Computer Applications in Electrical Engineering
Vol. 12 *2014*

Analysis of input shaping robustness for two-mass system

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Many mechanical objects are characterized by flexible joint. This systems are represented by cranes, conveyers drives, robotic arms, aircraft turbines and others. Many of this objects can be approximated as multi-mass system or, in the simplest cases, two-mass systems. The two mass system is very convenient in tests because of many reasons. First of all it is easy to model and simulate. Additionally changes in parameters to obtain specific targets are easy to calculate and design. The paper shows three input shapers used to control the two-mass system with variable moments of inertia. It was assumed, that the sum of moments of inertia of both masses is constant. All input shapers are analyzed for robustness. The research was made in Matlab/Simulink environment.

KEYWORDS: two mass system, input shaping, robustness

1. Introduction

Many practical mechanical systems are characterized by flexible joints. All of this objects have resonant characteristics. This systems are represented by the cranes [1], multi-mass systems [2],[3], the conveyers and others.

Avoiding the oscillations is often one of the main criteria for control systems. Among all of the methods used to reduce the oscillations, input shaping can be distinguished as one of the simplest in implementation and most effective methods. The dynamics parameters of driving system are maintained. This property is often highly decreased in other methods (exemplary jerk limitation or Closed Loop Filtering). The subject of the paper is the analysis of the robustness of this types of control methods for wide range of parameters deviations.

2. Simulation model

The base of the research is the two-mass system. Schematically the system is shown on Figure 1.

To build the simulation model of system, the equations describing it are required. The dynamic torque acting on the rotor is equal to:

$$T_m = T_e - T_s, \tag{2.1}$$

where T_e is the electromagnetic torque produced by the motor and T_S is the torque transmitted through the shaft.

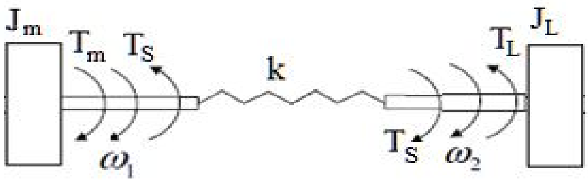

Fig. 1. Two-mass system

For the load side mass, analogically:

$$T_l = T_S - T_L, \tag{2.2}$$

where T_l is the dynamic torque acting on the second mass, T_L is the total torque of load including friction. The torque T_S can be calculated from equation

$$T_S = k_S \cdot (\theta_L - \theta_m) + \varsigma_S \cdot \left(\frac{\mathrm{d}\theta_L}{\mathrm{d}t} - \frac{\mathrm{d}\theta_m}{\mathrm{d}t} \right), \tag{2.3}$$

where θ_L and θ_m are the positions of both masses, and k_S and ς_S are the coefficients of springiness and damping of the shaft respectively. The motor side and load side position are determined from:

$$\frac{\mathrm{d}^2\theta_m}{\mathrm{d}t^2} = \frac{T_m}{J_m} \tag{2.4}$$

and

$$\frac{\mathrm{d}^2\theta_L}{\mathrm{d}t^2} = \frac{T_l}{J_L}. \tag{2.5}$$

3. Input shaping

3.1. Command generation

The research presented in following chapters are based on impulse commands for vibration reduction. Every input shaper is built using the convolution of the input reference signal with series of Dirac impulses with specific amplitudes and given in proper moments of time. The required parameters – amplitudes and moments of applications can be calculated, if the system's transfer function is known or can be read form response courses. If the ratio R of load inertia to motor inertia is defined as $R = J_L/J_m$ then the natural vibration pulsation is determined by

$$\omega_n = \omega_a \sqrt{1+R} = \sqrt{\frac{k_S}{J_L}}\sqrt{1+R} \tag{3.1}$$

where ω_a is the anti-resonance pulsation. The damping of the system is related to internal damping of the shaft d_s.

$$\varsigma = \frac{d_S(1+R)}{2\omega_n J_L} \tag{3.2}$$

According to equations presented in [1], the times and amplitudes of impulses can be presented as:

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{1+K} & \frac{K}{1+K} \\ 0 & 0{,}5T_d \end{bmatrix} \tag{3.3}$$

where K is

$$K = e^{(\frac{-\varsigma\pi}{\sqrt{1-\varsigma^2}})} \tag{3.4}$$

If the model of two-mass system is known, the ω_d and ω_n are possible to calculate [2]. Equations shown above will provide the time-optimal control for a specific T_d, but if any of parameters will be changed, the control will not be optimal any more. This problem can be solved by modifications of the control algorithm. The modifications are shown in chapters 3.2 and 3.3. Robustness of the solution is shown in part 4.

3.2. Robust input shaping

The solution shown in part 3.1 is working good if any uncertainties or inaccuracies are not existing in the model or the object is not changing its parameters during the work. Parameters change will influence on the resonant frequency and one of the Dirac impulses will be provided in wrong time what will result in oscillations. Additional problems, like uncertain parameters, will cause vibrations of whole system. To omit this problem the robust version of the control algorithm were designed.

The robust shaper is the simplest way to solve problem of uncertain parameters of the system. If all of the parameters are nearly constant during the system exploitation and research this solution is usually sufficient to control the system with insignificant vibrations[4].

The robust shaper can be described with equations:

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{(1+K)^2} & \frac{2K}{(1+K)^2} & \frac{K}{(1+K)^2} \\ 0 & 0{,}5T_d & T_d \end{bmatrix} \tag{3.5}$$

where τ_d is the period of modeling frequency.

The shaper described with equation (3.5) will cause vibrations at amplitude not bigger than 5% of input step for modeling frequency [1], but will provide robustness of the control.

3.3. Super-robust input shaping

The solution shown in part 3.2 works of the parameters of the system are not varying significantly during the experiment. To omit this limitation the super-robust shaper has to be used.

The situation, when parameters of the system are changing in a wide range can be found in various real systems. For example the moment of inertia of an robotic arm without of the load can be couple times lower than with the load. When the moment of inertia is varying also the resonant frequency is changed. For this case the shapers presented in 3.1 and 3.2 are insufficient. To reduce the oscillations in this example the super-robust algorithm was designed. The amplitudes and times of application of Dirac impulses are equal to:

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 0.125 & 0..375 & 0,375 & 0.125 \\ 0 & 0.5T_d & T_d & 1.5T_d \end{bmatrix} \tag{3.6}$$

4. Simulation test

The simulation test were designed to enable the analysis of the presented input shapers robustness. Because of this reason at least one of the parameters having influence on resonant frequencies has to vary. The moments of inertia were selected as variable parameters (the sum of moments is constant, equal to J_T). The moments of inertia are equal:

$$J_M = J_T \frac{1}{R+1} \tag{4.1}$$

$$J_L = J_T \frac{R}{R+1} \tag{4.2}$$

where J_L is the moment of inertia of load, J_M is the moment of inertia of the machine and J_T is sum of moments of inertia of both masses. R is the coefficient of mass division.

The torque was supplied to the system. The torque was selected to obtain the time-optimal movement for the reference system with non-flexible joint between the masses. The reference system was modeled as a single, rotating mass with the moment of inertia J_T. The control task was to rotate the mass by 40 rad, with speed limitation to 100 rad/s.

Figure 2 shows the input torque courses for presented input shapers. The shapers were calculated for the situation, where the moments of inertia of masses are equal (*R = 1*). Afterwards, the solutions robustness for mass division changes was checked in range between *R = 0.2* up to *R = 5*.

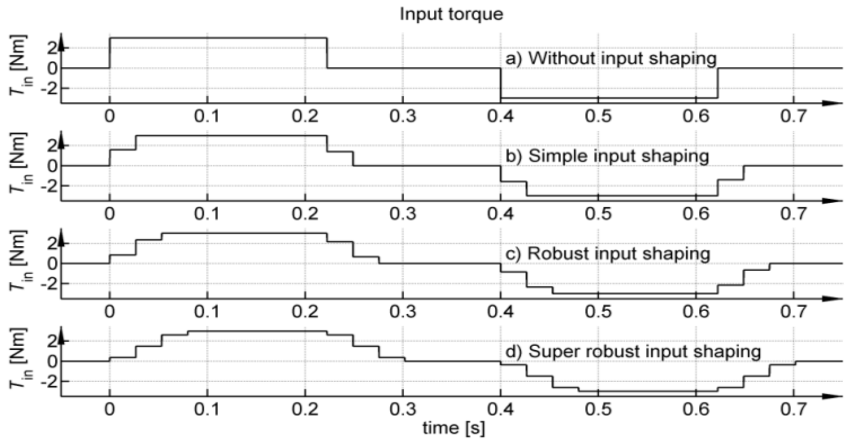

Fig. 2. Input torque for shapers

Figure 3 shows the characteristics for *R = 1.* Every input shaper works according to the expectations. Oscillations of speed not arise in the system. At right column the final phase of movement is presented. For the simple input shaper some small oscillations appear, for robust and super robust shaper oscillations are equal to zero.

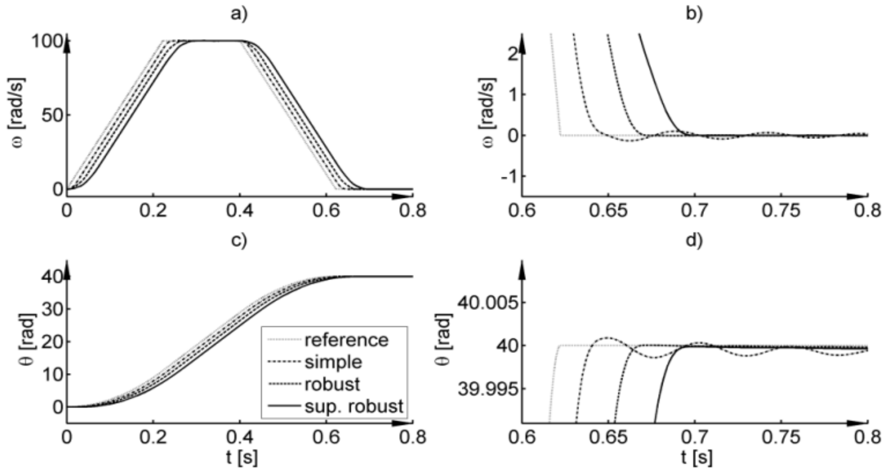

Fig. 3. Speed and position characteristics for various input shapers in case R = 1;
a) whole movement, b) final phase

Figure 4 presents the $R = 5$ case. For every shaper oscillations appear, but for the robust and super-robust shapers are not as significant as for the simple example. For $R = 0.2$ case the conclusion is identical.

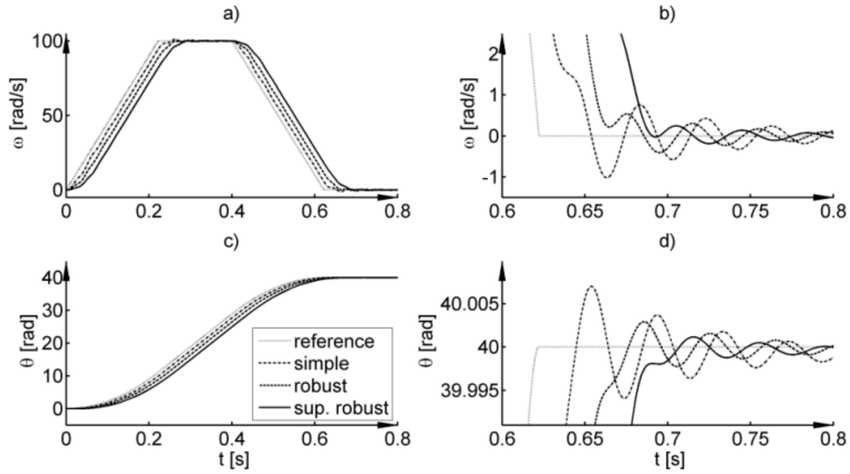

Fig. 4. Speed and position characteristics for various input shapers in case R = 5;
a) whole movement, b) final phase

The criteria of robustness was designed as the maximal difference of position between the single mass and load position and between the single mass and machine respectively. The plot is shown at Figure 5.

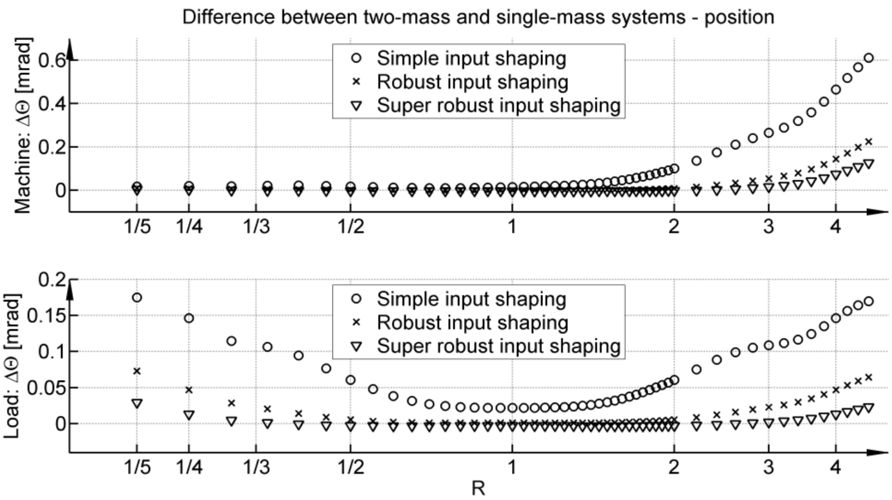

Fig. 5. Difference of the angular position between machine, load and single mass

The Figures 3 and 4 are proving the deliberations presented in part 3 – the super robust input shaper is working without of arising any unwanted oscillations. The robustness of this object has unfortunately negative influence for the control time. Every additional Dirac impulse will cause additional delays in control loop what will always affect with setting time enlargement.

5. Conclusion

The research has confirmed high robustness of presented methods of the input shaping. Those methods can be characterized by a small computational complexity and simple on-line realization. Approximated parameters of objects are sufficient to project the shaper. Shaping of closed-loop regulation signals for selected objects will be the subject of future research.

References

[1] W. Signhose and S. Warren, Command Generation for Dynamic Systems. 2011.
[2] K. Szabat, "Direct and indirect adaptive control of a two-mass drive system—a comparison," in Industrial Electronics, 2008. ISIE 2008. IEEE International Symposium on, 2008, pp. 564–569.
[3] N. Avdiu and S. Buza, "Analysis of the mutli-mass drive system synamics with induction motor," Prague, Sep. 2011.
[4] W. Signhose, N. Singer, and W. Seering, "Preshaping command inputs to reduce system vibration," vol. 1212, pp. 76–82, 1990.