

## GROWTH RATE AND CYCLICAL FLUCTUATIONS IN TRANSPORT. A SIMPLE MODEL

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**Abstract:** The paper contains a preliminary study of the transport market dynamics model, including its cyclical fluctuations. An additional goal is to enable a quantitative examination of the impact of transport congestion on the economic development and the effects of underinvestment in transport.

**Paper type:** Research Paper

**Published online:** 19 October 2018  
Vol. 8, No. 4, pp. 339–349  
DOI: 10.21008/j.2083-4950.2018.8.4.5

ISSN 2083-4942 (Print)  
ISSN 2083-4950 (Online)  
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**Keywords:** *Transport, Dynamics, Cyclical Fluctuations*

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## 1. INTRODUCTION

In this paper, we propose a model of growth and cyclical fluctuations of the freight transport market. Regarding the fact, that the transport activity accompanies most economic processes, it seems justified that the constructed model should include the whole of the real economy sphere with emphasis on its relations with the transport sector.

The presented construction refers to the general principles characterizing the family of dynamic stochastic models of general equilibrium. The basis of these models is the theory of the real business cycle, according to which the source of fluctuations in the economies is the occurrence of random disturbances leading to changes in production efficiency, the size of costs or preferences of economic entities and, consequently, to fluctuations in the level of production (Kydland & Prescott, 1982), (King & Rebelo, 1999), (Smets & Wouters, 2003), (Christiano, Eichenbaum & Evans, 2005). According to this methodology, we will assume that the economy and its individual elements adapt in an optimal way to these external shocks. As a result, not so much the smooth trajectories of the global product of the economy and its various components are observed, but the repeated fluctuations of those variables. This mechanism applies in a natural way to fluctuations of the volume of transport production. Sufficiently transparent microeconomic foundations of such kind of models allow to hope that they may be an additional source of forecasts for the transport sector (see Burniewicz, 2006).

The model refers to the economic system located in a number of spatially separated regions. The share of the transport sector is endogenous, as its production results from the volume of exchange of goods and services, which is a consequence of the assumptions made regarding the functioning of market entities. By the region in the model we mean the place where the considered types of entities (households and producers, including producers of transport services) are settled and where the economic activity is carried out. Without regarding the spatial structure of production within the region itself, it is obvious that the minimal structure for modelling the spatial flows of goods and services is the system consisted of two regions.

On a micro scale, these regions may have infinitesimal small area; in the macro scale they may correspond for example, the national economy and "the rest of the world". From a practical point of view, the most important thing seems to be the availability of data; their available resources allow to create models on a scale from voivodeships to countries or their groups.

As the way of construction of the models belonging to DSGE class seems to have been well-established since the paper of (Kydland & Prescott, 1982), in our remarks the emphasis has been placed on the formulation of microeconomic foundations for transport and its relation with the rest of the economy.

## 2. FORMULATION OF THE MODEL

We will consider a closed economic system consisting of two regions with a transport network with a number of connections enabling physical exchange of goods (Consideration only two regions does not cause the loss of generality. Moreover, the introduction of more regions to the model is immediate, however, resulting in increased complexity of formulas, it does not introduce any special new quality for the analysis). The production sphere of each region's economy is divided into sectors: the first one – conventionally referred to as industrial – and the second transport. The industrial sectors of both regions produce a large number of simple goods, from which, as a result of aggregation, a single complex good is created and then consumed. The part of the industrial production of a given region is, along with imported goods, consumed on-site, the remaining – exported to the neighbouring region. The carriers play an obvious role – they are responsible for transport and setting the shipping rates and, consequently, they play the most important role in shaping price relations in both regions.

The model consists of parts describing the functioning of individual categories of market entities: households, producers of industrial goods and carriers.

### 2.1. Households – consumers

The preferences, income level and structure of consumer spending determine the demand side of the industrial production sector. Modelling it in each of the regions (for simplicity we omit the region index) we will consider the case of a homogeneous population of consumers whose preferences are described by separable utility functions dependent on: consumption path  $C = (C_t)_{t=0}^{\infty}$  and supply of labour  $L = (L_t)_{t=0}^{\infty}$ . In the current period ( $t = 0$ ) utility has the following form:

$$U(C, L) = E [\sum_{t=0}^{\infty} \beta^t u(C_t, L_t)].$$

The level of consumption refers to a complex good constituting an aggregate of simple goods (intermediate goods numbered with the index  $i \in [0,1]$ ). Assuming a constant elasticity of substitution between them, we have (after the appropriate selection of units) To simplify the notation, we omit both the range of integration  $([0, 1])$  and the measure (Lebesgue,  $di$ ):

$$C_t = \left( \int_{[0,1]} c_t(i)^\rho di \right)^{1/\rho} = \left( \int c_t^\rho \right)^{1/\rho},$$

where  $0 < p < 1$  is constant. The elasticity of substitution of simple goods is in this situation equal to  $1/(1 - p)$ . The temporal utility  $u$  is increasing function with respect to its first argument and decreasing relative to the second.

The volume of consumer demand and labour supply ( $L^s$ ) is determined by solving the maximization problem (1) subject to a budget constraint. The result determines the relationship between the aggregate consumption demand and its components – this dependence is determined at every moment by the solution of the problem:

$$\int c_t^\rho \rightarrow \max,$$

where the maximum is calculated for given consumer expenditures ( $e_t$ ):

$$\int p_t c_t = e_t.$$

A simple calculation shows that for almost all (relative to Lebesgue measure)  $i \in [0,1]$  and all  $t = 0, 1, \dots$

$$c_t = c_t(i) = e_t P_t^{-\sigma} p_t(i)^{\sigma-1},$$

where  $\sigma = \rho/(1 - \rho)$ ,  $p_t(i)$  is the price of the partial  $i$ , and  $P_t$  denotes the price index:

$$P_t = \left( \int p_t^\sigma \right)^{1/\sigma}.$$

Consumption of goods includes goods produced on-site and imported. The producers' prices  $p_t(i)$  of imported goods include the cost of transport (with an additive or multiplicative form  $p_t(i) + k_t(i)$  or  $p_t(i)\tau_t(i)$  respectively). The coefficient specifies the amount of cargo that must be sent from the output region to guarantee that the unit of cargo reaches to the target region.  $\tau$  defines the technological aspects of transport, therefore it depends not only on the mode of transport used and the route chosen, but on all logistical aspects. The size of  $\tau$  ( $\tau \geq 1$ ) can be understood as material costs arising in the whole of the transport process. This is related to the Dixit's and Stiglitz's (Dixit & Stiglitz, 1977) concept of transport costs: the transport process absorbs  $1 - \tau$  units of good, which is a measure of transport costs expressed in physical units (a classic example is the transport of petroleum during which some petroleum is consumed. The size of is the unit cost of transport, i.e. the cost of delivering a unit cargo to the target region. In this work, it is more convenient to use this last approach. We will return to these issues when considering the transport services market.

The problem describing the household decision mechanism (and determining consumer demand and labour supply) has the form of maximizing the expression (1) with the budget constraint (for all  $t$ ):

$$\int_{I_1} p_t c_t + \int_{I_2} (p_t + \kappa_t) c_t + Inw_t + \text{other expenditures in the period } t \leq \text{total income up to the period } t,$$

$$c_t \geq 0,$$

the sets  $I_1, I_2$  refer respectively to goods manufactured in the region under consideration and imported to it. Each of the conditions (7) can be formulated using the aggregate consumption  $C_t^d, C_t^m$  of locally produced and imported goods and their price indices  $P_t^d, P_t^m$  see formula (6). Namely, for every  $t = 0, 1, \dots$  the following inequality occurs:

$$P_t^d C_t^d + P_t^m C_t^m + Inw_t + \text{other expenditures in the period } t \leq \text{total income up to the period } t.$$

The first two components in (7) are expenditures on consumer goods produced on-site and imported respectively. Next component ( $Inw_t$ ) denotes the investments that increase capital resources. The rest items of the left hand side correspond to the expenditures on purchase of a portfolio of financial instruments, insurance, etc. From the point of view of the basic purpose of the model, the detailed specification of these item does not appear necessary.

Sources of household income are as follows:

- the salary,  $w_t L_t^d$ , where  $w_t$  means unit pay and  $L_t^d$  – demand for work in the period  $[t, t + 1)$ ; Thus,  $1 - L_t^s$  is the amount of households' leisure time in the period  $[t, t + 1)$ ;
- the ownership of capital,  $r_t K_t^d$ , where  $K_t^d$  is the size of the capital involved in the production process,  $r_t$  – unit cost of capital;
- the ownership of companies that produce partial goods; the paid profit is an aggregate  $\Pi_t = \int \pi_t(i)$ ;
- other sources,  $\Pi_t^r$ .

The equation of dynamics of capital is stated by a standard mechanism: the size of investment ( $Inw$ ) excluding replacement investments and costs of capital installation increases the value of capital:

$$K_{t+1} = K_t + Inw_t - \delta K_t,$$

where  $\delta$  denotes the depreciation rate of capital.

## 2.2. Entrepreneurs

Modelling the sphere of industrial production, we will use the concept of monopolistic competition. According to this assumption production will be taken place in a two-phase manner. In the first phase, producers produce partial goods, which are then aggregated (under conditions of perfect competition), becoming the object of consumption.

The volume of production of the partial good  $i \in [0,1]$  is determined by its production function,  $y(i) = f(k(i), l(i))$ , where  $k(i)$ ,  $l(i)$ , denote the amount of capital and labour used. In the next stage, this production is divided into the part used on local market ( $d$ ) and the exported ( $e$ ) to other region:

$$y(i) = d(i) + e(i).$$

The producer, like any (quasi)monopolist can quite freely determine the price of his production setting the price for his product to obtain maximum expected profit. The mechanisms of such setting price can be different of course. In current model we will use a concept of bounded rationality. Namely, we will assume that the producers are procedurally rational, that is, perform a certain procedure (eg. compatible with the Calvo scheme, cf. (Calvo, 1983)) and setting prices as optimal from the point of view of the results of performed procedure.

We assume that the each producer maximizes his possible profit. It expresses the expected value of discounted (for a given period  $t$ ) future profits under the conditions determined by the used procedure. The component of this profit for the period  $s \geq t$  arises from the income  $p_s(i)y_s(i)$  decreased by the costs of rented capital and labour,  $w_s l_s(i) + r_s k_s(i)$ . Thus, the producer's decision criterion is

$$\pi_t(i) = E_t \left[ \sum_{s=t}^{\infty} \gamma_s (p_s y_s(i) - w_s l_s(i) + r_s k_s(i)) \right].$$

After integrating the last expression on an interval  $l = [0, 1]$ , we get a total profit that is part of the household income.

The aggregation of partial goods takes place in a market with perfect competition. The components of this aggregation are: production of  $d = d(i)$  for the local market and imported products  $m = m(i)$ ; mechanism analogous to (2). The result is a complex good in the amount of  $(\int (d^\rho + m^\rho))^{1/\rho}$  with the price determined by the prices of goods produced on the local market and, including transport costs, the prices of imported goods (see the formula (6)).

### 2.3. Carriers

In many countries, also in Poland, car transport of cargo is the dominant branch of transport. According to Central Statistical Office (GUS), the volume of transport in 2016. amounted to 1 836 652 thous. tons, of which 1 546 572 thou. tonnes (over 84%) belonged to road transport. A similar share (almost 79%) is recorded in the case of transport performance (tonne–kilometre). This market is strongly decentralized. Providers of transport services are a large number of relatively small transport companies. Burnewicz (2006) estimated their number at over 80,000. Other authors (Bentkowska-Senator & Kordel, 2007) based on various sources estimated it at around 51 thousand domestic transport companies and approximately 44 thousand companies performing international transport.

According to Eurostat database, in 2012 a total of 8,1893 transport companies were registered in Poland, of which 78 466 were employed up to 5 employees (95.8%), 1915 companies employed from 6 to 9 employees (2.3%), 2005 companies had from 10 up to 19 employees (less than 2.5%), while others 1512 (altogether slightly above 1.8%) employ more than 19 people. Despite constant changes (new registrations, bankruptcies and deregistration for other reasons), it seems that the total number remains at a more or less the same level. The natural consequence of these facts, given the large (over 2 million) fleet of trucks, is relatively small transport potential per transport company. In 2015, the number of companies employing from 9 persons and the fleet up to 5 lorries was 277 entities, 2442 companies had from 6 to 19 vehicles, while the remaining ones had up to 99 units, 1172 enterprises, 98 companies had a fleet of at least 100 vehicles. The most part of the population of carriers has the fleet concerned with the trucks with not very large carrying capacity. These premises lead to the conclusion that at the present time, the transport services market in Poland can be considered (almost) perfectly competitive.

Let us move on to a slightly more formal description of the transport services market. Obviously, this market provides the possibility of interregional exchange of goods – for simplicity, we omit transport within regions.

The main variables affecting the volume of transport production and its costs are in the first place: distance, working time, amount of cargo and transport capacity (Bentkowska-Senator, Kordel & Waśkiewicz, 2011; Tarski, 1976).

Assuming that the main factors of production in transport are capital and labour and considering that they are with a good approximation complementary, it is quite easy to obtain the function of production of transport services.

Multiplying the number of service employees (drivers) by the average unit time of their work, we will get the size of the labour factor ( $l^r$ ). Dividing it by the average time of a single course (passage with loading and unloading) and taking into account the average effective carrying capacity of the vehicle, we get the upper limit for the amount of transported cargo in a period  $[t, t + 1)$ . The second limitation of production comes from the boundedness of transport potential:

$$\frac{(\text{numbers of vehicles}) \cdot (\text{average effective capacity})}{(\text{average travel time})}$$

Assuming that the expression in the numerator sufficiently correctly describing the capital level used in the transport sector, we can write the production function as

$$f^{tr}(k^{tr}, l^{tr}) = \min(\tau_k k^{tr}, \tau_l l^{tr}).$$

The nonnegative coefficients  $\tau_k, \tau_l$  depend on the average frequency of travels. They are dependent on the speed of vehicle and the time of loading work, consequently they are strongly dependent on the level of transport congestion.

On the other hand, the volume of transport to the considered region is limited by the volume of imports ( $m$ ). Finally, the volume of transport is equal

$$\min(m\tau, \tau_k k^{tr}, \tau_l l^{tr}),$$

where  $\tau \geq 1$  is the coefficient determining the unit consumption of good during the transport. Formula (11) directly shows that provided a sufficiently large supply of labour and capital ( $k^{tr}, l^{tr}$ ) the cost of transport is equal

$$(rk^{tr}/\tau_k + wl^{tr}/\tau_l)m\tau.$$

The last two formulas refer to transport of each of the partial goods. The total amount of transport is the result of integration (11) over the set  $[0, 1]$  of all partial goods:

$$\int \min(m\tau, \tau_k k^{tr}, \tau_l l^{tr}).$$

Similarly, based on (12), one can determine the total cost of transport.

We assume that each carrier maximize his expected profit. The adoption of a mechanism of perfect competition on the transport market and the absence of enter/exit costs imply that the maximum profit is zero. The shipping rates,  $\kappa$ , determines the equation:

$$\int (\kappa \min(m\tau, \tau_k k^{tr}, \tau_l l^{tr})m\tau - (rk^{tr}/\tau_k + wl^{tr}/\tau_l)m\tau) - Inw^{tr} = 0,$$

where  $Inw^{tr}$  denotes the value of investments in the transport sector.

It is worth to emphasize that all factors having any impact on the transport cost also affect the volume of inter-regional exchange, production volume and the global



product of the economy. In particular, this effect is visible in the case of under-investment of transport when a value of capital  $k^{tr}$  is small (see formula (13)).

The rate of capital accumulation for transport sector is determined by the value of investments ( $Inw_t^{tr}$ ) and depreciation rate ( $\delta^{tr}$ ) in an analogous way as in equation (9):

$$k_{t+1}^{tr} = (1 - \delta^{tr})k_t^{tr} + Inw_t^{tr}.$$

At the end of this section, we briefly discuss the phenomenon of traffic congestion. In the first approximation, it can be assumed that the values of  $\tau_k, \tau_l$  in the formula (11) are fixed. However, especially when the traffic volume become high, one can observe noticeably faster than the linear (with respect to  $m$ ) increase in transport costs. The parameters of the cost and production functions are not constant, but depend on the volume of flows in transport network (equivalently imports  $m$  to the region). For a small inter-regional exchange, and therefore a small amount of traffic, it can be assumed that  $\tau_k, \tau_l$  are affine functions of  $m$ . In the case of congestion with a higher intensity, it may be advisable to use exponential functions or – in models with limited network capacity – formulas of the type

$$\tau_k, \tau_l \sim (1 - m/\lambda)^{-1}, \text{ where } \lambda > 0, \text{ and } 0 < m < \lambda.$$

#### 2.4. The equilibrium states in this model

The equilibrium in the considered model is the multifaceted notion. It covers the behaviour of business entities in different time horizons. Its short-term perspective concerns issues related to the process of adjusting both prices and costs, including prices of manufactured goods as well as transport costs.

The components of expenses and income are also subject to direct, relatively quick adjustment processes. Their relations are determined by consumer's preferences and the structure of production costs (including cost of transport). Summing up and formalize the last statements, in the short-term equilibrium the following conditions have to be satisfied in any period  $t$ :

- Consumers' income and expenses are balanced – conditions (7) are binding
- The balance of the labour market – the labour supply ( $L^s$  in model of households) is equal to the demand for labour (from both sectors: industrial and transport)  $L_t^s = \int (l_t^d + l_t^{d,tr})$ .
- The balance of the capital market (demand for capital equal its supply) analogously as in the previous case.

### 3. CONCLUSION

This paper attempts to propose a model belonging to models of DSGE-class, which is perhaps worth to be subjected to calibration and verification processes in subsequent stages. It also seems that such a way of including the interaction of transport with manufacturing sectors may be a natural starting point for the construction of a more advanced models of growth and cyclical fluctuations in transportation sector. It seems, however, that such a model, in order to better capture the long-term mechanisms responsible for cyclical fluctuations, in addition to changes in the productivity of the industrial and transport sectors, should take into account the processes of migration of production factors and hence changes in the spatial distribution of production factors.

It should therefore be expected that a more complete description of growth and cyclical fluctuation of transport market should contain the following additional elements:

- A model describing spatial distribution of capital flow as a function of the level of profit achieved by producers. In the simplest version, this means that as long as the expected profits from production in another region are sufficiently high compared to the current profits together with the cost of transfer of production activities, there are natural incentives to change the location of production.
- The model of the mobility on labour market. In its classical formulation, makes the intensity of migration dependent on the difference in standard of living (for example measured by the maximum values of consumers' utility in both regions).

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