TRANSNAV
http://www.transnav.eu
the International Journal on Marine Navigation and Safety of Sea Transportation
Volume 7 Number 4 December 2013

DOI: 10.12716/1001.07.04.01

Determination of Inertia Forces Acting on Break Bulk Cargo en Route

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ABSTRACT: The paper presents the analytical method of defining inertia forces that act on break bulk cargo as a result of the oscillatory motion of the vessel exposed to the effect of ambient forces. Considering that the linear models of roll, pitch and heave applicable in this case, the problem is solved by expressing the angle of heel, the angle of pitch, and the amplitude of heave. The obtained functions are differentiated and the inertia forces are determined by means of applying the Newton's second law.

1 INTRODUCTION

In the sea transportation of break bulk cargo, particularly that of the non-standard dimensions, both the weight of each load and the acting forces of inertia should be considered when designing the securing arrangement of the cargo. The inertia forces can be evaluated once the linear accelerations affecting the cargo are known, which depend on the laws of linear displacement changes for the cargo and the deck of the vessel relative to the reference coordinate system.

Generally the oscillatory motion of the ship is characterized by six degrees of freedom and is described by six differential equations. The ship oscillations are strongly coupled [1]. It is shown in [2] that one can apply the linear models of roll, pitch and heave to obtain the linear acceleration in the first approximation, i.e. use the corresponding isolated equations for the calculation.

To be able to find the inertia forces acting on a cargo item in an inertial reference frame it is enough to apply the Newton's second law, provided that the mass of the unit and the respective accelerations are known. The accelerations can be found as the second time derivatives of the angle of heel, the angle of pitch, and the amplitude of heave functions.

The expressions for the angles and the amplitude can be obtained by solving the equations for roll, pitch, and heave. This is done on the assertion that for the task of finding the forces of inertia the equations for roll, pitch, and heave can be considered decoupled [3].

2 PARAMETERS OF OSCILLATIONS

2.1 *Roll*

As roll is the governing factor and generates dominant forces of inertia further considerations proceed with the equation that define roll solving the problem so as to find the expression for the angle of heel θ. For this, as suggested in [4], we use the original second-order linear differential equation that defines the roll angle of a vessel θ:

$$\left(J_x + m_x\right)\ddot{\theta} + \mu_x\dot{\theta} + Dh_0\theta = \chi_\theta Dh_0 \sin\omega_k t \qquad (1)$$

where J_x = moment of inertia of the vessel about the longitudinal axis X-X; m_x = generalized added masses of water about the longitudinal axis X-X; μ_x = damping coefficient about the longitudinal axis X-X; D = displacement of the vessel (force of gravity); h_0 = transverse initial meta-centric height; χ_θ = reduction coefficient for the roll oscillations; ω_k = the apparent frequency of the waves.

After dividing the equation (1) by the coefficient of the highest derivative we obtain the normalised form of the equation:

$$\ddot{\theta} + 2h\dot{\theta} + \omega_0^2\theta = \chi_\theta\omega_0^2 \sin\omega_k t \qquad (2)$$

where roll damping coefficient h:

$$h = \frac{\mu_x}{2\left(J_x + m_x\right)}$$

and eigenfrequency of the rolling vessel ω_0:

$$\omega_0^2 = \frac{Dh_0}{\left(J_x + m_x\right)}$$

The expression (2) is a linear non-homogeneous differential equation with constant coefficients, and its solution is the sum of a particular solution θ_r, which describes the forced oscillation of the vessel about the axis X-X influenced by the regular waves, and the solutions of the corresponding homogeneous equation, which describes own damped oscillations of the ship.

Since the amplitude of the vessel's own damped oscillations turns to zero rather quickly, the equation of roll, as a stationary process, according to [4] can be described as forced oscillations only, i.e.:

$$\theta = \frac{\chi_\theta\omega_0^2}{\left[\left(\omega_0^2 - \omega_k^2\right)^2 + 4h^2\omega_k^2\right]^{0,5}} \times \sin\left[\omega_k t - arctg\left(\frac{2h\omega_k}{\omega_0^2 - \omega_k^2}\right)\right] \qquad (3)$$

2.2 *Pitch*

Similarly to the case of roll, as it was demonstrated in the works [3, 4] ship performs forced oscillations with the frequency of ω_k while pitching. The isolated equation of longitudinal pitching, as well as its solution, has structure similar to the structure of the transverse rolling equation, i.e. describes not only the vessel's own damped oscillations, but also the forced harmonic oscillations with the pitch frequency. This way the expression that defining the current angle of trim β is similarly characterized by the induced harmonious vibrations with the pitch frequency ω_k:

$$\beta = \frac{\chi_\beta\omega_{0\beta}^2}{\left[\left(\omega_{0\beta}^2 - \omega_k^2\right)^2 + 4h_\beta^2\omega_k^2\right]^{0,5}} \times \sin\left[\omega_k t - arctg\left(\frac{2h_\beta\omega_k}{\omega_{0\beta}^2 - \omega_k^2}\right)\right] \qquad (4)$$

where χ_θ = reduction coefficient for the roll oscillations; ω_k = eigenfrequency of the pitching vessel; h_β = pitch damping coefficient.

2.3 *Heave*

Finally, heave is the result of the orbital motion of the vessel on a radius equal to the half of the wave height [3, 5]. Heave motion ζ has harmonic character with the frequency of oscillations ω_k and can be described as follows:

$$\zeta = \zeta_0 \sin\left(\omega_k t\right) \qquad (5)$$

where ζ_0 = amplitude of the vertical motion induced by the waves with the height of h_w:

$$\zeta_0 = 0{,}5h_w \qquad (6)$$

3 FORMULATING THE INERTIA FORCES

The resulting expressions (3), (4) and (5) allow us to calculate the angular accelerations of the roll and pitch, the linear acceleration and inertia forces acting on the cargo. From this we find the inertia forces induced by roll, pitch and heave that act on a cargo unit with the mass m_c.

The most substantial is the lateral force of inertia of the roll F_θ. It is obvious that:

$$F_\theta = -m_c a_y$$

where a_y = linear acceleration due to roll.

In its turn, the linear acceleration a_y is the product of the angular acceleration $\ddot{\theta}$ by the radius of curvature r_y relative to the longitudinal axis passing through the center of gravity of the vessel G, i.e.:

$$a_y = r_y\ddot{\theta}$$

Thus finding the angular acceleration $\ddot{\theta}$ as the second derivative of the roll angle by differentiating twice the expression (3) yields:

$$\ddot{\theta} = -\theta_0\omega_k^2 \sin\left(\omega_k t - \psi\right)$$

where

$$\theta_0 = \frac{\chi_\theta \omega_0^2}{\left[\left(\omega_0^2 - \omega_k^2\right)^2 + 4h^2\omega_k^2\right]^{0,5}}$$

$$\psi = arctg\left(\frac{2h\omega_k}{\omega_0^2 - \omega_k^2}\right)$$

Successively the inertia force F_θ is defined as:

$$F_\theta = m_c r_y \theta_0 \omega_k^2 \sin\left(\omega_k t - \psi\right) \tag{7}$$

The longitudinal force F_θ can be derived in much the same way, i.e. $F_r = - m_c a_x$ where a_x = linear acceleration due to pitch.

The linear acceleration in this case is $a_x = r_x\ddot{\beta}$ where r_x = radius of curvature relative to the transverse axis. The angular acceleration β can be obtained by differentiating the expression (4) twice:

$$\ddot{\beta} = -\beta_0 \omega_k^2 \sin\left(\omega_k t - \psi_\beta\right)$$

$$\beta_0 = \frac{\chi_\beta \omega_{0\beta}^2}{\left[\left(\omega_{0\beta}^2 - \omega_k^2\right)^2 + 4h_\beta^2\omega_k^2\right]^{0,5}}$$

$$\psi_\beta = arctg\left(\frac{2h_\beta\omega_k}{\omega_{0\beta}^2 - \omega_k^2}\right)$$

Therefore the force of inertia F_θ is represented by the equation:

$$F_\beta = m_c r_x \beta_0 \omega_k^2 \sin\left(\omega_k t - \psi_\beta\right) \tag{8}$$

The heaving force of inertia F_ζ is formulated as

$F_\zeta = - m_c\zeta$

Linear acceleration ζ we get as the second derivative of the expression (5):

$$\ddot{\zeta} = -\zeta_0 \omega_k^2 \sin\left(\omega_k t\right)$$

Then, taking into account equation (6) we finally put F_ζ as:

$$F_\zeta = 0{,}5 h_w m_c \omega_k^2 \sin\left(\omega_k t\right) \tag{9}$$

It is to be noted that the inertia forces F_θ, F_β, and F_ζ were obtained with the reference to the unperturbed system of coordinates. Then in order to be able to calculate the reactions in lashings of the cargo these inertial forces and the force of gravity P_c must be projected on to the ship's frame of axes which is inclined by the angles of heel θ and trim β as shown in the Figure 1.

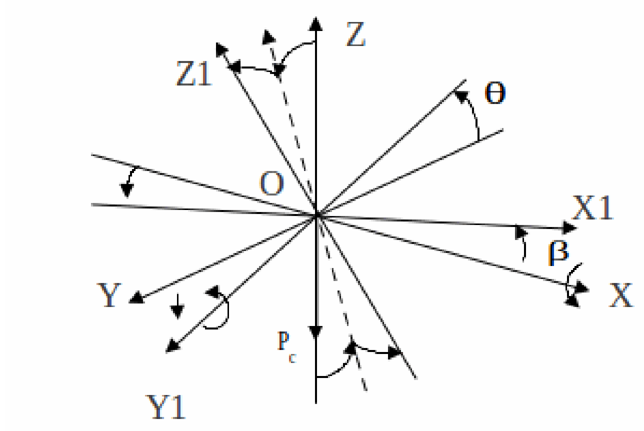

Figure 1. Frame of axes referenced to the unperturbed system of coordinates.

4 REMARKS AND CONCLUSIONS

The article describes the method of deriving the inertia forces acting on a cargo unit so these forces can be accounted for in further calculations to determine the maximum working load of lashings for the cargo. The method is based on the presumption that the linear models of roll, pitch and heave are sufficient for the case and can be considered independent within the scope of the problem. The resulting expressions of the angle of heel, the angle of trim and the amplitude of vertical motion induced by waves allow calculating the respective angular and linear accelerations. The inertia forces determined in the unperturbed reference frame can be easily ported to the ship's system of coordinates as the relation between the two systems of coordinates is known. The obtained functions are used by the author in his mathematical model describing the process of safe stowage and lashing of break bulk cargo on board a ship.

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