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The Vibrations and the Stability of a Flat Frame Type Γ Realizing the Euler's Load Taking Into Account the Vulnerability of the Structural Node Connecting the Pole and the Bolt of the System

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Abstract

The paper presents the results of theoretical research and numerical calculations of the vibration and the stability of a twin rod flat frame subjected to the Euler's load. Considering the total mechanical energy of the system and using the kinematic stability criterion (Hamilton's principle) is determined by the equations of motion and boundary conditions considered system. The results of numerical calculations are presented at selected geometrical and physical parameters in the system for selected values of the rotary spring stiffness modeling the structural rigidity of the node connecting bolt with the column of frame.

Keywords: flat frame, free vibrations, Euler's load

1. Introduction

In the scientific literature concerning the stability of slender elastic systems stands out conservative and non-conservative load. Euler load and a force directed to the pole are classified as a conservative load type [1]. However Beck's generalized load [2] and Reut's load [3] are the cases of non-conservative load. Euler's load is a load by the longitudinal force and have a fixed anchor point and the direction who does not change during buckling.

In the case the conservative load there are also the system realizing a specific load [4]. The cases of this load formulated by L. Tomski [4, 5] combine the features of generalized load [6] or tracking load [2] and the load with force directed to the pole [7].

The flat frames are classified as open or closed. At the ends of closed frame system [11, 12] has been installed the support or heads which are carrying the load. In case when one of the ends of the system is free this system is called an open frame [8]. Most of scientific publications are considered a simple frames type **Γ** who have got the form of angle [8], three-rod type **T** [10, 15] and portal systems which is built of several simple framework [14]. In many scientific papers many of the theoretical and numerical research of framework due to the type of system load and the criteria of loss of stability had been drawned. In paper were presented the range of variation of the natural frequencies of system as a function of the external force [9] and the changes in the value of the critical load [17] for the selected parameters of carrying load heads. In the studied issues of stability of flat frame also had been considered the initial inaccuracies of systems in the form of an eccentric load application [15] the elasticity of structural components (translational and rotational springs) for the method of connection the pole and bolt frame [16] or fixing these elements in the supports [8,12]. Shows the results of analysis of the influence of geometric imperfections in the form of right angle to the stability of the flat frames.

In this paper had been studied the impact of structural node connecting the bolt and the column to its own vibration and stability of twin rod closed flat frame type **Γ** treated Euler's load. Based on the kinetic stability criterion determined the equation of motion and the boundary conditions necessary to solve the boundary value problem. Taking into account the adopted geometric and physical parameters of the system the results theoretical and numerical calculations had been showed.

2. The physical model

Figure 1 shows the diagram of a flat frame type **Γ** subjected to the Euler's load.

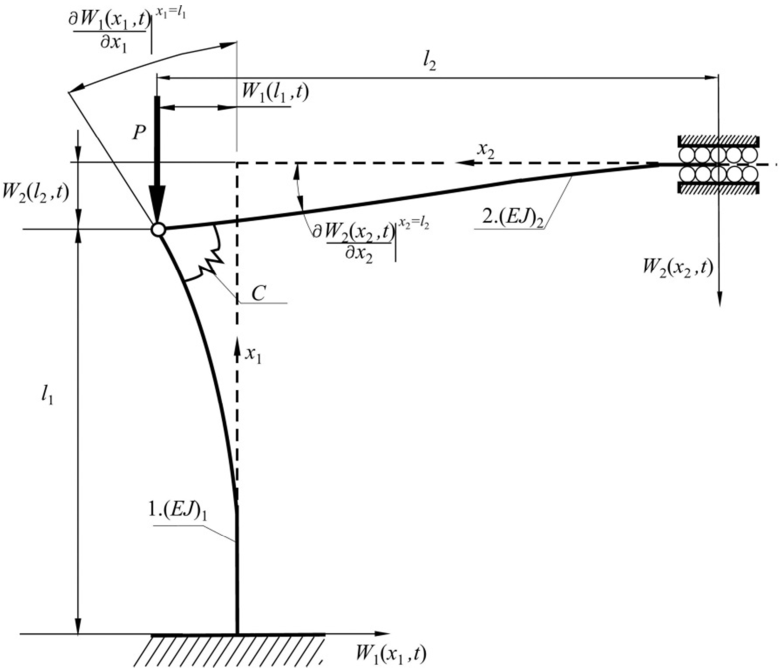

Figure 1. The physical model of frame type **Γ** subjected to Euler's load

The frame bolt of flexural rigidity $(EJ)_2$ was fixed rigidly but there is a possibility to longitudinal displacement however the pole of flexural rigidity $(EJ)_1$ was fixed rigidly without a possibility to longitudinal displacement. Both of them are connected by using

C - rigid spring. In the considered load case the pole of frame was charged by conservative force P which the direction of action passes through the pole and bolt connection.

3. Mechanical energy of systems, the equations of motion, boundary conditions

The kinetic energy T of contemplated flat frame is the sum of the kinetic energy of its individual bars:

$$T=\sum_{i=1}^{2}\frac{(\rho A)_i}{2}\int_0^{l_i}\left[\frac{\partial W_i(x_i,t)}{\partial t}\right]^2 dx_i \tag{1}$$

The V- potential energy recording takes into account the elasticity of bending of the individual rods the direction of the external load and susceptibility structural node of flatframe (C-spring stiffness):

$$V=\sum_{i=1}^{2}\frac{(EJ)_i}{2}\int_0^{l_i}\left[\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2}\right]^2 dx_i-\frac{P}{2}\int_0^{l_1}\left[\frac{\partial W_1(x_1,t)}{\partial x_1}\right]^2 dx$$
$$+\frac{1}{2}C\left[\left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|_{x_1=l_1}-\left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|_{x_2=l_2}\right]^2 \tag{2}$$

Considering the total mechanical energy of the system defined by (1), (2), the equation of motion and the boundary conditions of a frame were determined using principle of Hamilton [14]:

$$\delta\int_{t_1}^{t_2}(T-V)dt=0 \tag{3}$$

The equation of motion:

$$(EJ)_1\frac{\partial^4 W_1(x_1,t)}{\partial x_1^4}+P\frac{\partial^2 W_1(x_1,t)}{\partial x_1^2}+(\rho A)_1\frac{\partial^2 W_1(x_1,t)}{\partial t^2}=0$$
$$(EJ)_2\frac{\partial^4 W_2(x_2,t)}{\partial x_2^4}+(\rho A)_2\frac{\partial^2 W_1(x_1,t)}{\partial t^2}=0 \tag{4a,b}$$

Geometrical boundary conditions:

$$W_1(0,t)=W_2(0,t)=0$$
$$\left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|^{x_1=0}=\left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|^{x_2=0}=0 \tag{5a-d}$$

Natural boundary conditions:

$$\left.\frac{\partial^3 W_1(x_1,t)}{\partial x_1^3}\right|^{x_1=l_1} + \frac{P}{(EJ)_1}\left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|^{x_1=l_1} = 0$$

$$\left.\frac{\partial^3 W_2(x_2,t)}{\partial x_2^3}\right|^{x_2=l_2} = 0$$

(6a-d)

$$\left.\frac{\partial^2 W_1(x_1,t)}{\partial x_1^2}\right|_{x_1=0} + C_r\left[\left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|_{x_1=l_1} - \left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|_{x_2=l_2}\right] = 0$$

$$\left.\frac{\partial^2 W_2(x_2,t)}{\partial x_2^2}\right|_{x_2=0} + \frac{C_r}{\mu}\left[\left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|_{x_2=l_2} - \left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|_{x_1=l_1}\right] = 0$$

in which:

$$\mu = \frac{(EJ)_2}{(EJ)_1}, \quad C_r = \frac{C}{(EJ)_1} \qquad 7(a,b)$$

4. The results of numerical calculations

In this part of the paper the results of numerical calculations was presented. They were made on the basis of the solution of boundary value problem, while taking a constant flexural rigidity of the frame $(EJ)_1 + (EJ)_2 = const$ and a fixed sum of lengths of the bars of the $l_1 + l_2 = const$. The results were presented using the following dimensionless size:

$$\lambda^* = \frac{P_{kr}\,(l_1+l_2)^2}{(EJ)_1+(EJ)_2}, \quad \mu = \frac{(EJ)_2}{(EJ)_1}, \quad \phi = \frac{l_2}{l_1},$$

$$c_{1r}^* = \frac{C(l_1+l_2)}{(EJ)_1+(EJ)_2}, \quad \Omega^* = \frac{\left[(\rho A)_1+(\rho A)_2\right]\omega^2(l_1+l_2)^4}{(EJ)_1+(EJ)_2} \qquad (8a\text{-}e)$$

The results of numerical simulations concerning the course of changes in the critical load parameter λ^* as a function of the parameter μ was presented in relation to the parameter of elasticity of the structural node c^* (Fig. 2). Taking into account a variable value of the parameter μ and maximum value of critical parameter of load λ^* obtained with the rigid connection of the column and the bolt of frame ($1/c^* = 0$). The nature of the presented curves mainly due to the assumed condition of constant bending stiffness of the system.

Figure 3 shows the sequence of changes in the critical load parameter λ^* as a function of the parameter of elasticity of the structural node c^*. The results are shown for various asymmetry value of the bending stiffness of the column and the bolt of frame μ. In any case, you can determine the value of c^* above which the value of the critical load is only slightly modified. The graph curves 1.a - 4.a presents the stabilization of critical load λ^* with increasing rigidity of structural node. This occurs irrespective of the value of the asymmetry factor bending stiffness of the column and the bolt frame μ.

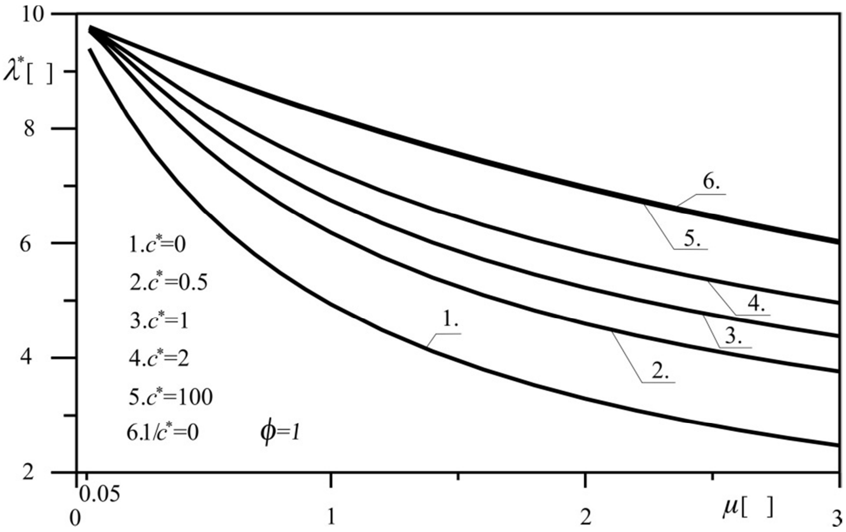

Figure 2. change the critical load parameter λ^* as a function of the parameter μ

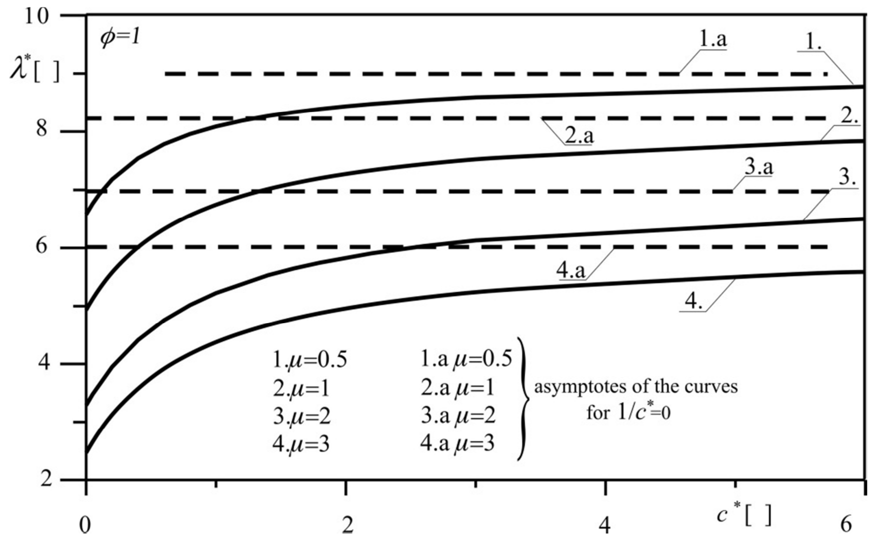

Figure 3. Change of the critical parameter of load λ^* as a function of c^* parameter

Figure 4 presents the results of numerical calculations for the free vibration of the frame. Illustrated are the relationship of the dimensionless external load parameter λ^* as a function of the dimensionless parameter frequency of free vibrations Ω^*. In terms of numerical calculations the nature of changes in the value of the first two fundamental natural frequencies Ω_1^*, Ω_2^* was determined. Constant asymmetry value of the bending stiffness of the column and lock the frame and the constant μ asymmetry value of the length of the bolt to the length of the pole frame φ was assumed. In the case of presented the course of changes in frequency of free vibrations, the value of the critical load λ^* obtained with the parameter frequency of free vibrations $\Omega_1^* = 0$ The results obtained parameter values of the critical load obtained on the basis of the kinetic stability criterion are the same as when using the static stability criterion.

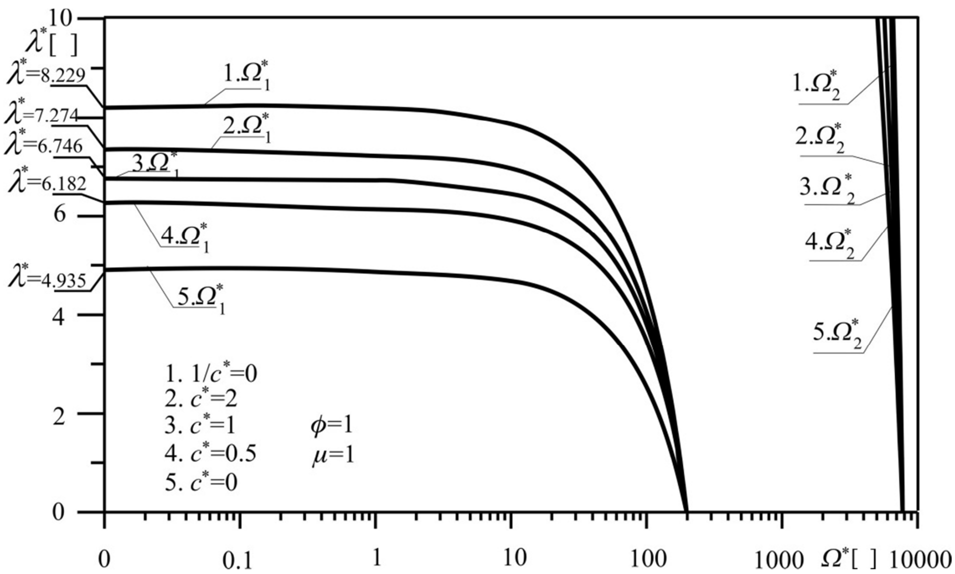

Figure 4. The curves in the plane: load parameter λ^*, the parameter resonance frequency Ω^* for a variable elasticity of connecting the pole with bolt of frame c^*

5. Conclusions

This paper presents the results of theoretical studies and numerical calculations on the twin rod flat frame type **Γ** vibration subjected to Euler's load. Taken into account the total mechanical energy of the system and based on kinematic stability criterion determined the equations of motion and boundary conditions considered system. Numerical calculations were performed at different valuesof the parameters under consideration, which include the asymmetry coefficient μ bending stiffness and rigidity of the structural node c^* connecting the pole with bolt frame. Taking into account the structural rigidity of the node connecting bolt to the column increases the critical load. The diagram changes in frequency of free vibrations corresponds to systems with a load of slender conservative (divergent type system).

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