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A general theoretical formulation of deformation of steel structures exposed to fire

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Abstract. This paper presents a proposal for the formulation of a mathematical model of a fire resistance testing process for steel beams subject to bending, carried out experimentally in a test furnace. The model for the formulation was created with a balance method by describing with the fundamental laws of thermomechanics the phenomena occurring during the experiment analysed for the formulation. The viscoplastic behave our of steel at high temperatures was described with constitutive relationships proposed by Perzyna (which form the so-called "overload model"). The general mathematical model of the experiment analysed, obtained in the form of a system of differential equations, is discussed. The authors considered the impact of known physical properties of the test material and the impact of the results of the observations made during the research on the behaviour of individual terms of the model's equations. Based on these considerations, simplifications were proposed which optimized the mathematical description and facilitated its adaptation to a special case. As a result of the investigation, a system of equation was obtained which described the processes present during the fire resistance test. It is postulated here to accept the system of equations as the mathematical model.
Keywords: thermomechanical analysis, fire resistance test, viscoplastic constitutive laws, Perzyna model
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Denotation:

t — time
T — temperature
T_R — reference temperature
f_i — density of mass forces
σ_{ij} — stress tensor
σ_{ij}^D — deviatoric stress tensor

σ^{ef} — effective stress (initial yield stress)
σ_{ij}^{rs} — residual stress tensor
σ_{ij}^{ls} — non-residual stress tensor
ε_{ij} — strain tensor
$\dot{\varepsilon}_{ij}^{e}$ — elastic part of the strain rate tensor
$\dot{\varepsilon}_{ij}^{vp}$ — non-elastic part of the strain rate tensor
u_i — displacement tensor
ρ — material density
ν — Poisson ratio
E — (Young's) elastic modulus
G — (Kirchhoff's) shear modulus
E_{ijkl} — elastic stiffness tensor
λ_T — thermal conductivity coefficient
c_p — steady-pressure specific heat
α_T — linear thermal expansion coefficient
η — thermodynamic constant
χ — Taylor-Quinney coefficient
$\dot{q}_i$ — heat flux density
$\dot{q}_V$ — internal heat source capacity
$\dot{q}_{Vw}$ — phase change heat source capacity
e — internal energy density per unit mass
λ — viscosity parameter
R — isotropic hardening parameter
n — calibration parameter
$\mathbf{X}$ — kinematic hardening tensor
$\langle \bullet \rangle$ — Macaulay brackets

1. Background

The high unpredictability and multitude of scenarios by which a fire event may proceed and affect the environment (e.g. the structure of a building) caused test furnace experiments to be the most reliable method for the determination of fire endurance of structural components. In a test furnace experiment, the test component, which is a physical model of the structure, is exposed to a mechanical load and the effect of hot combustion gases. The control over heating of the test furnace facilitates development of a combustion gas temperature compliant with the fire models described with temperature-time curves, established in the standard [27]. A schematic of the test experiment is shown in Fig. 1.

During the test experiment, the test component of a structure is subject to deformation (bending variable in time) by external effects. The test experiment continues until a conventional fire resistance limit state is achieved as expressed with the standard [26] for ultimate bending parameters. Fig. 2 presents the course and the result of the test experiment.

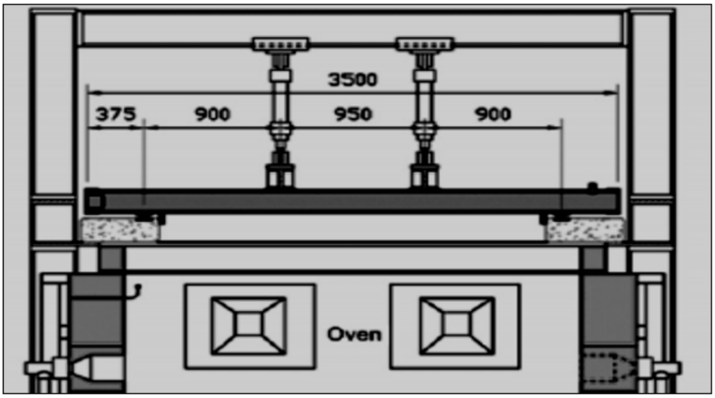

Fig. 1. Diagram of the test rig for steel beam fire resistance experiments [28]

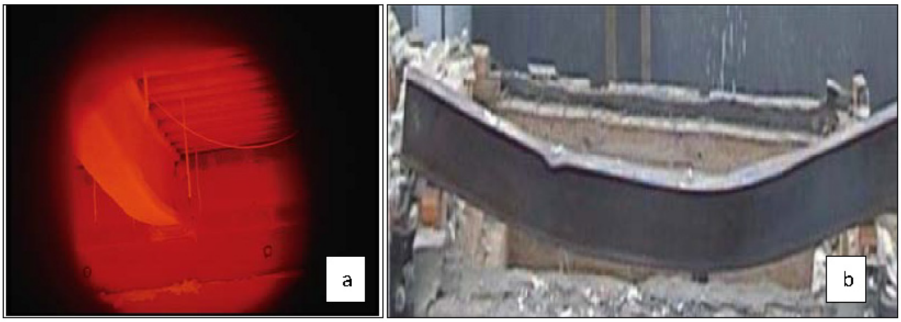

Fig. 2. Steel beam shown inside a test furnace (a) during and (b) after a test experiment [28]

The course (progression or evolution) of a process can be anticipated by testing a model of the process. Mathematical modelling is feasible for this purpose if a sufficient mathematical description of the process exists. Mathematical modelling thus combines the physical knowledge of the process with the mathematical knowledge of the subject [13].

This paper proposes a mathematical model which describes the reaction (behaviour) of structures to thermomechanical effects caused by fire. The proposed model can become an input for the formulation of an initial-boundary value problem a numerical solution of which facilitates simulation of deformation in, and determination the fire resistance of, the tested structural component.

2. Thermomechanical strain analysis of bodies

A balance method was applied to generate the mathematical model of the phenomenon considered in this work. The essence of rational modelling by application of the balance method is first to apply the principal laws of mechanics relevant to the balances of extensive values: mass, momentum, angular momentum, and energy. Next, the mathematical model must be complemented with equations or dependencies related to detailed laws of mechanics which specify what type the body is which takes part in the process described in the mathematical model. These equations and dependencies include sets of relations which describe the body (i.e. the dependencies between the characteristic constants of the material), the interactions (from external forces, temperature, etc.), and motion (the relations between displacements and strains) [2].

The reaction of a deformable body to external interactions can be described with a system of differential equations which represent the behaviour of a continuous medium.

A set of laws of thermodynamics formulated below [9] was applied for building of the system of equations required for the analysis of the problem:

— the principle of conservation of momentum (the equation of motion)

$$\sigma_{ij,j} + \rho f_i - \rho \ddot{u}_i = 0 \tag{1}$$

— the principle of conservation of angular momentum (the condition of symmetry of Cauchy's stress tensor)

$$\sigma_{ij} = \sigma_{ji} \tag{2}$$

— the first law of thermodynamics (conservation of energy)

$$\rho\dot{e} = \sigma_{ij}\dot{\varepsilon}_{ij} - \dot{q}_{i,i} + \dot{q}_V . \tag{3}$$

The equations required expansion with the following [9] to obtain the full assembly of equations which describe the mechanical phenomena occurring in the system:

— geometric relationships between strain and displacement

$$\varepsilon_{ij} = \frac{1}{2}\left[u_{i,j} + u_{j,i} - u_{k,i}u_{k,j}\right] \tag{4}$$

— constitutive relationships between the state of stress and the state of strain of the body

$$F(\sigma_{ij}, \dot{\sigma}_{ij}, \varepsilon_{ij}, \dot{\varepsilon}_{ij}, R, \mathbf{X}, T) = 0. \tag{5}$$

To solve the thermomechanical problems, this set required a complement of relations which described the laws of heat flow [17]:

— Fourier's law

$$\dot{q}_i = -\lambda_T T_{,i} \tag{6}$$

— Fourier-Kirchhoff equation

$$c_p \rho \dot{T} = -\dot{q}_{i,i} + \dot{q}_V . \tag{7}$$

The internal heat source in a body which is heated and deformed (strained) can originate from the processes of mechanical and mechanical strain and the processes concomitant to the phase changes within the body's material.

Many materials, including metals react to loads with elastic strain first (which means that the material returns to its original shape when the load is no longer applied); as the load increases or mechanical properties of the material change, the material is subject to plastic strain (deformation) (at which the removal of the load does not cause the material to return to its original state; hence, permanent deformation is present). The deformation process always changes the temperature field in the body subject to the strain; it is a macroscopic manifestation of the phenomena which occurs in the microstructure [20].

The principle of conservation of energy (3) and the heat flow laws (6) and (7), an equation of temperature evolution was derived in the form shown below [4] when the following was considered: the relationships between the internal energy and the Helmholtz free energy; and the dependencies of stress and temperature from the free energy, and without the effect of internal microfailure of the material:

$$c_p \rho \dot{T} = -\dot{q}_{i,i} + \dot{q}_{Vw} - \eta \dot{u}_{i,i} + \chi \sigma_{ij} \dot{\varepsilon}_{ij}^{vp} \tag{8}$$

The third and the fourth term on the right-hand side of the equation (8) describes the transformation of mechanical energy into heat, where the mechanical energy was generated during elastic strain and plastic strain, respectively. The elastic strain thermal energy was related to the strain caused by mechanical interactions and the thermal expansion of the material being heated [15]. The thermodynamic constant η was a material constant describing the heat caused by the strain rate at the state of stress [17].

The plastic strain thermal energy was related to the work of stress in the areas of plastic strain [24]. The conversion of work into thermal energy was only partial in the discussed case; the remainder of work was stored as the deformation energy. The Taylor-Quinney coefficient χ described the part of mechanical energy of the plastic strain which was dissipated (converted into heat).

3. Viscoplastic constitutive laws

A number of materials, especially those with a physical yield point, are sensitive to strain rate [14]. However, the effect of strain rate on the mechanical properties of steel, for example, is distinctly dependent on the material's temperature. Materials with physical yield points include low-carbon steel, a material widely used in the construction industry. The structural components used in standard fire tests are made from this steel grade. In the case contemplated here, adopting a constitutive model of a material which features elastic, plastic and viscous characteristics was justified.

The equations which describe the phenomenon of viscoplasticity generally include the laws of evolution of state variables; the state variables can be external or internal. External state variables are directly measurable and include stress, strain and temperature. Internal state variables cannot be directly observed; they are associated with structural changes in materials, including: plastic strain, kinematic or isotropic hardening, failure, etc.

The authors of [18] proposed a division of viscoplastic materials into two classes:
- elastic-viscoplastic materials: exhibit viscosity in the plastic and elastic ranges;
- elastic/viscoplastic materials: exhibit viscosity after plasticization only (according to the overload model).

Low-carbon steel conforms to the elastic/viscoplastic model; constitutive relationships typical of the overload model were adopted further in this work.

The fundamental postulate of the elastic/viscoplastic model theory when infinitesimal strain of the body is assumed is to adopt an additive decomposition of total strain rate [18]:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{\ e} + \dot{\varepsilon}_{ij}^{\ vp}. \tag{9}$$

The strain rate tensor in equation (9) is related by dependence with the stress rate according to Hooke's law [1]:

$$\sigma_{ij} = \sigma_{ij}^{rs} + E_{ijkl}\left[\varepsilon_{kl}^{\ e} - \alpha_T\,\delta_{kl}\left(T - T_R\right)\right]. \tag{10}$$

4. Perzyna's constitutive model

Many models of viscoplastic materials are featured in references for which constitutive equations are formulated to describe the behaviour of a plastic body. One of the first and most universal models is proposed by Perzyna [18]. Perzyna's constitutive model is useful for describing the viscoplasticity effect occurring at high temperatures at low or high strain velocities. The reliability proved with multiple tests, the universality and the relative simplicity of Perzyna's constitutive model, coupled with the availability of experimental test results qualified it for further consideration in the modelling of the fire resistance test process applied to steel structures.

Perzyna's constitutive model defines the evolution of strain rate in an isotropic viscoplastic material which is incompressible in its non-elastic range as follows [18]:

$$\dot{\varepsilon}_{ij}^{vp} = \frac{3}{2}\gamma(T)\left\langle \frac{\sigma^{ef}}{R(T)} - 1 \right\rangle^{n} \frac{\sigma_{ij}^{D}}{\sigma^{ef}}. \tag{11}$$

With the initial yield stress adopted following the Huber-Mises criterion, the effective stresses were defined with the following relation [8]:

$$\sigma^{ef} = \left(\frac{3}{2} \sigma_{ij}^{D} \sigma_{ij}^{D} \right)^{1/2}. \tag{12}$$

The constitutive equation in Perzyna's constitutive model does not describe the dependence between hardening or softening and temperature [23]. The dependencies for the state variables γ and R were determined by experimental testing. With the calibration parameter $n = 5$, the dependencies could be assumed in the following form for mild steels [10]:

$$R(T) = \sqrt{3} \cdot 119{,}51 \exp\left[0{,}45 \left(\frac{288}{T} - 1 \right) \right] \text{ [MPa]}, \tag{13}$$

$$\gamma(T) = \frac{\sqrt{3}}{3} \cdot 60{,}24 \left[1 + 2{,}6 \left(\frac{220 - T}{273} \right)^{2} \right] \text{ [s}^{-1}\text{]}. \tag{14}$$

5. Thermomechanical analysis of a bent steel beam: the mathematical model

To enable a computer simulation of a physical phenomenon or an industrial process, it is first necessary to develop a mathematical model being a function or a group of functions, binding different variables and describing the relationships between the quantities in the system [7].

The general mathematical model of a deformable body under thermomechanical loads can always be adapted to special conditions of the process being described. In this work it was also possible to adopt certain rational assumptions and simplifications which resulted from known material properties and the observations from experimental testing.

Condition 1: Small displacements

The displacement observed during the tests [25] was much lower than the geometrical dimensions, or the displacement gradients $\nabla_x u << 1$, of the test component. This justified the omission of the product of displacement gradients from equation (4) and the reduction of the strain measure to the dependencies relevant to infinitesimal strains:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \tag{15}$$

Condition 2: Isotropy of material

The test structural component was made from mild low-carbon steel, a polycrystalline material [6]. Polycrystalline bodies with a high grain count and devoid of a defined texture (i.e. a privileged grain orientation) could be considered to be quasi-isotropic [21]. Hence, mild low-carbon steel could be considered to be homogeneous and isotropic. This allowed a representation of Hooke's law as a Duhamel-Neumann relationship [16]:

$$\sigma_{ij} = \sigma_{ij}^{rs} + 2\mu\varepsilon_{ij}^{e} + \lambda\delta_{ij}\varepsilon_{kk}^{e} - \alpha_T\left(2\mu + 3\lambda\right)\delta_{ij}\left(T - T_R\right), \tag{16}$$

with λ and μ being Lame's constants defined as follows:

$$\lambda = \frac{\nu E}{\left(1+\nu\right)\left(1-2\nu\right)}, \quad \mu = G = \frac{E}{\nu\left(1+\nu\right)}. \tag{17}$$

General mathematical model

The considerations presented so far were concluded with the proposal for a general mathematical model to describe: (i) the thermomechanical effects on an isotropic body and (ii) the reaction of the isotropic body at a low strain. By combining equations (6) and (8) with (1), (15) and (16), the general mathematical model was reduced to this system of equations:

$$c_p\rho\dot{T} = \lambda_{T,i}T_{,ii} + \dot{q}_{Vw} - \eta\dot{u}_{i,i} + \chi\sigma_{ij}\dot{\varepsilon}_{ij}^{vp} \tag{18}$$

$$\begin{aligned} &\mu u_{i,jj} + \mu u_{j,ij} + \lambda\delta_{ij}u_{k,kj} - \alpha_T\left(3\lambda + 2\mu\right)\delta_{ij}T_{,j} = \\ &= \rho\ddot{u}_i - \rho f_i - \sigma_{,j}^{rs} - 2\mu\varepsilon_{ij,j}^{vp} + \lambda\delta_{ij}\varepsilon_{kk,j}^{vp}. \end{aligned} \tag{19}$$

The physical and thermal properties of the material (ρ, c_p, λ_T, μ, λ,α_T,) are functions of the temperature; hence, a system of non-linear coupled equations existed.

(Special) Condition 3: constant volume of the body

The constant volume of metallic solid bodies, as demonstrated in [11], under non-elastic strain at infinitely small strain caused the following:

$$\varepsilon_{kk}^{vp} = 0. \tag{20}$$

The assumed uniformity and isotropy of the material allowed Perzyna's constitutive equation to be applied in the form of (11).

(Special) Condition 4: displacement rate (quasi-static state)

The displacement (bending) increment rate observed in test [25] of the structural component was low enough it allowed an assumption that the process was quasi-static: there was a static equilibrium state at any point of time. Hence, it was possible to omit acceleration from equation (1) and construe the acceleration as equations of equilibrium [9]:

$$\sigma_{ij,j} + \rho f_i = 0. \tag{21}$$

(Special) Condition 5: elastic strain heat

The increase in the temperature of the body caused by the generation of heat related to the elastic strain rate was negligible when compared to the temperature increase from other factors; this applied only when the relative strain rate-was much lower than the relative temperature change rate [3], i.e.:

$$\dot{e}_V << \frac{1}{T_R}\dot{T}. \tag{22}$$

If the strain was only caused by the thermal expansion of the material, dependence (21) could be reduced to this condition [3]:

$$3\alpha_T T_R << 1, \tag{23}$$

The condition was met at the linear thermal expansion of carbon steel at an ambient temperature of approximately 11.9×10^{-6} K^{-1}. As for the strain caused by mechanical effects only, the application of conditions 1 and 4 provided the following:

$$\dot{e}_V \approx 0. \tag{24}$$

Hence, the effect of the thermal energy portion which originated from the elastic displacement of the body could be disregarded:

$$\eta \dot{u}_{i,i} \approx 0. \tag{25}$$

(Special) Condition 6: plastic strain heat

The Taylor-Quinney coefficient χ determined which part of plastic strain mechanical energy was dissipated and it was determined by experimental testing to be equal approx. 0.9 [22] for the steel.

High-rate deformation changed the isothermal conditions into adiabatic conditions, witnessed as an increase of the temperature of the body being deformed. Mechanical loads applied to the steel beam during the fire resistance test were constant in time. The magnitude and velocity of bending of the steel beam were limited with the assumed fire endurance limit state. The values were small enough to make the heat from the dissipation of plastic strain energy negligible, when compared to the heat input of the system from the temperature increase in the test furnace [19, 22]. Hence:

$$\chi\sigma_{ij}\dot{\varepsilon}_{ij}^{vp} \approx 0. \tag{26}$$

(Special) Condition 7: residual (internal) stresses

Steel structural components are subject to different heat and mechanical processing operations during manufacturing; the operations generate a certain state of internal stress which may exist in the material left at rest [18]. The determination of the stress tensor components requires considering the presence of the internal stresses, a.k.a. residual stresses. In general conditions the following should be assumed [12] to determine the function of plastic potential:

$$\sigma_{ij} = \sigma_{ij}^{ls} + \sigma_{ij}^{rs}. \tag{27}$$

Heating of a component which features residual stresses causes relaxation by which the residual stresses disappear. This process occurs only when the material has become plastic. Residual stresses remain unchanged in the elastic state, even if the temperature of the body is increased.

The residual stresses of flat structures calculated as bar structures can be omitted for the components subject to bending only due to the low impact on the accuracy of the mathematical model [5]. The steel beam was not compressed; hence:

$$\sigma_{ij} \approx \sigma_{ij}^{ls}. \tag{28}$$

Special mathematical model

When equations (18) and (19) included special conditions 3 to 7, described with the relationships (20), (21), (25), (26) and (28), the following system of equations was derived:

$$c_p\rho\dot{T} = \lambda_{T.i}T_{,ii} + \dot{q}_{Vw}, \tag{29}$$

$$\mu u_{i,jj} + \mu u_{j,ij} + \lambda \delta_{ij} u_{k,kj} - \alpha_T (3\lambda + 2\mu) \delta_{ij} T_{,j} = -\rho f_i - 2\mu \varepsilon^{vp}_{ij,j}, \tag{30}$$

$$\dot{\varepsilon}^{vp}_{ij} = \frac{3}{2} \gamma(T) \left\langle \frac{\sigma^{ef}}{R(T)} - 1 \right\rangle^n \frac{\sigma^D_{ij}}{\sigma^{ef}}. \tag{31}$$

The authors propose this system as a special mathematical model which describes the process of deformation in steel structures during fire resistance testing. Given the invariability of mechanical loads during the process, the special mathematical model defined the actual high-temperature creep.

The special mathematical model described with the system of equations (29) to (31) is a case in which thermomechanical couplings are unilateral: the heat flow and the temperature distribution determine the form of the equations which describe the mechanical reaction of the body. Hence, the system of thermomechanical equations can be solved alternately; the thermal (dominating) part can be solved first to solve the mechanical part.

6. Conclusion

This paper is an attempt at formulating a mathematical model which describes the processes occurring during laboratory testing of fire resistance of steel structural components. A balance method was applied to specify the group of functions which bound the variables representative of the interrelations between the variables in the process. This provided a coupled system of thermomechanical equations. The system was used to define a general mathematical model of the response of structures to simultaneous exposure to mechanical loads and high temperatures.

Further in the research, the conditions of fire tests were analysed with the measurements during an actual experimental fire test to determine if the assumptions for simplification of the model were feasible. The suppositions allowed the formulation of a special mathematical model which described the behaviour of the steel beam during a fire resistance test. The viscoplastic behaviour of the structural material was defined with Perzyna's constitutive model. By inclusion of initial and boundary conditions, an initial and boundary value problem can be formulated the numerical solution of which allows simulating the bending of the steel beam to determine its fire resistance.

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[28] Photographs and figures from available Internet resources.

A. BADOWSKI, W. DORNOWSKI

Ogólne sformułowanie teoretyczne zagadnień deformacji elementów konstrukcji stalowych w warunkach pożaru

Streszczenie. W artykule przedstawiono propozycję sformułowania modelu matematycznego procesu badania nośności ogniowej stalowych belek zginanych, przeprowadzanego w piecu ogniowym. W celu opracowania tego modelu posłużono się metodą bilansową, opisując zjawiska zachodzące w trakcie analizowanego eksperymentu za pomocą podstawowych praw termomechaniki. Zachowanie lepkoplastyczne stali w wysokich temperaturach zmiennych w czasie opisano, używając związków konstytutywnych zaproponowanych przez Perzynę (tzw. model „przeciążeniowy"). Uzyskany w postaci układu równań różniczkowych ogólny model matematyczny analizowanego eksperymentu poddano dyskusji. Rozważono wpływ znanych właściwości fizycznych materiału i wyników obserwacji dokonanych w czasie badań na zachowania poszczególnych składników równań modelu. W efekcie zaproponowano uproszczenia umożliwiające zoptymalizowanie opisu matematycznego i jego dostosowanie do rozważanego przypadku szczególnego. Jako rezultat przeprowadzonych dociekań uzyskano układ równań opisujący procesy zachodzące w badaniu ogniowym, który postuluje się przyjąć jako jego model matematyczny.

Słowa kluczowe: analiza termomechaniczna, badanie ogniowe, lepkoplastyczne prawa konstytutywne, model Perzyny

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