FIXED PASSIVE ARRAY ORIENTATION ESTIMATION BY BEARING ANALYSIS OF MOVING ACOUSTIC SOURCE

SŁAWOMIR HENCLIK

R&D Marine Technology Centre
ul. Dickmana 62, 81-109 Gdynia, Poland
shen@ctm.gdynia.pl

Precise source localization in passive sonar with linear, horizontal passive array and matched beam processing algorithms requires, among others, accurate determination of the fixed array position and orientation. Geographical coordinates of the array can be evaluated quite simple with precise GPS receiver. Especially important problem is then the accurate determination of the array orientation. It is not a trivial problem and simple compass measurements may produce precision troubles. At the paper a method that compares the known trajectory of a moving acoustic source and its bearings computed with the array system software is developed. The method is based on minimizing the function of square error between the true and candidate-estimated target positions. Another interesting feature of proposed algorithm is the possibility of optimizing the error function with respect to parameters that model in an assumed way signal propagation characteristics.

INTRODUCTION

Accurate bearings and positions determination of passive sonar targets is a complex problem as the precision is affected by many factors. It is a complex and separate problem, requiring much effort, to minimize measurement and model errors, especially affected by shallow water propagation. That is why the accurate determination of position and orientation of fixed passive array, as primary source of errors, is very important. It is relatively simple to determine the coordinates of the deployed at the sea bottom, array. Using GPS instrument allows determining array position with the accuracy better than 10 meters. It is enough for localizing far sources distant of hundreds and thousands of meters. More difficult task is to measure the array orientation relative to geographical directions. Required precision should be as high as possible and one may expect it to be not worst than 1^{o}. Simple compass measurements, because of many reasons (magnetic anomalies, underwater measurement conditions, non-perfect linearity of the array, etc.), may produce troubles in getting satisfactory precision. This was the experience of ours when trying to determine the orientation of fixed linear arrays developed at Marine Technology Centre and deployed at the bottom of Baltic Sea. As simple compass measurements were not satisfactory, the decision was to develop indirect method instead of improving the instrument measurements. One method based on GPS measurements is described in [1]. The average result of the orientation of one of the arrays (relative to North) was found to be -35.2° with standard deviation of ten measurements 2.0°. Another method is proposed at this paper and is based on acoustic measurements and signal processing for bearing estimation of moving acoustic target of known trajectory.

1. METHOD DESCRIPTION

Assume an acoustic target moving at the plane and its positions at moments $t_1, ..., t_K$ are known to be $\mathbf{r}_1, ..., \mathbf{r}_K$.

$$\mathbf{r}_i = [x_i \quad y_i] \tag{1}$$

The target is being "observed" by a linear array of hydrophones and its bearings (sine of the azimuth θ, determined relative to the array broadside) at subsequent time moments are computed by the array processing system. The scenario at a certain moment is presented at fig.1.

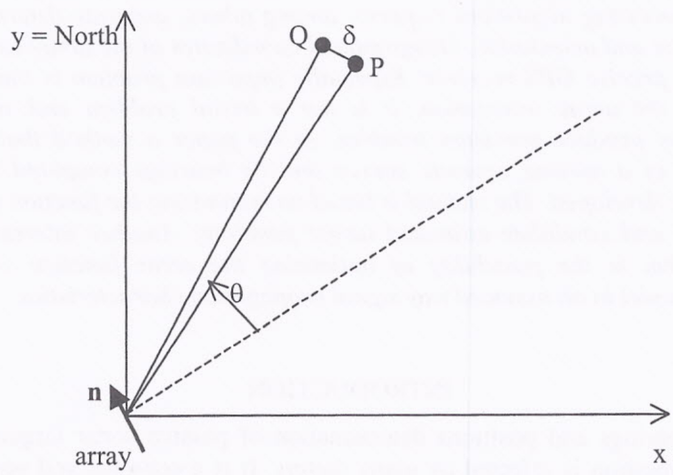

Fig.1. Target position relative to the array.
Q – the true target position, P – candidate-computed position.

The array position is assumed known and it is placed at the origin of the North-East oriented coordinate system. The array orientation is defined by unit vector **n** that points the direction of increasing number of sensors.

$$\mathbf{n} = [u \quad v] \tag{2}$$

Actually, the primary result of array processing system computations is the sine of signal seeming azimuth denoting as ξ. However, if the assumption of horizontal, straight-lines propagation is acceptable, ξ may be claimed to be equal to sine of the target azimuth θ.

$$\xi = \sin\theta \tag{3}$$

Denoting as **q** the unit vector pointing the true target position Q and as **p** the unit vector pointing the computed position P (that lies towards the DOA-direction_of_arrival of incoming signal) one can write (4) and (5), for τ defined with (6) [2].

$$\mathbf{q} = \frac{\mathbf{r}}{|\mathbf{r}|} = \frac{\mathbf{r}}{r} \tag{4}$$

$$\mathbf{p} = \begin{bmatrix} p_x & p_y \end{bmatrix} = \begin{bmatrix} u\xi + v\tau & v\xi - u\tau \end{bmatrix} \tag{5}$$

$$\tau = \varepsilon\sqrt{1-\xi^2} \tag{6}$$

ε means the sign and is equal +1 for target on the right side of the array (as in fig.1) and –1 for the left-side target. The square error δ_i^2 between the true and computed-candidate target positions at moment t_i can be found with formula (7).

$$\delta_i^2 = |\mathbf{r}_i - \mathbf{p}_i r_i|^2 \tag{7}$$

The average square error for the whole trajectory is than:

$$\delta^2 = \frac{1}{K}\sum_{i=1}^{K}\delta_i^2 = 2R - 2Au - 2Bv \tag{8}$$

The result in (8) was found after some manipulations with formulae (7), (5) and (1). The parameters R, A, B are given with (9) and it is also useful to define the parameter C.

$$\begin{aligned} R &= \frac{1}{K}\sum_{i=1}^{K} r_i^2 \\ A &= \frac{1}{K}\sum_{i=1}^{K} r_i x_i \xi_i - \frac{1}{K}\sum_{i=1}^{K} r_i y_i \tau_i \\ B &= \frac{1}{K}\sum_{i=1}^{K} r_i y_i \xi_i + \frac{1}{K}\sum_{i=1}^{K} r_i x_i \tau_i \\ C &= \sqrt{A^2 + B^2} \end{aligned} \tag{9}$$

For the known trajectory the goal is now to find such values of variables u, v (array orientation) that minimize the square error (8). Regarding $u^2+v^2=1$ one can found that the variables satisfying the minimum have to be calculated with formula (10) and the minimal value of the error is found as (11).

$$u_0 = \frac{A}{C} \qquad v_0 = \frac{B}{C} \tag{10}$$

$$\delta^2_{min} = 2(R - C) \tag{11}$$

The orientation of the array is the angle α measured clockwise relative to North, so it satisfies formula (12).

$$\sin\alpha = u_0 \qquad \cos\alpha = v_0 \tag{12}$$

The measure of the α accuracy can be the average angle error Δ between the true and computed positions Q_i and P_i of the target, calculated with formula (13).

$$\Delta = \sqrt{\frac{\delta^2_{min}}{R}} = \sqrt{2\left(1 - \frac{C}{R}\right)} \tag{13}$$

2. INFLUANCE OF PROPAGATION

The assumption of horizontal, straight-line propagation made in previous paragraph is obviously an approximation, especially in shallow water. The simple idea to make this model more adequate is presented below.

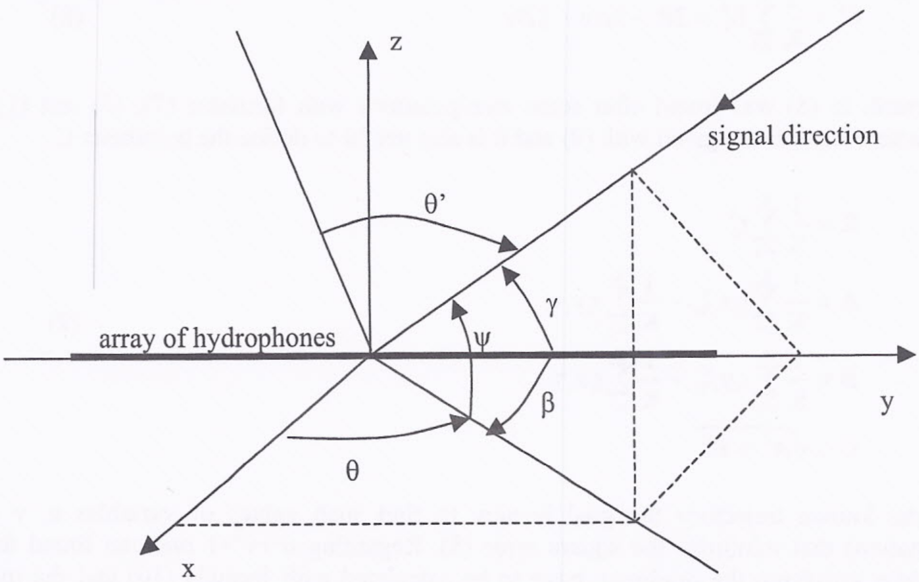

Fig.2. Signal acquiring in three dimensions.

Determining signal localization in three dimensions with linear (i.e. one-dimensional) array of sensors is not unambiguous. For far-field targets linear array may determine only the conic angle γ of receiving signal and distinguishing between azimuth θ and elevation ψ is not possible in direct manner.
For scenario presented at fig.2 formula (14) can be easily settle down.

$$\cos\gamma = \cos\beta\cos\psi \tag{14}$$

The true azimuth is θ, however what is computed with DOA processing system is the seeming azimuth θ' or actually ξ=sinθ'. Both θ and θ' satisfied formulae (15).

$$\begin{aligned}\theta &= \frac{\pi}{2} - \beta \\ \theta' &= \frac{\pi}{2} - \gamma\end{aligned} \tag{15}$$

Finally formula (16) is direct consequence of above.

$$\sin\theta = \frac{\xi}{\cos\psi} \tag{16}$$

Various phenomena [4] result in non-zero elevation angles of receiving wave. Though in many cases ψ is not large and the denominator in (16) is close to unity (e.g. for $\psi=20^{o}$, $\cos\psi=0.94$), the model given in (16) maybe applied instead of that defined with formula (3).
In above considerations θ is still signal, not target azimuth. The error between them (horizontal error) will not be modeled within current application, what is acceptable if it is zero mean and not large.

3. TARGET TRAJECTORY

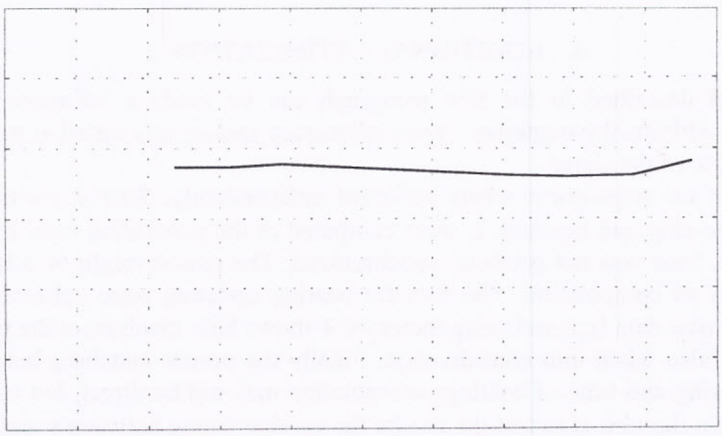

Fig.3. The route of the ship within tested period of time.

Using as the time unit the array processing system data updating time (usually 2÷16 sec.) the description of target trajectory is as follows. The target positions were determined with GPS receiver every 10 units and nine target positions measured at moments 0, 10, 20, ...,80 were taken into consideration. The schematic route of the ship is presented at fig.3. The array is positioned at the left-down corner of the diagram.
The bearings of the target were computed by the processing system with MUSIC [3] algorithm for a subarray of 23 uniformly spaced sensors. Narrowband processing was applied. Forward-backward averaging and spatial smoothing of rank 5 was used as preprocessing technique [5]. The bearings were determined with the array software at moments 0, 1, 2, 3, ...,86. The estimated bearing track of the target for frequency 702 Hz is shown at the fig.4.

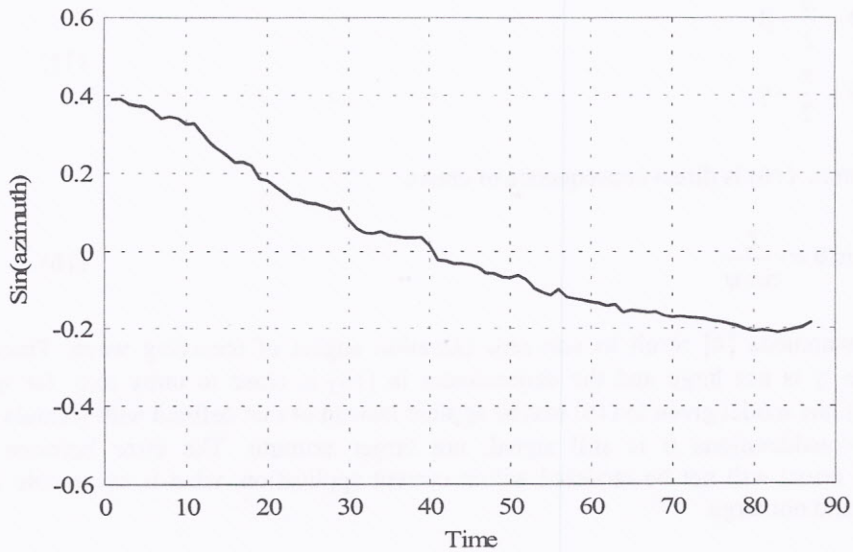

Fig.4. Computed bearings of moving target in time.

4. ADDITIONAL OPTIMIZATION

The model described in the first paragraph can be made a bit more complex by introducing two additional parameters – time inaccuracy and as was stated at paragraph two, the elevation angle of the signal.
Two data sets at the experiment where collected independently. Source positions $\mathbf{r}_i$ where determined at the ship and bearings ξ_i were computed at the processing unit. It may happen however that the time was not perfectly synchronized. The reason might be a human factor, but also the way of computation. The data for bearing updating were collected within one time unit. Some past data (remembering factor =0.4 shows how much from the past influence the present) are also taken into consideration. Finally the proper matching between time of positions measuring and time of bearings computation may not be direct, but may be shifted by a few units. So the idea is to test the results for varying fitting between $\mathbf{r}$ and ξ sets, i.e. $\mathbf{r}_t \leftrightarrow \xi_{t+k}$, for k=0, 1, 2, ...,6, and t=0, 10, 20, ...,80.
Another parameter is the cosine η of the elevation angle ψ.

$$\eta = \cos\psi \tag{17}$$

Assume that within the whole experiment the acquired, by the array, signal comes from the same elevation. In that case in formulae (6) and (9) the value of ξ_i that should be equal $\sin(\theta_i)$ have to be replaced with ξ_i/η, according to formula (16). One can expect the value of η to be close to unity, e.g. lying within the interval $1\leq\eta\leq 0.9$.
If such parameterization is made the final result of the array orientation and the minimal error given with formulae (10) and (13) are the function of k and η.

$$\begin{aligned} u_0 &= u_0(k,\eta) \\ v_0 &= v_0(k,\eta) \end{aligned} \tag{18}$$

$$\Delta = \Delta(k,\eta) \tag{19}$$

The optimization of the value of Δ, that is equivalent to looking for the minimum with respect to k and η, can be made numerically. Such process was performed for k varying through 0, 1, 2, 3, 4, 5, 6 and η varying from 1.05 down to 0.85 with step 0.001.

5. RESULTS

For the presented data set the optimization process, made numerically, have given (20) as the values of variables satisfying the minimum of Δ. The search of the minimum is illustrated at fig.5.

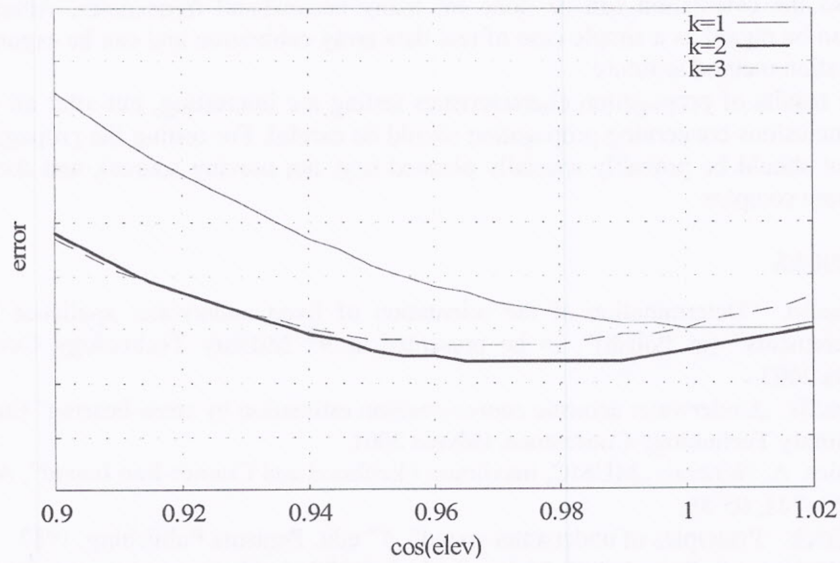

Fig.5. Searching for the minimum of $\Delta(k,\eta)$.

$$\begin{aligned} u_0 &= -0.575 \qquad & v_0 &= 0.818 \\ k_0 &= 2 \qquad & \eta_0 &= 0.976 \end{aligned} \tag{20}$$

The found values of u_0 and v_0 mean that the array orientation has to be -35.1^o. The found value of η (=0.976) means that the average elevation angle of the received signal within assumed model should be 12.6^o. Computations were made also for another frequencies. Selected minimums are as follows.

f = 444 Hz	k = 2	$\eta = 1.020$	$\alpha = -37.0^o$
f = 444 Hz	k = 4	$\eta = 0.928$	$\alpha = -36.9^o$
f = 444 Hz	k = 6	$\eta = 0.938$	$\alpha = -38.3^o$
f = 666 Hz	k = −1	$\eta = 0.993$	$\alpha = -33.9^o$
f = 666 Hz	k = 1	$\eta = 0.956$	$\alpha = -34.4^o$

The mean of above results taken for k=1÷2 is α=-35.5°. If the propagation model is simplifying to η=const=1.0 the average is α=-35.4°.

6. CONCLUSIONS

The presented acoustic method of estimating the array orientation is an interesting alternative for conventional compass measurements. The calculated with proposed method results are in very good consistency with measurements made with another method [1]. Presented method allows the array orientation to be checked with ease. Its advantage compare to direct instrument measurement is, it determines the mean orientation of the array so possible imperfect linearity of the array is not a problem. One experiment supplies wide set of data, so the calculation can be done for many narrowband frequencies. Actually this method can be regard as a simple case of real data array calibration and can be expand to the full calibration method in future.

The results of propagation characteristics testing are interesting, but after all drawing strong conclusions concerning propagation should be careful. For testing the propagation the experiment should be probably specially planned (e.g. not moving source), and the applied models more complex.

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