Paper

Network Dimensioning with Maximum Revenue Efficiency for the Fairness Index

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Abstract—Network dimensioning is a specific kind of the resource allocation problem. One of the tasks in the network optimization is to maximize the total flow on given pairs of nodes (so-called demands or paths between source and target). The task can be more complicated when different revenue/profit gained from each unit of traffic stream allocated on each demand is taken into account. When the total revenue is maximized the problem of starvation of less attractive paths can appear. Therefore, it is important to include some fairness criteria to preserve connections between all the demands on a given degree of quality, also for the least attractive paths. In this paper, a new bicriteria ratio optimization method which takes into account both, the revenue and the fairness is proposed. Mathematical model is built in a form of linear programming. The solutions are analyzed with some statistical measures to evaluate their quality, with respect to fairness and efficiency. In particular, the Gini's coefficient is used for this purpose.

Keywords—allocation problem, decision problems, dimensioning networks, fair optimization, linear programming, maximization, multi-criteria.

1. Introduction

A problem of fair allocation of some finite set of resources appears in various contexts, such as transport or other branches of economy. In general, network dimensioning could also be compared with the group of allocation problems. Let us consider the set of resources and set of possible allocations of them. Each allocation of the resource is more or less profitable. The main goal in fairness optimization is to treat equal each of these locations in some degree. Such a decision problems appear in society while distributing the public goods or allocation of public services. Interesting approach was proposed by Rawls [1] to treat justice as fairness in social problems and political decisions. The problem of equity is a complex idea encountered in society and many times it requires dedicated model of optimization [2]. While dimensioning the telecommunication wired networks, it is required to remember about a lot of restrictions. First of all, it is needed to obtain the highest possible value of total revenue, which is related to profit from each unit of allocated load on given demand. Demand can be called also as a path between source and target. There are resources such as bandwidth or traffic flow, which have to be allocated on given paths [3]. The main assumption in this kind of problems is to describe the path character and next to limit each connection with the value of capacity. Paths could be formulated in two ways. One is to define a set of single paths as chain of nodes or links. This approach does not allow for bifurcation of a path and requires some work to predefine the set of them. It is the so-called link-path approach. Another, the node-link approach does not need setting the path before the optimization process. It is based on maintaining the flow on source, target and transitive nodes. In this way the path is allowed to split up. The model choice is determined by established assumptions on traffic flow in the network. No matter which formulation has been chosen, there could appear a problem of blocking some of demands. When some path shares at least one link and has different unit revenue the solution maximizing the efficiency of the network will allocate whole bandwidth to more profitable paths. When the decision-maker is interested in keeping the same degree of quality on each demand, it is necessary to include the fairness criterion into the optimization problem [4]. There have been done a lot of work in area of fairness optimization. For example, it could be the Max-Min [5] or Lexicographic Max-Min [6], [7] concepts. These methods have high fairness index but in many cases they return not satisfying and sometimes even dominated solutions. Another concepts to gain a fair and more efficient solution are methods such as Proportional-Fairness (PF) [8], Reference Point Method (RPM) [9], Ordered Weighted Averages (OWA) [10], [11]. The latter methods ensure efficient solutions and, in general, allows to control the fairness degree by appropriate parameters.

In the paper a new method of fair optimization is proposed. The method is based on so-called ratio model. It allows to obtain the most satisfying solution with respect to two different and inversely proportional criteria. The model formulation guarantees to obtain the solution of maximum additional revenue from allocated traffic flow with minimal acceptable fairness. Considering the mean value of revenue obtained from allocated load on all demands as z, mean value of some percent of the most discriminated paths as z_0 and result of the solution which should be improved (for example Max-Min method) as τ the new ratio model of maximization could be written as follows:

$$\max\left\{\frac{z-\tau}{z_0}\right\}. \tag{1}$$

In the rest of the paper the model is described in details and the results of an example of intput data are shown. For verification there have been chosen the basic statistics and the Gini coefficient. Consider set D as set of given allocations (demands), h_d as value of allocated resource on d-th allocation and vector of revenues per unit gained from allocated resource p_d associated with d-th location. Typical function that is used to describe efficiency criterion in the easiest form is the total revenue gained from obtained solution:

$$T(\mathbf{y}) = \sum_{d \in D} h_d \cdot p_d, h_d \geq 0. \tag{2}$$

Just mentioned formulation of efficiency criterion, when $\bar{D}$ denotes the cardinality of set D, could be also written as the mean (average) outcome:

$$\mu(\mathbf{y}) = \frac{\sum_{d \in D} h_d \cdot p_d}{\bar{D}}. \tag{3}$$

Linear formulation maximizes one of mentioned objective functions (2), (3), always returning the most profitable solution but in the most cases it leads to unfair solution. The main reason of that is in using the rational model of preference like in the standard Pareto-optimal solution concept. The rational relation of strict preference is denoted with $\succ$, weak preference $\succeq$, indifference with $\cong$ [12]. It could be described using the vector inequalities denoted by $\leq$, $\leqq$ and $=$. In this notation the rational relation of preference is defined by the following formulas:

$$\mathbf{y}' \succeq \mathbf{y}'' \Leftrightarrow \mathbf{y}' \geqq \mathbf{y}'' \Leftrightarrow \mathbf{y}'_i \geqq \mathbf{y}''_i, \tag{4}$$

$$\mathbf{y}' \succ \mathbf{y}'' \Leftrightarrow \mathbf{y}' \geq \mathbf{y}'' \Leftrightarrow (\mathbf{y}'_i \geqq \mathbf{y}''_i \text{ and not } \mathbf{y}'_i \leqq \mathbf{y}''_i), \tag{5}$$

where $\mathbf{y}_i$ is the i-th vector value. Additionally it meets the base assumptions of rational preference:

- reversible
$$\mathbf{y} \succeq \mathbf{y}, \tag{6}$$
- transitivity
$$(\mathbf{y}' \succeq \mathbf{y}'') \wedge (\mathbf{y}'' \succeq \mathbf{y}''') \Rightarrow (\mathbf{y}' \succeq \mathbf{y}'''), \tag{7}$$
- strict monotonicity
$$\mathbf{y} + \varepsilon e_i \succ \mathbf{y}, \varepsilon > 0 \ , \ i = 1, 2, \ldots d, \tag{8}$$
where e_i is the i-th objective function unit vector on decision area Y.

Many times it can appear a problem of starvation some less attractive paths (with respect to revenue/profit). It is often not acceptable by the decision-maker and in this case Pareto-optimal solution is not good enough. One way to preserve the fairness is to add constraints, which enforce the model to treat impartially (anonymously) all the demands. Such a model of preferences is well-defined when for each vector of allocated resources is fulfilled $(h_{\tau(1)}, h_{\tau(2)}, \ldots, h_{\tau(d)}) \cong (h_1, h_2, \ldots, h_d)$, for various permutation τ of set $D = 1, 2, \ldots, d$. Fulfillment of the above assumptions allows us to obtain so-called anonymous rational relation of preferences. Additionally, fulfillment of another axiom, the Pigou-Dalton principle of transfers (9), leads to equitable relation of preferences.

$$y_{i'} > y_{i''} \Rightarrow y - \varepsilon e_{i'} + \varepsilon e_{i''} \succ y, \ 0 < \varepsilon < y_{i'} - y_{i''}. \tag{9}$$

Every optimal solution of anonymous and equitable aggregation of multiple criteria problem leads to a fairly efficient solution (or simply fair solution) [13]. The fairly efficient solution is also Pareto-optimal but not vice-versa.
To quantify the fairness of the system there are a lot of equality (or inequality) measures. According to work has been done by Lan and Chiang [14] in area of fairness optimization it should be noticed that there exist five fundamental axioms, which should not be omitted by fairness measure. Those axioms are:

- continuity,
- homogeneity,
- saturation,
- partition,
- starvation.

Over the years many of measures which meet those axioms have been proposed [15]–[18], [20], [21]. For example, there are:

- maximum absolute difference or the mean absolute difference,
- maximum absolute deviation or the mean absolute deviation
- standard deviation or the variance,
- the mean (downside), the standard (downside) or the maximum semi deviation,
- k-largest semi deviations,
- Gini coefficient,
- Jain's index.

The inequality measures take the value of 0 for perfectly equal outcomes and the higher positive values for more unequal solutions. The most frequently used is the Gini coefficient. It is formulated as relative mean difference and is given by the formula:

$$G(d) = \frac{\sum_{i \in D} \sum_{j \in D} |h_i - h_j|}{2\bar{D}^2 \mu(d)}. \tag{10}$$

Standard bicriteria mean-equity model takes into account both the efficiency with optimization of the mean outcome

$\mu(\mathbf{y})$ and the equity with minimization of an inequality measure (or maximization of the equality measure) $\rho(\mathbf{y})$. Each measure has its own characteristic features, unfortunately, sometimes not able to use it effectively in optimization process directly. However, for several inequality measures, the reward-inequality ratio optimization

$$\max\left\{\frac{\mu(d)-\tau}{\rho(d)}\ ,\ d\in Q\right\}, \tag{11}$$

guarantees fairness of the solutions. This applies, in particular, to the worst conditional semi deviation.

2. Mathematical Optimization Model

The optimization task refers to allocation the load (traffic flow) on the set of given demands D, and $d\in\{1,\ldots,\bar{D}\}$. Each demand is associated with vector of revenues per unit p_d. As example, the Polish backbone network is adopted. Each link existed in graph is included in the set L and is marked by $l\in\{1,\ldots,\bar{L}\}$. Similarly, the set N contains all nodes and each node is marked as $n\in\{1,\ldots,\bar{N}\}$. Traffic flow could come into each node and get out from it in the same value exact source and target nodes. The node-link approach was chosen for make the possibility of paths bifurcation. This requires additional parameters which values are included in the matrices of incidences between nodes and links. Parameter a_{nl} is an element of outcome links matrix and if l-th link comes out from n-th node takes value of 1 and 0 otherwise. The similar schema is in income links matrix case. Its parameter b_{nl} takes the value of 1 when l-th link comes into the n-th node and 0 otherwise. Such notation is frequently uses in directed graphs, like the model described in the paper. In undirected graph case the values of those matrices would be the same. In typical network dimensioning the goal is to maximize the total flow (traffic stream, volume of bandwidth allocation, etc.). This variable is described as h_d and refers to the volume of flow allocated on d-th demand. Ratio optimization mathematical model is formulated as follows:

$$\max\frac{z-\tau}{z_0}, \tag{12}$$

$$z-u+\frac{\sum_{d\in D}k_d}{\beta\cdot\bar{D}}=z_0, k_d\geq 0, \tag{13}$$

$$k_d+h_d\cdot p_d\geq u, \tag{14}$$

$$z=\frac{1}{\bar{D}}\sum_{d\in D}h_d\cdot p_d, \tag{15}$$

$$\sum_{l\in L}a_{nl}\cdot x_{ld}-\sum_{l\in L}b_{nl}\cdot x_{ld}=\begin{cases}h_d & \text{if}\quad n=s_d\\ 0 & \text{if}\quad n\neq s_d,t_d\\ -h_d & \text{if}\quad n=t_d\end{cases}, \tag{16}$$

$$\sum_{d\in D}x_{ld}\leq c_l\ ,\ \forall l\in L. \tag{17}$$

In the ratio model Eqs. (13) and (14) ensure the ordering of considered objective functions. They contain the non-negative k_d and unbounded u variables, which guarantee a best solution in refer to some fair degree. More precisely it guaranties a special treat of the least values of allocated traffic flow. In optimization process, when z_0 variable is tending to possibly minimal value, the u variable is in inversely proportional. Moreover, when the u parameter is getting greater values, the Eq. (14) maximizes some part of k_d variables, which could be treated as the most discriminated demands. In each iteration, as the given percentage of the most discriminated demands, a z_0 parameter is chosen. This percentage refers to one of the control parameter β [22]. Further, there are the mean of all of objective functions needed to improve, labeled as τ. It should be noted that τ in example included in article has been calculated using the Max-Min method but it is not necessary to do. Depending on the given β degree, τ could accept greater values then τ_{MaxMin} but never greater then maximal achievable mean value, which is obtained for the simple maximization of total system efficiency (τ_{MAX}). In other words decision maker could substitute as τ each value included in just mentioned interval. However, if the greater the τ parameter is, the less fair solution will be. To ensure the node-link model assumptions is formulated as (16) constraint. It is formulated for three cases of node types:

- source,
- target,
- and connection node.

It is also important to remember about maximum capacity of each link (c_l) in the network (17) – see Table 1.

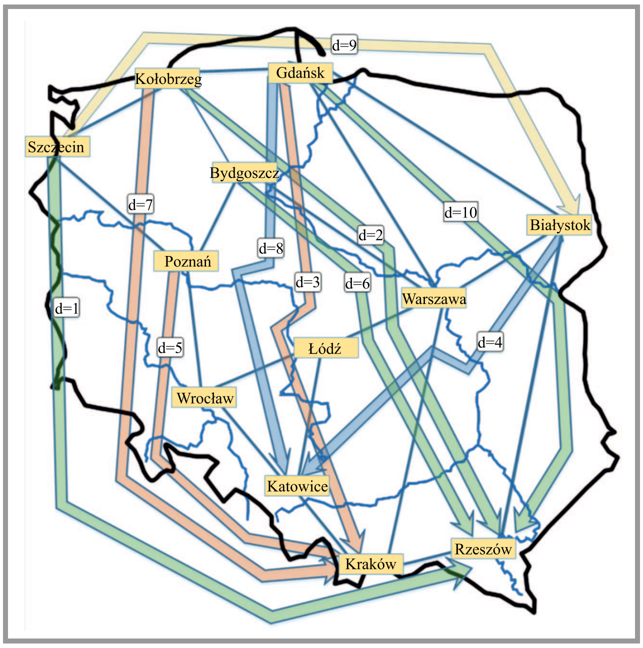

Fig. 1. An illustration of analyzed network.
(See color pictures online at www.nit.eu/publications/journal-jtit)

Table 1
Arc's characteristic

Arc ID	Start node	Target node	Capacity (c_l)
1	Kołobrzeg	Szczecin	150
2	Gdańsk	Kołobrzeg	100
3	Białystok	Gdańsk	100
4	Rzeszów	Białystok	150
5	Rzeszów	Kraków	100
6	Katowice	Kraków	80
7	Katowice	Wrocław	100
8	Wrocław	Poznań	150
9	Poznań	Szczecin	150
10	Bydgoszcz	Kołobrzeg	30
11	Warszawa	Gdańsk	80
12	Białystok	Warszawa	100
13	Warszawa	Kraków	100
14	Katowice	Łódź	80
15	Łódź	Wrocław	80
16	Poznań	Bydgoszcz	90
17	Warszawa	Bydgoszcz	200
18	Łódź	Warszawa	120
21	Szczecin	Kołobrzeg	150
22	Kołobrzeg	Gdańsk	100
23	Gdańsk	Białystok	100
24	Białystok	Rzeszów	150
25	Kraków	Rzeszów	100
26	Kraków	Katowice	80
27	Wrocław	Katowice	100
28	Poznań	Wrocław	150
29	Szczecin	Poznań	150
30	Kołobrzeg	Bydgoszcz	30
31	Gdańsk	Warszawa	80
32	Warszawa	Białystok	100
33	Kraków	Warszawa	100
34	Łódź	Katowice	80
35	Wrocław	Łódź	80
36	Bydgoszcz	Poznań	90
37	Bydgoszcz	Warszawa	90
38	Warszawa	Łódź	120

Model described above, unfortunately, cannot be used in the most of linear programming packages. It is caused by non-linear dependencies in the main objective function (12). To face the problem variables $v = z/z_0$ and $v_0 = 1/z_0$ have been introduced. Next, all the constraints were divined by z_0 and the following submissions have been made: $\widetilde{h_d} = \frac{h_d}{z_0}$, $\widetilde{k_d} = \frac{k_d}{z_0}$, $\widetilde{u} = \frac{u}{z_0}$, $\widetilde{x_{ld}} = \frac{x_{ld}}{z_0}$. After that the ratio optimization model is written as the following linear program:

$$\max \ v - \tau \cdot v_0 , \tag{18}$$

$$v = \frac{1}{\bar{D}} \sum_{d \in D} \widetilde{h_d} \cdot p_d , \tag{19}$$

$$v - \widetilde{u} + \frac{\sum_{d \in D} \widetilde{k_d}}{\beta \cdot \bar{D}} = 1 , \tag{20}$$

$$\widetilde{k_d} + \widetilde{h_d} \cdot p_d \geq \widetilde{u} \ , \ \forall d \in D , \tag{21}$$

$$\sum_{l \in L} a_{nl} \cdot \widetilde{x_{ld}} - \sum_{l \in L} b_{nl} \cdot \widetilde{x_{ld}} = \begin{cases} \widetilde{h_d} & \text{if } n = s_d \\ 0 & \text{if } n \neq s_d, t_d \\ -\widetilde{h_d} & \text{if } n = t_d \end{cases} , \tag{22}$$

$$\sum_{d \in D} \widetilde{x_{ld}} \leq c_l \cdot v_0 \ , \ \forall l \in L . \tag{23}$$

The experiments are performed for example of Polish backbone network [23]. Arrangement of given demands is presented in Fig. 1.

3. Results

In computations the CPLEX package was used as optimization environment. In the paper the results have been obtained for several configurations of control parameters (such as β and τ) and are presented in the Table 4. Considering the algorithm of described dimensioning problem optimization, there has been done several iterations as it is shown in Fig. 2. First, the solution of ratio model has been obtained τ = MAXMIN(H). Next the value of τ, were taking greater values, obtained from previous solutions of ratio model optimization – RM(H). The steps were repeated until the receive of maximal value of τ related to simple maximization concept solution – MAX(H).

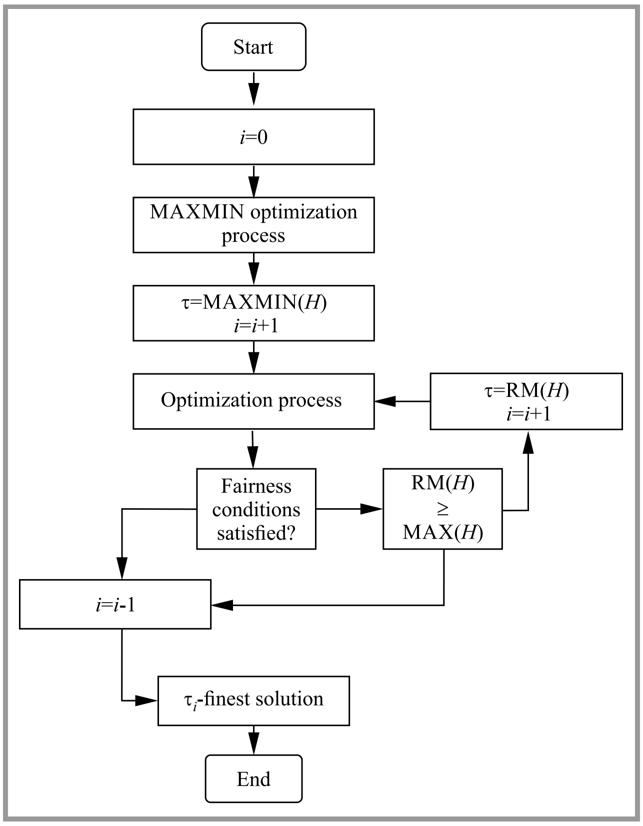

Fig. 2. An algorithm of decision process.

Table 2
Values of Gini coefficient for chosen control parameters

τ	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.5$	$\beta = 0.8$
3667	0.10	0.10	0.10	0.10	0.10
3950	0.11	0.11	0.11	0.09	0.11
4000	0.56	0.45	0.51	0.63	0.51
4100	0.56	0.48	0.43	0.54	0.43
4200	0.66	0.50	0.53	0.49	0.53
4300	0.62	0.63	0.52	0.53	0.52
5000	0.49	0.53	0.59	0.60	0.59
6500	0.66	0.51	0.62	0.64	0.62
7800	0.56	0.51	0.62	0.62	0.62

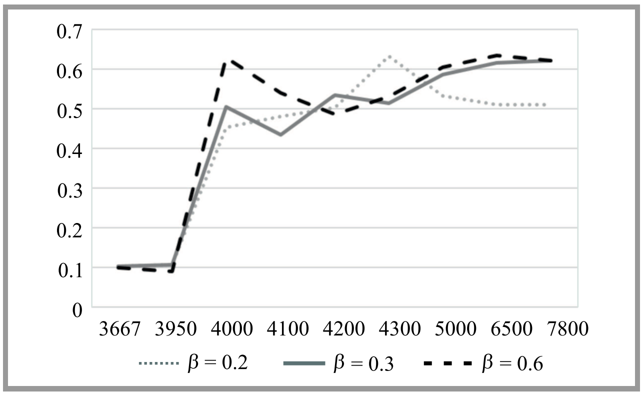

Fig. 3. A plot illustrating the course of Gini coefficient.

Table 3
Values of the mean traffic flow of obtained solutions

τ	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.5$	$\beta = 0.8$
3667	4258	4258	4258	4400	4400
3950	4288	4288	4288	5040	5040
4000	6269	6269	6269	5040	5040
4100	6269	6269	6269	5040	5040
4200	7900	7900	6823	7505	6840
4300	7900	7900	7900	7660	7560
5000	7900	7900	7900	7660	7560
6500	7900	7900	7900	7660	7640
7800	7900	7900	7900	7900	7900

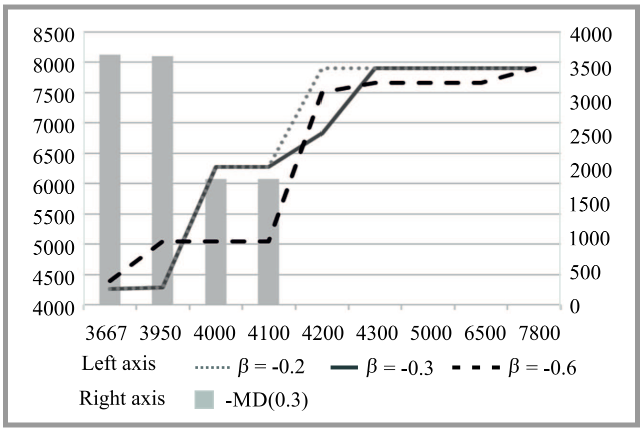

Fig. 4. A plot illustrating the course of mean value.

Considering the algorithm of described dimensioning problem optimization, there has been done several iterations. First, the solution of ratio model has been obtained τ = MAXMIN(H). Next the value of τ is taking greater values, obtained from previous solutions of ratio model optimization. The steps are repeated until the receive of maximal value of τ related to simple maximization concept solution MAX(H). Values presented in Tables 2 and 3 are in reference to several control parameters configurations. β parameter is related to percent of the most discriminated demands and it affects the dynamic of the mean value changes. Different situation is for the values of the Gini coefficient. The changes of β parameter have not impact for this inequality measure. Ratio model gives a capability of obtain some set of solutions in reference to chosen τ values and shows a spectrum of it, from more to less fair. The goal of each iteration of optimization process is to return the

Table 4
Detail table of solution obtained for $\beta = 0.3$ and given values of τ

d	p_d	$\tau = 3667$	$\tau = 3950$	$\tau = 4000$	$\tau = 4100$	$\tau = 4200$	$\tau = 4300$	$\tau = 5000$	$\tau = 6500$	$\tau = 7800$
1	200	3666.67	3647.06	2461.54	9076.92	16000	20000	12000	32000	16000
2	50	3666.67	3647.06	1846.15	1846.15	0	0	0	0	0
3	150	3666.67	3647.06	1846.15	1846.15	1846.15	12000	9000	9000	9000
4	100	6166.67	6000	11000	10692.3	7923.08	12000	8000	6000	6000
5	60	3666.67	3647.06	1846.15	1846.15	1846.15	0	0	0	0
6	200	3666.67	4000	24000	18000	24000	18000	34000	18000	34000
7	50	3666.67	3647.06	1846.15	1846.15	1846.15	0	0	0	0
8	150	5750	6000	6000	6000	6000	6000	9000	9000	9000
9	100	5000	5000	10000	9692.31	6923.08	11000	7000	5000	5000
10	60	3666.67	3647.06	1846.15	1846.15	1846.15	0	0	0	0

highest possible grow of efficiency of the system with the lowest possible loss of fairness. Figures 3 and 4 presents a graphs of changes of just mentioned values according to value of τ. Additional in Fig. 4 are added bars to visualize the changes of most discriminated value for value of $\beta = 0.3$ MD(0.3). Just mentioned degrees are suitable for the assessment of obtained solution but it not include the information about assigned 0 values to given demands. In some cases the situation such that (where at least one of objective function vector value gets 0) provides the non-acceptable judgment in the terms of justice. Considering solutions obtained for $\beta = 0.3$ the most visible growth of the mean value and the Gini index, according to increase of τ, is for the third iteration. Table 4 presents the solutions for this value of β in details. The outcome vectors which contains all non-zero values were assigned for the four first iterations of considered values of τ. It is necessary to decide if the solution which takes a 0 for at least one value of objective functions is automatically not fair. If such an assumption is made, according to Table 4 the solutions obtained for $\tau \geq 4200$ should be rejected in terms of fairness criterion. In calculations, the algorithm was stopped when τ parameter reached the value equal to simple maximization solution. Considering Figs. 3 and 4, the algorithm should be stopped some steps earlier. However, in the paper there was presented approach of finding the spectrum of solutions more or less fair, to demonstrate the range of options from the most fair to the most efficient. Next step belongs to the decision-maker who has his own preferences and may decide about the fairness degree of the selected solution.

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