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MACHINE VIBRATIONS WITH MORE DEGREES OF FREEDOM

Rigid mounting of machines to a solid base is one of the requirements for high performance and quality of production. Machine vibrations occur as a result of unbalanced revolving masses, loading forces and moments, starting and coasting of driving motors and other effects. We try to eliminate these undesirable vibrations by suitable mounting of the machine and its parts as much as possible. Suitable mounting of the machine is possible using components such as torsion springs, pneumatic springs, rubber pads or other components. The article describes numerical calculation of optimal displacements the machine for the given his parameters.

1. INTRODUCTION

It is possible to prevent of machines vibrations by suitable mounting of the machine to the base. Synchronous generators used for conversion of mechanic power to electric power represent one group of the machines requiring assessment from the viewpoint of dynamic effects on structures and bases. The following aspects belong among the principal ones for a reliable design of structures or bases under the machinery:

-effective rate of response of velocity in mm/s to the frequency of operational velocity (in transition conditions at machine starting or coasting when occurrence of resonance peaks of the response of the monitored parameter takes place as a result of excitation of significant own frequencies and shapes of oscillation),

-admissible level of oscillation of the base in the place of mounting of equipment (such as laboratory scales, spectrometers, etc.) – the level of the limit admissible amplitudes of offsets required by the manufacturer ranges in orders of units up to hundredths of µm,

-limit values at which it is possible to assume occurrence of visible defects on engineering structures – limit amplitudes of acceleration in mm/s^2 for dynamic effects with frequencies up to 10 Hz and amplitude of oscillation in mm/s for effects with frequencies within the range from 10 to 100 Hz for impact loading and for steady periodic vibration are specified,

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-base frames on which machinery is mounted should be in resonance with operational rotation speed of any machine part (e.g. of a generator with frequency of 25 Hz and a turbine with frequency of 50 Hz) – it is understood usually by this requirement that the dominant natural frequency should differ by at least ±20 % from the frequency of operational rotation speed; it is recommended in case of ceiling structures that their first bending own frequency at long-term and so also the most frequent operational loading should be at least 4 Hz,

-amplitudes of the response of internal forces or stress in the structure ascertained by dynamic calculation when considering maximum dynamic load – in case of rotational machines, maximum unbalanced masses on the revolving parts of the machine arising in case of rotor failure (such as turbine blade braking); when assessing the structure according to limit conditions of load bearing capacity, it is necessary to take into consideration amplitudes of the monitored response parameter (such as bending moment in cross section) within the complete frequency spectrum, i.e. not only at operational speed but also in transition conditions, i.e. also in resonance peaks or in the complete time section of the acting load (such as short-circuit moment),

-assessment of structure for highly cyclic fatigue stress – it should be carried out in the cases when the amplitude of stress or internal forces in the assessed cross sections of the structure at operational speed of the machine is significant when compared with the effect of static load acting for long-term period or if the dynamic effect causes oscillation of stress sign; a risk of occurrence of brittle fracture in case of welded steel structures but also fatigue damage of ferroconcrete structures are characteristic examples.

To vibrations generators happens at crossing critical turns and also because of drying of insulation material in grooves, formation of backlash and potentially as a result of other unexpected effects. We distinguish between high-speed generators (turbo-generators) driven with steam or gas turbines and low-speed generators driven with water turbines. Turbo-generators usually have a small diameter of the revolving parts (because of a reduction of centrifugal forces), long axial length and horizontal position. Two- and four-pole machines are used typically (in case of generators for the 50 Hz networks, speed is either 3,000 rpm or 1,500 rpm).

The low-speed generators usually have the speed on the level of 500 rpm and lower, corresponding number of poles, larger diameter and shorter axial length. All types have movable and static parts (stator and rotor) manufactured of a magnetic material.

The stator winding that powers the system is positioned in grooves distributed equidistantly along its internal periphery and consists of three identical parts pertaining to individual phases. The direct current excitation winding of turbo-generators is positioned in a similar way in the grooves on the rotor while in case of low-speed machines, on their protruding poles.

Moreover, the rotor is provided with damping winding the task of which is to dampen rotor mechanic oscillation. This winding is formed with conductive wedges in the grooves of the excitation winding in turbo-generators while in case of the low-speed generators, it is positioned in axial grooves in pole adapters.

Direct excitation current of the rotor induces revolving magnetic field in the machine the intensity of which is proportional to this current. The created magnetic flow then induces an electro-motoric force in each of three phases of the stator winding and as a result of this force, the current arisen and corresponding power will start to flow in the system.

The current flowing through the stator winding creates its own magnetic field that has a constant magnitude, however, it is revolving at the same speed like the rotor. The corresponding magnetic flow is superposed with the flow induced by the excitation winding. The arisen resulting flow has a stationary character with respect to the rotor, however, it revolves at a constant speed with respect to the stator.

The consequence is that the rotor can be massive (whirling currents are not induced in it so that corresponding losses do not also occur here) while the stator should be laminated because of opposite reasons. If the speed of the rotor deviates from the synchronous speed for any reason, the resulting flow with respect to the rotor will not have the stationary character, which will result in formation of currents in particular in the dampening winding. These currents will induce flow in the opposite direction according to the Lenz rule and prevent in a change of synchronous speed (and thus in oscillations).

Universal trend to increase the nominal power of the newly constructed generators and power plants existed in former years as relative investment and operational costs are reduced with growing power of the units (lower specific weight of the generator for unit power, smaller buildings, general built-up area and altogether all required equipment in the same sense). However, the opposite tendency has appeared in some countries recently, in particular where cheap natural gas is available. Gas turbines power plants with a combined cycle driving generators up to power of 250 MVA are produced. However, modern synchronous generators with power of 100 – 1300 MW and voltage of 10 – 32 kV are used commonly. In case of turbo-generator, we assume formation of vibrations as a result of unbalance of the rotor.

We will describe the model with movement equations in a matrix representation. After modification, it is possible to determine the transfer function *G(p)* from which it is then possible to determine the courses of amplitude frequency response curves for individual degrees of freedom using the Matlab program. The courses of velocity for individual degrees of freedom will be determined numerically then from the movement equations for the given stabilized generator displacements.

2. GENERATOR MODEL

Fig. 1 illustrates a simplified model of a three-phase two-pole generator (type 4A 275-02H, power 35 MVA, n=3000 rpm, product of ČKD Nové Energo a.s., Czech Republic) in which vibrations are caused as a consequence of unbalance U of the revolving part given by the equation:

$$U = m_n e, \tag{1}$$

where m_n is a mass of the revolving part and e is eccentricity. The generator is intended for connection with a steam turbine. The complete original set was firmly fixed to a concrete area. Requirement of a numerical design of spring and damping components in case of rotor unbalance that was detected in the former generator appeared.

In order to dampen vibrations, the generator will be mounted on 8 springs and dampers in the direction of *x*, *y* and *z* axes as indicated in Fig. 1. The components are represented by sets of torsion springs [1] with coefficient of rigidity k_x, k_y, k_z, and dampers with dampening coefficient $b_x=b_y=b_z$. The generator model represents a mechanic system with six degrees of freedom $x, y, z, \varphi_x, \varphi_y, \varphi_z$.

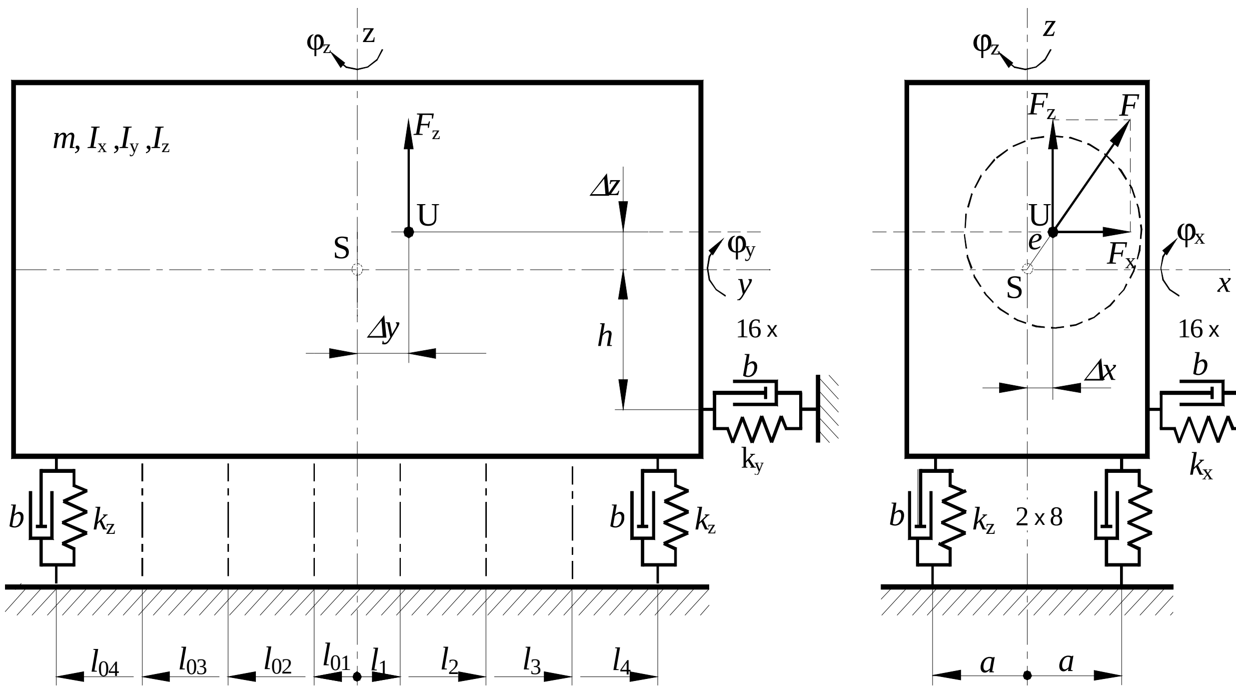

Fig. 1. Generator model

The system can be described with a movement equation in the matrix representation

$$\mathbf{m}\ddot{\overline{\mathbf{x}}} = \mathbf{k}\overline{\mathbf{x}} + \mathbf{b}\dot{\overline{\mathbf{x}}} + \mathbf{F}, \tag{2}$$

where

$$\mathbf{m} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x & 0 & 0 \\ 0 & 0 & 0 & 0 & I_y & 0 \\ 0 & 0 & 0 & 0 & 0 & I_z \end{bmatrix}, \qquad \overline{\mathbf{x}} = \begin{bmatrix} x \\ y \\ z \\ \varphi_x \\ \varphi_y \\ \varphi_z \end{bmatrix}, \qquad \mathbf{F} = \begin{bmatrix} m_n e\Omega^2 \cos \Omega t \\ 0 \\ m_n e\Omega^2 \sin \Omega t \\ m_n e\Delta y\Omega^2 \sin \Omega t \\ 0 \\ -m_n e\Delta y\Omega^2 \cos \Omega t \end{bmatrix}, \tag{3}$$

$$\mathbf{k} = \begin{bmatrix} -8k_x & 0 & 0 & 0 & 8k_x h & 0 \\ 0 & -8k_y & 0 & -8k_y h & 0 & 0 \\ 0 & 0 & -8k_z & 0 & 0 & 0 \\ 0 & -8k_y h & 0 & -8(\frac{\nu^2}{4}k_z + k_y h^2) & 0 & 0 \\ 8k_x h & 0 & 0 & 0 & -8(k_z a^2 + k_x h^2) & 0 \\ 0 & 0 & 0 & 0 & 0 & -8(k_x \frac{\nu^2}{4} + k_y a^2) \end{bmatrix},$$

$$\mathbf{b} = \begin{bmatrix} -8b & 0 & 0 & 0 & 8bh & 0 \\ 0 & -8b & 0 & -8bh & 0 & 0 \\ 0 & 0 & -8b & 0 & 0 & 0 \\ 0 & -8bh & 0 & -8b(\frac{\nu^2}{4} + h^2) & 0 & 0 \\ 8bh & 0 & 0 & 0 & -8b(a^2 + h^2) & 0 \\ 0 & 0 & 0 & 0 & 0 & -8b(\frac{\nu^2}{4} + a^2) \end{bmatrix}, \qquad (4)$$

where

$$\nu^2 = l_1^2 + l_2^2 + l_3^2 + l_4^2 + l_{01}^2 + l_{02}^2 + l_{03}^2 + l_{04}^2 .$$

If we introduce

$$\dot{\bar{\mathbf{x}}} = \bar{\mathbf{x}}_1, \qquad \bar{\mathbf{x}} = \bar{\mathbf{x}}_2, \qquad (5)$$

the equation (2) can be developed in the form

$$\mathbf{m}\dot{\bar{\mathbf{x}}}_1 = \mathbf{b}\bar{\mathbf{x}}_1 + \mathbf{k}\bar{\mathbf{x}}_2 + \mathbf{F}, \qquad \dot{\bar{\mathbf{x}}}_2 = \bar{\mathbf{x}}_{1,}. \qquad (6)$$

If we carry out modification of the equation (6)

$$\dot{\bar{\mathbf{x}}}_1 = \mathbf{m}^{-1}\mathbf{b}\bar{\mathbf{x}}_1 + \mathbf{m}^{-1}\mathbf{k}\bar{\mathbf{x}}_2 + \mathbf{F}, \qquad \dot{\bar{\mathbf{x}}}_2 = \bar{\mathbf{x}}_1, \qquad (7)$$

it is possible to rewrite the movement equation in the form

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{F}, \qquad \hat{\mathbf{y}} = \mathbf{C}\hat{\mathbf{x}}, \qquad (8)$$

where

$$\mathbf{A} = \begin{bmatrix} m^{-1}b & m^{-1}k \\ I_{6x6} & 0_{6x6} \end{bmatrix}_{12x12}, \qquad \widehat{\mathbf{x}} = \begin{bmatrix} \widehat{x}_1 \\ \widehat{x}_2 \end{bmatrix}_{12x1}, \qquad \mathbf{B} = \begin{bmatrix} m_{6x6}^{-1} \\ 0_{6x6} \end{bmatrix}_{12x6}, \tag{9}$$

$$\widehat{\mathbf{y}} = \begin{bmatrix} x \\ y \\ z \\ \varphi_x \\ \varphi_y \\ \varphi_z \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{6x12}.$$

The transfer function *G(p)* can be determined from the equations

$$p\widehat{\mathbf{x}} = \mathbf{A}\widehat{\mathbf{x}} + \mathbf{BF}, \qquad \widehat{\mathbf{y}} = \mathbf{C}\widehat{\mathbf{x}}. \tag{10}$$

From the equations (10), it is possible then to determine

$$\widehat{\mathbf{y}} = \mathbf{C}(p\mathbf{I} - \mathbf{A})^{-1}\mathbf{BF}, \tag{11}$$

where the transfer function can be expressed with the equation

$$\mathbf{G}(p) = \mathbf{C}(p\mathbf{I} - \mathbf{A})^{-1}. \tag{12}$$

Using the transfer function, it is then possible to determine the courses of resonance curves for individual degrees of freedom *x, y, z* by means of the Matlab program.

3. RESULTS OF THE SOLUTION

For the given values of coefficients of rigidity k_x, k_y, k_z and dampening coefficients $b=b_x=b_y=b_z$ of the individual components and the given value of mass *m* and inertia moments I_x, I_y, I_z of the generator (Fig. 1), we look for the amplitude frequency response curves (Fig. 3) using numeric calculation method with the Matlab-Simulink program (Fig. 2) by means of the transfer function. We also solve the courses of displacements $\mathrm{x, y, z}, \varphi_{\mathrm{x}}, \varphi_y, \varphi_z$. (Fig.4) using the numeric calculation. The parameters of the generator, its mounting and components are

$$m = 40430kg, \quad m_n = 14210kg, \quad I_x = 10kgm^2, \quad I_y = 947kgm^2, \quad I_z = 10kgm^2,$$
$$l_1 = 317{,}5mm, \quad l_4 = 952{,}5mm, \; l_{01} = 317{,}5mm, \; l_{04} = 952{,}5mm, \quad l_2 = l_3 = l_{02} = l_{03} = 0,$$
$$a = 1{,}17m, \quad h = 0{,}497m, \quad \Delta y = 0, \quad n = 3000\,rpm, \quad e = 1mm,$$
$$k_x = k_y = 1{,}5 \cdot 10^5\,Nm^{-1}, \quad k_z = 3 \cdot 10^5\,Nm^{-1}, \quad b = 2000Nsm^{-1}.$$

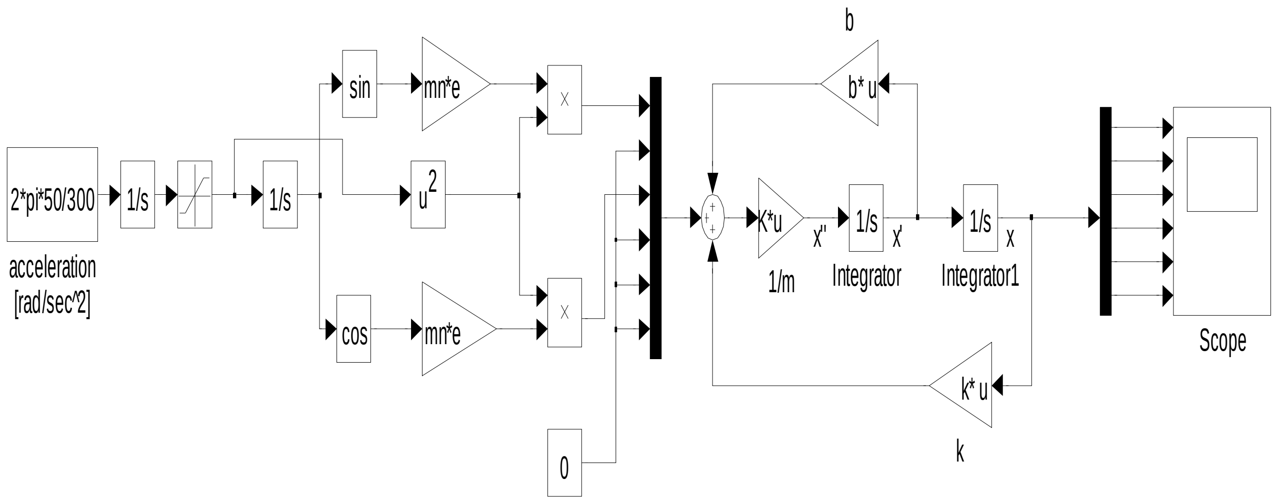

Fig. 2. Simulink scheme

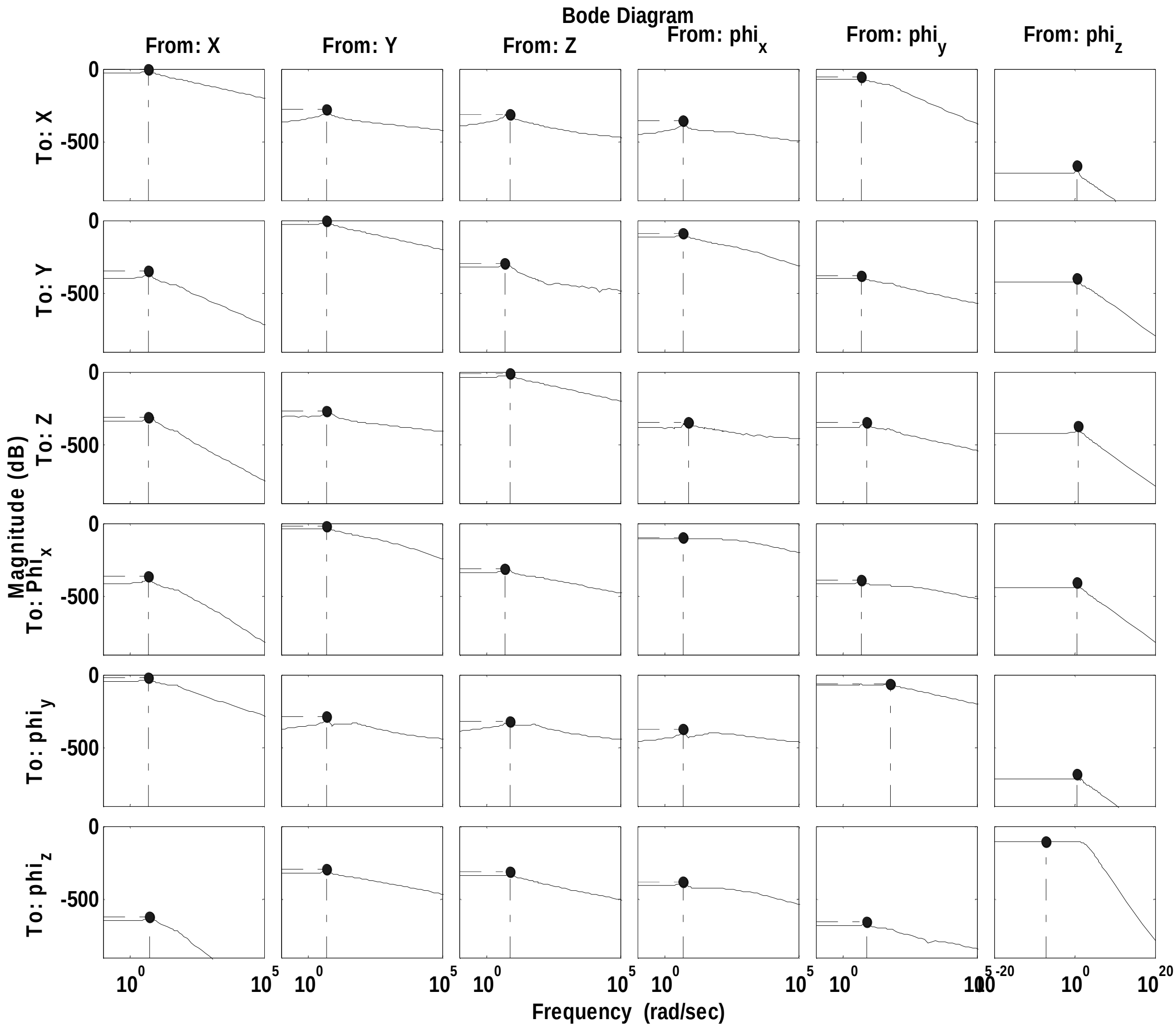

Fig. 3. Amplitude frequency response

4. CONCLUSION

In case that it is possible to describe the movement of a model of vibrating machine with a system of linear differential equations, it is then possible to express the transfer function *G(p)* (12) and the courses of amplitude frequency response curves (Fig. 3) from which it is apparent that maximum displacements by three resonant frequencies ω_1=5,14 s^{-1} (x_{max}=5,5 mm, φ_{ymax1}=0,95 10^{-3} rad), ω_2=7,7 s^{-1} (z_{max}=8,5 mm), ω_3=58,14 s^{-1} (φ_{ymax2}=0,61 10^{-3} rad) (Fig. 4). The courses of deviations $x, y, z, \varphi_x, \varphi_y, \varphi_z$ (Fig.4) at operational speed of the turbo-generator n=3000 rpm were determined numerically using the Matlab-Simulink (Fig. 2) program: x=1,1 mm, z=1,2 mm, φ_y=0,2 10^{-3} rad.

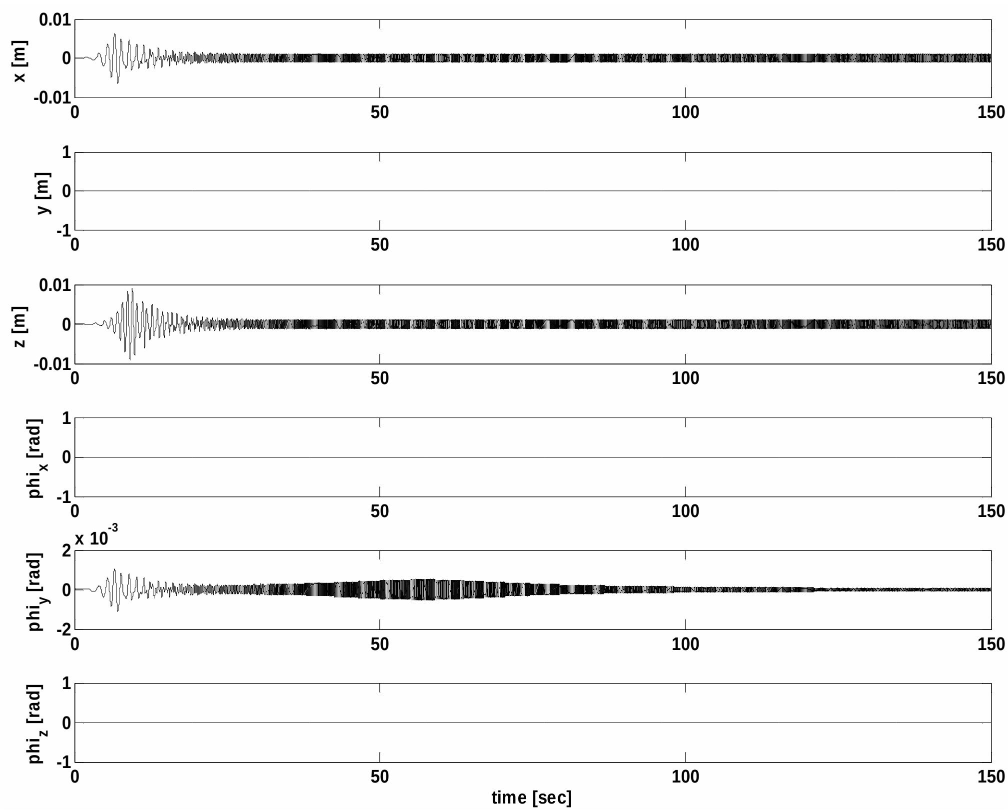

Fig. 4. Deviations $x, y, z, \varphi_x, \varphi_y, \varphi_z$. of the generator

To reduce vibrations considerably, it is suitable for a new set to mount it flexibly to a board that is mounted flexibly to a rigid base. The board will then act as an absorber accepting energy of oscillations of the set. It is then possible to determine the mass of the board and the coefficient of rigidity of the coefficient of damping using numerical calculation.

REFERENCES

[1] *Using Simulink, version 5*.The Math Works, Inc., Natic. July 2002, 484.