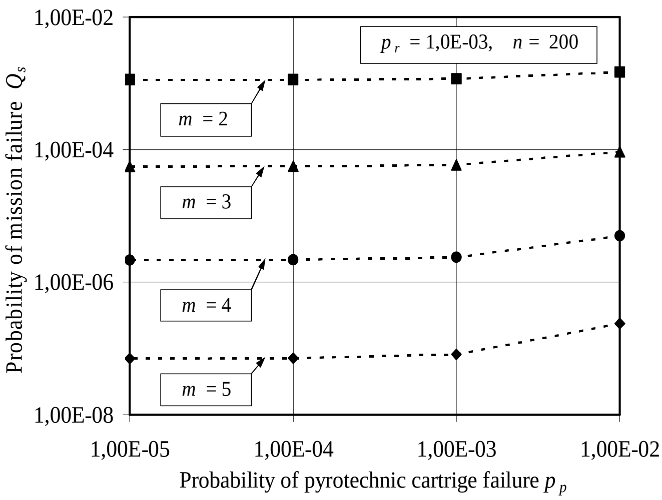


*Figure 5*. Influence of $p_p$ and m on probability of mission failure

In *Figure 6* there is a method on how to determine an optimum number of pyrotechnic cartridges which are necessary to achieve reliability of a system required.

Let´s presume that relating to the system defined with the parameters $R_w$= 0,999, $p_r$ = 0,001, $p_p$ = 0,001, $n$ = 200 it is required that $Q_s \leq$ 1,0E-04 means the probability of mission non-completion. The aim of the conception is to specify this number of pyrotechnic cartridges which guarantees that required reliability will be achieved.

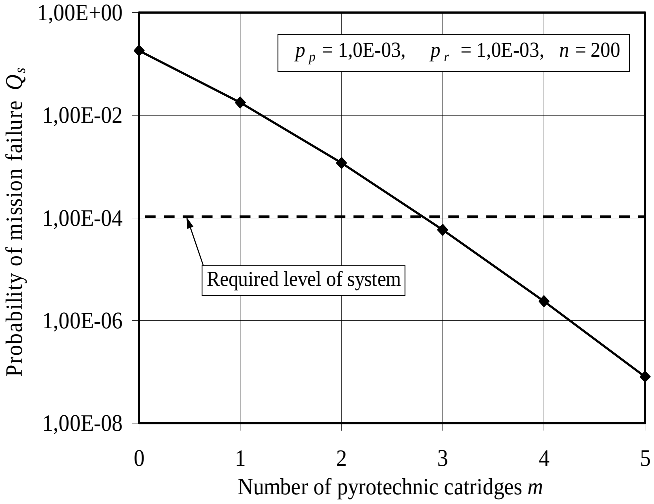


*Figure 6*. Determination of optimal number of pyrotechnic cartridges

By means of input quantities and equations (12), (13), and (14) we put into the graph the dependency of mission non-completion probability on the number of used pyrotechnic cartridges and the required probability will be put into the diagram. The diagram in *Figure 5* shows that the required level of reliability value will be achieved when using three pyrotechnic cartridges.

**Ad 3.2**

Given:

$p_n$ = 0,000 1 - round failure probability;

$n$ = 200 - maximum rounds´ number during one process;

$p_m$ = 0,01 - pyrotechnic cartridge failure probability;

$p_{mis}$ = 0,99 - probability of mission success.

Solution according to "Ad 1)": It has to be found a sufficient number of pyrotechnic cartridges used for removing a possible failure.

$$M = \frac{n.p_n}{1 - p_m} = \frac{200.0{,}0001}{1 - 0{,}01} \cong 0{,}02$$

The formula shows that having at least one pyrotechnic cartridge is enough to complete the mission successfully. However, it is not possible to quantify the probability for completing the mission.

Solution according to "Ad 2)" in section 3.2.2: It is necessary to find a level of mission completion probability $p_{mis}$ as well as the required number of pyrotechnic cartridges. The values described above are to be followed. The solution is put in the table.

*Table 1*. Results of example

| $\alpha$ | $x_\alpha$ | $\beta = \frac{p_{mis}}{\alpha}$ | $m$ |
|---|---|---|---|
| 0,991 | 1 | 0,998 991 | 2 |
| 0,992 | 1 | 0,997 984 | 2 |
| 0,993 | 1 | 0,996 979 | 2 |
| 0,994 | 1 | 0,995 976 | 2 |
| 0,995 | 1 | 0,994 975 | 2 |
| 0,996 | 1 | 0,993 976 | 2 |
| 0,997 | 1 | 0,992 979 | 2 |
| 0,998 | 1 | 0,991 984 | 2 |
| 0,999 | 1 | 0,990 991 | 2 |

If it is taken into account this solution and starting marginal conditions, two pyrotechnic cartridges will be enough to complete the mission successfully with 0,99 probability.

## 5. Conclusion

The results presented in this article, related partially to queuing processes, systems and reliability problems, are very recent. It is obvious that further advancement will require consideration of such a priori distributions of the values, and other traditional queuing systems as well the restorable devices input parameters that can be of practical interest. The distributions of the variables that characterize the functioning of different system types can be calculated after they have been randomized taking into account the given a priori distributions.

This contribution is an extension of the outputs of the work previously presented in several papers (e.g. [46], [47], [48] and [49]). It also serves as one of the alternatives for solving the problems connected with providing a function of an object whose function is redundant (backed up) because its failure is important to complete the mission. In order to solve the problem there have been methods chosen which are the most suitable for it. Other ways are also likely to be used in order to reach the aim but it is not the intention of this contribution.

The example presented represents the possibility of carrying out a precise analysis of the influence of one-shot items and their reliability on the reliability of the weapon as a whole. Easy determination of an optimal number of pyrotechnical cartridges which is supposed to guarantee a required level of weapon reliability represents a significant advantage.

One of the most interesting results of this analysis is the fact that the total level of weapon reliability is not influenced by the level of pyrotechnical cartridges reliability. The effort for increasing the level of pyrotechnical cartridges reliability does not increase the level of total weapon reliability.

On the other hand it can be observed that pyrotechnical cartridges application does increase the total value of weapon reliability and more over a relatively small amount of applied pyrotechnical cartridges does represent significant increase of the total reliability value of the weapon.

Due to concrete application of this developed model for a specific weapon type it is not impossible to utilise the procedures mentioned above (after modifications needed) in order to estimate different weapons´ construction reliability. Another possible way of this model application is the estimation of complex system reliability where one shot items are applied.

In such analysis it has to be taken into account a very sensitive thing which is the possible existence of failures dependencies (generally it means some events relationship). We presume in this analysis that some event occurrences are independent but when such a statement (we mean circumstance) is changed several consequences in analysis steps are supposed to be changed too.

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