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ESTIMATION OF THE MINING DAMAGE RISK IN THE HYPOTHETICAL IMPACT AREA OF THE CONCURRENT PROCESSES OF ROCK MASS DISORDERS

SZACOWANIE RYZYKA POWSTANIA SZKODY GÓRNICZEJ W OBSZARZE HIPOTETYCZNEGO ODDZIAŁYWANIA WSPÓŁBIEŻNYCH PROCESÓW ZABURZEŃ GÓROTWORU

The aim of this work is the estimation of the risk of mining damage occurrence, based on uncertain information regarding the impact of the concurrent processes of deformation and vibration. This problem concerns the experimental and theoretical description of the so-called critical phenomena occurring during the reaction mining area ↔ building object. Post-mining deformations of the rock mass medium and paraseismic vibrations can appear at a considerable distance from the sub-area of the mining operation – hence, the determination of the measures of their impacts is usually somewhat subjective, while the estimation of the mining damage based on deterministic methods is often insufficient. It is difficult to show the correlation between the local maximum of the impact of the velocity vector amplitude and the damage to the building – especially if the measures of interaction are not additive. The parameters of these impacts, as registered by measurements, form finite sets with a highly random character. Formally, it is adequate to the mapping from the probability space to the power set. For the purposes of the present study, the Dempster – Shafer model was used, where space is characterised by subadditive and superadditive measures. Regarding the application layer, the conclusions from the expert evaluations are assumed to be the values of random variables. The model was defined, and the risk of damage occurrence was estimated.

Keywords: mining area, risk of damage, stochastic process, uncertainty, model, expert inference, imprecise probability

Celem pracy jest szacowanie ryzyka powstania szkody górniczej, poprzez niepewne informacje dotyczące oddziaływania współbieżnych procesów deformacyjnych i drgań. Problem dotyczy doświadczalnego i teoretycznego opisu tak zwanych zjawisk krytycznych, zachodzących podczas reakcji teren górniczy ↔ obiekt budowlany. Pogórnicze deformacje ośrodka oraz drgania parasejsmiczne ujawniają się również w znacznej odległości od podobszaru eksploatacji – stąd też wyznaczenie miar tych oddziaływań z reguły jest nieco subiektywne a szacowanie szkody górniczej metodami deterministycznymi często jest niewystarczające. Trudno jest wykazać, że istnieje skorelowanie pomiędzy lokalnym maksimum oddziaływania amplitudy wektora prędkości a szkodą w obiekcie – zwłaszcza, jeśli miary oddziaływania

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nie są addytywne. Zarejestrowane w wyniku pomiaru parametry tych oddziaływań to zbiory skończone o charakterze silnie losowym. Formalnie jest to odwzorowanie z przestrzeni probabilistycznej do zbioru potęgowego. Dla celów niniejszej pracy wykorzystany został model Dempstera – Shafera, gdzie przestrzeń charakteryzują miary pod lub nadaddytywne. W warstwie aplikacyjnej skorzystano z konkluzji ocen eksperckich przyjmując je, jako wartości zmiennej losowej. Zdefiniowano model i oszacowano ryzyko wystąpienia szkody.

Słowa kluczowe: obszar górniczy, ryzyko powstania szkody, proces stochastyczny, niepewność, model, wnioskowanie eksperckie, nieprecyzyjne prawdopodobieństwo

1. Introduction

As in almost every business activity, implementing mining operations in technically urbanised areas is associated with risks and uncertainties. A precisely planned and implemented extraction of useful deposits located under building structures does not ensure the complete technical safety of the mining activities and may sometimes cause hazards (failure – mining damage) to facilities existing in the sub-area. An essential problem in estimating the risk of damage results from the fact that the measures of the impact on buildings caused by concurrent processes are subject to high uncertainty. The risk that can be variously defined, in this case, was estimated as a poorly measurable uncertainty.

2. Characteristics of the problem

Extraction of a deposit via underground mining causes a change in the primary stress in the rock mass, resulting in a translocation of the overlaid strata towards the post-mining void. The so-called supplementary pressure, generated by the gravitational field in the vicinity of the emptiness, often induces a change in the parameters of the physical system. Moreover, it is known that the physical system (single or multi-phase) remains in equilibrium as long as its parameters are constant within the entire system. Therefore, the change of the parameters of the system or one of its parts usually causes the flow of global quantities: mass, energy, and so on. The behaviour of the layers of the medium within the surroundings of an underground working mine formally results from the dynamic state of stress and strain.

A building (an object) and its substratum form a system with different physical characteristics. Therefore, post-mining changes in the ground surface via deformation and vibration have a particularly significant influence on the stability of the object structure (Firek, 2011).

The state of the deformation of the substratum is determined by the indices of deformation.

Generally, the two processes of the transformation of the medium $\{Y_t\}$ and the deformation of an object $\{X_t\}$ $t \in [a, b]$ can be analysed as stochastic processes (Piwowarski, 2006). Observation of the medium transformation $\{Y_t\}$ is performed in discrete time $(t_0, t_1, ..., t_n)$. Thus, the approximation $\{Y_t = Y(t) = Y\}$ is a stepped process $Y_i\Big|_{i=1}^{n}$.

The randomness and incomplete definiteness of deforming post-mining processes often cause the phenomena models or physical processes to not be formulated on the basis of real variables (Piwowarski, 2005). Stochastic processes are more appropriate for mapping the real processes. The mining process of rock mass translocation, which causes the risk of deformations to buildings located in the endangered area, can be mapped by means of appropriate stochastic models.

Seismic bumps induced by underground mining have an impact on the facilities located within the para-seismic area – therefore, as a result of a number of studies and assessments (Cianciara, 2005; Dubiński, 2002; Firek, 2011), their impact on buildings is regulated by the relevant standards. The allowable velocity and acceleration limits of vibrations – based on experience and research – have been established (Dubiński, 2009). As a result, for a certain class of buildings, if the values of those parameters are not exceeded, then no utilitarian damage of the considered structure should occur.

The results of extensive research and analysis indicate that the assessment of the effects of the vibrations of the mining origin, based on registered records of accelerations, is poorly correlated with the damage to buildings. A significant correlation, that is, cause ↔ effect, is related to the association of the amplitude of the vibrations' velocity (formula of energy and momentum) with the damage of the object. Very similar statements (an important parameter being velocity) have been accepted in the countries of Western Europe and have been developed in the form of relevant standards.

The observations of the physical vibrations indicate that this is a short duration process (a few seconds), which means that the frequency of vibration occurrence should not result in material fatigue; moreover, these vibrations are not able to activate a resonance phenomenon.

The material points of the substratum interact with other material points of an object via direct contact or via the field. The movement of the points of an object is ruled by the principles of cause and effect. Observations of the movements of the points of an object and the points of the ground are performed by measurements.

On the level of modelling, the current state of the object does not usually coincide with the state of the operator of the studied feature; in this case, the object generates a response that differs from the modelling results. If the mapping results are grouped in the vicinity of one value and are fairly repetitive, then they are appropriate for the given configuration of forces. The complex state of the process is, in general, a superposition of the interactions of the base vectors.

Analytical determination of the state of hazard to the object, as a result of "specific" effects of mining activities (deformation and vibration), is a risk exposure subject.

3. Formulation of the problem

Formally, an activity generating risk involves a class of decision problems, which is determined by the probability distribution of the possible results as the consequence of the decision or implementation of the technological process. An important problem is the estimation of the risk of mining damage in the case of the absence of sufficient knowledge or high uncertainty regarding the impact of concurrent stochastic mining processes on the object's structure. Substantively, the problem consists of the estimation of the cumulative impact of significantly different physical processes, in the case when each of them is characterised by uncertainty with respect to the established measures.

This study assumes that the risk of damage R is a random variable associated with the probability space (Ω,Σ,P) (Piwowarski, 2005, 2006).

If each object located in a mining area is assigned to a risk of damage r_i, then the obtained set of risks is:

$$R \overset{df}{=} (r_1, r_2, \ldots, r_n) \tag{1}$$

The risks of damage to facilities in the mining area can be proven to be a set of independent risks, while for a “very large” set R, the aggregate risk distribution can be approximated by a normal distribution $N(m_r,\sigma_r^2)$. In this case, the risk model can be expressed as:

$$R \sim N(m_r, \sigma_r^2) \tag{2}$$

while the individual risk is determined on the basis of (3)

$$G(r_i) = E(r_i) + c\sigma^2(r_i) \tag{3}$$

where: G — risk as a random variable

$$c = \frac{\kappa_{1-\eta}}{2\sigma(R)};\ \kappa_{1-\eta} \text{ denotes } 1-\eta \text{ quantile; } \eta \in [0,1]$$

Generally a structure (object) x_i^k located in the deformation field becomes subject to damage or is not damaged. The consideration of the problem indicates that it is necessary to create a measure in the probabilistic space (Ω,Σ,P) i.e., $R \supset P_R$ (P_R is a measure of R), to assign the distribution to the random variable R before proceeding according to the previous statements.

However, there are situations in which the two-argument assignment proves to be insufficient; thus, the measure P for (Ω,Σ,P) space requires modification. Determining the value of the index w_i, characterising the hazard z_j is possible if, and only if, the danger to the object occurs after the m-th value ($m > 1$) of the specific index of ground deformation, recorded by means of measurements and associated with the hazard to the object.

4. The process of deformation generating mining damage

The post-mining force is considered to be a stochastic process of continuous trajectories that generates the damage. Usually, in mining areas – in a given sub-area – a process of the medium displacement can be observed.

Let the analysed process be multi-periodic, i.e., X_t, $t = 1,2,...,T$, with a finite number of possible scenarios, so that the probabilistic space $\Omega = \{\omega_1,\omega_2,...,\omega_k\}$ is the finite set, the family of events is $\Sigma = 2^\Omega$, and the probabilistic measure is such that:

$$P(\{\omega_i\}) > 0 \qquad i = 1,2,...,k$$

Let us introduce σ – of Σ_t $t \in \{0,1,...,T\}$ Σ_t field, which is interpreted as the knowledge regarding the process acquired until the moment t. Observations of the process mean that $\Sigma_t \subset \Sigma_S$ for $t \leq s$, so the sequence Σ_t is the filtration. However, in accordance with current knowledge regarding the process, it is impossible to make a prediction for the moment t if we do not know the value of the process at the moment $s < t$.

If $s \leq t \Rightarrow \Sigma_S \subset \Sigma_t$, then the probabilistic space can be defined as: $(\Omega;\Sigma;\{\Sigma_t\}_{t\in T},P)$. In this space, we impose a stochastic process $\xi(t)$:

$$\left.\begin{aligned} d\xi(t) &= b(t;\xi(t))dt + \sigma(t,\xi(t))dW(t) \\ \xi(t=0) &= \xi_0 \end{aligned}\right\} \tag{4}$$

where: $W(t)$ — Wiener process.

(4) can be transformed to the form:

$$d\xi_t - \sigma(t)\xi_t dW_t = b(t,\xi_t)dt \tag{5}$$

Multiplication of (5) by the integrating factor (6)

$$F_t = e^{-\int_0^t \sigma(s)dW_S + \frac{1}{2}\int_0^t \sigma^2(s)ds} \tag{6}$$

leads to the result:

$$F_t \cdot d\xi_t - F_t \cdot \sigma(t)\xi_t dW_t = F_t b(t,\xi_t)dt \tag{7}$$

The left side of (7) is the differential F_t. ξ_t, so we can write:

$$d(F_t X_t) = F_t \cdot b(t,\xi_t) \tag{8}$$

Considering that:

$$Y_t := F_t \cdot \xi_t \tag{9}$$

then:

$$dY_t = F_t \cdot b\left(t, \frac{Y_t}{F_t}\right) \tag{10}$$

Equation (10) can be written as:

$$d\xi_t = b(t,\xi_t)dt \tag{11}$$

By directly integrating (4), we obtain:

$$\xi(t) = \xi_0 + \int_0^t b(s;\xi(s))dt + \int_0^t \sigma(s,\xi(s))dW(s) \tag{12}$$

Deformation or destruction of the structure continuity in some of rock mass layers, are the effect of exceeding the limits of their strength. To determine the boundary condition, as a general rule, the following formula (De Campos, 1994; Klir, 1994) is accepted:

$$\left.\begin{array}{l}\Omega_a = \{x : f(x) = 0\} \\ P(X \in \Omega_a) = 0\end{array}\right\} \tag{13}$$

The final process (deformation) occurs after the destruction of the structure of the layers in the immediate vicinity of the rock mass disorder (emptiness).

Deformation state of the medium

$$\Omega_b = \{x : f(x) > .0\} \tag{14}$$

$f(x)$ — resistance to the destruction of some layers of the rock mass structure.

Irregular trajectories of the displacement process are analysed here as mean values. The function $M : I \times I \to R$ is called the average if:

$$\min(x,y) \leq M(x,y) \leq \max(x,y), \quad x,y \in I$$

Internal extremes

Let us assume that $f : X \times Y \to R$. Let the points $\{(x1, y1), (x2, y2)\} \in X \times Y$; moreover, if the points of the surface (hypersurface) have the property that in their neighbourhood there exist points situated on both sides of the plane, then the considered points are the internal extremes.

The state of the rock mass deformation is identified by the so-called indices of deformation "Y", which are the simple functions that, for the probability space, can be written as follows:

$$Y = \sum_{i=1}^{n} y_i \; 1_{F_i} \tag{15}$$

where:

$y_i\Big|_{i=1}^{n}$ — the values taken by Y,

$F_i = \{\omega : X(\omega) = y_i\} \in F$

As a result, in the space of a measure (probabilistic), the following integral can be defined in a natural way (Walley, 1996):

$$\int_Y f(y)P(dy) \tag{16}$$

The impact of the completed mining extraction on an object – the subject of the study

The procedure of estimating the hazard based on the uncertain assessment of the impact of concurrent mining processes on the object was the essence of further analysis. The state of the ground surface deformation was estimated with the use of the geological and mining documentation of the observed sub-area and own computer applications.

Field of displacements in R^3

The estimated state of the terrain deformation in the surroundings of the studied object has been parametrically characterised. The analysis indicates that there is no cause-effect relationship between the state of the ground deformation, where the building has been embedded, and the "damage" to that object.

General features of the object

One-storey, semi-basement building,
Partial basement, ground floor and first floor, made using traditional technology,
Object dimensions: floor plan (77.39 [m] × 15.66 [m]) × height (23.42 [m]),
Foundations – concrete,

Load-bearing walls of basement and ground floor – ceramic brick,
Load-bearing walls of second floor – slag cement blocks,
Ceilings of first and second floor: reinforced concrete structure,
Basement ceiling: construction of concrete on steel beams,
Gabled roof,
Media: water, electricity, central heating, plumbing.

Fig. 1. Forms of failure in an object in a residual deformation field

The building presented here was not the subject of direct observation in its infinitesimal vicinity, with reference to ground deformations and vibrations (the described below registration of vibrations was performed in points significantly distant to it). As a result of the estimation, it could be concluded that only the residual deforming transformations of the area may have occurred in the close neighbourhood of the object. Thus, the following relationship practically does not exist:

$$\text{hazard to object} \xleftarrow{\textit{relationship}} \text{ground deformation (of medium)}$$

Because the parameters of vibrations were not recorded directly, the influence of para-seismic phenomena on the damage to the object was estimated by specialists – this is a subjective assessment. Due to the subjectivity of the estimates, it can be assumed that the result of expert assessment ϕ_{ij} is a random variable.

5. General characteristics of para-seismic vibrations

A mining plant retains a direct registration of ground vibrations at several points on the surface – however, beyond the sub-area of object location, the equipment for recording acceleration and velocity of vibrations is used. The sample parameters of the mining bumps are given as follows (Tab. 1):

TABLE 1

Characteristics of the spatial distribution of the mining bumps with parameters of energy $E \approx k \cdot 10^9$ [J]
PGV = $f(d$; E = const); PGA = $f(d$; E = const); d – epicentral distance

MINING BUMP $E = 3 \cdot 10^9$ [J]							
Measuring site	**Epicentral distance *d* [m]**	**Velocity**			**Acceleration**		
		PGV [m/s]	**Duration *t* [s]**	**Degree of intensity**	**PGA [m/s^2]**	**Duration *t* [s]**	**Degree of intensity**
A	4868	0.0075	2.85	1	0.2415	2.69	1
B	4630	0.0081	3.09	1	0.2333	2.80	1
C	3062	0.0119	2.91	2	0.3570	2.50	1/2
D	8500	0.0022	4.49	0	0.0564	4.38	0
E	4127	0.0087	2.95	1	0.2847	2.88	1
F	3393	0.0152	3.18	2	0.3948	2.79	2
G	3405	0.0257	2.07	3	0.7196	2.08	2
H	2421	0.0082	5.01	1	0.2202	4.04	1
I	3490	0.0087	4.90	1	0.1804	3.93	1
J	1206	0,0325	4.14	3	0.4336	2.86	2

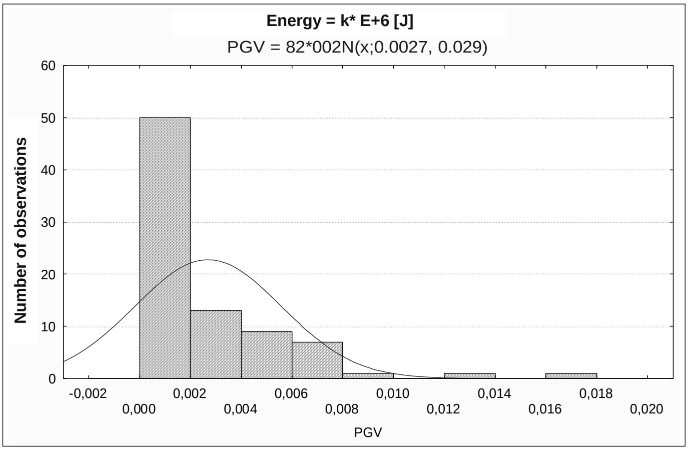

Fig. 2. Histogram of the spatial distribution of the vibration velocity amplitude of energy $E = k \times 10^6$ [J]

TABLE 2

Statistical parameters of the test, including measurements of the speed and acceleration of the vibrations

GROUPS	PGV MEAN	PGA MEAN	T STATISTICS	DF
	0.074121	0.002670	7.790262	162
PGV ~ PGA	σ_{PGV}	σ_{PGA}	SNEDECOR F TEST	P – VALUE
	0.083005	0.002875	833.5292	0.00

The performed statistical analyses indicate the independence of the variables PGV and PGA in the case when the $P_{VALUE} < \alpha$; where: $\alpha = 0.05$ (level of significance).

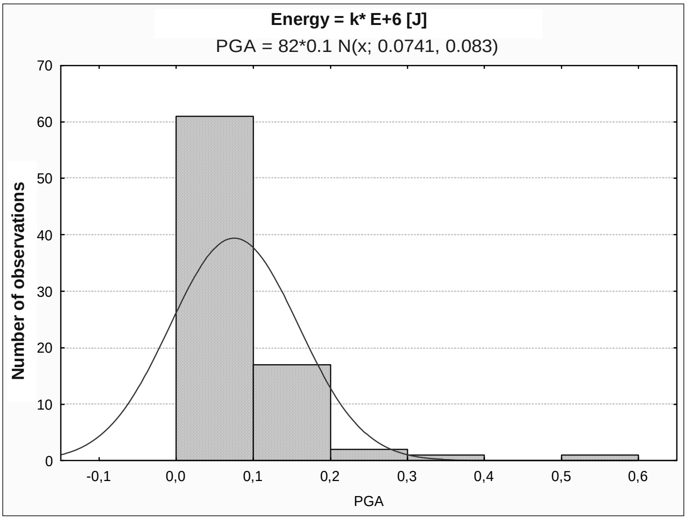

Fig. 3. Histogram of the spatial distribution of the vibration acceleration amplitude of energy $E = k \times 10^6$ [J]

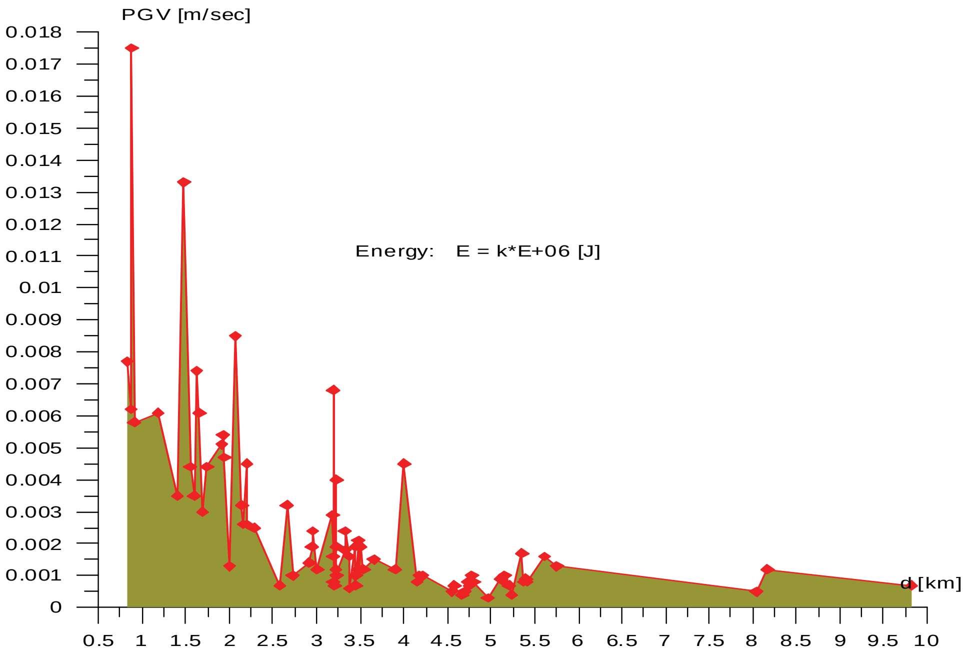

Fig. 4. Distribution of the vibration velocity amplitude as a function of the distance to the seismic bump source

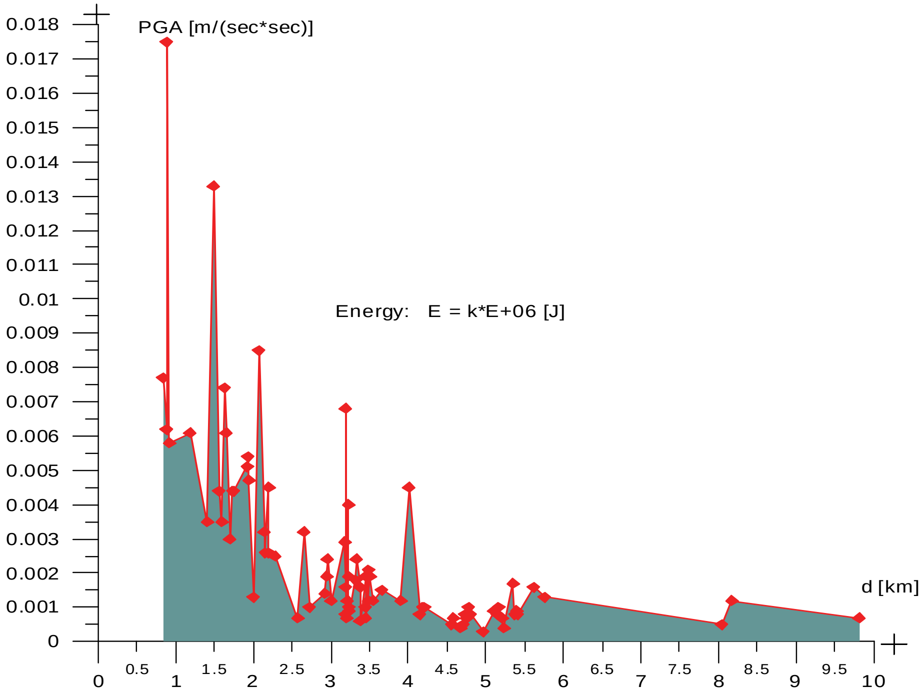

Fig. 5. Distribution of the vibration acceleration amplitude as a function of the distance to the seismic bump source

The mappings (measurements) indicate that the observed distributions of the parameters of vibrations are of a strongly irregular character.

6. Estimating the risk of mining damage occurrence

The parameters of the vibrations and the indices of deformations, estimated on the basis of modelling or the registered measurements, are finite sets of a highly random character. Formally, this estimation process corresponds to a mapping from a probability space to the power set. Therefore it is appropriate, regarding the uncertainty of the data, to provide systems for drawing conclusions of special processing mechanisms that allow for the characterisation of the nature and the degree of the imperfection of the knowledge from the detector as well as of the new knowledge derived on the basis of the system's inference.

Risk assessment is a difficult problem here because the value of the risk is a "combination" of the probability of an event occurrence and is dependent on the evaluation of the probabilities of all possible consequences as well as the values of the individual effects.

The definition of risk consists mainly of two components:

- the occurrence of adverse effects,
- the probability of their occurrence.

Formally, this definition can be written as risk $\subset$ probability $\cup$ measure of risk (hazard) and can be expressed in the form of functional dependencies:

$$r = f(p, \mu, S) \tag{17}$$

where:

p	—	probability of the event occurrence,
μ	—	measure of risk,
S	—	strategy of the event creation.

Many decision-making situations are dominated by uncertainty, defined as the subjective uncertainty (the use of incomplete knowledge). Hypotheses concerning the damage caused to the building located in the area of the random risk are formulated by experts. Hence, inference with the use of classical probability theory methods is not always possible. Therefore, there is a need of applying the inference methods that enable the formalisation of a subjective uncertainty (Parsons, 1994; Wang, 1994).

To define a mathematical model for estimating the parametric indicators of expertise in the field of the damage that occurred, a random variable (ϕ_{ij}) was introduced. The index i concerns the assessment of the i-th object, while the index j relates to the j-th expert. In this case, the random variable ϕ_{ij} is of a two-point distribution.

$$p\left(\sim \wp(1,p), \quad i = 1,\ldots,n, \ \ j = 1,2\right) \tag{18}$$

The correlation between a pair of expert judgment is $Corr(\phi_{i1}, \phi_{i2}) = \rho$.

Analytically, this correlation can be expressed by the formula (19):

$$Corr(\phi_{i1}, \phi_{i2}) = \frac{\mathrm{Cov}(\phi_{i1}\phi_{i2})}{\sqrt{Var(\phi_{i1}) \cdot Var(\phi_{i2})}} \tag{19}$$

and from (19), the following can be concluded:

$$E(\phi_{i1}\phi_{i2}) = Corr(\phi_{i1}\phi_{i2}) \cdot Var(\phi_{i1}) + E(\phi_{i1})^2 \tag{20}$$

Assuming that:

$$P(\phi_{ij} = 1) = p \text{ and } P(\phi_{ij} = 0) = q, \text{ where } q = 1 - p \tag{21}$$

Taking into account (21), (20) can be expressed as follows:

$$E(\phi_{i1}\phi_{i2}) = \rho \cdot p \cdot q + p^2 \tag{22}$$

The total distribution of random variables ϕ_{i1} and ϕ_{i2} is:

$$P(\phi_{i1} = 0, \phi_{i2} = 1) = P(\phi_{i1} = 1, \phi_{i2} = 0) = p \cdot q(1 - \rho) \tag{23}$$

Characteristics of the damage estimation by experts

The results obtained from expert estimation were: $p = 0.8$, $q = 0.2$ and $\rho(\phi_{i1}, \phi_{i2}) = 0.32$. On the basis of (23), we obtain $P(\phi_{i1} = 0, \phi_{i2} = 1) = 0.0512$. In this case, mining exploitation, which is indicated as a reason for the impact on the considered object, is unlikely. The object is physically located beyond the area of direct ground deformation at the terrain of the 0-th category of para-seismic hazards.

Methods of imprecise probability

Hazards induced by mining activity are characterised by a subjective uncertainty, making it impossible for the use of the classical theory of probability as the method analysing the process. Therefore, for the synthesis of inference on the basis of the expert assessment, the methods of imprecise probability (Dempster-Shafer method) (Parsons, 1994; Wang, 1994), as a generalisation of the Bayesian theory and the theory of Zadeh, were applied in this paper. These methods contain mechanisms for the synthesis of information from different sources and the creation of a coherent base knowledge based on various (often contradictory) information.

The base probability distribution satisfies the following conditions:

$$m : 2^{\Theta} \to [0,1];\ m[0] = 0;\ \sum_{A \in Q} m(A) = 1; \forall B \subseteq \Theta\ (B \notin Q \text{ we obtain } m(B) = 0) \tag{24}$$

Certainty (uncertainty) of the "A" hypothesis is represented by the interval:

belief ↔ *plausibility*

$$[Bel(A), Pl(A)] \quad \text{where: } Bel(A) = \sum_{B \subseteq A} m(B);\ \ Pl(A) = \sum_{B \subseteq A} m(B) \tag{25}$$

In the assessment process of the damage to the building in Figure 1, the experts established that the random variable (damage) can take values from the set {a, b, c}, where:

a – the impact of ground deformation,

b – the impact of ground vibrations induced by para-seismic bumps,

c – other causes.

According to (24), the set of all possible subsets interacting on the object is:

$$\Theta = \{(a),(b),(c),(a,b),(a,c),(b,c)(a,b,c)\} \tag{26}$$

The developed opinions result in the basic probability distribution of the variable *A*, as presented in Table 3.

TABLE 3

A	(a)	(b)	(c)	(a, b)	(a, c)	(b, c)	(a, b, c)
$m_1(A)$	0	0.7	0.1	0.1	0	0	0.1
$m_2(A)$	0.1	0.1	0.5	0	0.15	0.15	0

According to (25), function $Bel(a) = m(a)$; function $Bel(a,b) = \sum_{B \subseteq A} m(B)$.

As a result, we obtain:

TABLE 4

A	(a)	(b)	(c)	(a, b)	(a, c)	(b, c)	(a, b, c)
$Bel_1(A)$	0	0.7	0.1	0.8	0.1	0.8	1
$Bel_2(A)$	0.1	0.1	0.5	0.2	0.6	0.15	1

Function plausibility $P(A) = \sum_{B \cap A \neq 0} m(A)$

TABLE 5

A	(a)	(b)	(c)	(a, b)	(a, c)	(b, c)	(a, b, c)	$\sum_{i=1}^{n} (Pl_k - Bel_k)(A)$
$P_1(A)$	0.2	0.9	0.2	0.9	0.2	0.9	1	
$P_2(A)$	0.1	0.25	0.8	0.35	0.9	0.75	1	
$(Pl_1 - Bel_1)(A)$	0.2	0.2	0.1	0.1	0.1	0.1	0	0.8
$(Pl_2 - Bel_2)(A)$	0	0.15	0.3	0.15	0.3	0.6	0	1.5

Tables 3, 4, 5 contain the measures for estimating the mining interactions that generate damage to the object.

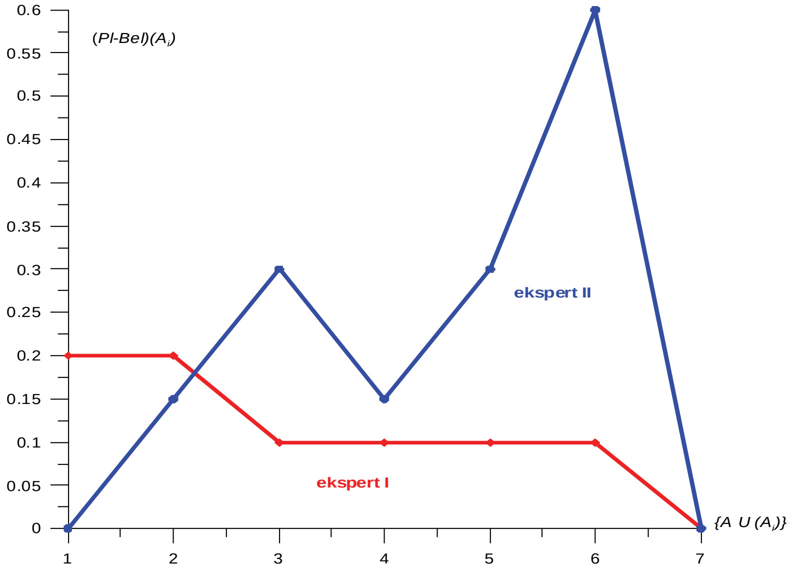

Fig. 6. Distribution of the impact of mining hazards subsets $\{a_i\} \subset A$ in the domain of mining damage risk

Introduction of the objective uncertainty associated with the random nature of the considered process and the concepts of the subjective uncertainty associated with uncertain knowledge allowed for the estimation of an effect of the declared impacts of the Θ set on the considered object. Table 5 indicates that a significant influence on the damage of the object is caused by vibrations and other reasons (the technical condition of the object).

7. Summary

The considerations in the domain of estimating an impact of individual mining hazards indicated that geology has a significant influence on the spatial distribution of the wave phenomena that are manifested as para-seismic vibrations in the space of deformation. Hazardous processes implied by mining activity are usually poorly defined in terms of modelling. The methods of imprecise probability make it possible to estimate the risk of mining damage occurrence.

Randomness and incomplete definiteness of destructive post-mining processes indicate that the phenomena models or physical processes should often not be formulated on the basis of real variables. Possibilities exist to apply appropriate stochastic models to the mapping of mining process of rock mass movements causing deformation to building site.

The risk, as a random variable, exhibits a complex nature of distribution. This distribution contains a variable describing the number of damage accidents and a variable describing the size of the damage. Hazards caused by underground mining – para-seismic vibrations (even at a considerable distance from the source) usually cause little damage to buildings. Therefore, it is necessary here to consider different spatial distributions of the propagation of vibrations.

The allocation of risk cannot be accepted a priori. The distribution should be determined on the basis of the relevant statistics. Hence, the hazard to the object caused by the impact of underground mining should be complemented by an estimation of the risk (both single risk and a set of risks) of the mining damage in a particular physical space.

The introduction of the method of imprecise probability (method of Dempster-Shafer) to the analysis of the post-mining risks can be the formal tool to control the spatiotemporal development of mining operations in terms of the minimisation of the local hazard.

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