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Research paper

Evaluation of some levelling techniques in surveying application

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Abstract: Applications in geodesy and engineering surveying require the determination of the heights of the vertical control points in the national and local networks using different techniques. These techniques can be classified as geometric, trigonometric, barometric and Global Positioning System (GPS) levelling. The aim of this study is to analyse height differences obtained from these three techniques using precise digital level and digital level, total station (trigonometric levelling) and GPS which collects phase and code observations (GPS levelling). The accuracies of these methods are analysed. The results obtained show that the precise digital levelling is more stable and reliable than the other two methods. The results of the three levelling methods agree with each other within a few millimetres. The different levelling methods are compared. Geometric levelling is usually accepted as being more accurate than the other methods. The discrepancy between geometric levelling and short range trigonometric levelling is at the level of 8 millimetres. The accuracy of the short range trigonometric levelling is due the reciprocal and simultaneous observations of the zenith angles and slope distances over relative short distances of 250 m. The difference between the ellipsoidal height differences obtained from the GPS levelling used without geoid and the orthometric height differences obtained from precise geometric levelling is 4 millimetres. The geoid model which is obtained from a fifth order polynomial fit of the project area is good enough in this study. The discrepancy between the precise geometric and GPS levelling (with geoid corrections) is 4 millimetres over 5 km.

Keywords: levelling, accuracy, height, GPS, precision

1. Introduction

Levelling is the general term applied to any process by which elevations of points or differences in elevation are determined. It is a vital operation in producing the necessary data for mapping, engineering design and construction. Differences in elevation have traditionally been determined by digital levelling, trigonometric levelling, barometric levelling and GPS levelling. The method selected depends primarily on the accuracy required, although the type of terrain over which the levelling is done is also a factor. Geometric levelling can produce the highest order of accuracy. In spite of the fact that trigonometric levelling produces a somewhat lower order of accuracy than digital levelling, the method is still suitable for many projects such as establishing vertical control for topographic mapping or for lower order construction setting-out. It is particularly convenient in hilly or mountainous terrain where large differences in elevations are encountered. Both methods are subject to systematic and random errors. The primary systematic errors include earth curvature, atmospheric refraction and calibration of the surveying tools. The Global Positioning System (GPS) can be used for low-order vertical control surveys. To get accurate elevations using this method, the geoidal undulations in the area must be known and applied. The ellipsoidal heights obtained from GPS cannot be used directly for practical surveying. The ellipsoidal heights have to be transformed to orthometric heights, being the distance measured along the plumb line between the geoid and a point on the Earth's surface and taken positive upward from the geoid. The difference between the ellipsoidal and the orthometric height is defined as the geoid height or geoid undulation. With the knowledge of the geoid (quasigeoid), it is possible to derive orthometric or normal heights from ellipsoidal heights. The most recent and advanced levelling approach is GPS levelling which is the combination of the GPS derived ellipsoidal heights and the geoid information in order to determine orthometric heights. The studies relating to the GPS technique need ellipsoidal heights with centimetre or sub-centimetre accuracy which are obtained from the GPS solution. This will only be possible with an interconnected network or reference stations (Ayan, 2001; Ceylan and Baykal, 2008; Schofield, 2001) assuming the Precise Digital Levelling (PDL) data as minimal error for the time being and equal weight of all sections (all sections assumed to be 250 m long). In this study, accuracy of the height determination techniques has been compared the twenty vertical control points were marked in the project area. In order to determine of height differences between these 20 points, three methods were used and analysed, namely, GPS levelling, reciprocal and simultaneous short range trigonometric levelling, and digital and precise digital levelling. The procedures and obtained results are explained in this paper.

2. Digital levelling

Recent advances in electronics now enable surveyors to perform digital levelling with an electronic (digital) level. The development of this type of level has become possible due to the advances in microchip technology and image processing. The attributes of self-levelling instrumentation coupled with digital array photography and electronic image processing have generated a digital level, which is very much close to being truly automatic. The digital level processes an electronic image of a bar coded staff for the determination of heights and distances with automatic recording of the data for future processing on a computer. The digital level is an automatic level (with a pendulum compensator) capable of normal optical levelling with normal graduated staffs. The level can also be used with the bar coded staff and rod readings obtained digitally with output to a display on the instrument. The measuring system of the digital levelling consists of a level comprising optics and compensator, a bar code scale mostly on an invar band fixed into a rod frame, a couple-charged device (CCD) linear array and software controlling all operations, procedures and processes of the digital level (Anderson and Mikhail, 1998; Ingesand, 1999; Kahmen, 2000; Rüeger and Brunner, 1982; 2000; Wolf and Ghilani, 2002; Woschitz and Brunner, 2002; Schofield, 2001).

3. Precise digital levelling

Precise digital levelling may be used in certain instances in construction such as in deformation monitoring, the provision of precise height control for large engineering projects like long-span bridges, dams and hydroelectric schemes and in mining subsidence measurements. Precise digital levelling staffs have an invar bar code kept under constant tension on the face. Invar has a very low coefficient of expansion, and the staffs are calibrated to an exact length. The maximum sight distances are (50 m, 60 m, 90 m) and allowable differences between backsight and foresight lengths (2–4 m, 5–10 m, 10–10 m) for first-second, and third-order levelling respectively. Rod persons can pace or count rail lengths or highway slab joints to set sight distances. Precise levelling demands good-quality turning points. Lines of sight should not pass closer than about 0.5 m from any surface, e.g., the ground, to minimise refraction. Readings at any set up must be completed in rapid succession; otherwise, changes in atmospheric conditions might significantly alter refraction characteristics between them (Assuming the PDL data as error free for the time being and equal weight of all sections (all sections assumed to be 250 m long), (Featherstone et al., 1998) assuming the PDL data as error free for the time being and equal weight of all sections (all sections assumed to be 250 m long) (Federal Geodetic Control Committee, 1984; 1998).

4. Short range simultaneous reciprocal trigonometric levelling

Trigonometric levelling is the method of obtaining height differences using measured slope distances and zenith angles. The targets in trigonometric levelling can always be placed at the same height above the ground. Thus, the sighting distances are not limited by the inclination of the terrain and systematic errors, e.g. refraction, because the back and foresight lines pass through similar layers of air. By extending the sighting distances to a few hundred metres, the number of set-ups per kilometre is minimised. The accuracy of trigonometric levelling mostly depends on the earth's curvature and refraction which directly affect the zenith angle and distance observations. Usually, the surveyor measures the reciprocal zenith angles and slope distances from both ends of the baseline. Besides, the surveyor should then use the mean value of the computed height difference in order to correct for earth curvature and refraction. However, this is not always practical and warranted. It should be remembered that in order to minimise the errors introduced by curvature and refraction, the distances between the instrument set-ups should be shorter than 250 m. The influence of the earth's curvature and refraction is given by $c\&r = 0.0675\ K^2$ metres, where K is the distance in kilometres. For 0.5 km the effect of c&r is 16.8 mm, for 0.2 km it is 2.7 mm, and so on (Anderson and Mikhail 1998). Figure 1 illustrates the short range trigonometric levelling with reciprocal and simultaneous measurement of zenith angles and slope distances. Field tests show that standard deviations of $\leq 2\ \mathrm{mm}/\sqrt{\mathrm{km}}$ are achievable, even along inclined terrain, at speed of 10 km/day when using sights no greater than 250 m for the reciprocal method (Ceylan and Baykal, 2008) assuming the PDL data as error free for the time being and equal weight of all sections (all sections assumed to be 250 m long), (Chrzanowski et al., 1985). The electronic total stations were set up on the 20 marked points of the 5 km run. The instruments were set up on the marks and the heights of instrument (iA and iB) were measured three times from the centre of the telescope (horizontal axis) above the station mark in the field, see Figure 1. The height difference between two points, namely, A and B can be written as,

$$\Delta H_{AB} = \sum_{A}^{B} \Delta h_{ij} + t_A - t_B \tag{1}$$

where ΔH_{AB} is the height difference between the terminals A and B, Δh_{ij} are the individual height differences, t_A and t_B are the height of the target at A and B from ground to the total station. The observations have been made reciprocally and simultaneously between two points (Figure 1). The zenith angles (Z'_{AB} and Z'_{BA}) and slope distances (C_{AB} and C_{BA}) were measured for the test scenario. The height difference between A and B points (ΔH_{AB}) are computed using the following section (Anderson, 1998; Ceylan and Baykal, 2008; Wolf and Ghilani, 2002).

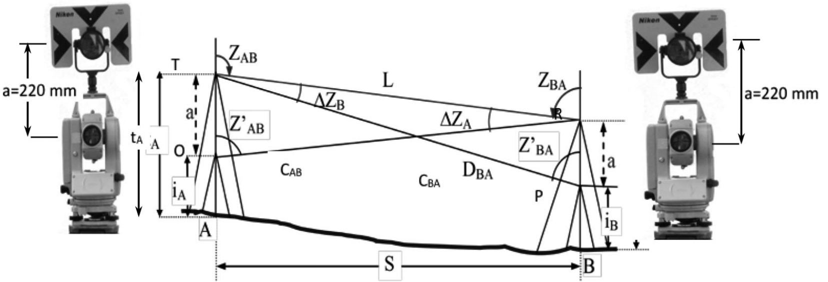

Fig. 1. Model of the reciprocal and simultaneous short range trigonometric levelling

In our method, the average height difference is free from the impact of the refraction and curvature of the Earth. However, this method has one disadvantage:

– the measurement of altitude i_A and i_B is not very precise. Another method of trigonometric leveling based on differential measurement from the inside can be used (Chrzanowski et al., 1985):
  - the instrument stands between the points: $(S_{AB}/2) = 125$ m – measurement in 2 face poles with supports, the poles are mounted with reflectors on the same height,
  - the instrument is not set on the point, so you do not need to measure its height,

– differential measurement eliminates the residual impact of refraction and curvature of the Earth,

– the measurement is carried out in the main and return direction, therefore it is possible to evaluate the internal accuracy as well as in geometric leveling.

5. GPS levelling

The geoid is defined as the gravity equipotential surface which best approximates mean sea level over the entire Earth. It has been defined as the datum for the orthometric height system. The irregular shape of the geoid, however, does not allow for an easy computation of the horizontal positions of points. Therefore, a reference surface of regular shape, usually a biaxial ellipsoid, is selected to best approximate the geoid either locally or globally. The geometric relationship between the geoid and the reference ellipsoid surface can be fully described by their separation and the slope of the geoid with respect to the reference ellipsoid. The former is known as the geoidal height (N), and the latter is known as the deviation of the vertical (θ). The deviation of the vertical is defined as the spatial angle between the ellipsoid normal and the actual gravity vector. With a large number of monuments where both the GPS ellipsoidal heights and the orthometric heights from digital levelling have been observed, the geoidal heights at these points can be approximated by using the following simple relation

$$N = h - H \tag{2}$$

where N is the geoidal height, h is the ellipsoidal height from GPS surveys, and H is the orthometric height from digital levelling (Kuang et al., 1996; Ayan, 2001; Featherstone et al., 1998; Ceylan and Baykal, 2008).

6. The United States Federal Geodetic Control Subcommittee (US FGCS) accuracy standards

The FGCS established accuracy standards and specifications for various orders of levelling. The (US) Federal Geodetic Control Subcommittee (FGCS) established accuracy standards and specifications for various orders of geometrical levelling. The FGCS recommends the following formula to compute the allowable misclosures (tolerances):

$$T_{Misc} = \pm m\sqrt{K}\ \text{mm} \tag{3}$$

where T_{Misc} is the allowable loop or section misclosure, in millimetres; mis a constant; and K is the total length levelled in kilometres. For loops (circuits that begin and end on the same bench mark), K is the total perimeter distance. The FGCS specifies the constants (m) of 4 (first-order class I), 5 (first-order class II), 6 (second-order class I), 8 (second-order class II), and 12 mm (third-order) for five classes of levelling. It is important to point out that meeting the FGCS misclosure criterion alone does not guarantee that a certain order of accuracy has been met (Assuming the PDL data as error free for the time being and equal weight of all sections (all sections assumed to be 250 m long; Featherstone et al, 1998; Federal Geodetic Control Committee, 1984; 1988).

The accuracy parameters of 1^{st} and 2^{nd} order levelling network for Turkey are $T\ [mm] \leq 12\ K\ (km)$.

7. Methods and data

The experiment was conducted in the Samandira region of Istanbul, see Figure 2. The GPS and terrestrial surveys (geometric and short range trigonometric levelling) were performed on a levelling route of about 5 kilometres length. In order to minimise the errors introduced by earth curvature and refraction, distances between the tests points had to be restricted to about 250 m. Figure 3 illustrates the distribution of the selected

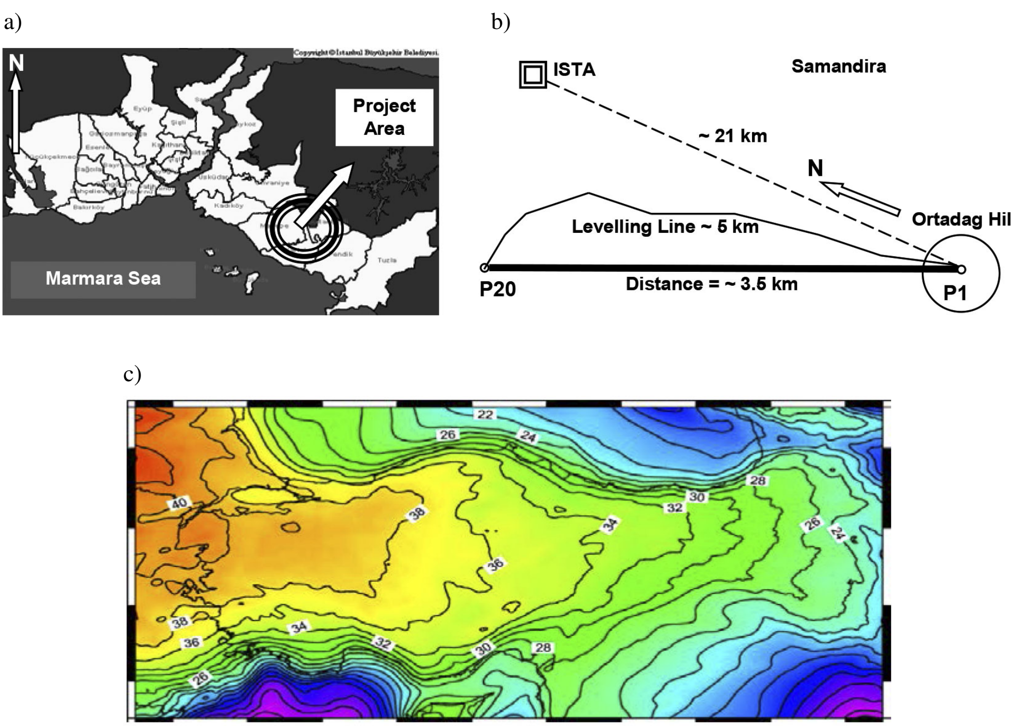

Fig. 2. Project Area (a) and GPS Network (b) and quasi-geoid isolines on the Turkey map (c)

Fig. 3. The distribution of the test points in the project area

points in the project area. The digital levelling (both digital and precise digital levelling) was carried out in order to assess the accuracy of the short range trigonometric levelling and the GPS levelling (Telci et al., 2006). The height differences between the 20 points were determined by geometric levelling performed as double run levelling. The instrument used for the precise digital levelling was a Leica DNA 03 precise digital level, together with two bar coded invar staffs of three metres length and the staffs were stabilised with struts. The Leica DNA 03 precise digital level provides rapid, accurate solutions for a wide range of levelling applications, from topographic and construction surveys to first-order levelling and monitoring. It provides 0.3 mm accuracy on a 1 km double run line with an invar staff and is ideal for first and second order levelling and high precision measurements. The levelling routine was performed double observance (BFFB, aBFFB) to increase the reliability of the measurement and to reduce possible errors caused by the staff sinking. Applying alternating observations procedures (aBFFB = BFFB FBBF) to eliminate horizontal tilt (residual error of the automatic compensator) (Assuming the PDL data as error free for the time being and equal weight of all sections (all sections assumed to be 250 m long). The instrument used for the digital levelling was a Topcon DL 102 digital level (0.8 mm/km with fibre glass staff) and with the two bar-coded aluminium rods of five metres length. The levelling routine was performed observance (BFFB). Minimum ground clearance of 0.5 m required to refractionary influences of ground proximity. Limit target distance, < 30 m. Levelling staffs with adjustable brace poles provide good stability. The staffs were stabilised with struts. Include equal backsights and foresights, maintaining a line of sight > 0.5 m above the ground and levelling the instrument to minimise any errors due to the obliquity of horizon problem. All these precautions were taken during the survey. Maximum allowable staff reading is 3 m. Four persons were performed in double run digital levelling. It took 6.5 hours on 5 km double run levelling. In the short range trigonometric levelling, the distances were measured by using two Nikon DTM 330 total stations with the (3 mm + 2 ppm) distance specification and 4.5″ zenith angle accuracy. The zenith angles and slope distances were reciprocally and simultaneously measured by using the same instruments four times in two faces. The heights of instruments, prisms, and targets for all points were measured three times to obtain mmlevel accuracy. The instruments were only set up on the 20 mark points. The instrument uses dual-axis compensation, and electronic level sensors, and it applies collimation, vertical index, and trunnion axis corrections automatically. Auto dual-axis compensation can ensure the accurate levelling of total station (Automatic dual-axis compensator with working range ±3′ (±55 mgon)). In the compensation range 3′, although the instrument is tilt, the horizontal and zenith angle can be measured precisely.

The GPS surveys were carried out in order to determine the ellipsoidal heights of the 20 points along a levelling route (~5 km). The GPS measurements were taken with four Ashtech Z Max GPS receivers using the static method. The GPS data were recorded in seven sessions. These sessions were measured on 10 May 2006 between 8:00–18:30 h (Local time). During this period, the satellite visibility varied between 6 and 9 satellites, and the PDOP values between 2.8 and 1.5. The GPS data were collected in 10 seconds epoch intervals. The station occupation time was ten hours and thirty minutes for the reference point (P1) and about sixty minutes for the remaining points. The reference station (P1) suffered no sky obstructions and was set up on Ortadag Hill which is the highest point in the project area, see Figure 3. The GPS data processing and adjustment for the reference point (P1) was conducted using the Bernese Software 4.2. In the adjustment procedure, the ITRF 2000 coordinates of ISTA (IGS (International GPS Service) Station) were held fixed. The GPS data for the rest of the points were processed by the Ashtech Solutions 2.60 Software using the reference point (P1) as fixed. The horizontal and vertical (height) positioning precision of the points is obtained, on average, as 1 mm and 5 mm, respectively. These results show that the GPS measurements are quite accurate and consistent (Telci et al., 2006).

8. Results and discussion

In order to compare the results of the levelling methods, the height differences between the points were separately determined, as are shown in Table 1. In these tables, ΔH_{Back} is backward levelling; ΔH_{Fore} is the forward levelling; ΔH_{Mean} is the mean of the backward and forward levelling; ΔH_{Trig} is the height difference from reciprocal and simultaneous short range trigonometric levelling and ΔH_{GPS} is the GPS levelling height differences between the points.

Table 1. Comparison of height differences obtained from different methods (Note: GPS data comparison of the height differences derived from digital, GPS and reciprocal and simultaneous short range trigonometric levelling method

	ΔH_{PL}	ΔH_{DL}	ΔH_{TL}	Δh_{GPS}	d_{PL-DL} [mm]	d_{PL-TL} [mm]	d_{PL-GPS} [mm]
1–2	−40.56827	−40.5695	−40.566	−40.565	1.23	−2.27	−3.27
2–3	−17.79039	−17.7892	−17.788	−17.787	−1.19	−2.39	−3.39
3–4	−18.65703	−18.6574	−18.658	−18.660	0.37	0.97	2.97
4–5	−0.34028	−0.3403	−0.339	−0.344	0.02	−1.28	3.72
5–6	−0.93113	−0.9311	−0.933	−0.939	−0.03	1.87	7.87
6–7	−0.74409	−0.7447	−0.742	−0.741	0.61	−2.09	−3.09
7–8	−1.12021	−1.1204	−1.120	−1.122	0.19	−0.21	1.79
8–9	−1.57150	−1.5714	−1.573	−1.575	−0.10	1.50	3.50
9–10	−3.82522	−3.8254	−3.822	−3.820	0.18	−3.22	−5.22
10–11	−1.79897	−1.7994	−1.796	−1.795	0.43	−2.97	−3.97
11–12	−13.39333	−13.3934	−13.391	−13.391	0.07	−2.33	−2.33
12–13	−9.63998	−9.6397	−9.641	−9.642	−0.28	1.02	2.00
13–14	−5.35411	−5.3538	−5.352	−5.347	−0.31	−2.11	−7.11
14–15	2.77112	2.7711	pmi 2.768	2.767	0.02	3.12	4.12
15–16	−1.42499	−1.4249	−1.425	−1.426	−0.09	0.01	1.01
16–17	−0.17022	−0.1706	−0.170	−0.172	0.38	−0.22	1.78
17–18	2.59111	2.5912	2.594	2.588	−0.09	−2.89	3.11
18–19	4.36295	4.3632	4.361	4.369	−0.25	1.95	−6.05
19–20	1.71840	1.7184	1.715	1.720	0	3.40	−1.60
	−105.88614	−105.8873	−105.878	−105.882			
				$\sum Pd^2 =$	14.43	362.22	1144.63
			$\sigma_{1KM} =$		±0.92 mm	±4.28 mm	±8.02 mm
			$\sigma_{5KM} =$		±2.06 mm	±9.57 mm	±17.93 mm

9. Accuracy analysis of height determination techniques

This section covers the basics of the statistical theory used to determine the level of accuracy for a survey. The standard deviations and the differences between paired observations for precise digital levelling and ordinary digital levelling are illustrated in Table 2. The precise digital levelling measurements were processed for the 5 km double run levelling; the standard deviation of a single run measurement of 250 m is ±0.25 mm/km; the standard deviations of double run levelling of 1 km is ±0.17 mm. The ordinary digital levelling measurements were being processed for about 5 km back and fore levelling range; the standard deviation of one measurement in 250 m is ±1.20 mm/km; the standard deviation of double run levelling in 1 km distance levelled is ±0.85 mm/km (these values were calculated according to the Levallois formula based on the differences from the main and the return measurement). The height differences determined by precise digital levelling were assumed as true values for the the comparison of the height differences derived from digital, GPS and reciprocal and simultaneous short range trigonometric levelling method. In Table 1, the d values could be considered the corrections (residuals) of the DL, TL, and GPS (corr.) data to the PDL data taken a reference.

Table 2. Accuracy analysis of two height determination methods (precise digital levelling with Leica DNA03 and (ordinary) digital levelling with TOPCON DL–102) and accuracy analysis and testing all height determination methods (Note: GPS data corrected with geoid), (Telci et al., 2006)

	Precise Digital Levelling		(Ordinary) Digital Levelling		Trigonometric Levelling	GPS Levelling
	ΔH_{Back} ΔH_{Fore}	ΔH_{Mean}	ΔH_{Back} ΔH_{Fore}	ΔH_{Mean}	ΔH_{Trig}	ΔH_{GPS}
$\sum \Delta H$	-105.88519 105.88706	4 -105.8861	-105.8903 105.8837	-105.8873	-105.878	-105.882
	Misclosure	= 1.87 mm	Misclosure	= 6 mm		
$T_{Misc} = 4\sqrt{K} = 9$ mm (FGCS First Order Class I) $T_{Misc} = 6\sqrt{K} = 13$ mm (FGCS Second Order Class I)						

Line	Length	Weight	Precise Digital Levelling	(Ordinary) Digital Levelling
	K [km]	P [1/km] [1/K]	D [mm]	d [mm]
$\sum =$	4.934 km	$\sum Pd^2 =$	2.31 [mm^2/km]	54.09 [mm^2/km]
Single run 1 km precision			± 0.25 mm	± 1.2 mm
Double run 1 km precision			± 0.17 mm	± 0.85 mm
Double run 5 km precision			± 0.55 mm	± 2.65 mm

$t = \dfrac{0.85}{\sqrt{0.55^2 + 2.65^2}} = 0.314$	$t = \dfrac{8.15}{\sqrt{0.55^2 + 6.86^2}} = 1.180$	$t = \dfrac{4.15}{\sqrt{0.55^2 + 12.22^2}} = 0.340$
$T\,\{\alpha = 0.05,\ f = 38\}$	2.024	

In Table 1, the d values could be considered the corrections (residuals) of the DL, TL, and GPS (corr.) data to the PDL data taken a reference deviation of the differences between precise digital levelling value and digital levelling value. P value is $\frac{1}{S}$ for precise and digital levelling, however P value is $\frac{1}{S^2}$ for precise and digital levelling. After that, we calculated the absolute t-value.

Assuming the PDL data as error free for the time being and equal weight of all sections (all sections assumed to be 250 m long), then the standard deviation of the

DL, TL and GPS (corr.) height differences (over 250 m) can be calculated approximately as $\sigma_{250\,\mathrm{m}} = \sqrt{\frac{[\mathrm{dd}]}{\mathrm{n}}}$ (n = 19). For Digital Levelling $\sigma_{\mathrm{DL}(250\,\mathrm{m})} = \pm 0.46$ mm, for short range trigonometric Levelling $\sigma_{\mathrm{TL}(250\,\mathrm{m})} = \pm 2.14$ mm, for GPS Levelling $\sigma_{\mathrm{GPS}(250\,\mathrm{m})} = \pm 4.01$ mm are calculated in Table 1. To get the approximate 1 km precision multiplying $\sqrt{4}$, that is 2. For DL $\sigma_{\mathrm{DL}} = \pm 0.92$ mm, for short range trigonometric levelling $\sigma_{\mathrm{TL}} = \pm 4.28$ mm, for GPS Levelling $\sigma_{\mathrm{GPS}} = \pm 8.02$ mm is calculated in Table 1.

In this study the t distribution is used to compare two different data test results. The following sections provide the mathematical equations to calculate the t–test statistic. We calculated the difference ($\mathrm{d_{PL}} - \mathrm{d_{DL}}$) between the precise digital levelling value and the ordinary digital levelling value for each set of test number, see Table 1. Then, we calculated the arithmetic average (mean, $\bar{\mathrm{d}}$) of the differences between the precise digital levelling value and the digital levelling value (last column) and the standard deviation of the differences between precise digital levelling value and digital levelling value. After that, we calculated the absolute t-value using Equation (4).

The sample size for observed differences is n = 19.

$$|\mathrm{t}| = \frac{\bar{\mathrm{d}}}{\sigma_{\mathrm{d}}} \tag{4}$$

where | | – absolute value (disregard sign), $\bar{\mathrm{d}}$ – average of the differences between test results, σ_{d} – standard deviation of the differences between test results, n – total number of sections (sample size).

The comparison among the precise digital levelling method, digital levelling method, trigonometric levelling and GPS levelling method has indicated no significant differences (the 95% confidence level) between accuracies the four techniques; refer to Table 2 (in the last rows) (Colorado Procedure, 2007).

The height differences can be determined with an accuracy of a few mm/km using the levelling method named "Precision Trigonometric Levelling". In Ceylan and Baykal (2008) and Chrzanowski et al. (1985) demonstrate that accuracy can be achieved trigonometric levelling better than 1 mm $\sqrt{\mathrm{K}}$.

The effects of errors in digital levelling can be reduced by taking equal backward and forward observation range, the round trip surveying, following BFFB or FBBF observation order or surveying calibration in lab and surveying additional parameters such as pressure, temperature and time at the survey moment. Trigonometric levelling is "low order" compared with conventional differential levelling. The reason for this is that during a 3D traverse, the height of every single instrument and target set-up must be measured. These heights are usually measured somewhat crudely with a folding ruler, and the accuracy of the results suffers directly from these imprecise height measurements. The total station used in trigonometric levelling had to be controlled and calibrated before the measurements. Short range trigonometric levelling methods use two total stations mounted on points observing simultaneous reciprocal angles, requiring numerous temperature and pressure measurements. In order to exploit the long range the modern instruments, maintaining the desired accuracy, reciprocal and simultaneous observations should be used so that errors due to the curvature and atmospheric refraction would be minimised. The effect of the vertical on the zenith angles should, also, be investigated in all cases. While the EDM Height Traversing technique is by no means new, the advancement of equipment technologies now makes the use of Total station (The Automatic Target Recognition (ATR)) demonstrates its full benefits during routine repeat measurements, e.g. monitoring, set measurements and measurement at two telescope faces. Using the sight, the observer aligns the telescope roughly with the target point and triggers a distance measurement. The total station automatically moves the telescope to the centre of the prism measures the distance and corrects the angle 1 mm with deviation to the centre of the prism (High constant accuracy independent of the observer, fatigue free and quick, no focusing necessary, measures using any standard prism) a practicable replacement for the normal spirit levelling. The Total station Levelling technique has a number of benefits over normal spirit levelling. The elimination of collimation errors and staff calibration errors and minimisation of refraction errors make the technique attractive to those undertaking first order levelling. The use of significantly longer sight lengths makes it attractive to everybody else. The technique does require slightly longer observation periods per standpoint; however this is offset by fewer instrument standpoints for the same length of run. The other significant advantage is that the Total station already exists on the surveyor's equipment list. There is no need for extra equipment to complete highly accurate level runs. It may be said that GPS-derived height determination techniques give effective solution and accurate results which will meet the most engineering projects' requirements. However, this method requires geoid undulation or geoid undulation differences.

10. Conclusions

In this study, four different levelling methods are compared. Geometric levelling is usually accepted as being more accurate than the other methods. The discrepancy between geometric levelling and short range trigonometric levelling is at the level of 8 millimetres. The accuracy of the short range trigonometric levelling is due the reciprocal and simultaneous observations of the zenith angles and slope distances over relative short distances of 250 m. The difference between the ellipsoidal height differences obtained from the GPS levelling used without geoid and the orthometric height differences obtained from precise geometric levelling is 4 millimetres. The geoid model which is obtained from a fifth order polynomial fit of the project area is good enough in this study. The discrepancy between the precise geometric and GPS levelling (with geoid corrections) is 4 millimetres over 5 km. This shows the necessity of an appropriate geoid model which for the study area. It was seen that the short range trigonometric levelling and the GPS levelling techniques give sufficiently accurate results when compared to geometric levelling. This study presented some practical solutions towards determination of the heights of vertical control points in engineering surveying applications using different techniques.

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