CHIRP-RATE ESTIMATION OF FM SIGNALS IN THE TIME-FREQUENCY DOMAIN

KRZYSZTOF CZARNECKI

Gdansk University of Technology
Faculty of Electronics, Telecommunications and Informatics
11/12 Gabriela Narutowicza Street, 80-233 Gdańsk-Wrzeszcz, Poland
krzycz@eti.pg.gda.pl

Novel dynamic representations of a complex signal in the time-frequency domain including: a channelized instantaneous complex frequency (CICF), a complex local group delay (CLGD) and a channelized instantaneous chirp-rate (CICR) are introduced. The proposed approach is based on the use of the gradient of the short-time Fourier transform complex phase. An interpretation of the newly-introduced distributions especially of the CICR is demonstrated by a chirp-rate estimation of mono- and multicomponent FM signals in the time-frequency domain. The paper corresponds to a part of the dissertation [1].

INTRODUCTION

The paper covers some issues related to the estimation of FM signal parameters in the time-frequency domain. The introduced signal representations are derived from the invertible short-time Fourier transform (STFT). This approach leads to non-stationary signal analysis as well as processing in the time-frequency domain that are also possible in real-time, where a delay is introduced mainly by an analyzing window. The paper is focused on the chirp-rate estimation of FM signal components. The above problem is considered among others in [3–6], where the chirp-rate is estimated by advanced algorithms based on the Hough transformation, statistical methods or using the Wigner conception of signal analysis. In this paper another point of view is presented: estimation in the joint time-frequency domain. An estimator is proposed that works in the time-frequency domain locally as much as possible. One limit is the Heisenberg-Gabor uncertainty principle and the ambiguity function of an analyzing window. Moreover, it is required that the introduced estimates are reversible in order to provide the ability of direct processing. In general, the main purpose is to show unknown properties of the STFT in order to estimate the chirp-rate of FM signals.

Linden [7] and Hahn [8, 9] proposed a new description of a band-limited finite-energy complex-valued, especially analytical, signal using the so-called instantaneous complex frequency (ICF). The complex signal with a real and an imaginary parts can be expressed in the following polar form:

$$u(t) = a(t)\exp\big(j\varphi(t)\big); \quad \forall_t\, u(t) \in \mathbb{C}; \tag{1}$$

In this way, the defined waveform marked by $u(t)$ should have non-zero values and has to be differentiable at every instant. $a(t)$ is the absolute value and $\varphi(t)$ represents the instantaneous phase of the signal $u(t)$. So its ICF is defined as a derivative of the instantaneous complex phase of the waveform with respect to time. The ICF consists of the instantaneous frequency (IF) and the signed instantaneous bandwidth (SIBW). Although the term ICF is not widely used in signals analysis and processing, their components, the IF and the SIBW independent of each other, are the subject of many papers, among others [10–12]. The definition of the ICF and an eduction into a Cartesian form can be formulated as follows:

$$\begin{aligned} S(t) &= \frac{\mathrm{d}}{\mathrm{d}t}\Phi(t) = \frac{\mathrm{d}}{\mathrm{d}t}\ln\big\{u(t)\big\} = \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\ln\big\{a(t)\exp\big(j\varphi(t)\big)\big\} = \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\Big\{\ln\big\{a(t)\big\} + \ln\big\{\exp\big(j\varphi(t)\big)\big\}\Big\} = \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\big\{\lambda(t) + j\varphi(t)\big\} = \frac{\mathrm{d}}{\mathrm{d}t}\lambda(t) + j\frac{\mathrm{d}}{\mathrm{d}t}\varphi(t) = \\ &= \Sigma(t) + j\Omega(t); \quad \forall_t\, S(t) \in \mathbb{C}; \quad \forall_t\, \Sigma(t), \Omega(t) \in \mathbb{R} \end{aligned} \tag{2}$$

or

$$S(t) = \frac{\mathrm{d}u(t)}{\mathrm{d}t}\Big/ u(t) \tag{3}$$

where t means time, $S(t)$ is the ICF in rad/s, $\ln\{\}$ represents the operator of a natural logarithm for complex-valued numbers, $\Omega(t)$ is the IF and $\Sigma(t)$ is the SIBW both in rad/s. Sets or real-valued and complex-valued numbers are denoted respectively by $\mathbb{R}$ and $\mathbb{C}$. The instantaneous bandwidth (IBW) can be calculated as the absolute value of the SIBW [12]. The sign of the SIBW expresses a local change of direction of the absolute value of the waveform. In Eq. (2) the instantaneous complex phase of the signal $u(t)$ is defined as follows:

$$\begin{aligned} \Phi(t) &= \ln\big\{u(t)\big\} = \ln\Big\{a(t)\exp\big(j\varphi(t)\big)\Big\} = \\ &= \lambda(t) + j\varphi(t); \quad \forall_t\, \Phi(t) \in \mathbb{C}; \quad \forall_t\, \lambda(t), \varphi(t) \in \mathbb{R} \end{aligned} \tag{4}$$

where $\lambda(t)$ is the instantaneous envelop of the waveform $u(t)$.

The complex group delay (CGD) can be defined dually as a derivative of the complex phase of the signal spectrum with respect to frequency. The definition of the CGD and an eduction into a Cartesian form can be formulated as follows:

$$
\begin{aligned}
Z(\omega) = \frac{\mathrm{d}}{\mathrm{d}\omega}\Psi(\omega) = \frac{\mathrm{d}}{\mathrm{d}\omega}\ln\big\{U(\omega)\big\} = \\
= \frac{\mathrm{d}}{\mathrm{d}\omega}\ln\big\{A(\omega)\exp\big(j\phi(\omega)\big)\big\} = \\
= \frac{\mathrm{d}}{\mathrm{d}\omega}\Big\{\ln\big\{A(\omega)\big\} + \ln\big\{\exp\big(j\phi(\omega)\big)\big\}\Big\} = \\
= \frac{\mathrm{d}}{\mathrm{d}\omega}\big\{\Lambda(\omega) + j\phi(\omega)\big\} = \frac{\mathrm{d}}{\mathrm{d}\omega}\Lambda(\omega) + j\frac{\mathrm{d}}{\mathrm{d}\omega}\phi(\omega) = \\
= \mho(\omega) - jD(\omega); \quad \forall_\omega\, Z(\omega) \in \mathbb{C}; \quad \forall_\omega\, D(\omega), \mho(\omega) \in \mathbb{R}
\end{aligned}
\tag{5}
$$

or

$$
Z(\omega) = \frac{\mathrm{d}U(\omega)}{\mathrm{d}\omega}\Big/ U(\omega) \tag{6}
$$

where $U(\omega)$ is the Fourier transform of the waveform $u(t)$, ω represents Fourier's frequency, $D(\omega)$ is the group delay (GD) and $\mho(\omega)$ represents the so-called signed group duration (SGDR) both in rad·s. The absolute value of the SGDR is suggested as being the group duration (GDR), while an SGDR sign expresses a local change of direction of the absolute value of the signal spectrum. $\Psi(\omega)$ is referred to as the complex phase of the Fourier transform $U(\omega)$ and is defined as follows:

$$
\begin{aligned}
\Psi(\omega) = \ln\big\{U(\omega)\big\} = \ln\Big\{A(\omega)\exp\big(j\phi(\omega)\big)\Big\} = \\
= \Lambda(\omega) + j\phi(\omega); \quad \forall_\omega\, \Psi(\omega) \in \mathbb{C}; \quad \forall_\omega\, \Lambda(\omega), \phi(\omega) \in \mathbb{R}
\end{aligned}
\tag{7}
$$

where $A(\omega)$ is the absolute value of the signal spectrum, or simultaneously the Fourier transform, $U(\omega) = A(\omega)\exp\big(j\phi(\omega)\big)$; $A(\omega) > 0$, $\Lambda(\omega)$ is its natural logarithm and $\phi(\omega)$ represents a phase of the Fourier transform. The CGD is rarely described in the scientific press, and one of the few publications is [13].

Some trends of modern time-frequency representations in scientific literature have been introduced since Gabor published his famous work: "Theory of communications" [14]. Many valuable papers and approaches in this subject were introduced by Rihaczek, Boashash, Kodera, Nelson, Auger, Flandrin, Stancovic, Fulop, Fitz, Cohen and others. The ICF and the CGD are useful representations in analyzing single-component signals. However, for multicomponent signals it is usually not sufficient and the use of, as we see in [11], time-frequency representations, such as Gabor's transforms, is recommended. Therefore in this paper the STFT is considered as a basis for signal analysis and processing [2].

1. TIME-FREQUENCY ANALYSIS

In order to investigate the multicomponent signal and obtain the properties of each component separately the use of a time-frequency analysis method is recommended, for example the short-time Fourier transformation. A multicomponent signal consisting of N components can be described in the following way:

$$
u(t) = \sum_{n=1}^{N} u_n(t) = \sum_{n=1}^{N} a_n(t)\exp\big(j\varphi_n(t)\big), \tag{8}
$$

where $u_n(t)$, $a_n(t)$ and $\varphi_n(t)$ denote respectively the n-th component, the envelop and the instantaneous phase of this component. Whereas the STFT of the signal $u(t)$, described in the time-frequency domain and calculated by using an analyzing window $h(t)$, can be defined as follows:

$$
\begin{aligned}
U(t,\omega) &= A(t,\omega)\exp\big(j\phi(t,\omega)\big) = \\
&= \int\limits_{-\infty}^{\infty} u(\tau+t)h^*(-\tau)\exp(-j\omega\tau)\mathrm{d}\tau; \quad \forall_t\forall_\omega\, U(t,\omega), h(t) \in \mathbb{C}
\end{aligned} \tag{9}
$$

In general, the window $h(t)$ can be complex for example in the chirplet transformation. The complex phase can be defined for the STFT as follows:

$$
\begin{aligned}
\Psi(t,\omega) &= \ln\big\{U(t,\omega)\big\} = \ln\Big\{A(t,\omega)\exp\big(j\phi(t,\omega)\big)\Big\} = \\
&= \Lambda(t,\omega) + j\phi(t,\omega); \quad \forall_t\forall_\omega\, \Psi(t,\omega) \in \mathbb{C}; \quad \forall_t\forall_\omega\, \Lambda(t,\omega), \phi(t,\omega) \in \mathbb{R}
\end{aligned} \tag{10}
$$

The CICF and CLGD both also described in the joint time-frequency domain are the analogical equivalents of the ICF and the CGD expressed respectively separately in the time and frequency domain. The CICF can be defined as a partial derivative of the complex phase of the STFT with respect to time, as follows:

$$
\begin{aligned}
S(t,\omega) &= \frac{\partial}{\partial t}\Psi(t,\omega) = \frac{\partial}{\partial t}\ln\big\{U(t,\omega)\big\} = \\
&= \frac{\partial}{\partial t}\ln\big\{A(t,\omega)\exp\big(j\phi(t,\omega)\big)\big\} = \\
&= \frac{\partial}{\partial t}\Big\{\ln\big\{A(t,\omega)\big\} + \ln\big\{\exp\big(j\phi(t,\omega)\big)\big\}\Big\} = \\
&= \frac{\partial}{\partial t}\big\{\Lambda(t,\omega) + j\phi(t,\omega)\big\} = \frac{\partial}{\partial t}\Lambda(t,\omega) + j\frac{\partial}{\partial t}\phi(t,\omega) = \\
&= \Sigma(t,\omega) + j\Omega(t,\omega); \quad \forall_t\forall_\omega\, S(t,\omega) \in \mathbb{C};\ \Sigma(t,\omega), \Omega(t,\omega) \in \mathbb{R}
\end{aligned} \tag{11}
$$

or

$$
S(t,\omega) = \frac{\partial U(t,\omega)}{\partial t} \Big/ U(t,\omega) \tag{12}
$$

where $\Omega(t,\omega)$ is the channelized instantaneous frequency (CIF), known for instance from Kodera's approach [15], referred to as the reassigned spectrogram. $\Sigma(t,\omega)$ represents the signed channelized instantaneous bandwidth (SCIBW), both real-valued and in rad/s. The absolute value of the SCIBW is known as the channelized instantaneous bandwidth (CIBW).

The CLGD can be defined dually as a partial derivative of the complex phase of the STFT with respect to frequency as follows:

$$
\begin{aligned}
Z(t,\omega) &= \frac{\partial}{\partial\omega}\Psi(t,\omega) = \frac{\partial}{\partial\omega}\ln\big\{U(t,\omega)\big\} = \\
&= \frac{\partial}{\partial\omega}\ln\big\{A(t,\omega)\exp\big(j\phi(t,\omega)\big)\big\} = \\
&= \frac{\partial}{\partial\omega}\Big\{\ln\big\{A(t,\omega)\big\} + \ln\big\{\exp\big(j\phi(t,\omega)\big)\big\}\Big\} = \\
&= \frac{\partial}{\partial\omega}\big\{\Lambda(t,\omega) + j\phi(t,\omega)\big\} = \frac{\partial}{\partial\omega}\Lambda(t,\omega) + j\frac{\partial}{\partial\omega}\phi(t,\omega) = \\
&= \mho(t,\omega) - jD(t,\omega); \quad \forall_t\forall_\omega\, Z(t,\omega)\in\mathbb{C};\ \mho(t,\omega), D(t,\omega)\in\mathbb{R}
\end{aligned} \tag{13}
$$

or

$$
Z(t,\omega) = \frac{\partial U(t,\omega)}{\partial\omega}\Big/ U(t,\omega) \tag{14}
$$

where $D(t,\omega)$ is the local group delay (LGD) also used in Kodera's approach in order to estimate the corrections of energy location in time. $\mho(t,\omega)$ is the novel representation that is suggested as being called the signed local group duration (SLGDR). Both the LGD and the SLGDR are expressed in rad·s. The absolute value of the SLGDR can be interpreted as the local group duration (LGDR) [2].

2. CHANNELIZED INSTANTANEOUS CHIRP-RATE

Nelson, as one of the first authors, described the second-order derivatives of the STFT phase in applications [16, 17]. He proposed the use of the second-order mixed derivative in order to develop indicator functions for testing excitations and resonances in acoustics. It seems that this task can be executed using the introduced CICR that is defined as follows:

$$
\text{Æ}(t,\omega) = \frac{\Sigma(t,\omega)}{\mho(t,\omega)} = \frac{\partial}{\partial t}\Lambda(t,\omega)\Big/\frac{\partial}{\partial\omega}\Lambda(t,\omega); \quad \forall_t\forall_\omega\,\Theta(t,\omega)\in\mathbb{R} \tag{15}
$$

where $\text{Æ}(t,\omega)$ is the CICR expressed in (rad/s) / (rad·s) = (1/s^2) = (Hz^2). If the values of the CICR are negative this represents an angular deceleration.

It is assumed that locally, in a very small region of the time-frequency plain, sized $\Delta t\times\Delta\omega$, occurs a linear change of signal parameters, especially the CIF, that can be expressed by a local chirp-rate estimate. According to the assumption: $\Delta t\to 0$ and $\Delta\omega\to 0$, the region converges to a point. For this region, the SLGDR and the SCIBW can be estimated that leads to a deduction of the local signal energy "stretch" in time and in frequency. Signs of the SLGDR and the SCIBW are used in order to designate the growth or decline of the CICR. If the SLGDR is interpreted as a range in time and the SCIBW as a range in frequency, Eq. (15) is consistent with the classical chirp-rate interpretation of a linear frequency modulated signal, that is expressed by the ratio $\Delta\omega/\Delta t$ [2].

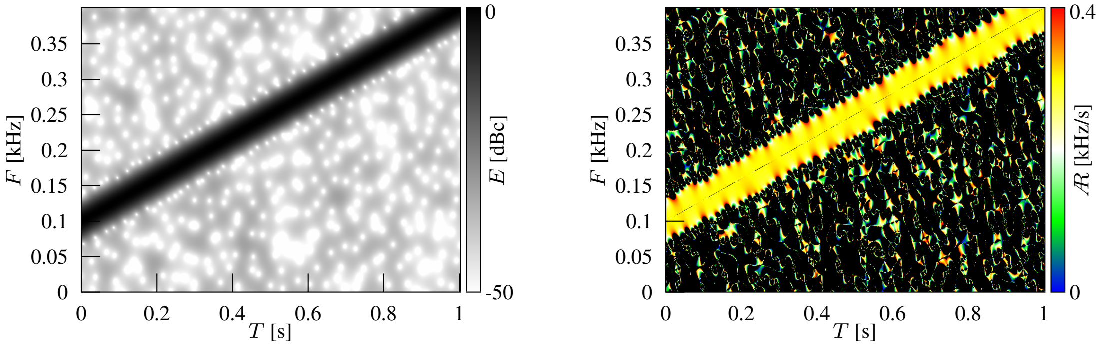

Fig. 1. The classical spectrogram and the corresponding accelerogram of the LFM chirp $u_1(t)$ defined by Eq. (16) in presence of the week additive white Gaussian noise where SNR is equal to approx. 25 dB.

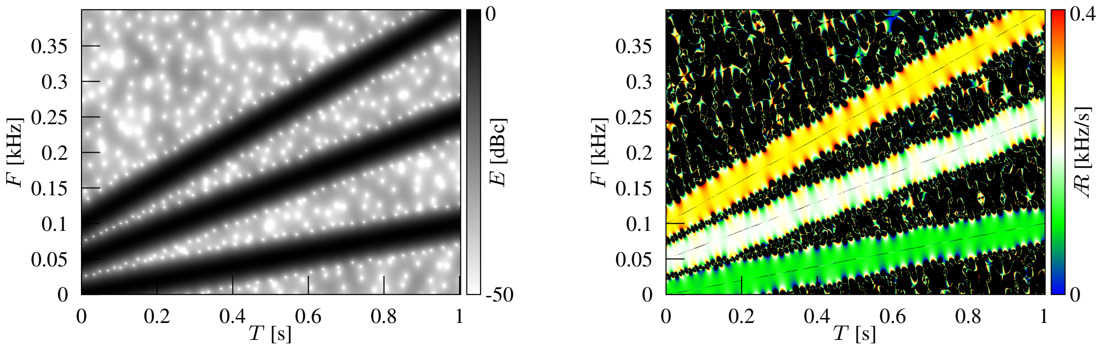

Fig. 2. The classical spectrogram and the corresponding accelerogram of the multicomponent signal $u_2(t)$ defined by Eq. (17) in presence of the week additive white Gaussian noise where SNR is equal to approx. 25 dB.

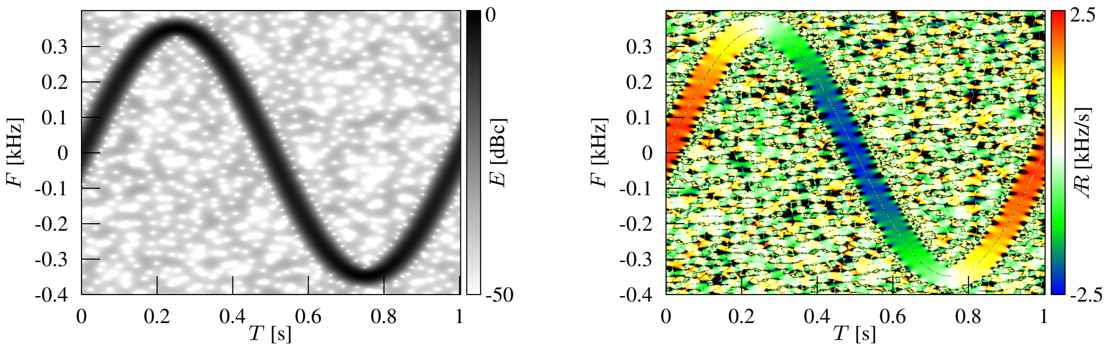

Fig. 3. The classical spectrogram and the corresponding accelerogram of the FM chirp $u_3(t)$ defined by Eq. (18) in presence of the week additive white Gaussian noise where SNR is equal to approx. 25 dB.

3. FM CHIRPS ANALYSIS

The CICR distribution can be applied in order to analyze mono- and multicomponent FM signals. The main purpose of this analysis is to obtain a chirp-rate (or frequency sweep) of each component. In this section, results of the analysis of three test signals are presented:

$$u_1(t) = \exp\big(j2\pi f_1 t + j\pi r_1 t^2\big), \tag{16}$$

$$u_2(t) = \sum_{n=1}^{3} \exp\big(j2\pi f_n t + j\pi r_n t^2\big), \tag{17}$$

$$u_3(t) = \exp\big(-j350\cos(2\pi t f_{\mathrm{u}})\big), \tag{18}$$

where, $f_1 = 100$ Hz, $f_2 = 50$ Hz, $f_3 = 0$ Hz, $f_u = 1$ Hz, $r_1 = 300$ Hz/s, $r_2 = 200$ Hz/s, $r_3 = 100$ Hz/s.

In Fig. 1, 2 and 3 the classical spectrograms and the accelerograms are presented. The CICR distributions are denoted by color. If at any point a value of the acceleration exceeds the range of the color scale, the point is colored black. The test signals are degraded by the additive white Gaussian noise where signal to noise ratio (SNR) is equal to approx. 25 dB.

4. CONCLUSION

A few representations of the FM signals in the time-frequency domain are presented, among others the GDR, SGDR, LGDR, LSGDR and LCSD. Special interest is focused on CICR distribution. The physical interpretation of the CICR is demonstrated through the execution of numerical experiments, in which FM testing chirp signals are examined. The development of the local chirp-rate estimator is proposed based on the use of the CICR distribution. Because the CICR is calculated using the SLGDR and the SCIBW, the utility of all these associated distributions is demonstrated. Moreover, the significance of SCIBW and SLGDR signs are underlined by estimations of the negative and positive values of the local chirp-rate. The paper may be a contribution in verifying the importance of the SIBW sign, which is usually neglected.

The presented parameters can be used not only in analysis but also in signal processing. Processing can be implemented by the reversibility of the STFT. This will be the subject of future work. Moreover, integration CICR, SCIBW and SLGDR representations with Kodera's approach can lead to designing a powerful and robust tool for non-stationary sparse signal processing in the time-frequency domain. Future work will also continue in this area. Newly introduced representations should be examined under many conditions: the uncertainty Heisenberg-Gabor principle and analyzing window influence, resistance against noise, the Cramer-Rao lower bound, a relation between CICR and second order partial derivatives of the phase of STFT etc. [2].

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