APPLICATION OF THE SKEW SURFACES IN DESIGN OF TIMBER STRUCTURES IN THE ELABORATION BY TEOFIL ŻEBRAWSKI „A FEW DESCRIPTIVE GEOMETRY PROBLEMS AS AN ADDITION TO THE WORK OF F. SAPALSKI WITH AN EXAMPLE OF APPLICATION OF SKEW SURFACES TO CARPENTRY" (1847), („KILKA ZADAŃ Z GEOMETRII WYKREŚLNEJ JAKO DODATKU DO DZIEŁA F. SAPALSKIEGO Z PRZYKŁADEM ZASTOSOWANIA POWIERZCHNI WICHROWATYCH W CIESIOŁCE") 1847R

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Abstract. In the article geometrical problems elaborated by Teofil Żebrawski and published in 1847 have been presented. The paper printed by J. Cypeer in Krakow was an extension and complement of the first textbook on descriptive geometry by Franciszek Sapalski entitled "*Descriptive Geometry with Application to Perspective, Shadows, Masonry, Carpentry and Other Constructions, Worked for the Use of Military Application School"*(1822). T. Żebrawski in his publication based on the theoretical part elaborated by F. Sapalski and developed the issue of tangency of a straight line to a given surface, assuming that the line passes through any point not lying on this surface. In addition, he supplemented the Sapalski's elaboration with the issues of intersections of developable and revolution surfaces.

Keywords: Teofil Żebrawski, Franciszek Sapalski, Descriptive Geometry, applications, cylindrical, conical,. skew surface, intersections, straight line

1 The life and works of Teofil Żebrawski

Teofil Żebrawski was born on April 5, 1800 in Wojnicz, Poland. In 1818 he graduated from Bartłomiej Nowodworski High School in Krakow. After that, he studied philosophy, nature, English philology, mathematics and astronomy at the Jagiellonian University in Krakow, as well as mining, building engineering and topography at the Academic and Mining School in Kielce. In 1832 he received his Ph.D. in philosophy and liberal science at the Jagiellonian University. In the years of 1832–1834 he worked as a lecturer and then adjunct professor in the Department of Natural History and Botanics at the Jagiellonian University. In 1837–1853 he worked as a land and water transport inspector of Kraków. In 1830s Żebrawski designed and built a road through the Free City of Kraków with two bridges over the Krzeszówka and Dłubnia rivers. Between 1825–1831 he made a triagulation of the Dobrzyń Land, the Pułtusk District, the Świętokrzyskie Mountains, the Sandomierz Region, the regions between Warsaw, Włodawa and the Bug river as well as of the Wilno Governorate region. Part of these measurements were used in the 1839 map of Kingdom of Poland.

Teofil Żebrawski was also an active architect and restaurateur of monuments. Among other things, he designed the rebuilding of the parish church in Ropczyce (1873), the neo – gothic parish church in Ustobna (1877), the annexe building of the Januszowiczowska

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tenement on the Main Square of Krakow and the Wadowice Gymnasium (1872) - the first school building located west of Krakow. After the fire (1850) he managed the reconstruction of the gothic Dominican Church dedicated to Holy Trinity in Krakow (1850–1872). He collaborated with F. Sapalski in the commission appointed by the Senate of the Free City of Krakow to build a new permanent bridge across the Vistula river (1830–1840). He converted the inside of the old chapter – house in The Clock Tower of the Wawel Castle into a chapel. He cooperated with Józef Łepkowski and Jan Matejko with renovation, organization and establishment of the Royal Crypts at the Wawel Castle (1872 – 1877). In 1880 he designed the Tombs of the Meritorious in the crypt of the Pauline Church on Skałka in Krakow. In 1866 he became a member of the Krakow Scientific Society and in 1872 a member of the Academy of Sciences. He participated in the November Uprising (1830–1831) as a First Lieutenant-Engineer of the Józef Dwernicki corps and in the Krakow Uprising (1846) as a captain. He died in Krakow on February 5, 1887.

Teofil Żebrawski was the creator of modern Polish mathematical bibliography *(Bibliography of Polish literature from the branch of mathematics and physics and their applications).* In the scientific work he dealt with descriptive geometry and its applications. He developed and published the following papers: *„A few descriptive geometry problems as an addition to the work of F. Sapalski with an example of application of skew surfaces to carpentry"* (1847), *"Initial knowledge of geometry for practical use. Part I. Planimetry"* (1849), *„The news of Adam Kochański and his mathematical writings"* (1862) [1].

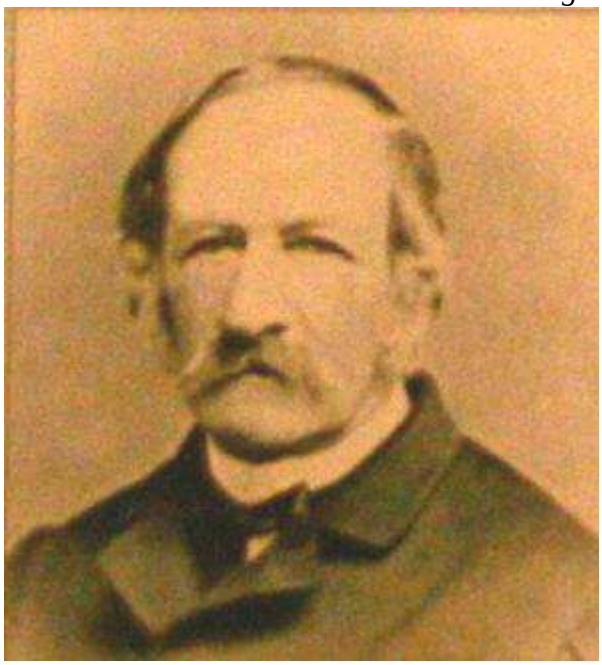

Figure1: Teofil Żebrawski [2]

2 Geometric issues in the problems developed by T. Żebrawski

In 1847 Teofil Żebrawski wrote a scientific paper entitled *„A few descriptive geometry problems as an addition to the work of F. Sapalski with an example of application of skew surfaces to carpentry*". The work was published by J. Cypeer in Krakow. This publication was an extension and supplement of the first text book on descriptive geometry by Franciszek Sapalski *„Descriptive Geometry with Application to Perspective, Shadows, Masonry, Carpentry and Other Constructions, Worked for the Use of Military Application School*"(1822).

Teofil Żebrawski based on the theoretical part developed by F. Sapalski and presented in his paper two geometrical problems:

2.1 Problem 1: Generalization of the construction of a line drawn through a given point and tangent to the conical surface (from the Sapalski's elaboration, chapter VI „On the intersections of surfaces and lines tangent to these intersections")

The problem consists on construction of the tangent line to any given surface, and drawn from any point not lying on this surface . It is shown in the textbook by F.Sapalski. and is a part of the construction of the line normal to a given conical surface and passing through any given point. F.Sapalski presents a construction where through all points of the given surface tangent planes are drawn and then from the given point lines perpendicular to the planes are found. The feet of the perpendiculars form a new auxiliary surface which intersects the given surface at points belonging to the sought for normal lines [5].

According to T. Żebrawski the solution presented by F. Sapalski is correct only for the auxiliary planes being in special positions and belonging to the line connecting the vertex of the cone with the given point. Teofil Żebrawski examines a general solution for developable surfaces and finds out that it is possible only for cylindrical ones for which the formed auxiliary surfaces (generated by the points that are feet of the lines passing by the given point and perpendicular to the planes tangent to the given surface) are also cylindrical. [3] According to T. Żebrawski implementation of the above construction for the conical and of revolution surfaces (developable – helix) cannot be generalized and the concept of Sapalski applies only to cylindrical surfaces. As a result of the analysis T. Żebrawski states that the auxiliary surface for the conical and of revolution surfaces is a skew surface. He suggests that in the text book by Sapalski the *auxiliary* surface should be named *skew in general* which would allow the presented construction to be used for the cylindrical, conical and developable surfaces.

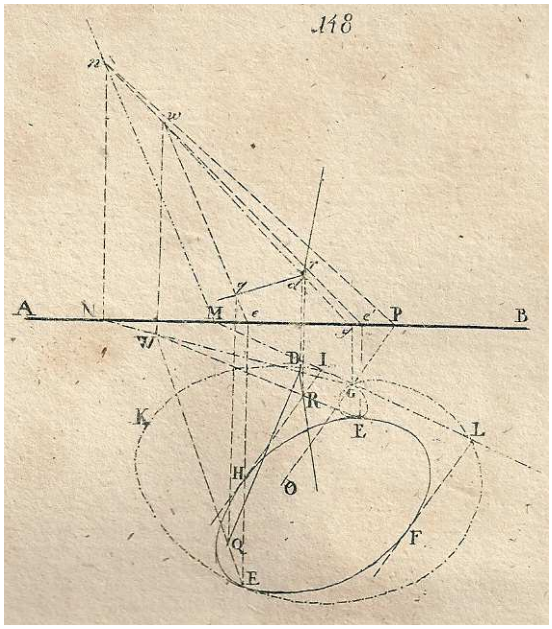

Figure 2: Construction of a line normal to given conical surface and passing through given point not belonging to the surface (F. Sapalski, Fig.148) [4]

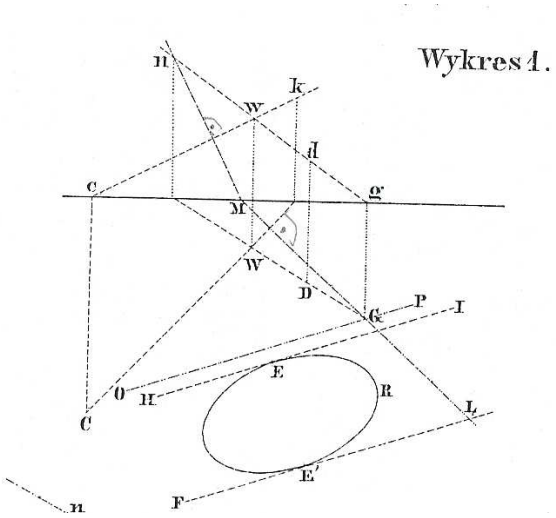

Figure 3: Construction of lines tangent to conical surface and passing through given point. Elaboration by T. Żebrawski – first phase of construction by F. Sapalski (acc. to elaboration by F. Sapalski, p.153) [3]

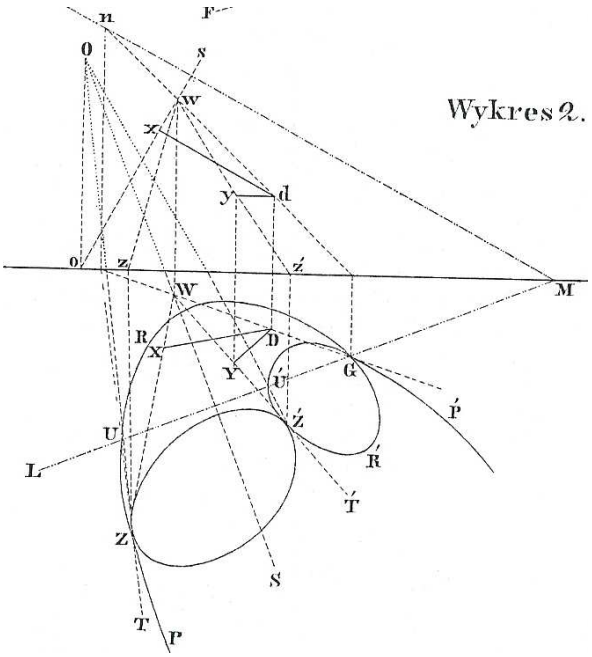

Figure 4: Proper construction of lines tangent to conical surface and passing through given point. Elaboration by T. Żebrawski) [3]

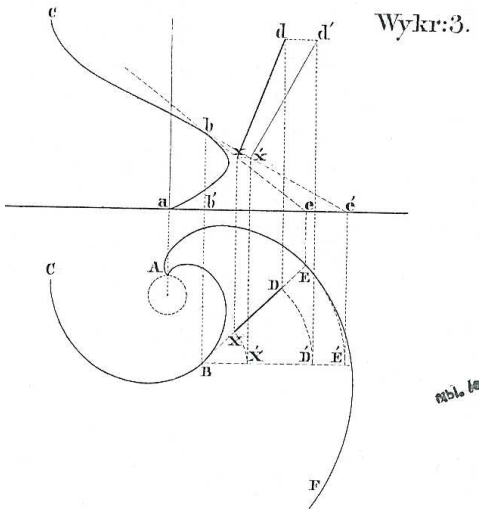

Figure 5: Construction of line perpendicular to developable surface through given point in space [3]

2.2 Problem 2: Supplement of chapter VI „On the intersections of surfaces and lines tangent to these intersections" (from the elaboration by Sapalski)

In the second part of his elaboration Teofil Żebrawski concentrates on the supplement of F. Sapalski's study. He presents examples of intersections of given surfaces i.e. cylindrical, conical and skew in general position and a developable surface of revolution, which - according to Sapalski (n.406 e, n.408) [5] - are difficult to solve. Sapalski in his study presents solutions of the above mentioned geometrical problems only in cases of special positions.

2.2.1 Find intersection of given cylindrical surface and developable surface

Construction :

Through each generating line of the developable surface pass an auxiliary plane parallel to the generating lines of the cylinder. This plane intersects the given surfaces in lines whose points of intersection are the points of the required curve.

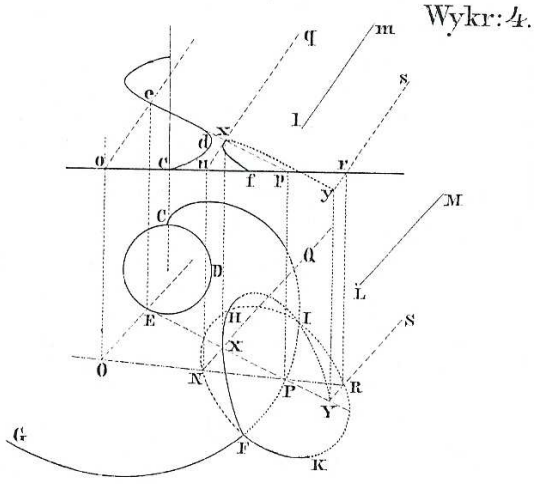

Figure 6: Construction of intersection of given cylindrical surface and developable surface [3]

2.2.2 Find intersection of conical and developable surfaces

Construction:

Through each generating line of the developable surface and through the vertex of the cone pass a plane. This plane intersects the given surfaces in lines whose points of intersection are the points of the required curve.

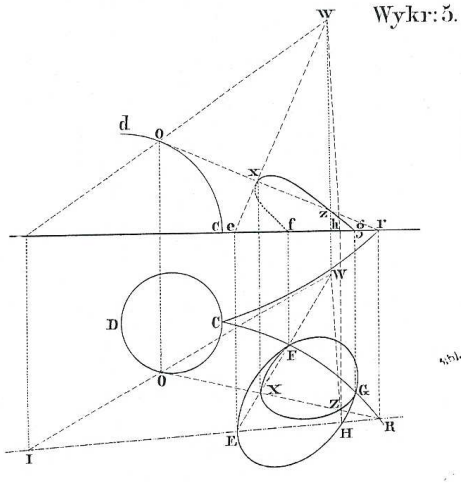

Figure 7: Construction of intersection of conical and developable surfaces [3]

2.2.3 Find intersection of given skew and cylindrical surfaces

Construction:

Through each of the generating line of the skew surface and parallel to the generating lines of the cylinder pass an auxiliary plane. This plane intersects the given surfaces in lines whose points of intersection are the points of the required curve.

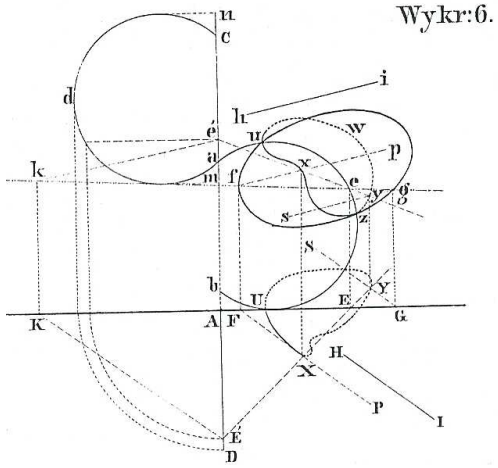

Figure 8: Construction of intersection of given skew and cylindrical surfaces [3]

3 Application of the selected problems in practice

Application of the construction of intersection of the skew and cylindrical surfaces was presented by Teofil Żebrawski on the example of a corner of timbered buildings of the framework construction in a district Słoboda in Kaunas where the fronts of the frameworks making the corner „in the lock style" are segments of circles. [3] It turned out that the consistent surface on which the frameworks touch each other was the portion of the skew surface described in the problem 5. In the figure shown below the author presented a geometric construction allowing for the practical implementation of the above mentioned connection of the frameworks.

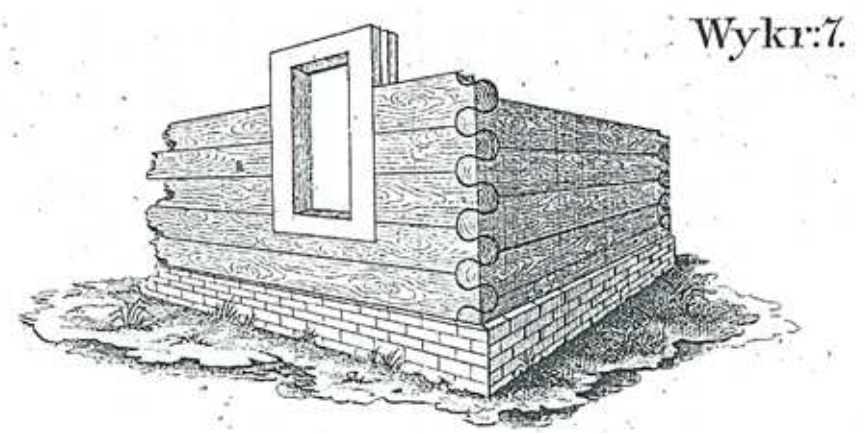

Figure 9: Framework construction of buildings in Kaunas (perspective) [3]

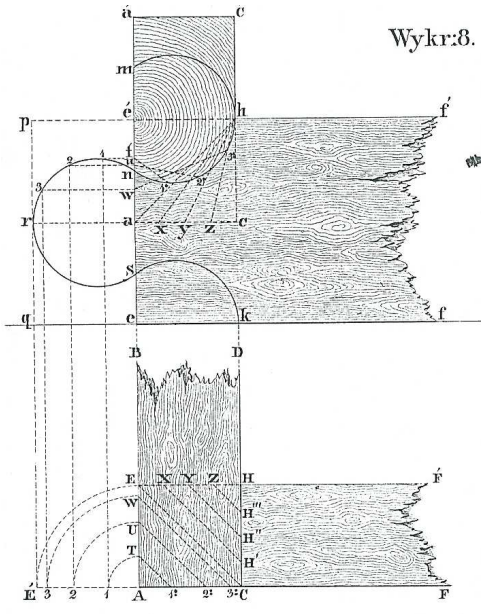

Figure 10: Construction elaborated by Teofil Żebrawski[3]

4 Conclusions

The above described geometrical problems are extension and complement of the first textbook on descriptive geometry by Franciszek Sapalski entitled *"Descriptive Geometry with Application to Perspective, Shadows, Masonry, Carpentry and other Constructions, Worked for the Use of Military Application School"* and published in 1822. Teofil Żebrawski based on the theoretical part shown in the Sapalski's textbook and examined selected issues presenting his own conclusions which completed chapter VI entitled *„On the intersections of surfaces and lines tangent to these intersections"*.

Teofil Żebrawski paid special attention to practical issues. He presented the theory and construction of intersection of the cylindrical and skew surfaces on the example of the geometrical construction of the framework connection in wooden buildings. The problem of descriptive geometry applications was for Franciszek Sapalski and Teofil Żebrawski an essential element of research and scientific analysis. Teofil Żebrawski, a friend of F.Sapalski, after his death continued his works in theoretical aspect and in practical applications. These were the first studies of descriptive geometry as a new field of science on the Polish territory.

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ZASTOSOWANIE POWIERZCHNI „WICHROWATYCH" W PROJEKTOWANIU KONSTRUKCJI DREWNIANYCH W OPRACOWANIU TEOFILA ŻEBRAWSKIEGO: „KILKA ZADAŃ Z GEOMETRII WYKREŚLNEJ JAKO DODATKU DO DZIEŁA F.SAPALSKIEGO Z PRZYKŁADEM ZASTOSOWANIA POWIERZCHNI WICHROWATYCH W CIESIOŁCE" 1847R

W artykule przedstawiono zagadnienia geometryczne opracowane przez Teofila Żebrawskiego i opublikowane w 1847r. Praca ukazała się nakładem J. Cypeera w Krakowie. Publikacja ta jest rozwinięciem i uzupełnieniem pierwszego podręcznika z geometrii wykreślnej autorstwa Franciszka Sapalskiego pt.*"Geometria wykreślna z zastosowaniem do perspektywy, cieniów, gnomoniki, kamieniarstwa, ciesiołki i innych konstrukcji, wypracowana dla użytku szkoły wojskowej aplikacyjnej"* z 1822 roku. Teofil Żebrawski w swojej publikacji, opierając się na części teoretycznej opracowanej przez F. Sapalskiego, rozwija zagadnienie styczności prostej do dowolnej zadanej powierzchni i wystawionej z dowolnego punktu nie leżącego na tej powierzchni. Ponadto uzupełnia opracowanie F. Sapalskiego o zagadnienia przenikania powierzchni obrotowych i rozwijalnych.