

THE JOURNAL BIULETYN OF POLISH SOCIETY

FOR GEOMETRY AND ENGINEERING GRAPHICS



POLSKIEGO TOWARZYSTWA
GEOMETRII I GRAFIKI INŻYNIERSKIEJ

VOLUME 31 / DECEMBER 2018

**THE JOURNAL
OF POLISH SOCIETY
FOR GEOMETRY AND
ENGINEERING GRAPHICS**

VOLUME 31

Gliwice, December 2018

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Editorial office address:

44-100 Gliwice, ul. Krzywoustego 7, POLAND
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ISSN 1644 - 9363

Publication date: December 2018 Circulation: 100 issues.

Retail price: 15 PLN (4 EU)

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GEOMETRIC MODELLING OF CONJUGATE RULED SURFACES WITH USING THE KINEMATIC SCREW DIAGRAM

Oksana NIKITENKO, Ivan KERNYTSKYI

Warsaw University of Life Sciences – SGGW
Department of Construction and Environmental Engineering,
ul. Nowoursynowska 166, Warszawa, POLAND
oksana_nikitenko@sggw.pl, ivan_kernytskyi@sggw.pl

Abstract. In gears with skew axes the theoretical initial or auxiliary surfaces are one-sheeted hyperboloids. Their individual parts that adjoin straight lines can be used to design various gears. One of the methods of geometric modeling of the conjugate initial surfaces of spatial gears is the method of applying of the kinematic screw diagram (KSD) proposed by A.F. Nikolaev in 1950. A number of problems based on Nikolaev's theory are presented and geometric models of conjugate ruled surfaces are developed with the using of KSD in this article. The surfaces of a one-sheeted hyperboloid and a convolute (or undevelopable) helicoid were taken for the analytical research. The conducted research proves graphically the proposed Nikolaev's theory and allows to design new gears train with spatial gearing.

Keywords: kinematic screw diagram, conjugate initial surfaces, one-sheeted hyperboloid, convolute helicoid

1 Introduction

Geometric modeling of conjugate ruled surfaces is an actual and perspective direction in applied geometry. Its main task is to create geometric, mathematical and computer models that can become a theoretical basis for the development of new types of gearing, tools, machines and processes.

2 History

The theory of plane gearing was developed by Euler in 1776. The founder of the theory of spatial gearing is the professor of descriptive geometry Theodore Olivier, who considered the formation of conjugate surfaces as the process of bending spatial forms and introduced the notion of an auxiliary (instrumental) surface. T. Olivier conducted analytical research using the methods of descriptive geometry and graphically performed all the constructions. Olivier accepted the fact that the theory of gearing is only the subject of the research of performance the geometry. Applied embodiment of the ideas of T. Olivier was carried out in 1886 by the Russian scientist mathematician and mechanic H.I. Gohman [1]. Using analytical methods, he transformed the theory of gearing into the field of analytical and differential geometry, eliminating the limitations imposed by applied geometry. One of the methods of geometric modeling of conjugated surfaces of spatial gearing is the using of the kinematic screw diagram (KSD), which was proposed by the mechanics Altman and Cormac in 1937, when they were researching spatial motion. Their work was continued and developed by Professor A.F. Nikolaev in 1950. He significantly expanded the circle of tasks associated with the motion around the crossing axes and what is he suggested using this diagram to construct conjugate ruled surfaces with a line tangency [2]. However being a mechanic he confined himself only to theoretical assumptions and did not develop this theory further.

3 Formulation of problem

At the present seems relevant and expedient the task to test Nikolaev's theory and to develop geometric models of conjugate ruled surfaces using KSD. An important conclusion in solving such problems is that only the conjugate surfaces constructed with the help of computer technologies can accurately show the zones of their touch or intersection. For our researching we take the surfaces of a one-sheeted hyperboloid and a convolute (or undevelopable) helicoid.

4 Methodology

The essence of the method developed by the authors is that by constructing the KSD with the given initial parameters of the spatial pair (the distance and the angle between the axes of the conjugate surfaces) and protraction one of the given parameters of the initial surface (the radius of the neck or the slope angle of the generator to the axis), one can determine the geometric parameters for another projected conjugate surface.

We consider the simplest and "classical" example – the conjugation of two hyperboloids. Let the distance $\Delta = O_1O_2$ between the axes i and j , crossing angle φ and the parameter of one of the conjugate surfaces – radius of neck a (Fig. 1). We construct the triangle AOB with base $AB = \Delta$ and angle $\angle AOB = \varphi$. We describe a circle around the triangle. Protracting the distance $AC = a$, dropping the perpendicular CK on a circle and connecting the point K with the vertex O , we obtain the diagram itself and the required parameters – the radius of the neck of the conjugate surface $b = \Delta - a$ and the slope angles of the generators of both surfaces to their axes $\alpha = \angle AOK$ and $\beta = \angle BOK$ (Fig. 2).

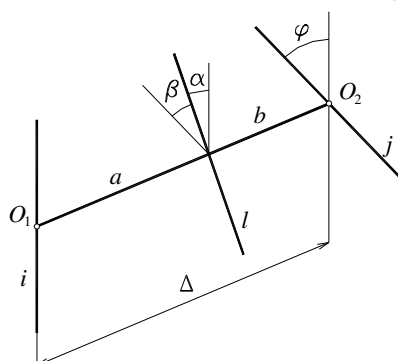


Figure 1: Position of the axes in the space of the conjugate surfaces

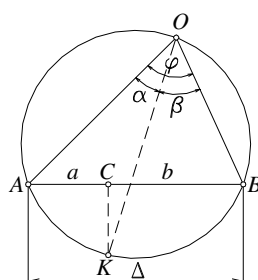


Figure 2: The kinematic screw diagram (KSD) for conjugate hyperboloids

We take $\Delta = 94$ mm, $\varphi = 71^\circ$ and $a = 38$ mm. After constructing KSD, we determine the missing parameters: $b = 56$ mm, $\alpha = 30^\circ$ and $\beta = 41^\circ$ (these parameters completely correspond to the diagram in Fig. 2). The obtained conjugate hyperboloids are shown in Fig. 3. We also add that the line of tangency of two surfaces in the process of their rotation makes a screw motion - in the diagram (see Fig. 2) the segment CK represents the pitch of the

helical surface. This screw motion occurs around the given axes, so we get two more conjugate convolute helicoids (Fig. 4). These helicoids will be conjugated to each other and associated with the corresponding hyperboloids (Fig. 5). Figures 3, 4, 5 show that four surfaces can participate in conjugation – two real and two imaginary surfaces, since they arise only during the rotation of two real surfaces.

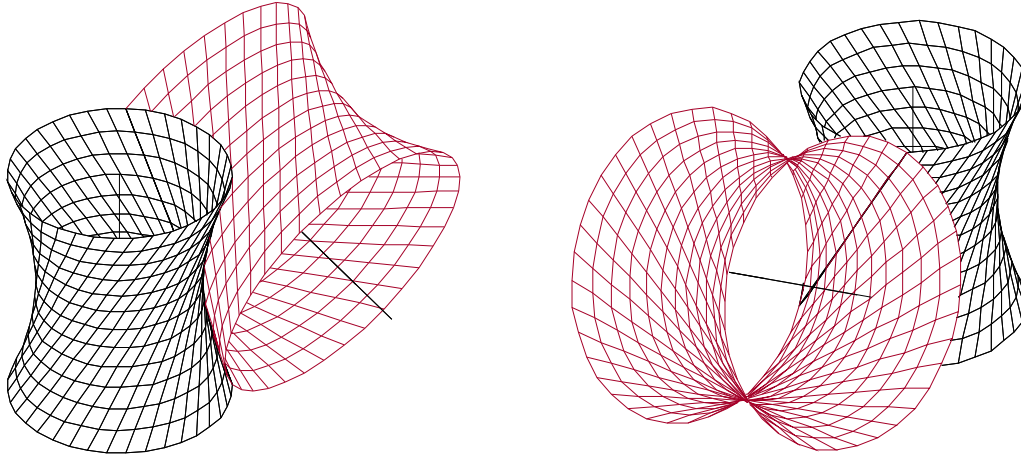


Figure 3: Spatial model of two conjugate hyperboloids of revolution

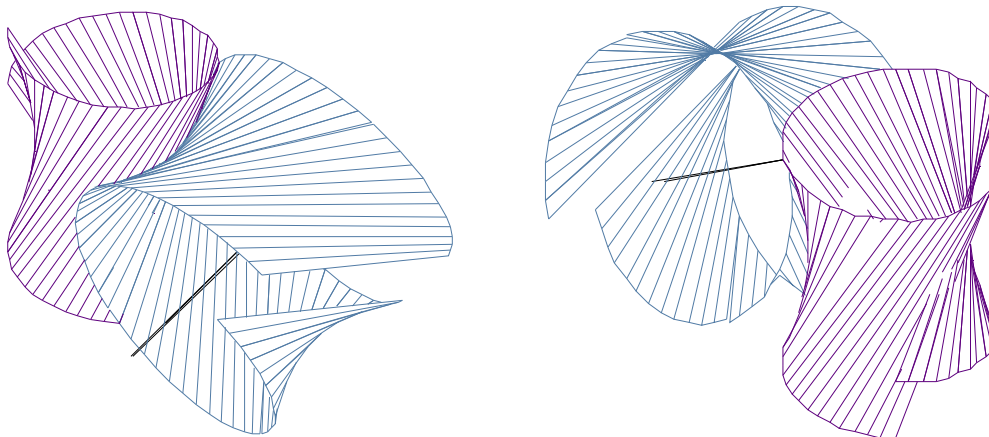


Figure 4: Spatial model of two imaginary conjugate convolute helicoids

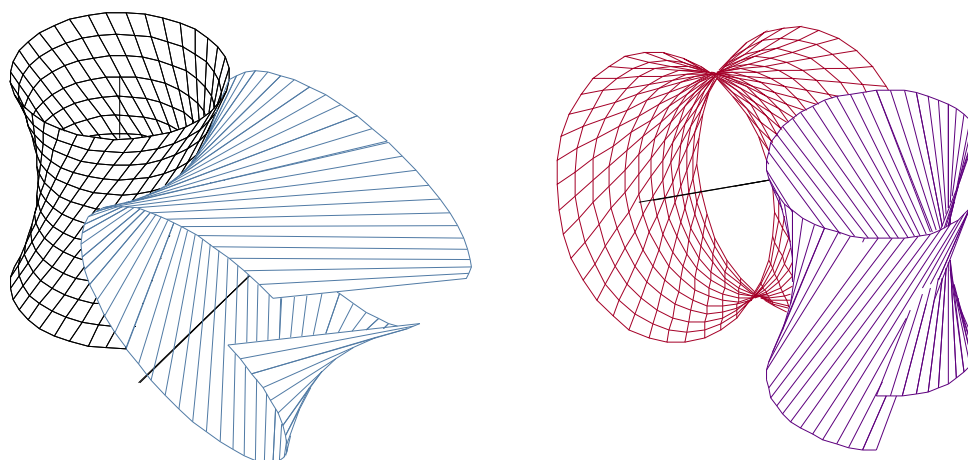


Figure 5: Spatial model of pairwise conjugate real and imaginary surfaces

Let's analyze the following diagram presented in Fig. 6. We form the conjugated pair of a hyperboloid and a helicoid. If we set the distance between the surfaces $\Delta = 70$ mm, angle of crossing $\varphi = 39^\circ$, pitch of helical surface $h = BK = 36$ mm, and the radius of the helix $BC = 42$ mm, then by the diagram we define the missing parameters: the radius of the neck of the hyperboloid $AC = 28$ mm and the slope angles of the generators to the corresponding axes $\alpha = 19^\circ$ and $\beta = 20^\circ$. The spatial images of the constructed real surfaces are shown in Fig. 7. Just as in the previous problem two imaginary conjugate convolute helicoid again appear in the mutual motion with pitch $h_1 = BK_1$ (Figs. 8, 9).

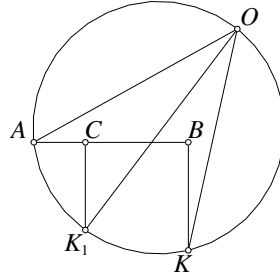


Figure 6: Diagram for conjugate hyperboloid and helicoids

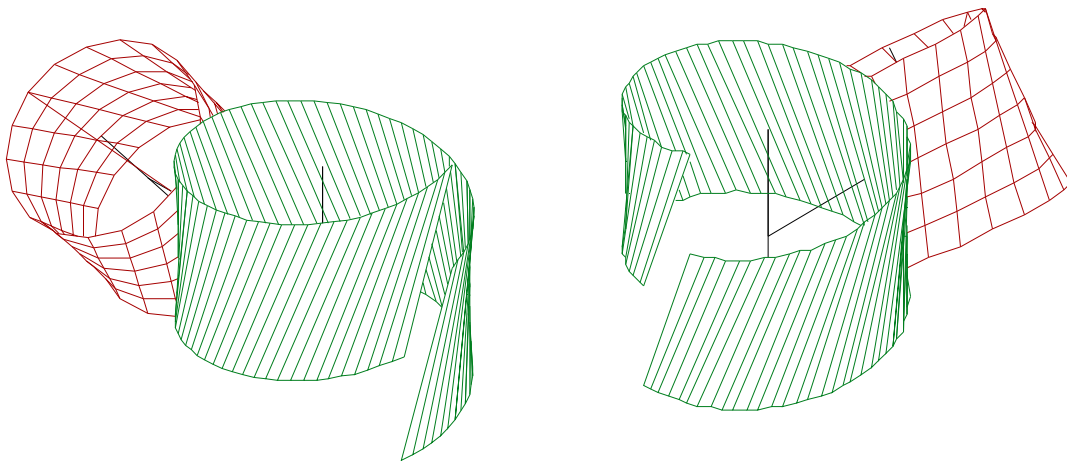


Figure 7: Spatial model of two conjugate real surfaces

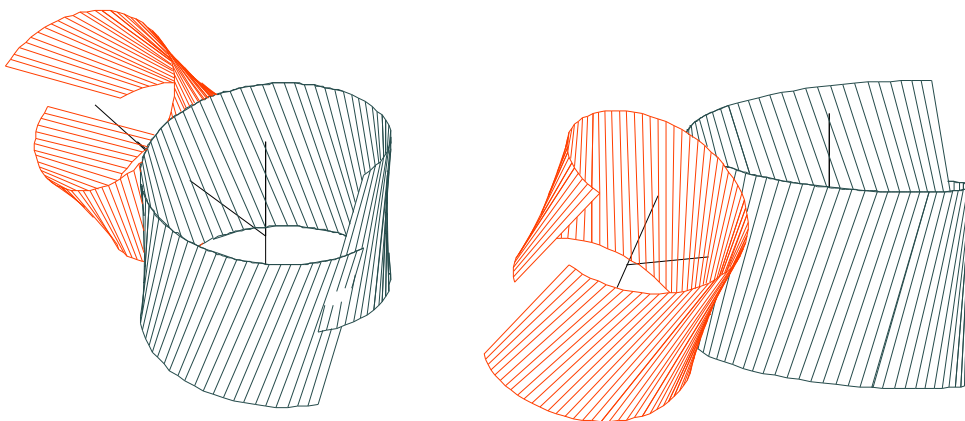


Figure 8: Spatial model of conjugate imaginary helicoids

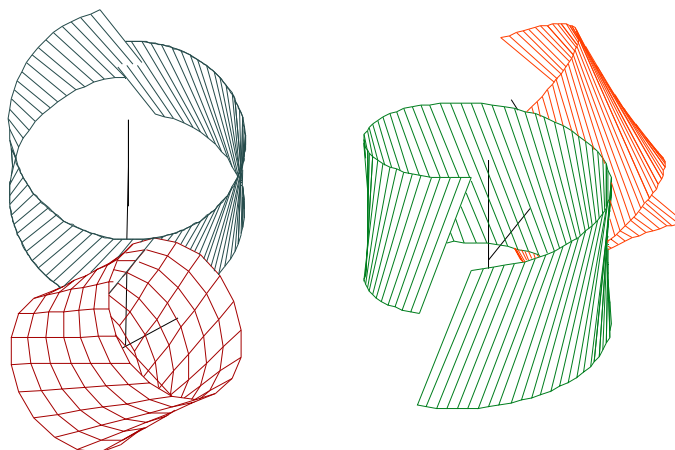


Figure 9: Spatial model of pairwise conjugate imaginary and real surfaces

In the next task we set the condition for determining conjugation parameters for two convolute helicoids. Let $\Delta = 120$ mm, $\varphi = 50^\circ$, pitch of the first helical surfaces $h_1 = AK_1 = 60$ mm, pitch of the second helical surface $h_2 = BK_2 = 36$ mm, helix radius of the first helicoid $AC = 50$ mm (Fig. 10).

We determine the missing parameters from the diagram: the helix radius of the second helicoid $BC = 70$ mm, the slope angles of the generators to the corresponding axes $\alpha = 19^\circ$ and $\beta = 31^\circ$. Parameter CK_2 – pitch of imaginary screw surfaces. In Fig. 11 shows the conjugation of real helicoids, in Fig. 12 - imaginary and in Fig. 13 are pairwise conjugate real and imaginary helicoids.

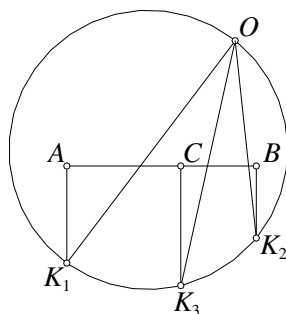


Figure 10: Diagram for two conjugate helicoids

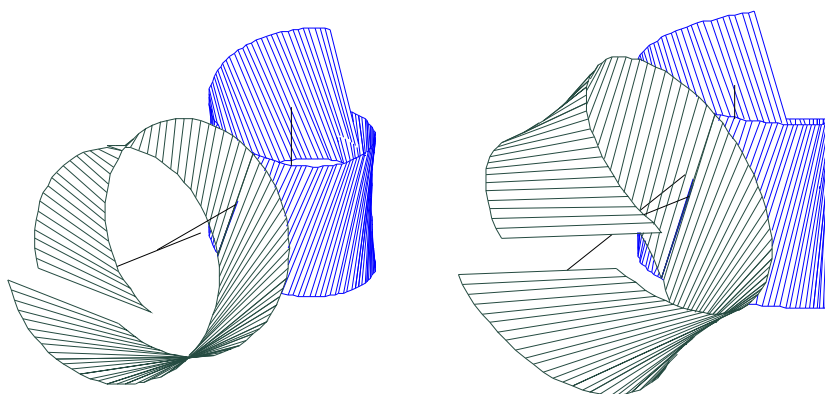


Figure 11: Spatial model of two conjugate convolute helicoids

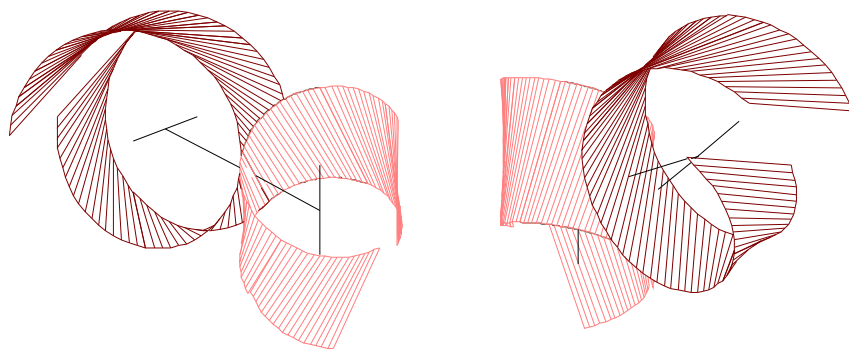


Figure 12: Spatial model of conjugate imaginary helicoids

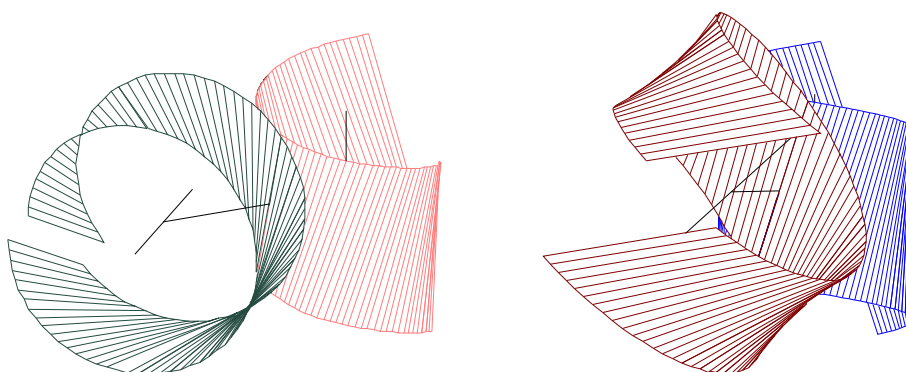


Figure 13: Spatial model of pairwise conjugate imaginary and real surfaces

5 Conclusions and acknowledgement

The theoretical initial (primary) surfaces in the gears with skew axes are rotational hyperboloids whose individual parts touching along straight lines can be used to design helical cylindrical gears (helical gears) and spiral bevel gears (hypoid gearing) [3, 4].

The conducted research graphically proves this accepted concept of auxiliary surfaces in transmissions with skew axes. Using KSD for conjugate hyperboloids it is possible to design new helical and hypoid gears. It is possible to design their initial surfaces in the graphical editor of AutoCAD which are shown in Fig. 14 in accordance with the diagram shown in Fig. 2. Cylindrical sections are located in the neck of the surface and conical - outside this zone.

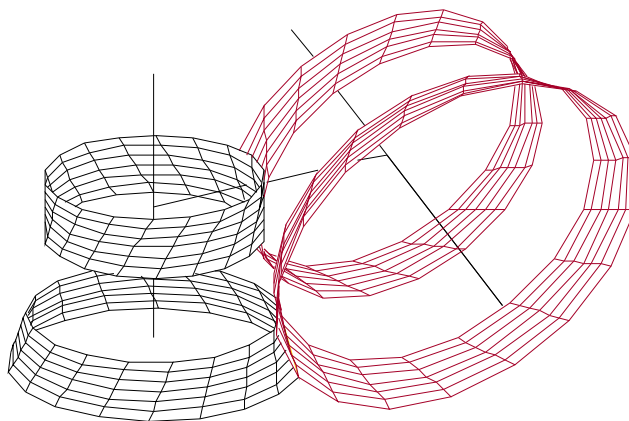


Figure 14: The conjugate initial surfaces constructed using the diagram shown in Fig. 2

Using the developed technique it is possible to design transmissions in which the initial surfaces are a hyperboloid and a convolute helicoid (Fig. 15) or two convolute helicoids (Fig. 16). In the case of diagrams for helicoids, it is necessary to construct a geometric model of conjugate surfaces since it is impossible to determine with the help of a diagram whether they will be interfaced or crossed.

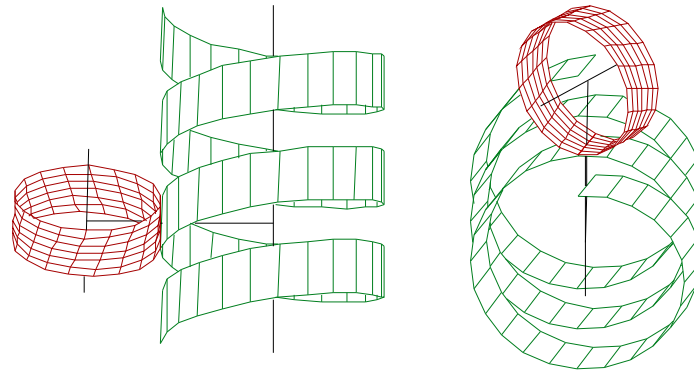


Figure 15: The conjugate initial surfaces constructed using the diagram shown in Fig. 6

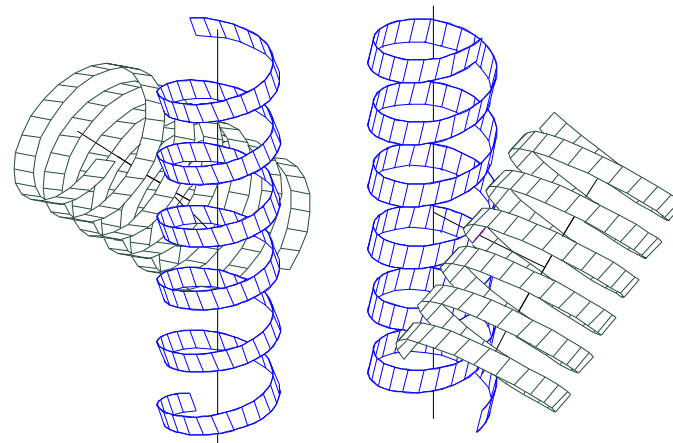


Figure 16: The conjugate initial surfaces constructed with using the diagram shown in Fig. 10

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MODELOWANIE GEOMETRYCZNE SPRĘŻONYCH POWIERZCHNI PROSTOKREŚLNYCH ZA POMOCĄ KINEMATYCZNEGO DIAGRAMU ŚRUBOWEGO

W przekładniach zębatych ze skośnymi osiami powierzchnie początkowe (generujące owe przekładnie) są hiperboloidami obrotowymi jednopowłokowymi. Parę powierzchni prostokreślnych stykających się wzdłuż linii prostych można wykorzystać do projektowania przekładni zębatych. Jednym ze sposobów modelowania geometrycznego sprzężonych początkowych powierzchni połączeń przestrzennych jest metoda zastosowania kinematycznego diagramu śrubowego, zaproponowanego przez A. Nikolaeva w 1950 roku. W artykule przedstawiono kilka problemów opartych na teorii Nikolaeva i opracowano geometryczne modele powierzchni o sprzężonych tworzących z wykorzystaniem diagramu. Przeprowadzone badania graficznie potwierdzają proponowaną teorię Nikolaeva i pozwalają na projektowanie nowych typów przekładni z przestrzennym zazębieniem.