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Oil transport in port
Part 3
Port oil piping transportation system safety and resilience impacted by the climate-weather change process and operation process

Keywords

piping system, operation and climate-weather change impacts, safety, resilience, cost analysis, optimization

Abstract

The paper is concerned with the application of the model of critical infrastructure safety prediction with considering its operation and climate-weather change impacts. The general approach to the prediction of critical infrastructure safety and resilience is proposed and the safety and resilience indicators are defined for a critical infrastructure impacted by its operation process and the climate-weather change process. Moreover, there is presented the model application for port oil piping transportation system safety and resilience prediction. Further, the cost analysis and optimisation of critical infrastructure operation process impacted by climate-weather change is proposed and applied to the considered piping system.

1. Introduction

This paper is the last part of the series of four papers proposed to comprehensive modelling and prediction of the safety and resilience of critical infrastructures with application to the port oil piping transportation system safety and resilience prediction in the scope of the EU-CIRCLE project Case Study 2, Storm and Sea Surge at Baltic Sea Port presented in JPSRA 2018, Vol. 9, No 2.

First, the operation process related to climate-weather change at the critical infrastructure operating area is considered, its parameters are introduced and its main characteristics are found. Next, the notions of the safety analysis of critical infrastructure impacted by operation process and to climate-weather change process are introduced, i.e. the conditional and unconditional safety function and the critical infrastructure risk function are defined.

Moreover, the critical infrastructure and its assets main safety characteristics and indicators are determined, i.e. the mean lifetime and standard deviation in the safety state subset, the intensities of degradation (ageing) and the indicator of critical infrastructure resilience to operation process and climate-weather change process impact.

Further, the IMCIS Model 4 created in [EU-CIRCLE Report D3.3-Part4, 2017] is applied to the port oil piping transportation system. Safety and resilience indicators are determined for the port oil piping transportation system, the operation cost analysis is performed and optimization of piping operation process is presented.

2. Critical infrastructure safety model related to climate-weather change process and operation process – IMCIS 4

In this section, we consider the critical infrastructure related to the operation process related to the climate-weather change process $ZC(t),\ t \in <0,\infty)$, impacted in a various way at this process states zc_{bl}, $b = 1,2,...,\nu,\quad l = 1,2,...,w.$ We assume that the changes of the states of operation process related to the climate-weather change process $ZC(t)$,

$t \in <0,\infty)$, at the critical infrastructure operating area have an influence on the critical infrastructure safety structure and on the safety of the critical infrastructure assets A_i, $i = 1,2,...,n$, as well.

We assume, as in [EU-CIRCLE Report D3.3-Part1, 2017], that the critical infrastructure during its operation process is taking $\nu, \nu \in N$, different operation states $z_1, z_2, \ldots, z_\nu$. Further, we define the critical infrastructure operation process $Z(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{z_1, z_2, \ldots, z_\nu\}$. Further, we assume that we have either calculated analytically or evaluated approximately by experts the vector of limit values of transient probabilities (OPC1)

$$p_b(t) = P(Z(t) = z_b),\ t \in <0,+\infty),\ b = 1,2,...,\nu,$$

of the critical infrastructure operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,...,\nu$,

$$[p_b]_{1\text{'x}\nu} = [p_1, p_2, ..., p_\nu].$$

Moreover, as in [EU-CIRCLE Report D3.3-Part3, 2017], we assume that the climate-weather change process $C(t)$, $t \in <0,+\infty)$, at the critical infrastructure operating area is taking w, $w \in N$, different climate-weather states $c_1, c_2, \ldots, c_w$. Further, we assume that we have either calculated analytically using the above parameters of the climate-weather change process or evaluated approximately by experts the vector of limit values of transient probabilities (C-WCPC1)

$$q_l(t) = P(C(t) = c_l),\ t \in <0,+\infty),\ b = 1,2,...,l,$$

of the climate-weather change process $C(t)$ at the particular climate-weather states c_l

$$[q_l]_{1\text{x}l} = [q_1, q_2, ..., q_w].$$

Under the assumption that the critical infrastructure operation process $Z(t)$, $t \in <0,+\infty)$, and the climate-weather change process $C(t)$ are independent, we introduce the joint process of critical infrastructure operation process and climate-weather change process called the critical infrastructure operation process related to climate-weather change marked by

$$ZC(t),\ t \in <0,+\infty),$$

and we assume that it can take $\nu w, \nu, w \in N$, different operation states

$$zc_{11}, zc_{12}, \ldots, zc_{\nu w},$$

We assume that the critical infrastructure operation process related to climate-weather change $ZC(t)$, at the moment $t \in <0,+\infty)$, is at the state zc_{bl}, $b = 1,2,..., \nu$, $l = 1,2,..., w$, if and only if at that moment, the operation process $Z(t)$ is at the operation states z_b, $b = 1,2,..., \nu$, and the climate-weather change process $C(t)$ is at the climate-weather state c_l, $l = 1,2,..., w$, what we mark as follows:

$$(ZC(t) = zc_{bl}) \Leftrightarrow (Z(t) = z_b \cap C(t) = c_l),$$
$$t \in <0,+\infty),\ b = 1,2,..., \nu,\ l = 1,2,..., w.$$

Further, the transient probabilities of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states zc_{bl}, $b = 1,2,..., \nu$, $l = 1,2,..., w$, can be defined by

$$pq_{bl}(t) = P(ZC(t) = zc_{bl}),\ t \in <0,+\infty),\ b = 1,2,..., \nu,$$
$$l = 1,2,..., w.$$

In the case when the processes $Z(t)$ and $C(t)$ are independent the expession for the transient probabilities can be expressed in the following way

$$pq_{bl}(t) = P(ZC(t) = zc_{bl}) = P(Z(t) = z_b \cap C(t) = c_l)$$
$$= P(Z(t) = z_b) \cdot P(C(t) = c_l) = p_b(t) \cdot q_l(t),$$
$$t \in <0,+\infty),\ b = 1,2,..., \nu,\ l = 1,2,..., w,$$

where $p_b(t)$, $b = 1,2,..., \nu$, are the transient probabilities of the operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,...,\nu$, and $q_l(t)$, $l = 1,2,..., w$, are the transient probabilities of the climate-weather change process $C(t)$ at the particular climate-weather states c_l, $b = 1,2,..., w$.

Hence the limit values of the transient probabilities of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states zc_{bl}, $b = 1,2,..., \nu$, $l = 1,2,..., w$,

$$pq_{bl} = \lim_{t \to \infty} pq_{bl}(t),\ b = 1,2,..., \nu,\ l = 1,2,..., w, \qquad (1)$$

can be found from

$$pq_{bl} = p_b q_l,\ b = 1,2,...,v,\ l = 1,2,...,w, \quad (2)$$

where p_b, $b = 1,2,...,v,$ are the limit transient probabilities of the operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,...,v,$ and q_l, $l = 1,2,...,w,$ are the limit transient probabilities of the climate-weather change process $C(t)$ at the particular climate-weather states c_l, $b = 1,2,...,w.$

Other interesting characteristics of the critical infrastructure operation process $ZC_{bl}(t)$ are its total sojourn times $\hat{\theta}\hat{C}_{bl}$, $b = 1,2,...,v,$ $l = 1,2,...,w,$ at the particular operation states zc_{bl}, $b = 1,2,...,v,$ $l = 1,2,...,w,$ during the fixed sufficiently large critical infrastructure operation time θ. They have approximately normal distributions with the expected values given by

$$\hat{M}\hat{N}_{bl} = E[\hat{\theta}\hat{C}_{bl}] = pq_{bl}\theta,\ b = 1,2,...,v,\ l = 1,2,...,w, \quad (3)$$

where pq_{bl}, $b = 1,2,...,v,$ $l = 1,2,...,w,$ are given above by (2).

2.1. Critical infrastructure safety indicators

We denote the critical infrastructure conditional lifetime in the safety state subset $\{u,u+1,...,z\}$, $u = 1,2,...,z,$ while the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, at the critical infrastructure operating area is at the state zc_{bl}, $b = 1,2,...,v,$ $l = 1,2,...,w,$ by $[T^4(u)]^{(bl)}$, $u = 1,2,...,z,$ $b = 1,2,...,v,$ $l = 1,2,...,w,$ and the conditional safety function of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, by the vector [EU-CIRCLE Report D3.3-Part4, 2017]

$$[\boldsymbol{S}^4(t,\cdot)]^{(bl)} = [1,\ [\boldsymbol{S}^4(t,1)]^{(bl)},\ ...,\ [\boldsymbol{S}^4(t,z)]^{(bl)}], \quad (5)$$

with the coordinates defined by

$$[\boldsymbol{S}^4(t,u)]^{(bl)} = P([T^4(u)]^{(bl)} > t \mid ZC(t) = zc_{bl}) \quad (6)$$

for $t \in <0,\infty)$, $u = 1,2,...,z,$ $b = 1,2,...,v,$ $l = 1,2,...,w.$

The safety function $[\boldsymbol{S}^4(t,u)]^{(bl)}$, $t \in <0,\infty)$, $u = 1,2,...,z,$ $b = 1,2,...,v,$ $l = 1,2,...,w,$ is the conditional probability that the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, lifetime $[T^4(u)]^{(bl)}$, $u = 1,2,...,z,$ in the safety state subset $\{u,u+1,...,z\}$, $u = 1,2,...,z,$ is greater than t, while the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, is at the climate-weather state zc_{bl}, $b = 1,2,...,v,$ $l = 1,2,...,w.$

Next, we denote the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, unconditional lifetime in the safety state subset $\{u,u+1,...,z\}$, $u = 1,2,...,z,$ by $T^4(u)$, $u = 1,2,...,z,$ and the unconditional safety function (SafI1) of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, by the vector

$$\boldsymbol{S}^4(t,\cdot) = [1,\ \boldsymbol{S}^4(t,1),...,\ \boldsymbol{S}^4(t,z)], \quad (7)$$

with the coordinates defined by

$$\boldsymbol{S}^4(t,u) = P(T^4(u) > t) \quad (8)$$

for $t \in <0,\infty)$, $u = 1,2,...,z.$

In the case when the system operation time θ is large enough, the coordinates of the unconditional safety function of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, defined by (8), are given by

$$\boldsymbol{S}^4(t,u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}[\boldsymbol{S}^4(t,u)]^{(bl)},\ t \geq 0,\ u = 1,2,...,z, \quad (9)$$

where $[\boldsymbol{S}^4(t,u)]^{(bl)}$, $u = 1,2,...,z,$ $b = 1,2,...,v,$ $l = 1,2,...,w,$ are the coordinates of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, conditional safety functions defined by (5)-(6) and pq_{bl}, $b = 1,2,...,v,$ $l = 1,2,...,w,$ are the operation process related to the climate-weather change process $ZC(t), t \in <0,\infty)$, limit transient probabilities at the critical infrastructure operating area limit transient probabilities at the states zc_{bl}, $b = 1,2,...,v,$ $l = 1,2,...,w,$ given by (1)-(2).

If r is the critical safety state, then the second safety indicator of the critical infrastructure related to the climate-weather change process $C(t)$, $t \in <0,\infty)$, the risk function (SafI2)

$$\boldsymbol{r}^3(t) = P(s(t) < r \mid s(0) = z) = P(T^3(r) \le t),\quad t \in <0,\infty), \tag{10}$$

is defined as a probability that the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, the risk function (SafI2)

$$\boldsymbol{r}^4(t) = P(s(t) < r \mid s(0) = z) = P(T^4(r) \le t),\quad t \in <0,\infty),$$

is defined as a probability that the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, is in the subset of safety states worse than the critical safety state r, $r \in \{1,...,z\}$ while it was in the best safety state z at the moment $t = 0$ and given by [EU-CIRCLE Report D3.3-Part4, 2017]

$$\boldsymbol{r}^4(t) = 1 - \boldsymbol{S}^4(t,r),\ t \in <0,\infty), \tag{11}$$

where $\boldsymbol{S}^4(t,r)$ is the coordinate of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, unconditional safety function given by (9) for $u = r$.

The graph of the critical infrastructure risk function $\boldsymbol{r}^4(t)$, $t \in <0,\infty)$, defined by (11), is the safety indicator called the fragility curve (SafI3) of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$.

Other practically useful safety indicators of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, are:

– the mean value of the critical infrastructure unconditional lifetime $T^4(r)$ up to exceeding critical safety state r (SafI4) given by

$$\boldsymbol{\mu}^4(r) = \int_0^\infty [\boldsymbol{S}^4(t,r)]dt \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}[\mu^4(r)]^{(bl)}, \tag{12}$$

where $[\mu^4(r)]^{(bl)}$, $b = 1,2,...,v$, $l = 1,2,...,w$, are the mean values of the critical infrastructure conditional lifetimes $[T^4(r)]^{(bl)}$, in the safety state subset $\{r, r+1,...,z\}$ at the operation process related to the climate-weather change process state zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, given by

$$[\mu^4(r)]^{(bl)} = \int_0^\infty [\boldsymbol{S}^4(t,r)]^{(bl)}dt,\ b = 1,2,...,v,\quad l = 1,2,...,w, \tag{13}$$

and $[\boldsymbol{S}^4(t,r)]^{(bl)}$, $b = 1,2,...,v$, $l = 1,2,...,w$, are defined by (5)-(6) and pq_{bl}, are given by (1)-(2),

– the standard deviation of the critical infrastructure lifetime $T^4(r)$ up to the exceeding the critical safety state r (SafI5) given by

$$\boldsymbol{\sigma}^4(r) = \sqrt{n^4(r) - [\boldsymbol{\mu}^4(r)]^2}, \tag{14}$$

where

$$n^4(r) = 2\int_0^\infty t\,\boldsymbol{S}^3(t,r)dt, \tag{15}$$

and $\boldsymbol{S}^4(t,r)$ is defined by (8) for $u = r$ and $\boldsymbol{\mu}^4(r)$ is given by (12);

– the moment τ^4 of exceeding acceptable value of critical infrastructure risk function level δ (SafI6) given by

$$\tau^4 = \boldsymbol{r}^{4\,-1}(\delta), \tag{16}$$

where $\boldsymbol{r}^{4\,-1}(\delta)$ is the inverse function of the risk function $\boldsymbol{r}^4(t)$ given by (10);

– the intensities of degradation of the critical infrastructure / the intensities of critical infrastructure departure from the safety state subset $\{u, u+1,...,z\}$, $u = 1,2,...,z$, (SafI9), i.e. the coordinates of the vector

$$\boldsymbol{\lambda}^4(t,\cdot) = [0,\ \boldsymbol{\lambda}^4(t,1),\ \ldots,\ \boldsymbol{\lambda}^4(t,z)],\quad t \in <0,+\infty), \tag{17}$$

where

$$\boldsymbol{\lambda}^4(t,u) = \frac{-\dfrac{d\boldsymbol{S}^4(t,u)}{dt}}{\boldsymbol{S}^4(t,u)},\ t \in <0,+\infty),\quad u = 1,2,...,z; \tag{18}$$

– the coefficients of operation process related to the climate-weather change process impact on the critical infrastructure intensities of degradation (the coefficients of operation process related to the climate-weather change process impact on critical infrastructure intensities of departure from the safety state subset $\{u, u+1, ..., z\}$) (SafI10), i.e. the coordinates of the vector

$$\boldsymbol{\rho}^4(t,\cdot) = [0,\ \boldsymbol{\rho}^4(t,1),\ \ldots,\ \boldsymbol{\rho}^4(t,z)\ ],$$
$$t \in <0,+\infty), \tag{19}$$

where

$$\boldsymbol{\lambda}^4(t,u) = \boldsymbol{\rho}^4(t,u) \cdot \boldsymbol{\lambda}^0(t,u),\ \ t \in <0,+\infty),$$
$$u = 1,2,...,z, \tag{20}$$

i.e.

$$\boldsymbol{\rho}^4(t,u) = \frac{\boldsymbol{\lambda}^4(t,u)}{\boldsymbol{\lambda}^0(t,u)},\ \ t \in <0,+\infty),$$
$$u = 1,2,...,z, \tag{21}$$

and $\boldsymbol{\lambda}^0(t,u)$, $t \in <0,+\infty)$, $u = 1,2,...,z$, are the intensities of degradation of the critical infrastructure without of the operation process related to the climate-weather change process impact, i.e. the coordinate of the vector

$$\boldsymbol{\lambda}^0(t,\cdot) = [0,\ \boldsymbol{\lambda}^0(t,1),\ \ldots, \boldsymbol{\lambda}^0(t,z)\ ],$$
$$t \in <0,+\infty), \tag{22}$$

and $\boldsymbol{\lambda}^4(t,u)$, $t \in <0,+\infty)$, $u = 1,2,...,z$, are the intensities of degradation of the critical infrastructure with the operation process related to the climate-weather change process impact, i.e. the coordinate of the vector

$$\boldsymbol{\lambda}^4(t,\cdot) = [0,\ \boldsymbol{\lambda}^4(t,1), \ldots,\ \boldsymbol{\lambda}^4(t,z)\ ],$$
$$t \in <0,+\infty), \tag{23}$$

– the indicator of critical infrastructure resilience to operation process related to the climate-weather change process impact (ResI1) defined by

$$\boldsymbol{RI}^4(t,r) = \frac{1}{\rho^4(t,r)},\ \ t \in <0,+\infty), \tag{24}$$

where $\boldsymbol{\rho}^4(t,r)$, $t \in <0,+\infty)$, is the coefficient of operation process related to the climate-weather change process impact on the critical infrastructure intensities of degradation given by (20) for $u = r$.

Further, we also will use the following critical infrastructure safety characteristics:

– the mean lifetime of the critical infrastructure in the safety state subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, given by

$$\boldsymbol{\mu}^4(u) = \int_0^\infty [\boldsymbol{S}^4(t,u)]dt \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}[\mu^4(u)]^{(bl)},$$
$$u = 1,2,...,z, \tag{25}$$

where $[\mu^4(u)]^{(bl)}$, are the mean values of the critical infrastructure conditional lifetimes $[T^4(u)]^{(bl)}$, in the safety state subset $\{u, u+1, ..., z\}$ at the operating process related to the climate-weather change process state zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, given by

$$[\mu^4(u)]^{(b)} = \int_0^\infty [\boldsymbol{S}^4(t,u)]^{(bl)} dt,\ \ u = 1,2,...,z,$$
$$b = 1,2,...,v,\ \ l = 1,2,...,w, \tag{26}$$

and $[\boldsymbol{S}^4(t,u)]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,...,v$, $l = 1,2,...,w$, are defined by (5)-(6) and pq_{bl}, are given by (1)-(2);

– the standard deviation of the critical infrastructure lifetime in the safety state subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, given by

$$\boldsymbol{\sigma}^4(u) = \sqrt{n^4(u) - [\boldsymbol{\mu}^4(u)]^2},\ u = 1,2,...,z, \tag{27}$$

where

$$n^4(u) = 2\int_0^\infty t\boldsymbol{S}^4(t,u)dt,\ u = 1,2,...,z, \tag{28}$$

– the mean lifetimes $\bar{\boldsymbol{\mu}}^4(u)$, $u = 1,2,...,z$, of the critical infrastructure in the particular safety states

$$\bar{\boldsymbol{\mu}}^4(u) = \boldsymbol{\mu}^4(u) - \boldsymbol{\mu}^4(u+1),\ \ u = 0,1,...,z-1,$$
$$\bar{\boldsymbol{\mu}}^4(z) = \boldsymbol{\mu}^4(z). \tag{29}$$

2.2. Critical infrastructure assets safety parameters

We denote the critical infrastructure asset A_i, $i = 1,2,...,n$, conditional lifetime in the safety state subset $\{u, u+1, ..., z\}$ while the operation process related to the climate-weather change process at the critical infrastructure operating area is at the climate-change state zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, by $[T_i^4(u)]^{(bl)}$, and its conditional safety function

(SafI1) by the vector [EU-CIRCLE Report D3.3-Part4, 2017]

$$[S_i^4(t,\cdot)]^{(bl)} = [1,\ [S_i^4(t,1)]^{(bl)}, ...,\ [S_i^4(t,z)]^{(bl)}\ ],$$
$$t \in <0,\infty),\ b = 1,2,...,v,\ l = 1,2,...,w,$$
$$i = 1,2,...,n, \quad (30)$$

with the coordinates defined by

$$[S_i^4(t,u)]^{(bl)} = P([T_i^4(u)]^{(bl)} > t | ZC(t) = zc_{bl}) \quad (31)$$

for $t \in <0,\infty)$, $u = 1,2,...,z$, $b = 1,2,...,v$, $l = 1,2,...,w$, $i = 1,2,...,n.$

The safety function $[S_i^4(t,u)]^{(bl)}$, is the conditional probability that the asset A_i lifetime $[T_i^4(u)]^{(bl)}$, in the safety state subset $\{u,u+1,...,z\}$ is greater than t, while the operation process related to the climate-weather change process at the critical infrastructure operating area is at the climate-weather state zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w.$

The conditional safety functions $[S_i^4(t,u)]^{(bl)}$, $t \in <0,\infty)$, u = 1, 2, ..., z, $b = 1,2,...,v$, $l = 1,2,...,w$, i = 1,2,...,n, defined by (31) are called the coordinates of the asset A_i, i = 1,2,...,n, conditional safety function $[S_i^4(t,\cdot)]^{(bl)}$, $t \in <0,\infty)$, $b = 1,2,...,v$, $l = 1,2,...,w$, i = 1,2,...,n, while the operation process related to the climate-weather change process $ZC(t)$ is at the operation state zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, given by (30). Thus, the relationship between the conditional distribution function $[F_i^4(t,u)]^{(bl)}$, $t \in <0,\infty)$, u = 1,2,...,z, $b = 1,2,...,v$, $l = 1,2,...,w$, i = 1,2,...,n, of the asset A_i, i = 1,2,...,n, lifetime $[T_i^4(u)]^{(bl)}$, u = 1,2,..., z, $b = 1,2,...,v$, $l = 1,2,...,w$, i = 1,2,...,n, in the safety state subset $\{u,u+1,...,z\}$, u = 1,2,...,z, and the coordinate $[S_i^4(t,u)]^{(bl)}$, $t \in <0,\infty)$, u = 1, 2, ..., z, $b = 1,2,...,v$, $l = 1,2,...,w$, i = 1,2,...,n, of its conditional safety function is given by

$$[F_i^4(t,u)]^{(bl)} = P([T_i^4(u)]^{(bl)} \le t)$$
$$= 1 - P([T_i^4(u)]^{(bl)} > t)$$
$$= 1 - [S_i^4(t,u)]^{(bl)},\ t \in <0,\infty),$$
$$u = 1,2,...,z, b = 1,2,...,v, l = 1,2,...,w, i = 1,2,...,n. \quad (32)$$

Thus, the function

$$[r_i^4(t)]^{(bl)} = 1 - [S_i^4(t,r)]^{(bl)},\ t \in <0,\infty),$$
$$b = 1,2,...,v,\ l = 1,2,...,w,\ i = 1,2,...,n, \quad (33)$$

is the asset A_i, i = 1,2,...,n, the conditional risk function (SafI2) and its graph is the asset A_i, i = 1,2,...,n, fragility curve (SafI3) while the operation process related to the climate-weather change process at the critical infrastructure operating area is at the climate-weather state zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w.$

Moreover, the conditional mean lifetime of the asset A_i in the safety state subset $\{u,u+1,...,z\}$, u = 1,2,...,z, while the operation process related to the climate-weather change process at the critical infrastructure operating area is at the climate-weather state zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, is given by

$$[\mu_i^4(u)]^{(bl)} = \int_0^\infty [S_i^4(t,u)]^{(bl)} dt,\ u = 1,2,...,z,$$
$$b = 1,2,...,v,\ l = 1,2,...,w,\ i = 1,2,...,n. \quad (34)$$

In the case, when the critical infrastructure assets A_i, $i = 1,2,...,n$, at the climate-weather states zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, have the exponential safety functions, the coordinates (31) of the vector (30) are given by

$$[S_i^4(t,u)]^{(bl)} = P([T_i^4(u)]^{(bl)} > t | ZC(t) = zc_{bl})$$
$$= \exp[-[\lambda_i^4(u)]^{(bl)} t],\ t \in <0,\infty),$$
$$u = 1,2,...,z,\ b = 1,2,...,v,\ l = 1,2,...,w,$$
$$i = 1,2,...,n. \quad (35)$$

Existing in (35) the intensities of degradation of the critical infrastructure asset A_i, $i = 1,2,...,n$, with the operation process related to the climate-weather change process at the critical infrastructure operating area impact at the states zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, (SafI7), i.e. the coordinates of the vector

$$[\lambda_i^4(\cdot)]^{(bl)} = [0,\ [\lambda_i^4(1)]^{(bl)}, \ldots, [\lambda_i^4(z)]^{(bl)}\ ],$$
$$t \in <0,+\infty),\ b = 1,2,...,v,\ l = 1,2,...,w,$$
$$i = 1,2,...,n, \quad (36)$$

are constant and given by

$$[\lambda_i^4(u)]^{(bl)} = \frac{1}{[\mu_i^4(u)]^{(bl)}},\ u = 1,2,...,z,\ b = 1,2,...,v,$$
$$l = 1,2,...,w,\ i = 1,2,...,n, \quad (37)$$

and moreover

$$[\lambda_i^4(u)]^{(bl)} = [\rho_i^4(u)]^{(bl)} \cdot \lambda_i^0(u),\ u = 1,2,...,z,$$
$$b = 1,2,...,\nu,\ l = 1,2,...,w,\ i = 1,2,...,n, \quad (38)$$

where $\lambda_i^0(u)$ are the intensities of degradation of the critical infrastructure asset A_i, $i = 1,2,...,n$, without any impact (SafI7), i.e. the coordinate of the vector

$$\lambda_i^0(\cdot) = [0,\ \lambda_i^0(1), \ldots, \lambda_i^0(z)\ ],\ i = 1,2,...,n, \quad (39)$$

and $[\rho_i^4(u)]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,...,\nu$, $i = 1,2,...,n$, are the coefficients of the operation process related to the climate-weather change process at the critical infrastructure operating area impact on the critical infrastructure asset A_i, $i = 1,2,...,n$, intensities of degradation at the zc_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, (SafI8), i.e. the coordinate of the vector

$$[\rho_i^4(\cdot)]^{(bl)} = [0,\ [\rho_i^4(1)]^{(bl)}, \ldots, [\rho_i^4(z)]^{(bl)}\ ],$$
$$b = 1,2,...,\nu,\ l = 1,2,...,w,\ i = 1,2,...,n. \quad (40)$$

where by (38)

$$[\rho_i^4(u)]^{(bl)} = \frac{[\lambda_i^4(u)]^{(bl)}}{\lambda_i^0(u)} = \frac{\mu_i^0(u)}{[\mu_i^4(u)]^{(bl)}},\ u = 1,2,...,z,$$
$$b = 1,2,...,\nu,\ l = 1,2,...,w,\ i = 1,2,...,n, \quad (41)$$

3. IMCIS 4 application to safety of port oil piping transportation system evaluation

In this section, we consider the port oil piping transportation system impacted by its operation process and the climate-weather change process in its operating area.

3.1. Joint parameters and characteristics of climate-weather change process at port oil piping transportation system operating area and its operation process

The port oil piping transportation system operation process related to climate-weather change process $ZC1(t)$, $t \in <0,+\infty)$, can take $\nu \cdot w1 = 7 \cdot 5 = 35$ different operation states $zc1_{1\,1}$, $zc1_{1\,2}$, $zc1_{1\,3}$, $zc1_{1\,5}$, $zc1_{1\,6}$; $zc1_{2\,1}$, $zc1_{2\,2}$, $zc1_{2\,3}$, $zc1_{2\,5}$, $zc1_{2\,6}$;…; $zc1_{7\,1}$, $zc1_{7\,2}$, $zc1_{7\,3}$, $zc1_{7\,5}$, $zc1_{7\,6}$.

The port oil piping transportation operation process related to climate-weather change process $ZC1(t)$ characteristics, determined on the basis of the operation process and the climate-weather change process data given in [GMU Safety Interactive Platform], are:

– the limit values of transient probabilities of operation process related to climate-weather change process $ZC1(t)$ at the particular operation states $zc1_{bl}$, $b = 1,2,\ldots,7$, $l = 1,2,3,5,6$,

$[pq1_{bl}]_{b = 1,2,\ldots,7,\ l = 1,2,3,5,6}$ = [0.338923, 0.060853, 0.000403, 0.002418, 0.000403, 0.046255, 0.008305, 0.000055, 0.00033, 0.000055, 0.002523, 0.000453, 0.000003, 0.000018, 0.000003, 0.001682, 0.000302, 0.000002, 0.000012, 0.000002, 0.167359, 0.030049, 0.000199, 0.001194, 0.000199, 0.047937, 0.008607, 0.000057, 0.000342, 0.000057, 0.236321, 0.042431, 0.000281, 0.001686, 0.000281]; (42)

– the expected values of the total sojourn times $ZC1_{bl}$, $b = 1,2,\ldots,7$, $l = 1,2,3,5,6$, of the operation process related to climate-weather change process $ZC1(t)$ at the particular operation states $zc1_{bl}$, $b = 1,2,\ldots,7$, $l = 1,2,3,5,6$, during the fixed operation time $ZC1$ = 1 year = 365 days:

$[\hat{MN}\ 1_{bl}]_{b = 1,2,\ldots,7,\ l = 1,2,3,5,6}$ = [123.7069, 22.21134, 0.147095, 0.88257, 0.147095, 16.88307, 3.031325, 0.020075, 0.12045, 0.020075, 0.920895, 0.165345, 0.001095, 0.00657, 0.001095, 0.61393, 0.11023, 0.00073, 0.00438, 0.00073, 61.08604, 10.96788, 0.072635, 0.43581, 0.072635, 17.49701, 3.141555, 0.020805, 0.12483, 0.020805, 86.25717, 15.48732, 0.102565, 0.61539, 0.102565].

The port oil piping transportation system operation process related to climate-weather change process $ZC2(t)$, $t \in <0,+\infty)$, can take $\nu \cdot w2 = 7 \cdot 7 = 49$ different operation states $zc2_{1\,2}$, $zc2_{1\,6}$, $zc2_{1\,7}$, $zc2_{1\,10}$, $zc2_{1\,11}$, $zc2_{1\,15}$, $zc2_{1\,16}$; $zc2_{2\,2}$, $zc2_{2\,6}$, $zc2_{2\,7}$, $zc2_{2\,10}$, $zc2_{2\,11}$, $zc2_{2\,15}$, $zc2_{2\,16}$;…; $zc2_{7\,2}$, $zc2_{7\,6}$, $zc2_{7\,7}$, $zc2_{7\,10}$, $zc2_{7\,11}$, $zc2_{7\,15}$, $zc2_{7\,16}$.

The port oil piping transportation operation process related to climate-weather change process $ZC2(t)$ characteristics, determined on the basis of the operation process and the climate-weather change process data given in [GMU Safety Interactive Platform], are:

– the limit values of transient probabilities of operation process related to climate-weather change process $ZC2(t)$ at the particular operation states $zc2_{bl}$, $b = 1,2,\ldots,7$, $l = 2,6,7,10,11,15,16$,

$[pq2_{bl}]_{b = 1,2,\ldots,7,\ l = 2,6,7,10,11,15,16}$ = [0.010478, 0.111631, 0.005642, 0.003224, 0.246636, 0.024986, 0.000403, 0.00143, 0.015235, 0.00077, 0.00044, 0.03366, 0.00341, 0.000055, 0.000078, 0.000831, 0.000042, 0.000024, 0.001836, 0.000186, 0.000003, 0.000052, 0.000554, 0.000028, 0.000016, 0.001224, 0.000124, 0.000002, 0.005174, 0.055123, 0.002786, 0.001592, 0.121788, 0.012338, 0.000199, 0.001482, 0.015789, 0.000798, 0.000456, 0.034884, 0.003534, 0.000057, 0.007306, 0.077837, 0.003934, 0.002248, 0.171972, 0.017422, 0.000281]; (43)

– the expected values of the total sojourn times $ZC2_{bl}$, b = 1,2,…,7, l = 2,6,7,10,11,15,16, of the operation process related to climate-weather change process $ZC2(t)$ at the particular operation states $zc2_{bl}$, b = 1,2,…,7, l = 2,6,7,10,11,15,16, during the fixed operation time $ZC2$ = 1 year = 365 days:

$[\hat{M}N\,2_{bl}]_{b=1,2,\ldots,7,\ l=2,6,7,10,11,15,16}$ = [3.82447, 40.74531, 2.05933, 1.17676, 90.02214, 9.11989, 0.147095, 0.52195, 5.560775, 0.28105, 0.1606, 12.2859, 1.24465, 0.020075, 0.02847, 0.303315, 0.01533, 0.00876, 0.67014, 0.06789, 0.001095, 0.01898, 0.20221, 0.01022, 0.00584, 0.44676, 0.04526, 0.00073, 1.88851, 20.11989, 1.01689, 0.58108, 44.45262, 4.50337, 0.072635, 0.54093, 5.762985, 0.29127, 0.16644, 12.73266, 1.28991, 0.020805, 2.66669, 28.41051, 1.43591, 0.82052, 62.76978, 6.35903, 0.102565].

3.2. Joint parameters of climate-weather change process and operation process impact on port oil piping transportation system safety

Since according to (3.3), we have

$[\rho 1^4_{ij}(u)]^{(bl)} = [\rho^1_{ij}(u)]^{(b)} \cdot [\rho 1^3_{ij}(u)]^{(l)},\ u=1,2,$
$b=1,2,\ldots,7,\ l=1,2,3,5,6,\ i=1,2,\ j=1,2;\ i=3,$
$j=1,2,3,$ (44)

$[\rho 2^4_{ij}(u)]^{(bl)} = [\rho^1_{ij}(u)]^{(b)} \cdot [\rho 2^3_{ij}(u)]^{(l)},\ u=1,2,$
$b=1,2,\ldots,7,\ l=2,6,7,10,11,15,16,\ i=1,2,$
$j=1,2;\ i=3,\ j=1,2,3,$ (45)

then applying the above formula to the parameters defined in [EU-CIRCLE Report D3.3-Part4, 2017], we get the intensities of ageing of the critical infrastructure assets A_{ij}, i = 1,2, j =1,2, i = 3, j =1,2,3, / the intensities of critical infrastructure assets A_{ij}, i = 1,2, j =1,2, i = 3, j =1,2,3, departure from the safety state subset {1,2} and {2}impacted by the port oil piping transportation system operation process related to the climate-weather change process

$[\rho 1^4_{11}(1)]^{(bl)} = 1.00$, $[\rho 1^4_{11}(2)]^{(bl)} = 1.00$,
b = 1,2,7, l = 1,2,
$[\rho 1^4_{11}(1)]^{(bl)} = 1.036$, $[\rho 1^4_{11}(2)]^{(bl)} = 1.048$,
b = 1,2,7, l = 3,5,6,
$[\rho 2^4_{11}(1)]^{(bl)} = 1.00$, $[\rho 2^4_{11}(2)]^{(bl)} = 1.00$,
b = 1,2,7, l = 2,6,7,10,11,15,16,
$[\rho 1^4_{11}(1)]^{(bl)} = 1.20$, $[\rho 1^4_{11}(2)]^{(bl)} = 1.20$,
b = 3,4,5,6, l = 1,2,
$[\rho 1^4_{11}(1)]^{(bl)} = 1.2432$, $[\rho 1^4_{11}(2)]^{(bl)} = 1.2576$,
b = 3,4,5,6, l = 3,5,6,
$[\rho 2^4_{11}(1)]^{(bl)} = 1.20$, $[\rho 2^4_{11}(2)]^{(bl)} = 1.20$,
b = 3,4,5,6, l = 2,6,7,10,11,15,16,

$[\rho 1^4_{12}(1)]^{(bl)} = 1.00$, $[\rho 1^4_{12}(2)]^{(bl)} = 1.00$,
b = 1,2,7, l = 1,2,
$[\rho 1^4_{12}(1)]^{(bl)} = 1.036$, $[\rho 1^4_{12}(2)]^{(bl)} = 1.048$,
b = 1,2,7, l = 3,5,6,
$[\rho 2^4_{12}(1)]^{(bl)} = 1.00$, $[\rho 2^4_{12}(2)]^{(bl)} = 1.00$,
b = 1,2,7, l = 2,6,7,10,11,15,16,
$[\rho 1^4_{12}(1)]^{(bl)} = 1.20$, $[\rho 1^4_{12}(2)]^{(bl)} = 1.20$,
b = 3,4,5,6, l = 1,2,
$[\rho 1^4_{12}(1)]^{(bl)} = 1.2432$, $[\rho 1^4_{12}(2)]^{(bl)} = 1.2576$,
b = 3,4,5,6, l = 3,5,6,
$[\rho 2^4_{12}(1)]^{(bl)} = 1.20$, $[\rho 2^4_{12}(2)]^{(bl)} = 1.20$,
b = 3,4,5,6, l = 2,6,7,10,11,15,16,

$[\rho 1^4_{21}(1)]^{(bl)} = 1.00$, $[\rho 1^4_{21}(2)]^{(bl)} = 1.00$,
b = 1,2,7, l = 1,2,3,5,6,
$[\rho 2^4_{21}(1)]^{(bl)} = 1.00$, $[\rho 2^4_{21}(2)]^{(bl)} = 1.00$,
b = 1,2,7, l = 6,7,10,11,
$[\rho 2^4_{21}(1)]^{(bl)} = 1.004$, $[\rho 2^4_{21}(2)]^{(bl)} = 1.007$,
b = 1,2,7, l = 2,15,16,
$[\rho 1^4_{21}(1)]^{(bl)} = 1.20$, $[\rho 1^4_{21}(2)]^{(bl)} = 1.20$,
b = 3,4,5,6, l = 1,2,3,5,6,
$[\rho 2^4_{21}(1)]^{(bl)} = 1.20$, $[\rho 2^4_{21}(2)]^{(bl)} = 1.20$,
b = 3,4,5,6, l = 6,7,10,11,
$[\rho 2^4_{21}(1)]^{(bl)} = 1.2048$, $[\rho 2^4_{21}(2)]^{(bl)} = 1.2084$,
b = 3,4,5,6, l = 2,15,16,

$[\rho 1^4_{21}(1)]^{(bl)} = 1.00$, $[\rho 1^4_{21}(2)]^{(bl)} = 1.00$,
b = 1,2,7, l = 1,2,3,5,6,
$[\rho 2^4_{21}(1)]^{(bl)} = 1.00$, $[\rho 2^4_{21}(2)]^{(bl)} = 1.00$,
b = 1,2,7, l = 6,7,10,11,
$[\rho 2^4_{21}(1)]^{(bl)} = 1.004$, $[\rho 2^4_{21}(2)]^{(bl)} = 1.007$,
b = 1,2,7, l = 2,15,16,
$[\rho 1^4_{21}(1)]^{(bl)} = 1.20$, $[\rho 1^4_{21}(2)]^{(bl)} = 1.20$,
b = 3,4,5,6, l = 1,2,3,5,6,
$[\rho 2^4_{21}(1)]^{(bl)} = 1.20$, $[\rho 2^4_{21}(2)]^{(bl)} = 1.20$,
b = 3,4,5,6, l = 6,7,10,11,
$[\rho 2^4_{21}(1)]^{(bl)} = 1.2048$, $[\rho 2^4_{21}(2)]^{(bl)} = 1.2084$,
b = 3,4,5,6, l = 2,15,16,
$[\rho 1^4_{31}(1)]^{(bl)} = 1.00$, $[\rho 1^4_{31}(2)]^{(bl)} = 1.00$,

$b = 3,5$, $l = 1,2,3,5,6$,
$[\rho 2^4{}_{31}(1)]^{(bl)} = 1.00$, $[\rho 2^4{}_{31}(2)]^{(bl)} = 1.00$,
$b = 3,5$, $l = 2,6,7,10,11,15,16$,
$[\rho 1^4{}_{31}(1)]^{(bl)} = 1.20$, $[\rho 1^4{}_{31}(2)]^{(bl)} = 1.20$,
$b = 1,2,4,6,7$, $l = 1,2,3,5,6$,
$[\rho 2^4{}_{31}(1)]^{(bl)} = 1.20$, $[\rho 2^4{}_{31}(2)]^{(bl)} = 1.20$,
$b = 1,2,4,6,7$, $l = 2,6,7,10,11,15,16$,

$[\rho 1^4{}_{31}(1)]^{(bl)} = 1.00$, $[\rho 1^4{}_{31}(2)]^{(bl)} = 1.00$,
$b = 3,5$, $l = 1,2,3,5,6$,
$[\rho 2^4{}_{31}(1)]^{(bl)} = 1.00$, $[\rho 2^4{}_{31}(2)]^{(bl)} = 1.00$,
$b = 3,5$, $l = 2,6,7,10,11,15,16$,
$[\rho 1^4{}_{31}(1)]^{(bl)} = 1.20$, $[\rho 1^4{}_{31}(2)]^{(bl)} = 1.20$,
$b = 1,2,4,6,7$, $l = 1,2,3,5,6$,
$[\rho 2^4{}_{31}(1)]^{(bl)} = 1.20$, $[\rho 2^4{}_{31}(2)]^{(bl)} = 1.20$,
$b = 1,2,4,6,7$, $l = 2,6,7,10,11,15,16$,

$[\rho 1^4{}_{31}(1)]^{(bl)} = 1.00$, $[\rho 1^4{}_{31}(2)]^{(bl)} = 1.00$,
$b = 3,5$, $l = 1,2,3,5,6$,
$[\rho 2^4{}_{31}(1)]^{(bl)} = 1.00$, $[\rho 2^4{}_{31}(2)]^{(bl)} = 1.00$,
$b = 3,5$, $l = 2,6,7,10,11,15,16$,
$[\rho 1^4{}_{31}(1)]^{(bl)} = 1.20$, $[\rho 1^4{}_{31}(2)]^{(bl)} = 1.20$,
$b = 1,2,4,6,7$, $l = 1,2,3,5,6$,
$[\rho 2^4{}_{31}(1)]^{(bl)} = 1.20$, $[\rho 2^4{}_{31}(2)]^{(bl)} = 1.20$,
$b = 1,2,4,6,7$, $l = 2,6,7,10,11,15,16$. (46)

3.3. Safety parameters of port oil piping transportation system assets impacted by climate-weather change process and operation process

Since according to (38), we have

$$[\lambda 1^4{}_{ij}(u)]^{(bl)} = [\rho 1^4{}_{ij}(u)]^{(bl)} \cdot \lambda^0{}_{ij}(u),\ u = 1,2,\ b = 1,2,...,7,\ l = 1,2,3,5,6,\ i = 1,2,\ j = 1,2;\ i = 3,\ j = 1,2,3, \quad (47)$$

$$[\lambda 2^4{}_{ij}(u)]^{(bl)} = [\rho 2^4{}_{ij}(u)]^{(bl)} \cdot \lambda^0{}_{ij}(u),\ u = 1,2,\ b = 1,2,...,7,\ l = 1,2,3,5,6,\ i = 1,2,\ j = 1,2;\ i = 3,\ j = 1,2,3, \quad (48)$$

then applying the above formula to the parameters defined in [EU-CIRCLE Report for D6.4-Part 0, 2017] and (46), we get the intensities of ageing of the critical infrastructure assets A_{ij}, $i = 1,2$, $j = 1,2$, $i = 3$, $j = 1,2,3$, / the intensities of critical infrastructure assets A_{ij}, $i = 1,2$, $j = 1,2$, $i = 3$, $j = 1,2,3$, departure from the safety state subset {1,2} and {2} impacted by the port oil piping transportation system operation process related to the climate-weather change process, i.e. the coordinates of the vector

$$[\lambda 1^4{}_{ij}(\cdot)]^{(bl)} = [0, [\lambda 1^4{}_{ij}(1)]^{(bl)}, [\lambda 1^4{}_{ij}(2)]^{(bl)}],\ l = 1,2,3,5,6,\ i = 1,2,\ j = 1,2;\ i = 3,\ j = 1,2,3, \quad (49)$$

$$[\lambda 2^3{}_{ij}(\cdot)]^{(l)} = [0, [\lambda 2^3{}_{ij}(1)]^{(l)}, [\lambda 2^3{}_{ij}(2)]^{(l)}],\ l = 2,6,7,10,11,15,16,\ i = 1,2,\ j = 1,2;\ i = 3,\ j = 1,2,3, \quad (50)$$

follows:

– the intensities of departure of the asset A_{11} and A_{12}
  - for safety state subset {1,2}

    $[\lambda 1^4_{11}(1)]^{(bl)} = [\lambda 1^4_{12}(1)]^{(bl)} = 0.00362$,
    $b = 1,2,7$, $l = 1,2$,
    $[\lambda 1^4_{11}(1)]^{(bl)} = [\lambda 1^4_{12}(1)]^{(bl)} = 0.00375032$,
    $b = 1,2,7$, $l = 3,5,6$,
    $[\lambda 2^4_{11}(1)]^{(bl)} = [\lambda 2^4_{12}(1)]^{(bl)} = 0.00362$,
    $b = 1,2,7$, $l = 2,6,7,10,11,15,16$,
    $[\lambda 1^4_{11}(1)]^{(bl)} = [\lambda 1^4_{12}(1)]^{(bl)} = 0.004344$,
    $b = 3,4,5,6$, $l = 1,2$,
    $[\lambda 1^4_{11}(1)]^{(bl)} = [\lambda 1^4_{12}(1)]^{(bl)} = 0.004500384$,
    $b = 3,4,5,6$, $l = 3,5,6$,
    $[\lambda 2^4_{11}(1)]^{(bl)} = [\lambda 2^4_{12}(1)]^{(bl)} = 0.004344$,
    $b = 3,4,5,6$, $l = 2,6,7,10,11,15,16$;

  - for safety state subset {2}

    $[\lambda 1^4_{11}(2)]^{(bl)} = [\lambda 1^4_{12}(2)]^{(bl)} = 0.00540$,
    $b = 1,2,7$, $l = 1,2$,
    $[\lambda 1^4_{11}(2)]^{(bl)} = [\lambda 1^4_{12}(2)]^{(bl)} = 0.0056592$,
    $b = 1,2,7$, $l = 3,5,6$,
    $[\lambda 2^4_{11}(2)]^{(bl)} = [\lambda 2^4_{12}(2)]^{(bl)} = 0.00540$,
    $b = 1,2,7$, $l = 2,6,7,10,11,15,16$,
    $[\lambda 1^4_{11}(2)]^{(bl)} = [\lambda 1^4_{12}(2)]^{(bl)} = 0.00648$,
    $b = 3,4,5,6$, $l = 1,2$,
    $[\lambda 1^4_{11}(2)]^{(bl)} = [\lambda 1^4_{12}(2)]^{(bl)} = 0.00679104$,
    $b = 3,4,5,6$, $l = 3,5,6$,
    $[\lambda 2^4_{11}(2)]^{(bl)} = [\lambda 2^4_{12}(2)]^{(bl)} = 0.00648$,
    $b = 3,4,5,6$, $l = 2,6,7,10,11,15,16$;

– the intensities of departure of the assets A_{21} and A_{22}
  - for safety state subset {1,2}

    $[\lambda 1^4_{21}(1)]^{(bl)} = [\lambda 1^4_{22}(1)]^{(bl)} = 0.01444$, $b = 1,2,7$, $l = 1,2,3,5,6$,
    $[\lambda 2^4_{21}(1)]^{(bl)} = [\lambda 2^4_{22}(1)]^{(bl)} = 0.01449776$,
    $b = 1,2,7$, $l = 2,15,16$,
    $[\lambda 2^4_{21}(1)]^{(bl)} = [\lambda 2^4_{22}(1)]^{(bl)} = 0.01444$,
    $b = 1,2,7$, $l = 6,7,10,11$,
    $[\lambda 1^4_{21}(1)]^{(bl)} = [\lambda 1^4_{22}(1)]^{(bl)} = 0.017328$,
    $b = 3,4,5,6$, $l = 1,2,3,5,6$,

$[\lambda 2^{4}_{21}(1)]^{(bl)} = [\lambda 2^{4}_{22}(1)]^{(bl)} = 0.01739731$,
$b = 3,4,5,6$, $l = 2,15,16$,
$[\lambda 2^{4}_{21}(1)]^{(bl)} = [\lambda 2^{4}_{22}(1)]^{(bl)} = 0.017328$,
$b = 3,4,5,6$, $l = 6,7,10,11$;

- for safety state subset {2}

$[\lambda 1^{4}_{21}(2)]^{(bl)} = [\lambda 1^{4}_{22}(2)]^{(bl)} = 0.02163$,
$b = 1,2,7$, $l = 1,2,3,5,6$,
$[\lambda 2^{4}_{21}(2)]^{(bl)} = [\lambda 2^{4}_{22}(2)]^{(bl)} = 0.02178141$,
$b = 1,2,7$, $l = 2,15,16$,
$[\lambda 2^{4}_{21}(2)]^{(bl)} = [\lambda 2^{4}_{22}(2)]^{(bl)} = 0.02163$,
$b = 1,2,7$, $l = 6,7,10,11$,
$[\lambda 1^{4}_{21}(2)]^{(bl)} = [\lambda 1^{4}_{22}(2)]^{(bl)} = 0.025956$,
$b = 3,4,5,6$, $l = 1,2,3,5,6$,
$[\lambda 2^{4}_{21}(2)]^{(bl)} = [\lambda 2^{4}_{22}(2)]^{(bl)} = 0.02613769$,
$b = 3,4,5,6$, $l = 2,15,16$,
$[\lambda 2^{4}_{21}(2)]^{(bl)} = [\lambda 2^{4}_{22}(2)]^{(bl)} = 0.025956$,
$b = 3,4,5,6$, $l = 6,7,10,11$;

– the intensities of departure of the assets A_{31} and A_{32}
  - for safety state subset {1,2}

$[\lambda 1^{4}_{31}(1)]^{(bl)} = [\lambda 1^{4}_{32}(1)]^{(bl)} = 0.00730$, $b = 3,5$,
$l = 1,2,3,5,6$,
$[\lambda 2^{4}_{31}(1)]^{(bl)} = [\lambda 2^{4}_{32}(1)]^{(bl)} = 0.00730$, $b = 3,5$,
$l = 2,6,7,10,11,15,16$,
$[\lambda 1^{4}_{31}(1)]^{(bl)} = [\lambda 1^{4}_{32}(1)]^{(bl)} = 0.00876$,
$b = 1,2,4,6,7$, $l = 1,2,3,5,6$,
$[\lambda 2^{4}_{31}(1)]^{(bl)} = [\lambda 2^{4}_{32}(1)]^{(bl)} = 0.00876$,
$b = 1,2,4,6,7$, $l = 2,6,7,10,11,15,16$;

  - for safety state subset {2}

$[\lambda 1^{4}_{31}(2)]^{(bl)} = [\lambda 1^{4}_{32}(2)]^{(bl)} = 0.00912$, $b = 3,5$,
$l = 1,2,3,5,6$,
$[\lambda 2^{4}_{31}(2)]^{(bl)} = [\lambda 2^{4}_{32}(2)]^{(bl)} = 0.00912$, $b = 3,5$,
$l = 2,6,7,10,11,15,16$,
$[\lambda 1^{4}_{31}(2)]^{(bl)} = [\lambda 1^{4}_{32}(2)]^{(bl)} = 0.010944$,
$b = 1,2,4,6,7$, $l = 1,2,3,5,6$,
$[\lambda 2^{4}_{31}(2)]^{(bl)} = [\lambda 2^{4}_{32}(2)]^{(bl)} = 0.010944$,
$b = 1,2,4,6,7$, $l = 2,6,7,10,11,15,16$;

– the intensities of departure of the asset A_{33}
  - for safety state subset {1,2}

$[\lambda 1^{4}_{33}(1)]^{(bl)} = [\lambda 1^{4}_{33}(1)]^{(bl)} = 0.00874$, $b = 3,5$,
$l = 1,2,3,5,6$,
$[\lambda 2^{4}_{33}(1)]^{(bl)} = [\lambda 2^{4}_{33}(1)]^{(bl)} = 0.00874$, $b = 3,5$,
$l = 2,6,7,10,11,15,16$,
$[\lambda 1^{4}_{33}(1)]^{(bl)} = [\lambda 1^{4}_{33}(1)]^{(bl)} = 0.010488$,
$b = 1,2,4,6,7$, $l = 1,2,3,5,6$,
$[\lambda 2^{4}_{33}(1)]^{(bl)} = [\lambda 2^{4}_{33}(1)]^{(bl)} = 0.010488$,
$b = 1,2,4,6,7$, $l = 2,6,7,10,11,15,16$;

  - for safety state subset {2}

$[\lambda 1^{4}_{33}(2)]^{(bl)} = [\lambda 1^{4}_{33}(2)]^{(bl)} = 0.00984$, $b = 3,5$,
$l = 1,2,3,5,6$,
$[\lambda 2^{4}_{33}(2)]^{(bl)} = [\lambda 2^{4}_{33}(2)]^{(bl)} = 0.00984$, $b = 3,5$,
$l = 2,6,7,10,11,15,16$,
$[\lambda 1^{4}_{33}(2)]^{(bl)} = [\lambda 1^{4}_{33}(2)]^{(bl)} = 0.011808$,
$b = 1,2,4,6,7$, $l = 1,2,3,5,6$,
$[\lambda 2^{4}_{33}(2)]^{(bl)} = [\lambda 2^{4}_{33}(2)]^{(bl)} = 0.011808$,
$b = 1,2,4,6,7$, $l = 2,6,7,10,11,15,16$.

3.4. Characteristics of port oil piping transportation system safety impacted by its operation process related to climate-weather change process

After applying formulae for the safety function of the "m_i out of l_i"-series critical infrastructure from [EU-CIRCLE Report D3.3-Part 3, 2017], we get the safety function of the port oil piping transportation system

$$\boldsymbol{S}^4(t, \cdot) = [1, \boldsymbol{S}^4(t,1), \boldsymbol{S}^4(t, 2)], \ t \geq 0, \qquad (51)$$

where the coordinates are given by (4.88)-(4.89) in [EU-CIRCLE Report D3.3-Part 3, 2017].
The graph of the safety function of the port oil piping transportation system is given in *Figure 1*.

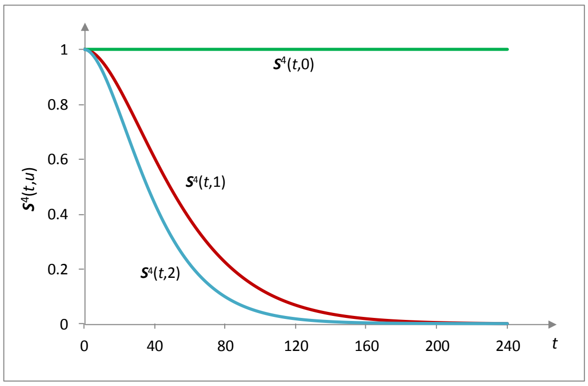

Figure 1. The graphs of the port oil piping transportation system safety function coordinates

According to (26), the conditional expected values of the port oil piping transportation system are:

– in the safety state subset {1,2}:

$[\boldsymbol{\mu}^4(1)]^{(b\ l1\ l2)}{}_{1\ \mathrm{x}\ 245} =$
[57.132024998, 57.229759661, 57.229759661, 57.229759661, 57.229759661, 57.132024998, 57.132024998, 57.132024998, 57.229759661, 57.229759661, 57.229759661, 57.229759661, 57.132024998, 57.132024998, 57.024501171, 57.121749744, 57.121749744, 57.121749744, 57.121749744, 57.024501171, 57.024501171, 57.024501171, 57.121749744, 57.121749744, 57.121749744, 57.121749744, 57.024501171, 57.024501171, 57.024501171, 57.121749744, 57.121749744, 57.121749744, 57.121749744, 57.024501171, 57.024501171;

57.132024998, 57.229759661, 57.229759661, 57.229759661, 57.229759661, 57.132024998, 57.132024998, 57.132024998, 57.229759661, 57.229759661, 57.229759661, 57.229759661, 57.132024998, 57.132024998, 57.024501171, 57.121749744, 57.121749744, 57.121749744, 57.121749744, 57.024501171, 57.024501171, 57.024501171, 57.121749744, 57.121749744, 57.121749744, 57.121749744, 57.024501171, 57.024501171, 57.024501171, 57.121749744, 57.121749744, 57.121749744, 57.121749744, 57.024501171, 57.024501171;

56.242817505, 56.363180572, 56.363180572, 56.363180572, 56.363180572, 56.242817505, 56.242817505, 56.242817505, 56.363180572, 56.363180572, 56.363180572, 56.363180572, 56.242817505, 56.242817505, 56.10318052, 56.222778787, 56.222778787, 56.222778787, 56.222778787, 56.10318052, 56.10318052, 56.10318052, 56.222778787, 56.222778787, 56.222778787, 56.222778787, 56.10318052, 56.10318052, 56.10318052, 56.222778787, 56.222778787, 56.222778787, 56.222778787, 56.10318052, 56.10318052;

52.040651851, 52.140976656, 52.140976656, 52.140976656, 52.140976656, 52.040651851, 52.040651851, 52.040651851, 52.140976656, 52.140976656, 52.140976656, 52.140976656, 52.040651851, 52.040651851, 51.927150651, 52.026907303, 52.026907303, 52.026907303, 52.026907303, 51.927150651, 51.927150651, 51.927150651, 52.026907303, 52.026907303, 52.026907303, 52.026907303, 51.927150651, 51.927150651, 51.927150651, 52.026907303, 52.026907303, 52.026907303, 52.026907303, 51.927150651, 51.927150651;

56.242817505, 56.363180572, 56.363180572, 56.363180572, 56.363180572, 56.242817505, 56.242817505, 56.242817505, 56.363180572, 56.363180572, 56.363180572, 56.363180572, 56.242817505, 56.242817505, 56.10318052, 56.222778787, 56.222778787, 56.222778787, 56.222778787, 56.10318052, 56.10318052, 56.10318052, 56.222778787, 56.222778787, 56.222778787, 56.222778787, 56.10318052, 56.10318052, 56.10318052, 56.222778787, 56.222778787, 56.222778787, 56.222778787, 56.10318052, 56.10318052;

52.040651851, 52.140976656, 52.140976656, 52.140976656, 52.140976656, 52.040651851, 52.040651851, 52.040651851, 52.140976656, 52.140976656, 52.140976656, 52.140976656, 52.040651851, 52.040651851, 51.927150651, 52.026907303, 52.026907303, 52.026907303, 52.026907303, 51.927150651, 51.927150651, 51.927150651, 52.026907303, 52.026907303, 52.026907303, 52.026907303, 51.927150651, 51.927150651, 51.927150651, 52.026907303, 52.026907303, 52.026907303, 52.026907303, 51.927150651, 51.927150651;

57.132024998, 57.229759661, 57.229759661, 57.229759661, 57.229759661, 57.132024998, 57.132024998, 57.132024998, 57.229759661, 57.229759661, 57.229759661, 57.229759661, 57.132024998, 57.132024998, 57.024501171, 57.121749744, 57.121749744, 57.121749744, 57.121749744, 57.024501171, 57.024501171, 57.024501171, 57.121749744, 57.121749744, 57.121749744, 57.121749744, 57.024501171, 57.024501171, 57.024501171, 57.121749744, 57.121749744, 57.121749744, 57.121749744, 57.024501171, 57.024501171], (52)

– in the safety state subset {2}:

$[\boldsymbol{\mu}^4(2)]^{(b\ l1\ l2)}{}_{1\ \mathrm{x}\ 245} =$
[42.34486146, 42.491353447, 42.491353447, 42.491353447, 42.491353447, 42.34486146, 42.34486146, 42.34486146, 42.491353447, 42.491353447, 42.491353447, 42.491353447, 42.34486146, 42.34486146, 42.21865764, 42.364019901, 42.364019901, 42.364019901, 42.364019901, 42.21865764, 42.21865764, 42.21865764, 42.364019901, 42.364019901, 42.364019901, 42.364019901, 42.21865764, 42.21865764, 42.21865764, 42.364019901, 42.364019901, 42.364019901, 42.364019901, 42.21865764, 42.21865764;

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40.559333949, 40.728773621, 40.728773621, 40.728773621, 40.728773621, 40.559333949, 40.559333949, 40.559333949, 40.728773621, 40.728773621, 40.728773621, 40.728773621, 40.559333949, 40.559333949, 40.406553219, 40.574381113, 40.574381113, 40.574381113, 40.574381113, 40.406553219, 40.406553219, 40.406553219, 40.574381113, 40.574381113, 40.574381113, 40.574381113, 40.406553219, 40.406553219, 40.406553219, 40.574381113, 40.574381113, 40.574381113, 40.574381113, 40.406553219, 40.406553219;

38.037573353, 38.183194221, 38.183194221, 38.183194221, 38.183194221, 38.037573353, 38.037573353, 38.037573353, 38.183194221, 38.183194221, 38.183194221, 38.183194221, 38.037573353, 38.037573353, 37.909045176, 38.053410056, 38.053410056, 38.053410056, 38.053410056, 37.909045176, 37.909045176, 37.909045176, 38.053410056, 38.053410056, 38.053410056, 38.053410056, 37.909045176, 37.909045176, 37.909045176, 38.053410056, 38.053410056, 38.053410056, 38.053410056, 37.909045176, 37.909045176;

40.559333949, 40.728773621, 40.728773621, 40.728773621, 40.728773621, 40.559333949, 40.559333949, 40.559333949, 40.728773621, 40.728773621, 40.728773621, 40.728773621, 40.559333949, 40.559333949, 40.406553219, 40.574381113, 40.574381113, 40.574381113, 40.574381113, 40.406553219, 40.406553219, 40.406553219, 40.574381113, 40.574381113, 40.574381113, 40.574381113, 40.406553219, 40.406553219, 40.406553219, 40.574381113, 40.574381113, 40.574381113, 40.574381113, 40.406553219, 40.406553219;

38.037573353, 38.183194221, 38.183194221, 38.183194221, 38.183194221, 38.037573353, 38.037573353, 38.037573353, 38.183194221, 38.183194221, 38.183194221, 38.183194221, 38.037573353, 38.037573353, 37.909045176, 38.053410056, 38.053410056, 38.053410056, 38.053410056, 37.909045176, 37.909045176, 37.909045176, 38.053410056, 38.053410056, 38.053410056, 38.053410056, 37.909045176, 37.909045176, 37.909045176, 38.053410056, 38.053410056, 38.053410056, 38.053410056, 37.909045176, 37.909045176;

42.34486146, 42.491353447, 42.491353447, 42.491353447, 42.491353447, 42.34486146, 42.34486146, 42.34486146, 42.491353447, 42.491353447, 42.491353447, 42.491353447, 42.34486146, 42.34486146, 42.21865764, 42.364019901, 42.364019901, 42.364019901, 42.364019901, 42.21865764, 42.21865764, 42.21865764, 42.364019901, 42.364019901, 42.364019901, 42.364019901, 42.21865764, 42.21865764, 42.21865764, 42.364019901, 42.364019901, 42.364019901, 42.364019901, 42.21865764, 42.21865764]. (53)

After applying (25) and (13)-(15) to (42) and (52)-(53), the mean values and standard deviations of the unconditional lifetimes of the port oil piping transportation system are:

– in the safety state subset: {1,2}

$$\boldsymbol{\mu}^4(1) \cong \sum_{b=1}^{7}\sum_{l1=1}^{6}\sum_{l2=1}^{16} pq_{b\,l1\,l2}[\boldsymbol{\mu}^4(1)]^{(b\,l1\,l2)} = 56.7439 \text{ years}, \quad (54)$$

$$\boldsymbol{\sigma}^4(1) = 38.0292 \text{ years},$$

– in the safety state subset {2}

$$\boldsymbol{\mu}^4(2) \cong \sum_{b=1}^{7}\sum_{l1=1}^{6}\sum_{l2=1}^{16} pq_{b\,l1\,l2}[\boldsymbol{\mu}^4(2)]^{(b\,l1\,l2)} = 41.8663 \text{ years}, \quad (55)$$

$$\boldsymbol{\sigma}^4(2) = 28.0922 \text{ years}.$$

From (54)-(55), applying (29), the mean lifetimes $\bar{\boldsymbol{\mu}}^4(u)$, $u = 1,2$, of the port oil piping transportation system in the particular safety states are:

$$\bar{\boldsymbol{\mu}}^4(1) = \boldsymbol{\mu}^4(1) - \boldsymbol{\mu}^4(2) = 14.8776 \text{ years},$$
$$\bar{\boldsymbol{\mu}}^4(2) = \boldsymbol{\mu}^4(2) = 41.8663 \text{ years}. \quad (56)$$

As the critical safety state is $r = 1$, then by (4), the port oil piping transportation system risk function is

$$\boldsymbol{r}^4(t) = 1 - \boldsymbol{S}^4(t, 1), \qquad (57)$$

where $\boldsymbol{S}^4(t, 1)$ is given by (50). By (16), the moment τ^4 of exceeding acceptable value of critical infrastructure risk function level $\delta = 0.05$ is

$$\tau^4 = (\boldsymbol{r}^4)^{-1}(0.05) = 10.972 \text{ years}. \qquad (58)$$

The graph of the port oil piping transportation system risk function is presented in *Figure 2*.

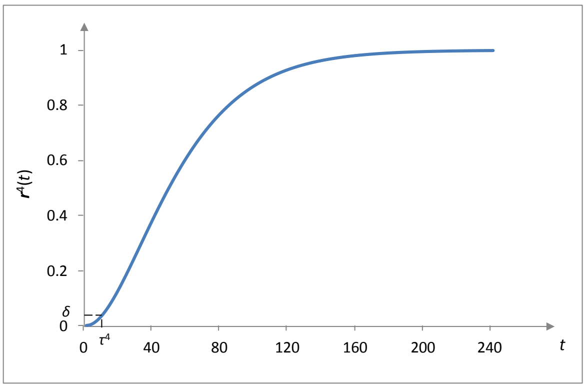

Figure 2. The graph of the port oil piping transportation system risk function

The intensities of degradation (ageing) of the port oil piping transportation system / the intensities the port oil piping transportation system departure from the safety state subset $\{1,2\}$, $\{2\}$, i.e. the coordinates of the vector

$$\boldsymbol{\lambda}^4(t,\cdot) = [0,\ \boldsymbol{\lambda}^4(t,1), \boldsymbol{\lambda}^0(t,2)\,],\ t \in <0,+\infty), \qquad (59)$$

where

$$\boldsymbol{\lambda}^4(t,u) = \frac{-\dfrac{d\boldsymbol{S}^4(t,u)}{dt}}{\boldsymbol{S}^4(t,u)},\ u = 1,2,\ t \in <0,+\infty), \qquad (60)$$

and $\boldsymbol{S}^4(t, u)$, $u = 1,2$, are given by (50)-(51)
The values of the intensities of degradation given by (60) stabilize for large time and approximately amounts

$$\boldsymbol{\lambda}^4(1) = \lim_{t \to +\infty} \boldsymbol{\lambda}^4(t,1) \cong 0.03563,$$
$$\boldsymbol{\lambda}^4(2) = \lim_{t \to +\infty} \boldsymbol{\lambda}^4(t,2) \cong 0.048966. \qquad (61)$$

The graphs of the intensities of degradation of the port oil piping transportation system are given in *Figure 3*.

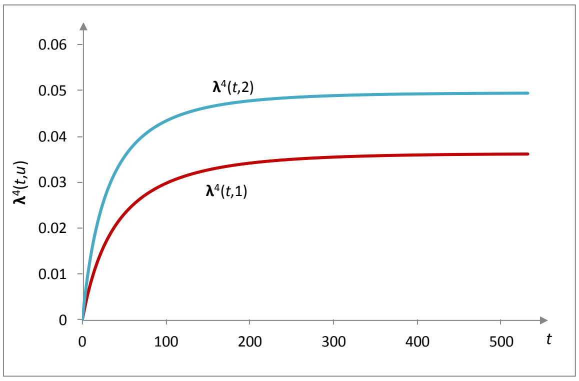

Figure 3. The graphs of the intensities of ageing of the port oil piping transportation system

According to (21) and (24), considering (4.42) from [EU-CIRCLE Report for D6.4-Part 0, 2017] and (61), the limit value of the indicator of critical infrastructure resilience to operation and climate-weather change process impact is given by

$$\boldsymbol{RI}^4(1) = \lim_{t \to +\infty} \boldsymbol{RI}^4(t,1) = \lim_{t \to +\infty} \frac{\boldsymbol{\lambda}^0(t,1)}{\boldsymbol{\lambda}^4(t,1)}$$
$$\cong 0.03271/0.035630 \cong 0.92 = 92\%. \qquad (62)$$

If we replace in the above formula the intensities of degradation by the appropriate mean values, assuming

$$\boldsymbol{\lambda}^0(t,1) \cong 1/\boldsymbol{\mu}^0(1),\ \boldsymbol{\lambda}^4(t,1) \cong 1/\boldsymbol{\mu}^4(1), \qquad (63)$$

then by (21), considering (4.36) from [EU-CIRCLE Report for D6.4 - Part 0, 2017] and (54), the approximate mean value of the indicator of critical infrastructure resilience to operation process impact is given by

$$\boldsymbol{RI}^4(1) \cong \frac{\boldsymbol{\mu}^4(1)}{\boldsymbol{\mu}^0(1)} \cong 56.7439/62.5692 \cong 0.9069$$
$$= 90.69\%. \qquad (64)$$

4. Cost analysis of critical infrastructure operation process

We consider the critical infrastructure impacted by the operation process related to climate-weather change $ZC(t)$ consisted of n components and we assume that the operation costs of its single basic components at the climate-weather state zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, during the system operation time θ, $\theta \geq 0$, amount

$$k_i^4(\theta, b, l),\ b = 1,2,\ldots,v,\ l = 1,2,\ldots w,\ i = 1,2,\ldots,n.$$

First, we suppose that the system is non-repairable, i.e. the system during the operation has not exceeded the critical safety state r. In this case, the total cost of the non-repairable system during the operation time θ, $\theta \geq 0$, is given by

$$\boldsymbol{K}^4(\theta) = \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{i=1}^{n} k_i^4(\theta,b,l), \; \theta \geq 0, \tag{65}$$

where pq_{bl}, $b = 1,2,...,v$, $l = 1,2,...w$, are the transient probabilities defined by (1)-(2).

Further, we additionally assume that the system is repairable after exceeding the critical safety state r, its renovation time is ignored and the cost of the system singular renovation is k_{ig}^4.

Then, the approximate total operation cost of the repairable system with ignored its renovation time during the operation time θ, $\theta \geq 0$, amounts

$$\boldsymbol{K}_{ig}^4(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{i=1}^{n} k_i^4(\theta,b,l) + k_{ig}^4 H^4(\theta,r),$$
$$\theta \geq 0, \tag{66}$$

where pq_{bl}, $b = 1,2,...,v$, $l = 1,2,...w$, are the transient probabilities defined by (1)-(2) and $H^4(\theta,r)$ is the mean value of the number of exceeding the critical reliability state r by the system operating at the variable conditions during the operation time θ defined by (3.58) in [Kołowrocki, Soszyńska-Budny, 2011].

Now, we assume that the system is repairable after exceeding the critical safety state r and its renewal time is non-ignored and have distribution function with the mean value $\mu_0^4(r)$ and the standard deviation $\sigma_0^4(r)$ and the cost of the system singular renovation is k_{nig}^4.

Then, the approximate total operation cost of the repairable system with non-ignored its renovation time during the operation time θ, $\theta \geq 0$, amounts

$$\boldsymbol{K}_{nig}^4(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{i=1}^{n} k_i^4(\theta,b,l) + k_{nig}^4 \; \overline{\overline{H}}^4(\theta,r),$$
$$\theta \geq 0, \tag{67}$$

where pq_{bl}, $b = 1,2,...,v$, $l = 1,2,...w$, are the transient probabilities defined by (1)-(2) and $\overline{\overline{H}}^4(\theta,r)$ is the mean value of the number of renovations of the system operating at the variable conditions during the operation time θ defined by (3.92) in [Kołowrocki, Soszyńska-Budny, 2011].

The particular expressions for the mean values $H^4(\theta,r)$ and $\overline{\overline{H}}^4(\theta,r)$ for the repairable systems with ignored and non-ignored renovation times existing in the formulae (66) and (67), respectively defined by (3.58) and (3.92), are determined in Chapter 3 in [Kołowrocki, Soszyńska-Budny, 2011] for typical repairable critical infrastructures, i.e. for multistate series, parallel, "*m* out of *n*", consecutive "*m* out of *n*: F", series-parallel, parallel-series, series-"*m* out of *k*", "m_i out of l_i"-series, series-consecutive "*m* out of *k*: F" and consecutive "m_i out of l_i: F"-series critical infrastructures operating at the variable operation conditions.

5. Cost analysis of port oil piping transportation system operation process

The port oil piping transportation system is composed of n = 2880 components and according to the information coming from experts, the approximate mean operation costs of its single basic components during the operation time is θ = 1 year, independently of the climate-weather states zc_{bl}, $b = 1,2,...,7$, $l = 1,2,...,w$, amount

$$k_i^4(\theta,b,l) \cong 9.6 \text{ PLN}, \; b = 1,2,...,7, \; l = 1,2,...,w,$$
$$i = 1,2,...,2880,$$

and it is equal to 0 in the component is not used. Thus, according to (65), if the non-repairable port oil piping transportation system during the operation is θ = 1 year has not exceeded the critical safety state r = 1, then its total operation cost during the operation time θ = 1 year is approximately given by

$$\boldsymbol{K}^4(1) \cong \sum_{b=1}^{7}\sum_{l1=1}^{6}\sum_{l2=1}^{16} pq_{bl} \sum_{i=1}^{n} k_i^4(1,b,l) \cong 0.403 \cdot 1086 \cdot 9.6$$
$$+ 0.055 \cdot 1086 \cdot 9.6 + 0.003 \cdot 1794 \cdot 9.6$$
$$+ 0.002 \cdot 2880 \cdot 9.6 + 0.199 \cdot 1794 \cdot 9.6$$
$$+ 0.057 \cdot 2880 \cdot 9.6 + 0.281 \cdot 1086 \cdot 9.6$$
$$= 12814.68 \text{ PLN}. \tag{68}$$

Further, we assume that the considered the port oil piping transportation system is repairable after exceeding the critical safety state r = 1, its renovation time is ignored and the approximate mean cost of the system singular renovation is

$$k_{ig}^4 = 88\,500 \text{ PLN}.$$

In this case, since the expected number of exceeding the critical reliability state r = 1, according to (3.58) in [Kołowrocki, Soszyńska-Budny, 2011], amounts

$H^4(1{,}1) = 1/56.75439 = 0.017623,$

the total operation cost of the repairable system with ignored its renovation time during the operation time θ = 1 year approximately amounts

$$\boldsymbol{K}_{ig}^4(1) \cong \sum_{b=1}^{7}\sum_{l1=1}^{6}\sum_{l2=1}^{16} pq_{bl}\sum_{i=1}^{n} k_i^4(1,b,l) + k_{ig}^4 H^4(1{,}1)$$
$$= 12\ 814.68 + 88\ 500 \cdot 0.017623$$
$$\cong 14\ 374.32 \text{ PLN}. \quad (69)$$

If the port oil piping transportation system is repairable after exceeding the critical safety state r = 1 and its renewal time is non-ignored and have distribution function with the mean value

$\mu_0^4(1)$ = 0.2 year

and the cost of the system singular renovation is

k_{nig}^4 = 90 000 PLN

then, since the number of exceeding the critical reliability state r = 1, according to (3.92) in [Kołowrocki, Soszyńska-Budny, 2011], amounts

$\overline{\overline{H}}{}^4(1{,}1) = 1/(56.7439 + 0.2) = 0.017561,$

the total operation cost of the repairable the port oil piping transportation system with non-ignored its renovation time during the operation time θ = 1 approximately amounts

$$\boldsymbol{K}_{nig}^4(1) \cong \sum_{b=1}^{7}\sum_{l=1}^{w} pq_{bl}\sum_{i=1}^{n} k_i^4(1,b,l) + k_{nig}^4\ \overline{\overline{H}}{}^4(1{,}1)$$
$$= 12\ 814.68 + 90\ 000 \cdot 0.017561$$
$$\cong 14395.17 \text{ PLN}. \quad (70)$$

6. Optimization of operation and safety of port oil piping transportation system

6.1. Optimization problem formulation

Considering the equation (9), it is natural to assume that the critical infrastructure operation process has a significant influence on the critical infrastructure safety. This influence is also clearly expressed in the equation (25) for the mean values of the critical infrastructure unconditional lifetimes in the safety state subsets.

From the linear equation (25), we can see that the mean value of the critical infrastructure unconditional lifetime $\boldsymbol{\mu}^4(u)$, $u = 1,2,\ldots,z,$ is determined by the limit values of transient probabilities pq_{bl}, $b=1,2,...,v$, $l=1,2,...,w$, of the critical infrastructure operation process related to climate-weather change at the operation states and the mean values $[\boldsymbol{\mu}^4(u)]^{(bl)}$, $b=1,2,...,v$, $l=1,2,...,w$, $u=1,2,...,z$, of the critical infrastructure conditional lifetimes in the safety state subsets $\{u,u+1,...,z\}$, $u=1,2,...,z$, given by (26). Therefore, the critical infrastructure lifetime optimization approach based on the linear programming [EU-CIRCLE Report D3.5-GMU, 2017] can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}q_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$, of the transient probabilities pq_{bl}, $b=1,2,...,v$, $l=1,2,...,w$, of the critical infrastructure operation process at the operation states zc_{bl}, $b=1,2,...,v$, $l=1,2,...,w$, to maximize the mean value $\boldsymbol{\mu}^4(u)$ of the unconditional critical infrastructure lifetimes in the safety state subsets $\{u,u+1,...,z\}$, $u=1,2,...,z$, under the assumption that the mean values $[\boldsymbol{\mu}^4(u)]^{(bl)}$, $b=1,2,...,v$, $l=1,2,...,w$, $u=1,2,...,z$, of the system conditional lifetimes in the safety state subsets are fixed. As a special case of the above formulated system lifetime optimization problem, if r, $r=1,2,...,z$, is a critical infrastructure critical safety state, we want to find the optimal values $\dot{p}q_{bl}$, $b=1,2,...,v$, $l=1,2,...,w$, of the transient probabilities pq_{bl}, $b=1,2,...,v$, $l=1,2,...,w$, of the critical infrastructure operation process at the critical infrastructure operation states zc_{bl}, $b=1,2,...,v$, $l=1,2,...,w$, to maximize the mean value $\boldsymbol{\mu}^4(r)$ of the unconditional critical infrastructure lifetime in the critical infrastructure state subset $\{r,r+1,...,z\}$ of the states not worse than the critical stare r, given by (12), under the assumption that the mean values $[\boldsymbol{\mu}^4(r)]^{(bl)}$, $b=1,2,...,v$, $l=1,2,...,w$, of the critical infrastructure conditional lifetimes in this safety state subset, given by (13), are fixed. More exactly, we may formulate the optimization problem as a linear programming model with the objective function of the following form

$$\boldsymbol{\mu}^4(r) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}[\boldsymbol{\mu}^4(r)]^{(bl)}, \quad (71)$$

for a fixed $r \in \{1,2,...,z\}$ and with the following bound constraints

$$\breve{p}q_{bl} \le pq_{bl} \le \hat{p}q_{bl},\ b=1,2,...,v,\ l=1,2,...,w,$$
$$\sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} = 1, \quad (72)$$

where $[\boldsymbol{\mu}^4(r)]^{(bl)}$, $[\boldsymbol{\mu}^4(r)]^{(bl)} \geq 0$, $b = 1,2,...,\nu$, $l = 1,2,...,w$, are fixed mean values of the critical infrastructure conditional lifetimes in the safety state subset $\{r, r+1,...,z\}$ and

$$\breve{p}q_{bl},\ 0 \leq \breve{p}q_{bl} \leq 1 \text{ and } \hat{p}q_{bl},\ 0 \leq \hat{p}q_{bl} \leq 1,$$
$$\breve{p}q_{bl} \leq \hat{p}q_{bl},\ b = 1,2,...,\nu,\ l = 1,2,...,w, \quad (73)$$

are lower and upper bounds of the unknown transient probabilities pq_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, respectively.

The procedure of finding the optimal values $\dot{p}q_{bl}$, $b = 1,2,...,\nu$, $l = 1,2,...,w$, of the transient probabilities pq_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, that will be applied in the next section can be found in [EU-CIRCLE Report D3.5-GMU, 2017].

6.2. Optimization of port oil piping transportation system operation process

The objective function defined by (71), in this case, as the port oil piping transportation system critical state is $r = 1$, considering (46)-(51) takes the form

$$\boldsymbol{\mu}^4(1) = \sum_{b=1}^{7} \sum_{l1=1}^{6} \sum_{l2=1}^{16} pq_{b\,l1\,l2}[\boldsymbol{\mu}^4(1)]^{(b\,l1\,l2)}. \quad (74)$$

The lower $\breve{p}_b$ and upper $\hat{p}_b$ bounds of the unknown transient probabilities p_b, $b = 1,2,...,7$, coming from experts respectively are:

$$\breve{p}_1 = 0.31,\ \breve{p}_2 = 0.04,\ \breve{p}_3 = 0.002,\ \breve{p}_4 = 0.001,$$
$$\breve{p}_5 = 0.15,\ \breve{p}_6 = 0.04,\ \breve{p}_7 = 0.25;$$
$$\hat{p}_1 = 0.46,\ \hat{p}_2 = 0.08,\ \hat{p}_3 = 0.006,\ \hat{p}_4 = 0.004,$$
$$\hat{p}_5 = 0.26,\ \hat{p}_6 = 0.08,\ \hat{p}_7 = 0.40. \quad (75)$$

Then, using formulae

$$\breve{p}q_{b\,l1\,l2} = \breve{p}_b \cdot q1_{l1} \cdot q2_{l2},$$
$$\hat{p}q_{b\,l1\,l2} = \hat{p}_b \cdot q1_{l1} \cdot q2_{l2},\ b = 1,2,\ldots,7,$$
$$l_1 = 1,2,3,5,6,\ l_2 = 2,6,7,10,11,15,16, \quad (76)$$

we calculate lower $\breve{p}q_{b\ l1\ l2}$ and upper $\hat{p}q_{b\ l1\ l2}$ bounds of the unknown transient probabilities, which are given in [EU-CIRCLE Report for D6.4-Part 4, 2017].

Therefore, according to (72)-(73) and (75), we assume the following bound constraints

$$\breve{p}q_{b\,l1\,l2} \leq pq_{b\,l1\,l2} \leq \hat{p}q_{b\,l1\,l2},\ b = 1,2,\ldots,7,$$
$$l_1 = 1,2,3,5,6,\ l_2 = 2,6,7,10,11,15,16,$$
$$\sum_{b=1}^{7} \sum_{l1=1}^{6} \sum_{l2=1}^{16} pq_{b\,l1\,l2} = 1. \quad (77)$$

Now, before we find optimal values $\dot{p}q_{bl}$, of the transient probabilities pq_{bl}, $b = 1,2,...,7$, $l1 = 1,2,3,5,6$, $l2 = 2,6,7,10,11,15,16$, that maximize the objective function (74), w arrange the system conditional lifetime mean values $[\boldsymbol{\mu}^4(1)]^{(b\ l1\ l2)}$ $b = 1,2,\ldots,7$, $l1 = 1,2,3,5,6$, $l2 = 2,6,7,10,11,15,16$, in non-increasing order. Next, according to procedure given in [EU-CIRCLE Report D3.5-GMU, 2017], we maximize with respect to x_i, $i = 1,2,\ldots, 245$, the linear form (7.4) with the following bound constraints

$$\breve{x}_b \leq x_b \leq \hat{x}_b,\ b = 1,2,\ldots,245,\ \sum_{i=1}^{245} x_i = 1. \quad (78)$$

Finally, according to the procedure given in [EU-CIRCLE Report D3.5-GMU,2017], we get the optimal transient probabilities

$\dot{p}q_{1\,1\,2} = 0.006778460$, $\dot{p}q_{1\,1\,6} = 0.107160220$,
$\dot{p}q_{1\,1\,7} = 0.005416040$, $\dot{p}q_{1\,1\,10} = 0.003094880$,
$\dot{p}q_{1\,1\,11} = 0.236758320$, $\dot{p}q_{1\,1\,15} = 0.016164020$,
$\dot{p}q_{1\,1\,16} = 0.000260710$, $\dot{p}q_{1\,2\,2} = 0.001217060$,
$\dot{p}q_{1\,2\,6} = 0.019240420$, $\dot{p}q_{1\,2\,7} = 0.000972440$,
$\dot{p}q_{1\,2\,10} = 0.000555680$, $\dot{p}q_{1\,2\,11} = 0.042509520$,
$\dot{p}q_{1\,2\,15} = 0.002902220$, $\dot{p}q_{1\,2\,16} = 0.000046810$,
$\dot{p}q_{1\,3\,2} = 0.000008060$, $\dot{p}q_{1\,3\,6} = 0.000085870$,
$\dot{p}q_{1\,3\,7} = 0.000004340$, $\dot{p}q_{1\,3\,10} = 0.000002480$,
$\dot{p}q_{1\,3\,11} = 0.000189720$, $\dot{p}q_{1\,3\,15} = 0.000019220$,
$\dot{p}q_{1\,3\,16} = 0.000000310$, $\dot{p}q_{1\,5\,2} = 0.000048360$,
$\dot{p}q_{1\,5\,6} = 0.000515220$, $\dot{p}q_{1\,5\,7} = 0.000026040$,
$\dot{p}q_{1\,5\,10} = 0.000014880$, $\dot{p}q_{1\,5\,11} = 0.001138320$,
$\dot{p}q_{1\,5\,15} = 0.000115320$, $\dot{p}q_{1\,5\,16} = 0.000001860$,
$\dot{p}q_{1\,6\,2} = 0.000008060$, $\dot{p}q_{1\,6\,6} = 0.000085870$,
$\dot{p}q_{1\,6\,7} = 0.000004340$, $\dot{p}q_{1\,6\,10} = 0.000002480$,
$\dot{p}q_{1\,6\,11} = 0.000189720$, $\dot{p}q_{1\,6\,15} = 0.000019220$,
$\dot{p}q_{1\,6\,16} = 0.000000310$, $\dot{p}q_{2\,1\,2} = 0.000874640$,
$\dot{p}q_{2\,1\,6} = 0.018636560$, $\dot{p}q_{2\,1\,7} = 0.000941920$,
$\dot{p}q_{2\,1\,10} = 0.000538240$, $\dot{p}q_{2\,1\,11} = 0.041175360$,
$\dot{p}q_{2\,1\,15} = 0.002085680$, $\dot{p}q_{2\,1\,16} = 0.000033640$,
$\dot{p}q_{2\,2\,2} = 0.000157040$, $\dot{p}q_{2\,2\,6} = 0.003346160$,
$\dot{p}q_{2\,2\,7} = 0.000169120$, $\dot{p}q_{2\,2\,10} = 0.000096640$,
$\dot{p}q_{2\,2\,11} = 0.007392960$, $\dot{p}q_{2\,2\,15} = 0.000374480$,
$\dot{p}q_{2\,2\,16} = 0.000006040$, $\dot{p}q_{2\,3\,2} = 0.000001040$,
$\dot{p}q_{2\,3\,6} = 0.000011080$, $\dot{p}q_{2\,3\,7} = 0.000000560$,
$\dot{p}q_{2\,3\,10} = 0.000000320$, $\dot{p}q_{2\,3\,11} = 0.000024480$,
$\dot{p}q_{2\,3\,15} = 0.000002480$, $\dot{p}q_{2\,3\,16} = 0.000000040$,
$\dot{p}q_{2\,5\,2} = 0.000006240$, $\dot{p}q_{2\,5\,6} = 0.000066480$,

$\dot{p}q_{2\,5\,7} = 0.000003360$, $\dot{p}q_{2\,5\,10} = 0.000001920$,
$\dot{p}q_{2\,5\,11} = 0.000146880$, $\dot{p}q_{2\,5\,15} = 0.000014880$,
$\dot{p}q_{2\,5\,16} = 0.000000240$, $\dot{p}q_{2\,6\,2} = 0.000001040$,
$\dot{p}q_{2\,6\,6} = 0.000011080$, $\dot{p}q_{2\,6\,7} = 0.000000560$,
$\dot{p}q_{2\,6\,10} = 0.000000320$, $\dot{p}q_{2\,6\,11} = 0.000024480$,
$\dot{p}q_{2\,6\,15} = 0.000002480$, $\dot{p}q_{2\,6\,16} = 0.000000040$,
$\dot{p}q_{3\,1\,2} = 0.000043732$, $\dot{p}q_{3\,1\,6} = 0.000465914$,
$\dot{p}q_{3\,1\,7} = 0.000023548$, $\dot{p}q_{3\,1\,10} = 0.000013456$,
$\dot{p}q_{3\,1\,11} = 0.001029384$, $\dot{p}q_{3\,1\,15} = 0.000104284$,
$\dot{p}q_{3\,1\,16} = 0.000001682$, $\dot{p}q_{3\,2\,2} = 0.000007852$,
$\dot{p}q_{3\,2\,6} = 0.000083654$, $\dot{p}q_{3\,2\,7} = 0.000004228$,
$\dot{p}q_{3\,2\,10} = 0.000002416$, $\dot{p}q_{3\,2\,11} = 0.000184824$,
$\dot{p}q_{3\,2\,15} = 0.000018724$, $\dot{p}q_{3\,2\,16} = 0.000000302$,
$\dot{p}q_{3\,3\,2} = 0.000000052$, $\dot{p}q_{3\,3\,6} = 0.000000554$,
$\dot{p}q_{3\,3\,7} = 0.000000028$, $\dot{p}q_{3\,3\,10} = 0.000000016$,
$\dot{p}q_{3\,3\,11} = 0.000001224$, $\dot{p}q_{3\,3\,15} = 0.000000124$,
$\dot{p}q_{3\,3\,16} = 0.000000002$, $\dot{p}q_{3\,5\,2} = 0.000000312$,
$\dot{p}q_{3\,5\,6} = 0.000003324$, $\dot{p}q_{3\,5\,7} = 0.000000168$,
$\dot{p}q_{3\,5\,10} = 0.000000096$, $\dot{p}q_{3\,5\,11} = 0.000007344$,
$\dot{p}q_{3\,5\,15} = 0.000000744$, $\dot{p}q_{3\,5\,16} = 0.000000012$,
$\dot{p}q_{3\,6\,2} = 0.000000052$, $\dot{p}q_{3\,6\,6} = 0.000000554$,
$\dot{p}q_{3\,6\,7} = 0.000000028$, $\dot{p}q_{3\,6\,10} = 0.000000016$,
$\dot{p}q_{3\,6\,11} = 0.000001224$, $\dot{p}q_{3\,6\,15} = 0.000000124$,
$\dot{p}q_{3\,6\,16} = 0.000000002$, $\dot{p}q_{4\,1\,2} = 0.000021866$,
$\dot{p}q_{4\,1\,6} = 0.000232957$, $\dot{p}q_{4\,1\,7} = 0.000011774$,
$\dot{p}q_{4\,1\,10} = 0.000006728$, $\dot{p}q_{4\,1\,11} = 0.000514692$,
$\dot{p}q_{4\,1\,15} = 0.000052142$, $\dot{p}q_{4\,1\,16} = 0.000000841$,
$\dot{p}q_{4\,2\,2} = 0.000003926$, $\dot{p}q_{4\,2\,6} = 0.000041827$,
$\dot{p}q_{4\,2\,7} = 0.000002114$, $\dot{p}q_{4\,2\,10} = 0.000001208$,
$\dot{p}q_{4\,2\,11} = 0.000092412$, $\dot{p}q_{4\,2\,15} = 0.000009362$,
$\dot{p}q_{4\,2\,16} = 0.000000151$, $\dot{p}q_{4\,3\,2} = 0.000000026$,
$\dot{p}q_{4\,3\,6} = 0.000000277$, $\dot{p}q_{4\,3\,7} = 0.000000014$,
$\dot{p}q_{4\,3\,10} = 0.000000008$, $\dot{p}q_{4\,3\,11} = 0.000000612$,
$\dot{p}q_{4\,3\,15} = 0.000000062$, $\dot{p}q_{4\,3\,16} = 0.000000001$,
$\dot{p}q_{4\,5\,2} = 0.000000156$, $\dot{p}q_{4\,5\,6} = 0.000001662$,
$\dot{p}q_{4\,5\,7} = 0.000000084$, $\dot{p}q_{4\,5\,10} = 0.000000048$,
$\dot{p}q_{4\,5\,11} = 0.000003672$, $\dot{p}q_{4\,5\,15} = 0.000000372$,
$\dot{p}q_{4\,5\,16} = 0.000000006$, $\dot{p}q_{4\,6\,2} = 0.000000026$,
$\dot{p}q_{4\,6\,6} = 0.000000277$, $\dot{p}q_{4\,6\,7} = 0.000000014$,
$\dot{p}q_{4\,6\,10} = 0.000000008$, $\dot{p}q_{4\,6\,11} = 0.000000612$,
$\dot{p}q_{4\,6\,15} = 0.000000062$, $\dot{p}q_{4\,6\,16} = 0.000000001$,
$\dot{p}q_{5\,1\,2} = 0.003279900$, $\dot{p}q_{5\,1\,6} = 0.034943550$,
$\dot{p}q_{5\,1\,7} = 0.001766100$, $\dot{p}q_{5\,1\,10} = 0.001009200$,
$\dot{p}q_{5\,1\,11} = 0.077203800$, $\dot{p}q_{5\,1\,15} = 0.007821300$,
$\dot{p}q_{5\,1\,16} = 0.000126150$, $\dot{p}q_{5\,2\,2} = 0.000588900$,
$\dot{p}q_{5\,2\,6} = 0.006274050$, $\dot{p}q_{5\,2\,7} = 0.000317100$,
$\dot{p}q_{5\,2\,10} = 0.000181200$, $\dot{p}q_{5\,2\,11} = 0.013861800$,
$\dot{p}q_{5\,2\,15} = 0.001404300$, $\dot{p}q_{5\,2\,16} = 0.000022650$,
$\dot{p}q_{5\,3\,2} = 0.000003900$, $\dot{p}q_{5\,3\,6} = 0.000041550$,
$\dot{p}q_{5\,3\,7} = 0.000002100$, $\dot{p}q_{5\,3\,10} = 0.000001200$,
$\dot{p}q_{5\,3\,11} = 0.000091800$, $\dot{p}q_{5\,3\,15} = 0.000009300$,
$\dot{p}q_{5\,3\,16} = 0.000000150$, $\dot{p}q_{5\,5\,2} = 0.000023400$,
$\dot{p}q_{5\,5\,6} = 0.000249300$, $\dot{p}q_{5\,5\,7} = 0.000012600$,
$\dot{p}q_{5\,5\,10} = 0.000007200$, $\dot{p}q_{5\,5\,11} = 0.000550800$,
$\dot{p}q_{5\,5\,15} = 0.000055800$, $\dot{p}q_{5\,5\,16} = 0.000000900$,
$\dot{p}q_{5\,6\,2} = 0.000003900$, $\dot{p}q_{5\,6\,6} = 0.000041550$,
$\dot{p}q_{5\,6\,7} = 0.000002100$, $\dot{p}q_{5\,6\,10} = 0.000001200$,
$\dot{p}q_{5\,6\,11} = 0.000091800$, $\dot{p}q_{5\,6\,15} = 0.000009300$,
$\dot{p}q_{5\,6\,16} = 0.000000150$, $\dot{p}q_{6\,1\,2} = 0.000874640$,
$\dot{p}q_{6\,1\,6} = 0.009318280$, $\dot{p}q_{6\,1\,7} = 0.000470960$,
$\dot{p}q_{6\,1\,10} = 0.000269120$, $\dot{p}q_{6\,1\,11} = 0.020587680$,
$\dot{p}q_{6\,1\,15} = 0.002085680$, $\dot{p}q_{6\,1\,16} = 0.000033640$,
$\dot{p}q_{6\,2\,2} = 0.000157040$, $\dot{p}q_{6\,2\,6} = 0.001673080$,
$\dot{p}q_{6\,2\,7} = 0.000084560$, $\dot{p}q_{6\,2\,10} = 0.000048320$,
$\dot{p}q_{6\,2\,11} = 0.003696480$, $\dot{p}q_{6\,2\,15} = 0.000374480$,
$\dot{p}q_{6\,2\,16} = 0.000006040$, $\dot{p}q_{6\,3\,2} = 0.000001040$,
$\dot{p}q_{6\,3\,6} = 0.000011080$, $\dot{p}q_{6\,3\,7} = 0.000000560$,
$\dot{p}q_{6\,3\,10} = 0.000000320$, $\dot{p}q_{6\,3\,11} = 0.000024480$,
$\dot{p}q_{6\,3\,15} = 0.000002480$, $\dot{p}q_{6\,3\,16} = 0.000000040$,
$\dot{p}q_{6\,5\,2} = 0.000006240$, $\dot{p}q_{6\,5\,6} = 0.000066480$,
$\dot{p}q_{6\,5\,7} = 0.000003360$, $\dot{p}q_{6\,5\,10} = 0.000001920$,
$\dot{p}q_{6\,5\,11} = 0.000146880$, $\dot{p}q_{6\,5\,15} = 0.000014880$,
$\dot{p}q_{6\,5\,16} = 0.000000240$, $\dot{p}q_{6\,6\,2} = 0.000001040$,
$\dot{p}q_{6\,6\,6} = 0.000011080$, $\dot{p}q_{6\,6\,7} = 0.000000560$,
$\dot{p}q_{6\,6\,10} = 0.000000320$, $\dot{p}q_{6\,6\,11} = 0.000024480$,
$\dot{p}q_{6\,6\,15} = 0.000002480$, $\dot{p}q_{6\,6\,16} = 0.000000040$,
$\dot{p}q_{7\,1\,2} = 0.005466500$, $\dot{p}q_{7\,1\,6} = 0.093182800$,
$\dot{p}q_{7\,1\,7} = 0.003294670$, $\dot{p}q_{7\,1\,10} = 0.001682000$,
$\dot{p}q_{7\,1\,11} = 0.128673000$, $\dot{p}q_{7\,1\,15} = 0.013035500$,
$\dot{p}q_{7\,1\,16} = 0.000210250$, $\dot{p}q_{7\,2\,2} = 0.000981500$,
$\dot{p}q_{7\,2\,6} = 0.010456750$, $\dot{p}q_{7\,2\,7} = 0.000528500$,
$\dot{p}q_{7\,2\,10} = 0.000302000$, $\dot{p}q_{7\,2\,11} = 0.023103000$,
$\dot{p}q_{7\,2\,15} = 0.002340500$, $\dot{p}q_{7\,2\,16} = 0.000037750$,
$\dot{p}q_{7\,3\,2} = 0.000006500$, $\dot{p}q_{7\,3\,6} = 0.000069250$,
$\dot{p}q_{7\,3\,7} = 0.000003500$, $\dot{p}q_{7\,3\,10} = 0.000002000$,
$\dot{p}q_{7\,3\,11} = 0.000153000$, $\dot{p}q_{7\,3\,15} = 0.000015500$,
$\dot{p}q_{7\,3\,16} = 0.000000250$, $\dot{p}q_{7\,5\,2} = 0.000039000$,
$\dot{p}q_{7\,5\,6} = 0.000415500$, $\dot{p}q_{7\,5\,7} = 0.000021000$,
$\dot{p}q_{7\,5\,10} = 0.000012000$, $\dot{p}q_{7\,5\,11} = 0.000918000$,
$\dot{p}q_{7\,5\,15} = 0.000093000$, $\dot{p}q_{7\,5\,16} = 0.000001500$,
$\dot{p}q_{7\,6\,2} = 0.000006500$, $\dot{p}q_{7\,6\,6} = 0.000069250$,
$\dot{p}q_{7\,6\,7} = 0.000003500$, $\dot{p}q_{7\,6\,10} = 0.000002000$,
$\dot{p}q_{7\,6\,11} = 0.000153000$, $\dot{p}q_{7\,6\,15} = 0.000015500$,
$\dot{p}q_{7\,6\,16} = 0.000000250$, (79)

that maximize the pipeline system mean lifetime $\boldsymbol{\mu}^4(1)$ in the safety state subset $\{1,2\}$ expressed by the linear form (74).

Considering (79), and assuming as in Section 3.2 the system operation time $\theta = 1$ year = 365 days, after appropriate formula from [EU-CIRCLE Report D3.5-GMU, 2017], we get the optimal mean values of the total sojourn times at the particular operation states during this operation time, given by (7.16) in [EU-CIRCLE Report for D6.4-Part 4, 2017].

6.3 Optimal safety characteristics of port oil piping transportation system

Thus, as a result of Section 7.2 analysis, the optimal value of the port oil piping transportation system lifetime $\boldsymbol{\mu}^4(1)$ in the safety state subset $\{1,2\}$, according to (74) and (79), is

$$\dot{\boldsymbol{\mu}}^4(1) \cong 56.8814. \tag{80}$$

Further, we obtain the optimal solution for the mean value of the port oil piping transportation system unconditional lifetime in the safety state subset $\{2\}$

$$\dot{\boldsymbol{\mu}}^4(2) \cong 42.0353, \tag{81}$$

and according to (6.23) in [Kołowrocki, Soszyńska-Budny, 2011], the optimal values of the mean values of the port oil piping transportation system unconditional lifetimes in the particular safety states 1 and 2, respectively are

$$\dot{\bar{\boldsymbol{\mu}}}^4(1) = \dot{\boldsymbol{\mu}}^4(1) - \dot{\boldsymbol{\mu}}^4(2) = 14.8461$$
$$\dot{\bar{\boldsymbol{\mu}}}^4(2) = \dot{\boldsymbol{\mu}}^4(2) = 42.0353. \tag{82}$$

Moreover, according to (6.20)-(6.21) from [Kołowrocki, Soszyńska-Budny, 2011], considering the intensities of departure of the assets from Section 3.3, the corresponding optimal unconditional multistate safety function of the port oil piping transportation system (Saf1) is of the form

$$\dot{\boldsymbol{S}}^4(t,\cdot) = [1,\ \dot{\boldsymbol{S}}^4(t,1), \dot{\boldsymbol{S}}^4(t,2)], \tag{83}$$

with the coordinates given by (7.21)-(7.22) in [EU-CIRCLE Report for D6.4-Part 4, 2017].
Further, by (6.22) from [Kołowrocki, Soszyńska-Budny, 2011], considering (80)-(81) and (90), the corresponding optimal standard deviations of the port oil piping transportation system unconditional lifetime in the state subsets are

$$\dot{\boldsymbol{\sigma}}^4(1) \cong 38.1107, \tag{84}$$

$$\dot{\boldsymbol{\sigma}}^4(2) \cong 28.1875. \tag{85}$$

As the port oil piping transportation system critical safety state is r =1, then its optimal system risk function, according to (6.24) in [Kołowrocki, Soszyńska-Budny, 2011], considering (90), is given by

$$\dot{\boldsymbol{r}}^4(t) = 1 - \dot{\boldsymbol{S}}^4(t,1),\ t \geq 0, \tag{86}$$

where $\dot{\boldsymbol{S}}^4(t,1)$ is given by (86). Hence, and considering (6.25) in [Kołowrocki, Soszyńska-Budny, 2011], the moment when the optimal system risk function exceeds a permitted level (SafI6), for instance δ = 0.05, is

$$\dot{\boldsymbol{\tau}}^4 = (\dot{\boldsymbol{r}}^4)^{-1}(0.05) \cong 10.9986 \text{ year}. \tag{87}$$

By (80) and (84), the port oil piping transportation system optimal mean lifetime up to exceeding critical safety state r = 1 (SafI4) is

$$\dot{\boldsymbol{\mu}}^4(1) \cong 56.8814 \text{ years}, \tag{88}$$

and the optimal standard deviation of the port oil piping transportation system lifetime up to exceeding critical safety state r = 1 (SafI5) is

$$\dot{\boldsymbol{\sigma}}^4(1) \cong 38.1107. \tag{89}$$

By (90), applying (60), the port oil piping transportation system optimal intensities of ageing (SafI7) are:

$$\dot{\boldsymbol{\lambda}}^4(t,1) \cong 0.035630 \text{ for large } t, \tag{90}$$

$$\dot{\boldsymbol{\lambda}}^4(t,2) \cong 0.048966 \text{ for large } t. \tag{91}$$

Considering (90)-(91) and the values of the port oil piping transportation system intensities of ageing without of operation impact from [EU-CIRCLE Report for D6.4-Part 0, 2017] and applying (41), the optimal coefficients of the operation process impact on the port oil piping transportation system intensities of ageing (SafI8) are:

$$\dot{\boldsymbol{\rho}}^4(t,1) = \frac{\dot{\boldsymbol{\lambda}}^4(t,1)}{\boldsymbol{\lambda}^0(t,1)} = \frac{0.035630}{0.032710} \cong 1.089, \tag{92}$$

$$\dot{\boldsymbol{\rho}}^4(t,2) = \frac{\dot{\boldsymbol{\lambda}}^4(t,2)}{\boldsymbol{\lambda}^0(t,2)} = \frac{0.048966}{0.045330} \cong 1.080. \tag{93}$$

Finally, by (62) and (92), the optimal port oil piping transportation system resilience indicator (RI1), i.e. the coefficient of the port oil piping transportation system resilience to operation process impact, is

$$\boldsymbol{R}\dot{\boldsymbol{I}}(t) = 1/\dot{\boldsymbol{\rho}}(t,1) \cong 0.918 \cong 92\%,\ t \in < 0,+\infty). \tag{94}$$

If we replace in the above formula the intensities of degradation by the appropriate mean values, assuming

$$\boldsymbol{\lambda}^0(t, 1) \cong 1/\boldsymbol{\mu}^0(1), \ \dot{\boldsymbol{\lambda}}^4(t,1) \cong 1/\dot{\boldsymbol{\mu}}^4(1), \quad (95)$$

then by (21), considering (4.36) from [EU-CIRCLE Report for D6.4, Part 0, GMU-V1.0, 2017] and (80), the approximate mean value of the indicator of critical infrastructure resilience to operation process impact is given by

$$\boldsymbol{R}\dot{\boldsymbol{I}}^4(t) = \dot{\boldsymbol{\mu}}^4(1) / \boldsymbol{\mu}^0(1) = 56.8814 / 62.5692 \cong 0.9091 = 90.91\%.$$

6.4. Port oil piping transportation system operation strategy

Using formula

$$\dot{p}_{b|l1\,l2} = P(\dot{Z}(t) = z_b \mid C1(t) = c1_{l1}, C2(t) = c2_{l2}) = \dot{p}q_{b\,l1\,l2} / (q1_{l1} \cdot q2_{l2}),\ b = 1,2,\ldots,7,$$
$$l_1 = 1,2,3,5,6,\ l_2 = 2,6,7,10,11,15,16,$$

we receive

$\dot{p}_{1|l1\,l2} = 0.310$, $l_1 = 1,2$, $l_2 = 2,15,16$,
$\dot{p}_{1|l1\,l2} = 0.460$, $l_1 = 1,2$, $l_2 = 6,7,10,11$,
$\dot{p}_{1|l1\,l2} = 0.310$, $l_1 = 3,5,6$, $l_2 = 2,6,7,10,11,15,16$,

$\dot{p}_{2|l1\,l2} = 0.040$, $l_1 = 1,2$, $l_2 = 2,15,16$,
$\dot{p}_{2|l1\,l2} = 0.080$, $l_1 = 1,2$, $l_2 = 6,7,10,11$,
$\dot{p}_{2|l1\,l2} = 0.040$, $l_1 = 3,5,6$, $l_2 = 2,6,7,10,11,15,16$,

$\dot{p}_{3|l1\,l2} = 0.002$, $l_1 = 1,2,3,5,6$, $l_2 = 2,6,7,10,11,15,16$,
$\dot{p}_{4|l1\,l2} = 0.001$, $l_1 = 1,2,3,5,6$, $l_2 = 2,6,7,10,11,15,16$,
$\dot{p}_{5|l1\,l2} = 0.150$, $l_1 = 1,2,3,5,6$, $l_2 = 2,6,7,10,11,15,16$,
$\dot{p}_{6|l1\,l2} = 0.040$, $l_1 = 1,2,3,5,6$, $l_2 = 2,6,7,10,11,15,16$,

$\dot{p}_{7|l1\,l2} = 0.025$, $l_1 = 1$, $l_2 = 2,10,11,15,16$,
$\dot{p}_{7|l1\,l2} = 0.400$, $l_1 = 1$, $l_2 = 6$,
$\dot{p}_{7|l1\,l2} = 0.280$, $l_1 = 1$, $l_2 = 7$,
$\dot{p}_{7|l1\,l2} = 0.025$, $l_1 = 2,3,5,6$, $l_2 = 2,6,7,10,11,15,16$.

As we can see above, the climate-weather change process does not have any influence on the optimised transient probabilities of operation states $\dot{p}_3$, $\dot{p}_4$, $\dot{p}_5$, $\dot{p}_6$, but it has an influence on the optimised transient probabilities of operation states $\dot{p}_1$, $\dot{p}_2$, $\dot{p}_7$.

The knowledge of optimal transient probabilities $\dot{p}_{b\,l1\,l2}$, $b = 1,2,\ldots,7$, $l1 = 1,2,3,5,6$, $l2 = 2,6,7,10,11,15,16$, at the particular operation states given by (79), may be the basis to improving the port oil piping transportation system safety indicators before its operation process optimization determined in Section 3.1 to that determined after its operation process optimization determined in Section 6.2. This justifies the sensibility of the performed operation process optimization, and some suggestions on new strategy of the port oil piping transportation system operation process organizing should be proposed.

The first suggestion is to organize intuitively the operation process in the way that makes the transient probabilities $[pq1_{bl}]_{b\,=\,1,2,\ldots,7,\ l\,=\,1,2,3,5,6}$ and $[pq2_{bl}]_{b\,=\,1,2,\ldots,7,\ l\,=\,2,6,7,10,11,15,16}$ at the particular operation states before the optimization, given by (42) and (43), approximately convergent to their optimal values $\dot{p}_{b\,l1\,l2}$, $b = 1,2,\ldots,7$, $l1 = 1,2,3,5,6$, $l2 = 2,6,7,10,11,15,16$, given by (79).

The easiest way of the port oil piping transportation system operation process reorganizing is that leading to the approaching the values of its total sojourn times $\hat{M}N_{bl}$, $b = 1,2,\ldots,7$, $l = 1,2,3,5,6$, at the particular operation states during the fixed operation time for instance $\theta = 1$ year, before the optimization given in Section 3.1 to the values of its optimal total sojourn times $\dot{\hat{M}}N_{b\,l1\,l2}$, $b = 1,2,\ldots,7$, $l1 = 1,2,3,5,6$, $l2 = 2,6,7,10,11,15,16$, after the operation pocess optimization given by (7.16) in [EU-CIRCLE Report for D6.4-Part 4, 2017].

More complicated way of the complex system operation process reorganization after its optimization is proposed in [Kołowrocki, Soszyńska-Budny, 2011].

7. Critical infrastructure operation cost optimization

7.1. Critical infrastructure optimal operation cost after its operation optimization with respect to its safety maximization

After the optimization of the critical infrastructure operation process and safety, the critical infrastructure total operation costs given by (68)-(70) assume their optimal values expressed by the appropriate formulae given in this section.

The total optimal cost of the non-repairable critical infrastructure during the operation time θ, $\theta \geq 0$, is given by

$$\dot{\boldsymbol{K}}^4(\theta) = \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl} \sum_{i=1}^{n} k_i^4(\theta,b,l),\ \ \theta \geq 0, \quad (96)$$

where $\dot{p}q_{bl}$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are the optimal transient probabilities defined in Section 6.1.

The optimal total operation cost of the repairable system with ignored its renovation time during the operation time θ, $\theta \geq 0$, amounts

$$\dot{\boldsymbol{K}}_{ig}^4(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl} \sum_{i=1}^{n} k_i^4(\theta,b,l) + k_{ig}^4 \dot{H}^4(\theta, r),$$
$$\theta \geq 0, \quad (97)$$

where $\dot{p}q_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, are the optimal transient probabilities defined in Section 6.1 and $\dot{H}^4(\theta, r)$ is the mean value of the optimal number of exceeding the critical reliability state r by the system operating at the variable conditions during the operation time θ defined by (6.29) in [Kołowrocki, Soszyńska-Budny, 2011].

The optimal total operation cost of the repairable system with non-ignored its renovation time during the operation time θ, $\theta \ge 0$, amounts

$$\dot{\boldsymbol{K}}^4_{nig}(\theta) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl} \sum_{i=1}^{n} k_i^4(\theta, b, l) + k^4_{nig} \ \dot{\overline{\overline{H}}}{}^4(\theta, r),$$
$$\theta \ge 0, \quad (98)$$

where $\dot{p}q_{bl}$, $b = 1,2,...,v$, $l = 1,2,...,w$, are the optimal transient probabilities defined in Section 6.1 and $\dot{\overline{\overline{H}}}{}^4(\theta, r)$ is the mean value of the optimal number of renovations of the system operating at the variable conditions during the operation time θ defined by (6.37) in [Kołowrocki, Soszyńska-Budny, 2011].

The particular expressions for the mean values $\dot{H}^4(\theta, r)$ and $\dot{\overline{\overline{H}}}{}^4(\theta, r)$ for the repairable systems with ignored and non-ignored renovation times existing in the formulae (97) and (98), respectively defined by (6.29) and (6.37), are determined in Chapter 6 in [Kołowrocki, Soszyńska-Budny, 2011] for typical repairable critical infrastructures, i.e. for multistate series, parallel, "*m* out of *n*", consecutive "*m* out of *n*: F", series-parallel, parallel-series, series-"*m* out of *k*", "m_i out of l_i"-series, series-consecutive "*m* out of *k*: F" and consecutive "m_i out of l_i: F"-series critical infrastructures operating at the variable operation conditions.

7.2. Port oil piping transportation system operation cost optimization

7.2.1. Port oil piping transportation system optimal operation cost after its operation optimization with respect to its safety maximization

In this section, we will analyze the port oil piping transportation system operation cost after its operation process optimization.

Thus, according to (96), if the non-repairable port oil piping transportation system during the operation is θ = 1 year has not exceeded the critical safety state r = 1, then its optimal total operation cost during the operation time θ = 1 year is approximately given by

$$\dot{\boldsymbol{K}}^4(1) \cong \sum_{b=1}^{7}\sum_{l=1}^{w} \dot{p}q_{bl} \sum_{i=1}^{n} k_i^4(1, b, l) \cong 0.46 \cdot 1086 \cdot 9.6$$
$$+ 0.08 \cdot 1086 \cdot 9.6 + 0.002 \cdot 1794 \cdot 9.6$$
$$+ 0.001 \cdot 2880 \cdot 9.6 + 0.15 \cdot 1794 \cdot 9.6$$
$$+ 0.04 \cdot 2880 \cdot 9.6 + 0.267 \cdot 1086 \cdot 9.6$$
$$= 12\ 164.83 \text{ PLN.} \quad (99)$$

Further, as the expected optimal number of exceeding the critical reliability state r = 1 amounts

$\dot{H}^4(1,1)$ = 1/56.8894 = 0.01758,

then according to (97), the optimal total operation cost of the repairable system with ignored its renovation time during the operation time θ = 1 year approximately amounts

$$\dot{\boldsymbol{K}}^4_{ig}(1) \cong \sum_{b=1}^{7}\sum_{l=1}^{w} \dot{p}q_{bl} \sum_{i=1}^{n} k_i^4(1, b, l) + k^4_{ig} \dot{H}^4(1,1)$$
$$= 12164.83 + 88\ 500 \cdot 0.01758$$
$$= 12164.83 + 1555.83 \cong 13\ 721 \text{ PLN.} \quad (100)$$

Since the expected optimal number of exceeding the critical reliability state r = 1 amounts

$\dot{\overline{\overline{H}}}{}^4(1,1)$ = 1/(56.8894 +0.2) =0.01752,

the total optimal operation cost of the repairable the port oil piping transportation system with non-ignored its renovation time during the operation time θ = 1 approximately amounts

$$\dot{\boldsymbol{K}}^4_{nig}(1) \cong \sum_{b=1}^{7}\sum_{l=1}^{w} \dot{p}q_{bl} \sum_{i=1}^{n} k_i^4(1, b, l) + k^4_{nig} \ \dot{\overline{\overline{H}}}{}^4(1,1)$$
$$= 12164.83 + 90\ 000 \cdot 0.01752$$
$$= 12164.83 + 1576.8 \cong 13\ 742 \text{ PLN.} \quad (101)$$

8. Conclusions

The proposed in [EU-CIRCLE Report D3.3-Part 3, 2017] Model 4 of critical infrastructure safety was applied to safety analysis of the port oil piping transportation system related to climate-weather change process and operation process. The application of this model is supported by suitable computer software that is placed at the GMU Safety Interactive Platform http://gmu.safety.am.gdynia.pl/.

The results of this application are the generalizations of the three earlier parts of the series of 4 papers concerned with the EU-CIRCLE project Case Study 2, Storm and Sea Surge at Baltic Sea Port presented in this issue of JPSRA, applied to the safety and resilience analysis of port oil piping transportation system impacted by its operation process related to climate-weather change.

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