







# An optimal hybrid quadcopter control technique with MPC-based backstepping

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Quadcopter unmanned aerial vehicle is a multivariable, coupled, unstable, and underactuated system with inherent nonlinearity. It is gaining popularity in various applications and has been the subject of numerous research studies. However, modelling and controlling a quadcopter to follow a trajectory is a challenging issue for which there is no unique solution. This study proposes an optimal hybrid quadcopter control with MPC-based backstepping control for following a reference trajectory. The outer-loop controller (backstepping controller) regulates the quadcopter's position, whereas the inner-loop controller (Model Predictive Control) regulates its attitude. The translational and rotational dynamics of the quadcopter are analyzed utilizing the Newton-Euler method. After that, the backstepping controller (BC) is created, which is a recurrent control method according to Lyapunov's theory that utilizes a genetic algorithm (GA) to choose the controller parameters automatically. In order to apply a linear control technique in the presence of nonlinearities in the quadcopter dynamics, Linear Parameter Varying (LPV) Model Predictive Control (MPC) structure is developed. Simulation validated the dynamic performance of the proposed optimal hybrid MPC-based backstepping controller of the quadcopter in following a given reference trajectory. The simulations demonstrate the fact that using a command control input in trajectory tracking, the proposed control algorithm offers suitable tracking

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over the assigned position references with maximum appropriate tracking errors of 0.1 m for the  $X$  and  $Y$  positions and 0.15 m for the  $Z$  position.

**Key words:** UAV, quadcopter, Model Predictive Control, backstepping control, linear parameter varying, genetic algorithm

## 1. Introduction

Research in the engineering field pertaining to designing and manufacturing of either remotely operated or fully autonomous unmanned aerial vehicles (UAVs), particularly quadcopter models [1, 2], has been the subject of considerable attention and emphasis in the area of embedded control systems. Due to their configuration [3], quadcopters are widely used to perform tasks in a wide range of applications that include military [4], parcel delivery [5, 6], conveyance of medical samples [7], mapping [8, 9], monitoring [10], precision agriculture [11, 12] and greenhouses [10]. As established in [3], quadcopters have been widely studied and applied as Unmanned Aerial Vehicles (UAVs) due to their simplified mechanical structure, enhanced closed-loop equilibrium compared to alternative designs, and substantial flexibility in both indoor and outdoor environments. Future alternatives to UAVs will consist of compact Vertical Take-Off and Landing (VTOL) aircraft, which are capable of vertical takeoff and landing as well [13]. Control of mechatronic systems, such as quadcopters, is challenging due to the nonlinear, multivariable, unsteady, coupled, and underactuated features found in these systems. Also, precise trajectory tracking is a demanding feature for any required task to be performed successfully and is important when operating in realistic environments where the flight performance is heavily affected by external disturbances.

Literature reviews such as [14–17] present a thorough evaluation of the essential state-of-the-art control techniques which could be efficiently employed to quadcopters, such as Backstepping control (BC), MPC, Sliding Mode Control (SMC), Linear–Quadratic Regulator (LQR), H-infinity, Proportional–Integral–Derivative (PID), Adaptive control, Fuzzy logic and Neural Network control, Feedback Linearization (FL) control. Since the quadcopter is a nonlinear system, a few nonlinear control methods have obtained good results in trajectory tracking difficulties such as sliding mode control, nonlinear model predictive control (NMPC), backstepping control design and state feedback linearization control as seen in [12, 18–20]. The term “model-based predictive control” (MPC) implies a class of sophisticated control methods that forecast the behavior of the system being controlled using a process model [21] and can even control systems that conventional feedback controllers are unable to control. MPC strategies for trajectory tracking of quadcopter have been presented

in [2, 19, 22]. In [22] MPC was applied to mixed logical dynamical (MLD) model and solved using mixed integer quadratic programming (MIQP) optimization for a small-scale helicopter's obstacle avoidance as a hybrid system for the UAV to choose the best trajectory and avoid obstacles. The simulation's findings show that the helicopter can avoid the obstruction by following the best possible path from its starting position to its destination. In [23], a Nonlinear Model Predictive Controller (NLMPC) was designed to track a reference trajectory while preserving system stability and ensuring minimal energy usage. Linear and Nonlinear MPC approaches were compared to demonstrate the effectiveness of the NLMPC, and it was demonstrated that the NLMPC offers better results than Linear MPC (LMPC). In [19], the utilization of some approaches such as SMC, Fuzzy Logic, and GA have been examined to enhance the performance of the modelled Unmanned Aerial Vehicle (UAV) robot sample. A Ducted Fan UAV (DFUAV) trajectory tracking control problem is discussed in [24] using an offset-free MPC approach in the presence of various inconsistencies and external disruptions. This technique ensured adequate flight strength and stability while addressing the effects of time delays, linear uncertainties, and disorders. In [25], MPC scheme was used to stabilize the Quadcopter Unmanned Aerial Vehicle's (Q-UAV) desired altitude and attitude. When this controller is compared to PID, it shows to have the least steady state errors and the fastest error convergence even when model uncertainties are present. Control input was used as a reference to track the referred trajectory. In [26] the MPC approach for an unmanned quadcopter trajectory tracking is offered, and the quadcopter's dynamics are modelled employing the approach of hybrid systems. This dual control method showed robustness in calculating optimal control actions against atmospheric issues.

Nonlinear system uses specific theories and methods like backstepping control to ensure stability without regard to the inner dynamics of the system [27]. [28] combined the backstepping technique and a nonlinear robust PI controller for the attitude control of a vertical take-off and landing (VTOL) quadcopter aircraft based on the Euler angles parametrization. In [29], an efficient backstepping Dynamic Surface Control (DSC) scheme is proposed to address the trajectory tracking control of a quadcopter UAV, which considers control input saturation in the presence of model uncertainties and unidentified external disturbances. The results show that the proposed control system can minimize the limitation of control input and meet the desired steady-state and transient tracking performance indices. The paper [19] proposes a Disturbance Observer-Based Control (DOBC) with backstepping for the multirotor Unmanned Aerial Vehicles (UAVs) trajectory tracking motion control. This improved tracking performance and robustness. Also, in [12], a robust control technique was developed depending on the Lyapunov strategy, and the backstepping approach to attain quadcopter stability

against unmodeled matching disintegrating dynamics and the robust backstepping control was observed to demonstrate its efficiency in enhancing the vehicle trajectory tracking as it captures the crop images. [30] developed a new control method for an autonomous quadrotor helicopter, utilizing the combination of fuzzy logic and backstepping sliding mode control. This method aimed to address the chattering phenomenon often encountered in sliding mode control. The performances of this method demonstrate the effectiveness of the control strategy for the quadrotor helicopter in vertical flights. Similarly, [31] designed a fault-tolerant control scheme for a 2 degrees of freedom (DOF) helicopter, focusing on addressing faults in pitch and yaw motion control using sliding mode control. This approach used both reaching and super-twisting laws to manage faults effectively, and its efficacy was validated through simulation tests, demonstrating better control with reduced chattering. [32] used a nonlinear control theory to propose two different methods for a quadcopter system's trajectory tracking control. The initial approach depends on the fundamental backstepping method, and the second approach is an optimal controller of the Linear Quadratic Integral (LQI) using feedback linearization to manage the variability and the connecting elements of the quadcopter state variables. This controller showed some robustness. Nonlinear control of a quadcopter involving altitude and lateral motion is proposed by [18] utilizing the integral BC approach and SMC technique. The Lyapunov theory is applied to prove the stability of the BC-SMC approach, which is based on a high-order extended state observer for quadcopters exposed to external wind disturbances. There was an effectiveness of the quadcopter of the proposed scheme [33]. In [34], a cascade control strategy backstepping approach based on the dynamic modelling of a quadcopter was developed, and it successfully guided the quadcopter in both altitude and attitude to ensure trajectory tracking tasks. A robust adaptive development and trajectory tracking control of several quadcopter UAVs was recommended in [35] using the super twisting sliding mode control (STSMC) approach but its stability was ensured using Lyapunov theorems showing an adaptive STSMC exhibit improved robustness than the fixed gain STSMC. Using a new supporting system to deal with the control input saturation, [36] also introduced a new robust BC for a quadcopter with input saturation. Numerical simulations provided evidence for the efficacy of the suggested algorithm.

In this study, an optimal hybrid quadcopter control technique is proposed. The hybrid control system uses an MPC-based controller (inner loop control) for attitude and an optimal backstepping control for position controller that serves as the reference generator for the inner-loop controller in tracking a given trajectory. Tuning of the backstepping controller is based on a genetic algorithm. Due to the high computational nature of the MPC, this work adopts a linear parameter-

varying-based MPC technique. The proposed methodology is computationally cost-effective and could feasibly be implemented on real-time autonomous systems to realize the desired outcome. The structure of this work is as follows: The formulation of the quadcopter model is presented in Section 2. The optimal hybrid control technique proposed is described in Section 3, while simulation results and analyses are provided in Section 4. In Section 5, concluding remarks are presented.

## 2. Materials and methods

In this work, an X-shaped quadcopter is proposed as the unmanned aerial vehicle (UAV) [37]. To generate maneuvering within the space, the rotors M1 and M3 are to be rotated clockwise, while M2 and M4 are to be rotated counterclockwise, as represented in Figure 1. Quadcopter operations such as vertical take-off and landing (VTOL), roll, pitch, and yaw are made possible by varying the speed of the rotor [38]. The quadcopter has 6 degrees of freedom (DOF), three of which specify where a reference point is located within the body ( $x$ ,  $y$ , and  $z$ ). The remaining three define the trajectory of the body (roll, pitch, and yaw) angles [11]. The body frame of the quadcopter is located at the centre of gravity, and its Earth-Fixed frame (E) or inertial frame is orientated with respect to the World coordinate system, with  $x$ ,  $y$ ,  $z$  axes corresponding to  $X$ ,  $Y$ , and  $Z$ , respectively, with the origin being positioned at the center of gravity [39].  $E_1 = [\phi, \theta, \psi]^T$ , represents the orientation of the quadcopter, where  $\phi$  is the roll,  $\theta$  is the pitch and  $\psi$  is the yaw angles [10]. In an inertial frame of reference, the generalized

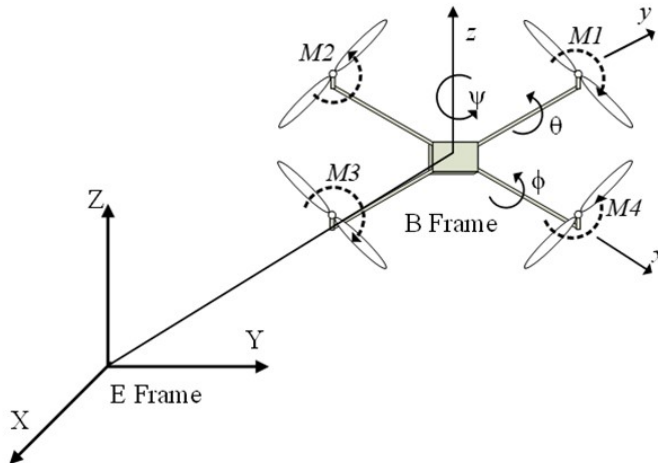


Figure 1: The quadcopter configuration

coordinates of the quadcopter, represented by the vector  $E_2 = [X, Y, Z]^T$ , can be used to determine its position, with the altitude ( $Z$ ) denoting the vertical position. The quadcopter attitude is created by this orientation. A primary force ( $U1$ ) and three moments ( $U2$ ,  $U3$ , and  $U4$ ) that make up the control inputs are applied to the quadcopter.

The parameterization of the quadcopter attitude using Euler angles is formulated in (1) with the  $R_{zyx}$  representation as presented in [14]. The orientation of the quadcopter, defined by the parameters  $C$  (cosine) and  $S$  (sine), can be uniquely determined for each instance.

$$R_{zyx} = \begin{bmatrix} C\psi C\theta & C\psi S\theta S\phi - S\phi S\psi & C\psi C\phi S\theta + S\phi S\theta \\ C\theta S\psi & S\phi S\psi S\theta + C\phi C\psi & S\psi S\theta C\psi - C\psi S\phi \\ -S\theta & C\theta S\phi & C\phi C\theta \end{bmatrix}. \quad (1)$$

### 2.1. Quadcopter kinematics

The UAV uses a synergy of two kinematics (angular and translational velocities) to track a specified reference trajectory in space. The rotational kinematics of the quadcopter can be presented in (2) and computed with utilization of the relationship between the angular velocity vectors of the body frame ( $p, q, r$ ) and the inertial frame ( $\dot{\phi}, \dot{\theta}, \dot{\psi}$ ) through the inversion of the transformation matrix,  $T(\phi, \theta)$  [14]

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = T(\phi, \theta)^{-1} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}, \quad (2)$$

where

$$T(\phi, \theta) = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi \sec \theta & \cos \phi \sec \theta \end{bmatrix}. \quad (3)$$

In (4), the translational kinematics of the quadcopter is defined as the relationship between the translational velocity vectors of the body frame ( $u, v, w$ ) and that of the inertia frame ( $X, Y, Z$ ) through the inverse of (1).

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = (R_{zyx})^{-1} \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix}. \quad (4)$$

## 2.2. Quadcopter dynamics

Using the Newton-Euler method, a dynamic model of the quadcopter is developed. The translational and rotational dynamic equations of the body frame of the quadcopter are expressed with respect to the Earth-fixed frame in [40]. The forces involved in the translational dynamics of the quadcopter are thrust, and the force applied along the  $z$ -axis against gravity [41]. Newton-Euler's second law defines the translational dynamic equations of the quadcopter as defined in (5).

$$F = m \vec{a} = -mg \vec{p} + U_1 R_{zyx} \vec{p}, \quad (5)$$

where  $m$  is the mass of the quadcopter,  $\vec{a} = [\dot{u} \ \dot{v} \ \dot{w}]^T$  is the body frame translational acceleration vector,  $g$  is the gravitational acceleration,  $\vec{p} = [0 \ 0 \ 1]^T$  denotes the earth-fixed frame unit vector, and  $U_1$  is the amount of thrust generated by the rotors and is derived from the relationship between the total force,  $\sum F_i$  and the total speed  $\sum \Omega_i^2$  as defined in (6):

$$U_1 = \sum_{i=1}^4 F_i = b \sum_{i=1}^4 \Omega_i^2, \quad (6)$$

where  $b$  is the thrust factor,  $\sum_{i=1}^4 F_i = (F_1 + F_2 + F_3 + F_4)$  and  $\sum_{i=1}^4 \Omega_i^2 = (\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2)$ .

The dynamic model of the quadcopter in terms of position is defined in (7) by recalling, factoring out the highest order, and expanding (5) into vector-matrix form:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + \begin{bmatrix} C\psi C\phi S\theta + S\phi S\theta \\ S\psi S\theta C\psi - C\psi S\phi \\ C\phi C\theta \end{bmatrix} \frac{U_1}{m}, \quad (7)$$

where  $[\dot{u} \ \dot{v} \ \dot{w}]^T = [\ddot{x} \ \ddot{y} \ \ddot{z}]^T$ . The rotational dynamic of the quadcopter according to the Newton-Euler second law of motion,  $M$  is the sum of the quadcopter torque vector,  $\vec{U}_\tau$  and the quadcopter gyroscopic effects as defined in (8) [40]:

$$M = I \vec{\omega} = -(\vec{\omega} \times I \vec{\omega} + G_e) + \vec{U}_\tau, \quad (8)$$

where  $I$  represents an inertia matrix,  $\vec{\omega} = [p \ q \ r]^T$  is the angular velocity defined in (2),  $\vec{\dot{\omega}} = [\dot{p} \ \dot{q} \ \dot{r}]^T$  is the angular acceleration,  $\vec{\omega} \times I \vec{\omega}$  is the gyroscopic effect caused by the rotation of the rigid body,  $G_e$  is the propeller orientation

change and  $\vec{U}_\tau = [U_1 \ U_2 \ U_3 \ U_4]^T$ .  $G_e$  and  $\vec{U}_\tau$  are defined as follows [37]:

$$G_e = \sum_{i=1}^4 J_i \left( \vec{\omega} \vec{p} (-1)^{i+1} \Omega_i \right), \quad (9)$$

$$\begin{bmatrix} U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} lb(\Omega_4^2 - \Omega_2^2) \\ lb(\Omega_3^2 - \Omega_1^2) \\ d(\Omega_4^2 + \Omega_2^2 - \Omega_3^2 - \Omega_1^2) \end{bmatrix}, \quad (10)$$

where the inertia of the rotor is represented by  $J_i$ , the distance between the rotors and the mass center is denoted by  $l$ , and the drag factor is denoted by  $d$ . Furthermore, the control input is given by  $U_1 = lb(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2)$ . By recalling, factoring out the highest order and reformulating (8), the following describes the quadcopter's dynamic model in terms of rotation:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} qr \frac{(I_{yy} - I_{zz})}{I_{xx}} - \frac{J_i}{I_{xx}} q \Omega_d \\ pr \frac{(I_{zz} - I_{xx})}{I_{yy}} + \frac{J_i}{I_{yy}} p \Omega_d \\ pq \frac{(I_{xx} - I_{yy})}{I_{zz}} \end{bmatrix} + \begin{bmatrix} \frac{1}{I_{xx}} U_2 \\ \frac{1}{I_{yy}} U_3 \\ \frac{1}{I_{zz}} U_4 \end{bmatrix}, \quad (11)$$

where  $[\dot{p} \ \dot{q} \ \dot{r}]^T = [\ddot{\phi} \ \ddot{\theta} \ \ddot{\psi}]^T$  and  $\Omega_d = (\Omega_4 + \Omega_2 - \Omega_3 - \Omega_1)$ .

Finally, the dynamic model of the quadcopter is written as follows [42]:

$$\ddot{x} = \frac{U_1}{m} (C\psi C\phi S\theta + S\phi S\theta), \quad (12)$$

$$\ddot{y} = \frac{U_1}{m} (S\psi S\theta C\psi - C\psi S\phi), \quad (13)$$

$$\ddot{z} = -g + \frac{U_1}{m} (C\phi C\theta), \quad (14)$$

$$\ddot{\phi} = qr \frac{(I_{yy} - I_{zz})}{I_{xx}} - \frac{J_i}{I_{xx}} q \Omega_d + \frac{1}{I_{xx}} U_2, \quad (15)$$

$$\ddot{\theta} = pr \frac{(I_{zz} - I_{xx})}{I_{yy}} + \frac{J_i}{I_{yy}} p \Omega_d + \frac{1}{I_{yy}} U_3, \quad (16)$$

$$\ddot{\psi} = pq \frac{(I_{xx} - I_{yy})}{I_{zz}} + \frac{1}{I_{zz}} U_4. \quad (17)$$



### 3. Control model formulation

The proposed control strategy for the quadcopter has a multiloop structure, as shown in Figure 2. The backstepping controller which is optimal for altitude,  $U_1$ , and the position control of the outer loop is utilized as a reference generator for the inner loop control (MPC). The MPC is responsible for controlling the angular position of the quadcopter and augmenting stability with the control input  $U_2$ ,  $U_3$ , and  $U_4$  respectively. The reference trajectory generator produces reference position commands that are thereafter sent to the BC, and it also produces the reference yaw angle,  $\psi_d$ , for the MPC algorithm. The BC outputs reference values of pitch angle  $\theta_d$ , roll angle  $\phi_d$ , which are then transmitted to the MPC. The BC outputs reference values of pitch angle  $\theta_d$ , roll angle  $\phi_d$ , which are then transmitted to the MPC.

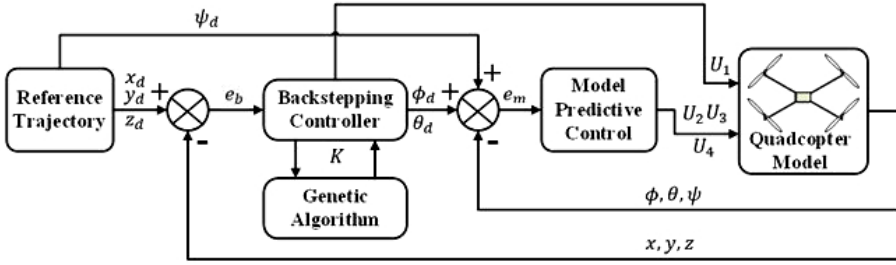


Figure 2: The proposed hybrid control scheme for the quadcopter

#### 3.1. Backstepping control structure and design

The backstepping control technique generates the desired roll, pitch, and altitude control inputs for the quadcopter in space. This is accomplished firstly by reformulating the position dynamics (12) and (13) using the desired yaw angle, as shown in Figure 2. This yields the desired roll and pitch angles. Next, these generated angles, along with the altitude control in (14) and attitude from (15) to (17), are used to create the quadcopter control inputs. First, however, to apply the backstepping control [43], a nonlinear dynamic equation is used to represent the dynamic position model of (12) to (14) as:

$$\vec{\dot{P}} = f(\vec{x}_p) + g(\vec{x}_p)\vec{u}, \quad (18)$$

where

$\vec{x}_p = [x_1 \ x_2 \ x_3]^T = [x \ y \ z]^T$  is the position state vector,

$\vec{u} = [U_x \ U_y \ U_z]^T$  is the input vector,

$f(\vec{x}_p) = [0 \ 0 \ -g]^T$  is the nonlinear dynamic function,

$g(\vec{x}_p) = [r_x \ r_y \ r_z]^T$  is the control input function,

$$r_x = \frac{(C\psi C\phi S\theta + S\phi S\theta)}{m}, \quad r_y = \frac{(S\psi S\theta C\psi - C\psi S\phi)}{m}, \quad r_z = \frac{(C\phi C\theta)}{m},$$

$U_x$  is the longitudinal control input,  $U_y$  is the latitude control input and  $U_x = U_1$  is the altitude control input.

The backstepping control system aims to formulate an appropriate control law such that the position state trajectory,  $\vec{x}_p$ , can track the reference trajectory,  $\vec{x}_d = [x_{xd} \ x_{yd} \ x_{zd}]^T$ .

### 3.1.1. Altitude control design

For simplicity, only the  $z$  of the three DOF position dynamics in (14) will be considered because the design of the control system for the quadcopter is similar for each of the three DOF of the position dynamics. As the result, the following step-by-step description of the backstepping control may be formulated:

*Step 1:* Define the tracking error,  $e_{z1}$ :

$$e_{z1} = z_{1d} - z_1, \quad (19)$$

where  $z_{1d}$  is the desired trajectory that the reference model specifies, and  $z_1 = z$ . The tracking error derivative can then be expressed as:

$$\dot{e}_{z1} = \dot{z}_{1d} - \dot{z}_1. \quad (20)$$

The first Lyapunov function is taken as [39]:

$$V_1(e_{z1}) = \frac{1}{2}e_{z1}^2. \quad (21)$$

The derivative of (21) is:

$$\dot{V}_1(e_{z1}) = e_{z1}\dot{e}_{z1} = e_{z1}(\dot{z}_{1d} - \dot{z}_1). \quad (22)$$

The virtual control is comparable to  $\dot{z}_1$ . The desired value for the virtual control, known as the stabilising function, can be defined as follows:

$$\alpha_{z1} = \dot{z}_{1d} + k_1 e_{z1}, \quad (23)$$

Take  $k_1$  to denote a positive constant which is to be specified by the GA. When the virtual control is substituted with the desired value, the expression (22) becomes:

$$\dot{V}_1(e_{z1}) = -k_1 e_{z1}^2 \leq 0. \quad (24)$$

*Step 2:* Define the difference between the virtual control and the desired deviation value as:

$$e_{z2} = \dot{z}_1 - \alpha_{z1} = \dot{z}_1 - \dot{z}_{1d} - k_1 e_{z1}. \quad (25)$$

The derivative of (25) is expressed as:

$$\dot{e}_{z2} = \ddot{z}_1 - \alpha'_{z1}, \quad (26)$$

$\ddot{z}_1$  in (26) is substituted with the corresponding nonlinear position dynamics of (18) as follows:

$$\dot{e}_{z2} = f(z) + g(z)U_1 - \ddot{z}_{1d} - k_1\dot{e}_{z1}. \quad (27)$$

The second Lyapunov function is chosen as:

$$V_2(e_{z1}, e_{z2}) = \frac{1}{2}e_{z1}^2 + \frac{1}{2}e_{z2}^2. \quad (28)$$

Finding the derivative of (28) yields:

$$\begin{aligned} \dot{V}_2(e_{z1}, e_{z1}) &= e_{z1}\dot{e}_{z1} + e_{z2}\dot{e}_{z2} \\ &= e_{z1}(\dot{z}_{1d} - \dot{z}_1) + e_{z2}(\ddot{z}_1 - \alpha'_{z1}) \\ &= e_{z1}(-e_{z2} - k_1e_{z1}) + e_{z2}(f(z) + g(z)U_1 - \ddot{z}_{1d} - k_1\dot{e}_{z1}) \\ &= -k_1e_{z1}^2 + e_{z2}(-e_{z1} + f(z) + g(z)U_1 - \ddot{z}_{1d} - k_1\dot{e}_{z1}). \end{aligned} \quad (29)$$

*Step 3:* For satisfying  $\dot{V}_2(e_{z1}, e_{z1}) \leq 0$ , the control input,  $U_1$  is selected as:

$$U_1 = \frac{1}{g(z)}(e_{z1} + k_1\dot{e}_{z1} + \ddot{z}_{1d} - k_2e_{z2}). \quad (30)$$

The GA should also determine the positive constant  $k_2$ . To stabilize the tracking error,  $e_1$ , the term  $k_2e_2$  is added.

When (30) is substituted for (29), the following equation is obtained:

$$\dot{V}_2(e_{z1}, e_{z2}) = -(k_1e_{z1}^2 + k_2e_{z2}^2) = -E^TKE \leq 0, \quad (31)$$

where  $E = [e_{z1} \ e_{z2}]^T$  and  $K = \text{diag}(k_1, k_2)$ .

Since  $\dot{V}_2(e_{z1}, e_{z1}) \leq 0$ ,  $\dot{V}_2(e_{z1}, e_{z1})$  is a negative semi-definite. Then, the control law in (30) will stabilize the quadcopter asymptotically.

### 3.2. Position control design

Equations (12) and (13) define the trajectory of the quadcopter in a two-dimensional ( $2 - D$ ) plane ( $x$  and  $y$ ) with the only application of thrust  $U_1$ . The Cartesian motion of the quadcopter in this plane is contingent upon the angles  $\theta$  and  $\phi$  [44]. Consequently, the angles of  $\theta$  and  $\phi$  are viewed as the results of the  $2 - D$  control laws. As the result,  $U_x$  and  $U_y$  will serve as the control

signals used to determine the desired roll,  $\phi_d$  angle, and pitch,  $\theta_d$  required to move the quadcopter in the 2 –  $D$  plane. These control signals should also satisfy the conditions  $\dot{V}_2(e_{x1}, e_{x2}) \leq 0$  and  $\dot{V}_2(e_{y1}, e_{y2}) \leq 0$  respectively [39]. By applying the backstepping control procedure derived in the section on altitude control design to the quadcopter's position equations, one can derive the following control laws:

$$\theta_d = \arcsin \left( \frac{m}{C\phi C\theta U_1} \left( e_{x1} + \dot{\alpha}_{x1} - \frac{S\psi S\phi}{m} U_1 - k_4 e_{x2} \right) \right), \quad (32)$$

$$\phi_d = -\arcsin \left( \frac{m}{C\psi U_1} \left( e_{y1} + \dot{\alpha}_{y1} - \frac{S\psi C\phi}{m} U_1 - k_6 e_{y2} \right) \right), \quad (33)$$

where  $\dot{\alpha}_{x1} = (\ddot{x}_{1d} + k_3 \dot{e}_{x1})$  is the derivative of the  $x$  position stabilizing function, and  $\dot{\alpha}_{y1} = (\ddot{y}_{1d} + k_5 \dot{e}_{y1})$  is the derivative of the  $y$  position stabilizing function.

### 3.3. Model Predictive Control structure and design

MPC is an optimization-based technique for controlling the attitude of a quadcopter [45]. It requires the solution of a finite-horizon optimal control challenge subjected to the quadcopter rotational dynamics established in the previous section, reference inputs from BC, and trajectory and state limitations introduced on the quadcopter. Using the quadcopter model, the future output states (attitude controls) are estimated at each time step  $T_s$  for a selected predictions horizon. The predicted inputs can be obtained by solving the optimization problem with the specified optimality criterion for a designated control horizon. The process then receives the initial input for this optimal sequence. The subsequent step involves resolving the optimization problem and obtaining the new input sequence.

#### 3.3.1. Linear MPC formulation

The quadcopter rotational dynamics is a nonlinear model that requires a nonlinear controller such as a nonlinear MPC. Nonlinear MPC is computationally expensive [46], so a linear MPC is adopted for the control of the quadcopter attitude. The nonlinear rotational dynamic model is reformulated to Linear Parameter varying (LPV) and treated as the linear time invariant system (LTI) [47].

**Assumption 1.** *To achieve a hovering condition for the quadcopter,  $\phi$  and  $\theta$  in (3) are equal to zero. Hence (3) becomes an identity matrix [1].*

LPV is one of the powerful techniques of representing a nonlinear model into linear structure and it is represented here as follows:

$$\begin{aligned}
 \begin{bmatrix} \dot{\phi} \\ \ddot{\phi} \\ \dot{\theta} \\ \ddot{\theta} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{J_i}{I_{xx}}\Omega_d & 0 & q\frac{(I_{yy}-I_{zz})}{I_{xx}} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{J_i}{I_{xx}}\Omega_d & 0 & 0 & 0 & p\frac{(I_{yy}-I_{zz})}{I_{xx}} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{p}{2}\frac{(I_{yy}-I_{zz})}{I_{xx}} & 0 & \frac{q}{2}\frac{(I_{yy}-I_{zz})}{I_{xx}} & 0 & 0 \end{bmatrix} \begin{bmatrix} \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix} \\
 &+ \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{I_{xx}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{I_{yy}} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_{zz}} \end{bmatrix} \begin{bmatrix} U_2 \\ U_3 \\ U_4 \end{bmatrix}. \tag{34}
 \end{aligned}$$

The reformulated LPV form is discretized since it will be validated using computer as follows:

$$x_{k+1} = A_d \vec{x}_k + B_d \vec{u}_k, \tag{35}$$

$$y_k = C_d \vec{x}_k, \tag{36}$$

where  $A_d = I + A_x T_s$  is the discretised state matrix,  $B_d = B_x T_s$  is the discretised input vector,  $y_k$  is this state output,  $C_d = C$  is the discretised output matrix,  $I$  is identity matrix, and  $T_s$  is sampling time.

### 3.3.2. MPC cost function

**Assumption 2.** For each sample time  $T_s$ , the receding horizon regulator is defined as a solution to an optimization problem determined by full-state feedback [1].

The cost function is used to select the optimal control trajectory to bring the predicted output as close as possible to the set-point. The cost is therefore defined as follows [1]:

$$J(x_k, U) = \vec{x}_{(k+N_p)}^T P \vec{x}_{(k+N_p)} + \sum_{k=0}^{N_p-1} \vec{x}_{(k+i)}^T Q \vec{x}_{(k+i)} + \sum_{k=0}^{N_u-1} \vec{u}_{(k+i)}^T R \vec{u}_{(k+i)}. \tag{37}$$

Cost function minimization:

$$\min_{U \triangleq (u_k, \dots, u_{k+N_u})} = J(x_k, U). \quad (38)$$

Subject to:

$$\begin{aligned} \vec{x}_{(k+i+1)} &= A_d \vec{x}_{(k+i)} + B_d \vec{u}_{(k+i)}, & k &\geq 0, \\ y_{(k+i)} &= C_d \vec{x}_{(k+i)}, & k &\geq 0, \\ \vec{u}_{(k+i)} &= K \vec{x}_{(k+i)}, & N_p &< K \leq N_u, \\ y_{\min} &\leq y_{(k+i)} \leq y_{\max}, & i &= 1, \dots, N_c, \\ \vec{u}_{\min} &\leq y_{(k+i)} \leq \vec{u}_{\max}, & i &= 0, 1, \dots, N_c. \end{aligned}$$

When the input sequence  $(u_k, \dots, u_{k+N_u})$  is applied to the state model, the predicted state vector at sampling time,  $T_s$ , is obtained as  $\vec{x}_{(k+i)}$ . The total number of input control horizons is  $N_u$ , the total number of output prediction horizons is  $N_p$ , and the total number of constraint horizons is  $N_c$ , with  $N_u \leq N_p$  and  $N_c \leq N_p$ .  $\vec{x}_{(k+N_p)} = [\phi_{\text{ref}(k+N_p)} \ \theta_{\text{ref}(k+N_p)} \ \psi_{\text{ref}(k+N_p)}]^T - [\phi_{(k+N_p)} \ \theta_{(k+N_p)} \ \psi_{(k+N_p)}]^T$  is the last  $N_p$  predicted term,

$\vec{x}_{(k+i)} = [\phi_{\text{ref}(k+i)} \ \theta_{\text{ref}(k+i)} \ \psi_{\text{ref}(k+i)}]^T - [\phi_{(k+i)} \ \theta_{(k+i)} \ \psi_{(k+i)}]^T$  is the first predicted term to  $(N_p - 1)$  predicted term of  $N_p$ ,  $\vec{u}_{(k+i)}$  is the predicted input vector with respect to  $N_u$ .

It has been determined by [1, 48] that the most efficient approach to the optimization problem in (38), at a sampling time,  $T_s$ , is defined as follows:

$$U^*(k) \triangleq \{\vec{u}_k^*, \dots, \vec{u}_{k+N_u-1}^*\}. \quad (39)$$

The first input from the optimal command sequence (39) is then applied to the rotational dynamic equations (35) of the quadcopter as follows:

$$\vec{u}_k = \vec{u}_k^*. \quad (40)$$

### 3.4. Genetic algorithm

A backstepping controller was designed in the previous section to stabilize the position dynamics of the quadcopter. The response of the controller is highly dependent on the values of its gains,  $k_1, k_2, k_3, k_4, k_5$ , and  $k_6$ . These coefficients are the altitude, longitude, and latitude control parameters for the quadcopter, and they must be positive to meet stability criteria [43]. In conventional backstepping methods, there is no predefined approach for tuning the gains of the backstepping controller [49]. Thus such parameters are generally selected arbitrarily. Although it is not guaranteed that the parameters have been selected to achieve the best results, it is also conceivable that they have been chosen appropriately.

The efficacy of the backstepping control system is dependent upon the gain parameters that are adjusted. This study employs one of the metaheuristic algorithms known as the GA to determine the optimal gains for the BC parameters to overcome this drawback. The GA is biologically inspired by how nature selects the strongest genes [50]. The Fixed-state GA, which implements a generational paradigm similar to that of long-lived species such as mammals, whereby parents and their issues concurrently continue to exist yet experience competition as time progresses, is used. In this model, two-parent individuals must be selected to propagate their genetic information to the next generation, with the requirement that the previous population is eliminated to create space for the new offspring, thus allowing the offspring to be immediately ready for propagation. This enables the model to capitalize on the qualities of a promising individual quickly. In terms of genetics, the strongest genes are typically considered to be the fittest. For evaluation purposes, the system is partitioned into three subsystems, one for each axis: the  $X$  subsystem has two associated gains ( $k_3$  and  $k_4$ ), the  $Y$  subsystem has two associated gains ( $k_5$  and  $k_6$ ), and the  $Z$  subsystem also has two associated gains ( $k_1$  and  $k_2$ ). Rather than being globally tuned, the system utilises the inherent segmentation of the system. The algorithm, which is based on the flowchart in Figure 3, includes the following operations: population creation, selection, reproduction, mutation, and population re-integration.

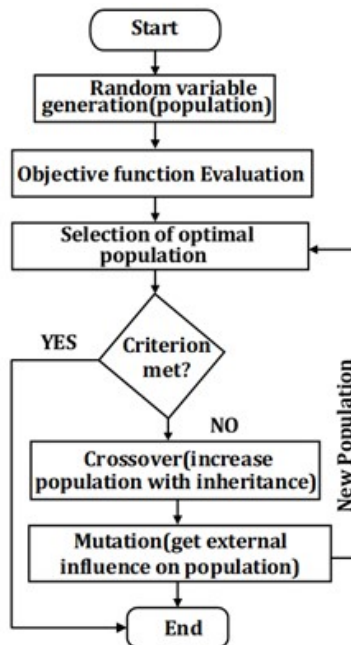


Figure 3: Flowchart of the genetic algorithm

#### 4. Results

In this section, the effectiveness of the proposed method is evaluated. Implemented in MATLAB 2021a are the algorithms for the dynamics of the quadcopter, the two controllers, and the GA. First, the values of the model parameters for the proposed quadcopter, the design parameters for the MPC controller which are selected to achieve a satisfactory attitude, and the results of the GA are summarized in Table 1.

Table 1: The initialized model parameters

Quadcopter parameters	
Parameter	Value
Quadcopter mass, $m$	1.8 kg
Acceleration due to gravity, $g$	9.81 m/s <sup>2</sup>
Distance from the rotors to the center of mass, $l$	0.225 m
X moment of inertia, $I_{xx}$	0.022 kg.m <sup>2</sup>
Y moment of inertia, $I_{yy}$	0.022 kg.m <sup>2</sup>
Z moment of inertia, $I_{zz}$	0.0439 kg.m <sup>2</sup>
Rotor inertia, $J_i$	$1.38 \times 10^{-5}$ N.s <sup>2</sup>
Thrust factor, $b$	$3.7634 \times 10^{-5}$ kg.m <sup>2</sup>
Drag factor, $d$	1
Sampling time, $T_s$	0.1 seconds
MPC tuned parameters	
Prediction horizon, $N_p$	6
Control input horizon, $N_u$	5
Weights for the final horizon outputs, $P$	diag(5, 5, 5)
Weights for the $(N_p - 1)$ horizon outputs, $Q$	diag(5, 5, 5)
Weights for the inputs, $R$	diag(10, 10, 10)
GA tuned parameters	
$k_1$	13.24
$k_2$	12.54
$k_3$	14.02
$k_4$	13.30
$k_5$	16.43
$k_6$	15.98



Figure 4, shows an irregular helical type of trajectory denoted by a blue line tracked by the proposed quadcopter model denoted by a red dashed line. The initial point of the desired trajectory is  $(2, 0, 2)$ , while the initial position of the quadcopter is  $(0, 0, -1)$ . It can be observed that the initial position of the quadcopter deviates slightly from the initial position of the desired trajectory, whereas the actual trajectory of the quadcopter quickly converged to the desired trajectory.

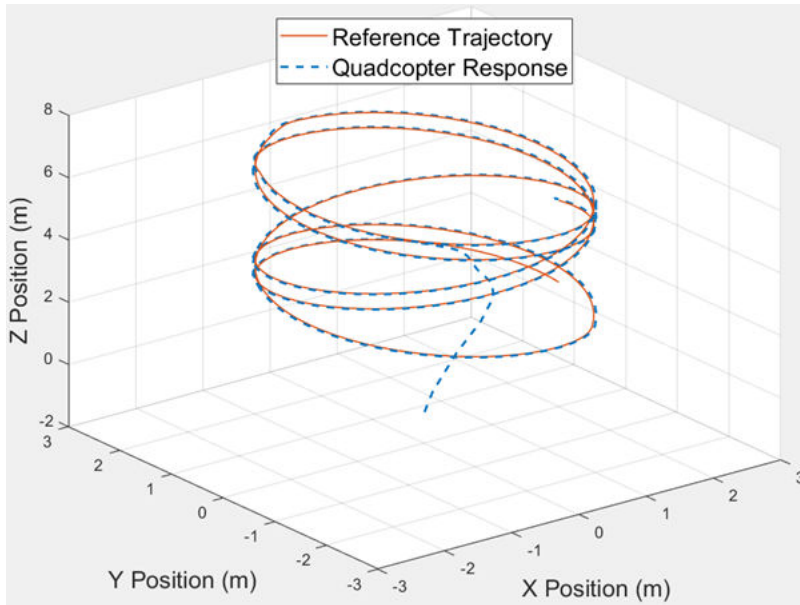
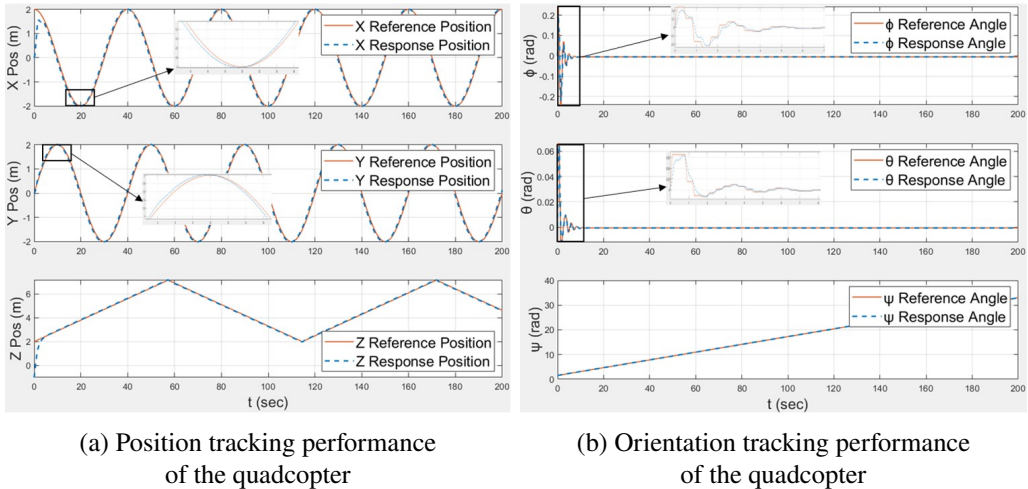


Figure 4: 3D reference trajectory and quadcopter actual trajectory using the proposed hybrid MPC based backstepping control technique

The performance of the reference position trajectory tracking by the quadcopter at 200 seconds of simulation time is shown in Figure 5a. At zero initial positions, the quadcopter's  $x$  and  $y$  positions tracked a 2 m magnitude sinusoidal reference with the least amount of tracking error visible at the peak and trough of the wave. A more reliable and stable altitude tracking performance was achieved by the  $z$  position, which began tracking the  $z$  reference position at  $-1$  m. Figure 5b displays the quadcopter's performance in terms of orientation tracking. The pitch and roll angles had very little tracking error at the beginning of the simulation before steering at zero radians, but the yaw angle tracked the reference yaw angle more accurately. Figure 6a depicts the quadcopter's ability to track translational velocity. The quadcopter had to operate at a faster speed of 1.2 m/s  $X$  velocity, 0.58 m/s  $Y$  velocity, and 1.8 m/s  $Z$  velocity because it began tracking the reference trajectory at initial conditions that were different from those of

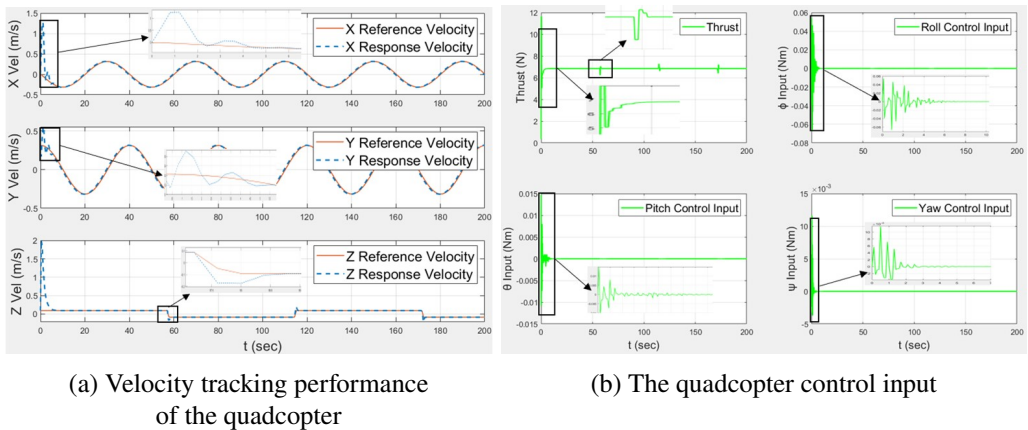
the reference trajectory  $(2, 0, 2)$ . These diagrams demonstrate how the suggested hybrid MPC-based backstepping controller technique produces an input control command shown in Figure 6b is effective in tracking the specified reference with minimal tracking error.



(a) Position tracking performance of the quadcopter

(b) Orientation tracking performance of the quadcopter

Figure 5: Output state performance of the quadcopter



(a) Velocity tracking performance of the quadcopter

(b) The quadcopter control input

Figure 6: Velocity and control input performance of the quadcopter

Figure 7a and Figure 7b shows the tracking performance error of the six state outputs (Position and Orientation) of the quadcopter. At a range of 0 to 5 seconds of simulation time, the maximum tracking error achieved by the X and Z position states are 2 m and 3 m, before settling at a steady sinusoidal error of

0.1 m magnitude. The Y position maintained a regular sinusoidal tracking error of 0.15 m. It is also observed that the tracking error of motion on orientation converged to zero with a minor divergence of 0.18 rad pitch angle, 0.042 rad roll angle, and 0.008 rad yaw angle at initial simulation time of 5 seconds. The high error values of the six states of the quadcopter at initial simulation time are because of the quadcopter tracking the reference trajectory at different initial conditions which are different from that of the reference trajectory.

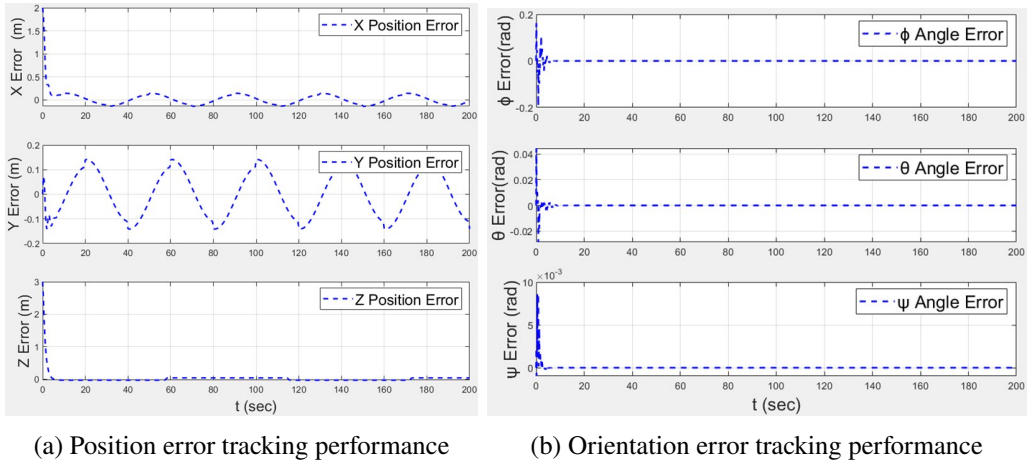


Figure 7: Output states error tracking performance of the quadcopter

## 5. Conclusion

This study successfully demonstrates the utilization of a hybrid MPC-based backstepping control system to maneuver a quadcopter along an established trajectory. At first, the translational and rotational dynamics of the quadcopter are developed using the Newton-Euler method. These dynamics are utilized to design a hybrid controller with a multiloop structure incorporating an optimal backstepping control technique and the MPC method. The outer-loop optimal backstepping controller is responsible for controlling the position of the quadcopter, with the inner-loop MPC controller maintaining the attitude of the quadcopter. Applying Lyapunov's theory, a recursive control procedure known as optimal backstepping control has been constructed, incorporating the genetic algorithm to determine the controller parameters autonomously. Furthermore, the MPC structure employs linear parameter varying to implement a linear control approach in the face of the nonlinearities present in the dynamics of the quadcopter. The MPC can operate with reduced computational demand through the resolution of the quadratic

optimization problem in an offline mode. Hence, the proposed technique has considerable capabilities for real-time applications with constrained computational power.

Through simulation, the proposed optimal hybrid MPC-based backstepping controller of the quadcopter was verified for its dynamic performance in following an established reference trajectory. The simulations show that when a command control input is employed for trajectory tracking, the proposed hybrid control algorithm ensures satisfactory tracking of the assigned position references while ensuring maximum tracking errors of 0.1 m for the  $X$  and  $Y$  positions and 0.15 m for the  $Z$  position are not exceeded. Nevertheless, the next stage of this research will involve the creation of an algorithm aimed at improving the inner-loop controller (MPC) in disturbance-rich environments, as well as its actual physical implementation on a quadcopter and the carrying out of actual flight tests.

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