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SPATIAL CYCLOGRAPHIC MODELING ON NAUMOVICH HYPERDRAWING

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Abstract. The present paper shows the capability of application of three-dimensional drawing of space R^4 proposed by N.V. Naumovich in 1958, in the field of cyclographic modeling of non-linear objects: curves and surfaces of space R^4, models of which are widely applied in modern CAD/CAM and Computer Aided Geometric Design (CAGD) systems. The possibility of implementing spatial cyclography on the Naumovich hyperdrawing is illustrated by the example of modeling a canal surface as a cyclographic image of a spatial curve of R^4 space.

Keywords: cyclography, curves and surfaces, multidimensional space, Naumovich hyperdrawing

1 Introdction

Cyclographic modeling of space R^3 (planar cyclography) is based upon the works of well-known West European geometers of the XIX-XX centuries – W. Fiedler [1], L. Eckhart [2], E. Muller, J. L. Krames [3]. In these works the cyclographic models of linear objects (straight lines and planes) of space R^3 were studied in depth and the foundations of cyclographic modeling of non-linear objects (curves and surfaces) were laid. The models of curves were given much more consideration than the models of surfaces. In the XX-XXI centuries the cyclographic modeling of non-linear objects of Euclidean space has seen further development mainly in theoretical research in the works [4-6, 7-11]. At the same time, the areas of practical application of cyclographic modeling were established. They include the traditional area of application – geometric optics [3, 8, 10], as well as Global Positioning System (GPS) [11], highway design [12], pocket machining [13], multi-layer 3D printing. Cyclographic models of non-linear objects of Euclidean space are relevant in the area of Computer Aided Geometric Design (CAGD) [7].

In contemporary cyclographic modeling the direction of research called "spatial cyclography" devoted to research and development of models of objects of space R^n, $n>3$ has been formed. It is shown that a channel surface of space R^3 serves as a cyclographic model of a curve of space R^4 [6-9] and 2-surfaces of space R^4 correspond to a two-parameter set of spheres of space R^3 [10]. Research and development of said cyclographic models is performed analytically. With that, constructive models of Euclidean multi-dimensional spaces that allow analytic representation are known in higher descriptive geometry [14]. One of such models is Naumovich hyperdrawing, which allows us to perform R^4 space modeling. On the example of the curve of the space R^4 is shown the possibility of computer realization of the spatial cyclography on the Naumovich hyperdrawing.

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2 Theory

2.1 Elements of spatial cyclography

It is known that an isomorphic correspondence can be established between the set of points of space R^4 and the set of spheres of space R^3 [1, 2, 3]. Let us show that the said correspondence is conveniently realized by the use of Naumovich hyperdrawing [14], which represents a three-dimensional two-projection drawing of space R^4, where the projection hyperplanes (xyz), (xyv) and axial plane (xy), act correspondingly as the projection planes Π_2, Π_1, and their common projection axis x of the known Monge drawing (Fig.1).

The convenience of Naumovich hyperdrawing application consists in the capability of its realization in the form of three-dimensional drawing in virtual 3D space by means of modern graphic CAD [15]. On the hyperdrawing a point of space R^4 is modeled by the use of a pair of projection points: $A_2(x,y,v)$ and $A_1(x,y,z)$. On the analogy with planar cyclography [1, 2, 3, 8, 16], we bring a 2-sphere S^2_{xyz} of direction (⊚ or ⊚) and radius r=|x| embedded in projection hyperplane (xyv) into correspondence with the point $A_2(x,y,z,v)$. A directed sphere $\vec{S}^2_{xyz}$ is called a two-dimensional cycle. Thereby, the cyclographic image of a point $A_2(x,y,z,v)$ on projection hyperplane (xyz) is a two-dimensional cycle $\vec{S}^2_{xyz}$ with orthogonal projection S^2_{xy} on the axial plane represented by a disk of radius r=|v| featuring a directed border - a one-dimensional cycle. The direction of the two-dimensional cycle is defined by the sign of the coordinate v. The given approach to cyclographic modeling of points of space R^4 corresponds to algebraic mapping C_A, while the mapping itself is spatial cyclographic [8, 11].

Let us now consider a geometric mapping C_G in the context of cyclographic modeling of points of space R^4. Let us accept a sphere S^2_{xyz} in the projection hyperplane (xyz) as a bottom of α-hypercone of revolution K^3_α with vertex in point (x,y,z,v), axis perpendicular to axial plane (xy), and body half-angle at the vertex α equal to 45°. Then, the "frontal" projection of α-hypercone K^3_α is the usual three-dimensional α-cone $K^3_{\alpha(xyv)}$ with all its interior points, while the "horizontal" projection is a ball S^3_{xyz} of radius r=|v| centered at the point (x,y,z) and featuring a directed spherical shell – a two-dimensional cycle $\vec{S}^2_{xyz}$.

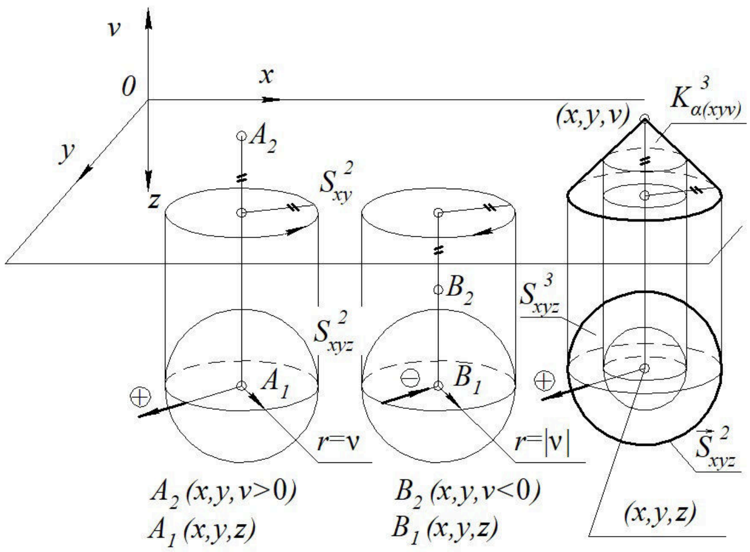

Figure 1: Cyclographic modeling of points of space R^4 on Naumovich hyperdrawing

On the analogy of cyclographic modeling of linear objects of space R^3 [1, 2, 3, 16] and on the basis of cyclographic model of a point of space R^4 it is possible to construct cyclographic models of straight lines, planes and hyperplanes of space R^4 in space R^3. These models represent the basis of linear spatial cyclography of space R^4 realized of Naumovich hyperdrawing.

Let us consider cyclographic modeling of a curve $\overline{P}(t)$ of space $R^4(E^4)$ on the basis of mappings C_A and C_G. In the case of mapping C_A, in projection hyperplane (xyz) the cyclographic projection of a curve $\overline{P}(t) \subset R^4$ is represented by channel surface – an envelope of all two-dimensional cycles, in turn representing cyclographic images of points of the curve $\overline{P}(t)$ (Fig. 2). Orthogonal projection of this surface on the axial plane is represented by an area, the border of which envelops a one-parameter set of one-dimensional cycles, in turn representing directed borders of set of disks S^3_{xy}. Disks with directed borders (cycles) represent orthogonal projections of two-dimensional cycles $\vec{S}^2_{xyz}$ on axial plane (xy).

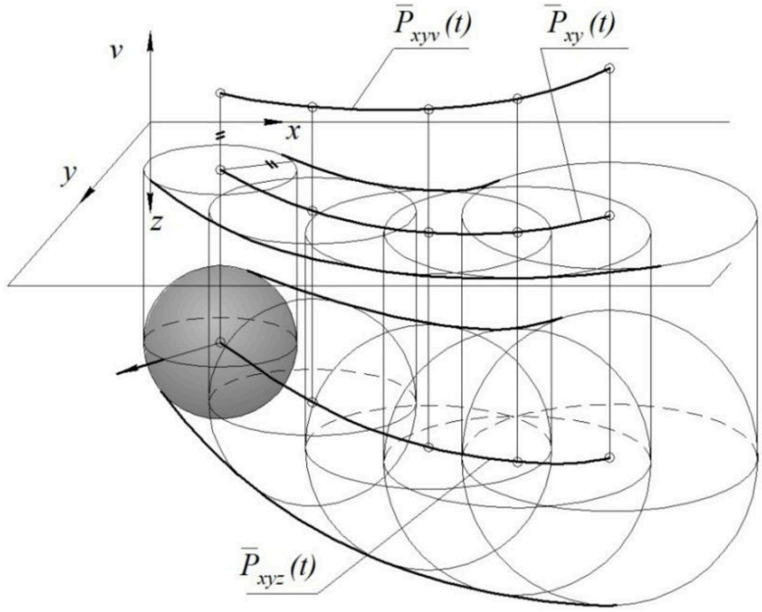

Figure 2: Cyclographic modeling of a curve of space R^4 on Naumovich hyperdrawing on the basis of mapping C_A

In the case of mapping C_G, in projection hyperplane (xyv) we acquire a three-dimensional ruled α-surface $\Phi^3_{\alpha(xyv)}$ with its interior points, which envelopes a one-parameter set of α-cones $K^3_{\alpha(xyv)}$. The surface in question constitutes the “frontal” projection of a ruled α-hypersurface Φ^3_{α} enveloping a one-parameter set of α-cones of revolution K^3_{α}. In projection hyperplane (xyz) we acquire a three-dimensional surface Φ^3_{xyz} with its interior points, which envelopes a one-parameter set of the balls S^3_{xyz}. The two-dimensional area in axial projection plane (xy) with a border enveloping the borders of disks representing the bottoms of three-dimensional cones of revolution $K^3_{\alpha(xyv)}$ in projection plane (xy) is correspondent in hyperplane of projection (xyz) to the envelope of two-dimensional cycles $\vec{S}^2_{xyz}$, i.e. the very same channel surface as in the case of representation C_A (Fig.3).

2.2 Cyclographic modeling of a curve

Let us now consider a task of cyclographic modeling of a curve of space R^4 on Naumovich hyperdrawing. Let us accept the mapping C_A, without excepting the capability of applying mapping C_G.

Let us suppose that the given curve is of form

$$P(t)=\{ x(t), y(t), z(t), v(t)\}, t\in G\subset R,$$

where G represents an interval on real numerical axis R; x, y, z, v represent functions differentiable about a point $P(t)=(x(t_0), y(t_0), z(t_0), v(t_0))$, $t_0\in G$ having continuous derivatives at the point. The curve $\overline{P}(t)$ consists of regular points meeting the condition

$$(x'(t))^2+(y'(t))^2+(z'(t))^2+(v'(t))^2\neq 0.$$

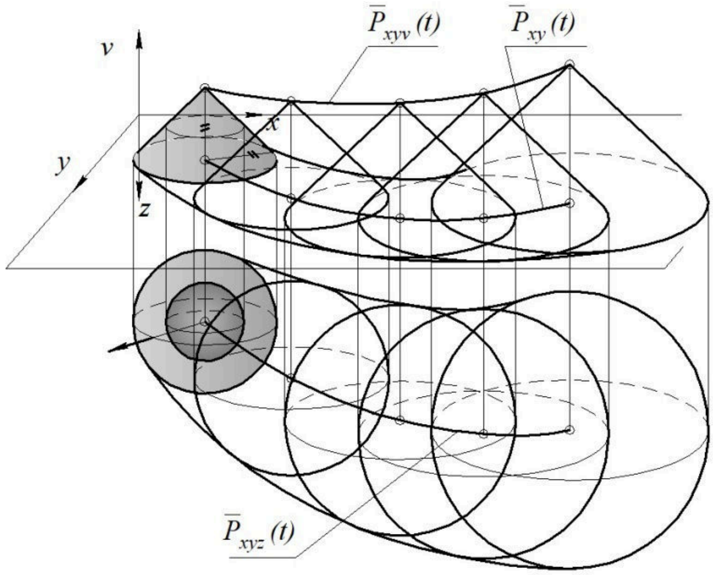

Figure 3. Cyclographic modeling of a curve of space R^4 on Naumovich hyperdrawing on the basis of mapping C_G

Let us perform orthogonal projection of the curve $\overline{P}(t)$ on coordinate hyperplane R^3_{xyz}. Geometrically such projection is performed by means of hyper bunch of straight lines with infinitely remote center V^∞ representing an infinitely remote point of coordinate axis v. Algebraically the same projection is performed by accepting the coordinate v equal to zero. In such case, curves $\overline{P}_{xyz}(t)\subset R^3_{xyz}$ appear in hyperplane (xyz) having the following form:

$$\overline{P}_{xyz}(t) = \{ x(t), y(t), z(t)\}. \qquad (1)$$

Let us accept every point of curve $\overline{P}_{xyz}(t)$ as center of a directed sphere of radius $r=|v(t)|$. Then a bijective correspondence is established between the set of points of curve $\overline{P}(t)\subset R^4$ and a one-parameter set of directed spheres centered at the curve $\overline{P}_{xyz}(t)$. Such correspondence is a subset of the more general continuous bijective correspondence between the set of points of space R^4 and the set of directed spheres of space R^3.

The envelope $\overline{P}_c = \{x_1(t,\varphi), y_1(t,\varphi), z_1(t,\varphi)\}$ of a one-parameter family $\Phi(t)$ of directed spheres of radius $r=|v(t)|$ centered at the curve $\overline{P}_{xyz}(t)$ represents a channel surface, which is a cyclographic image of the given curve $\overline{P}(t)\subset R^4$ in hyperplane R^3_{xyz}.

The equation of one-parameter family $\Phi(t)$ of spheres can be presented in the following form:

$$\Phi(t): \langle \overline{P}_c - \overline{P}_{xyz}(t),\ \overline{P}_c - \overline{P}_{xyz}(t) \rangle_{R^4} - \overline{r}(t)^2 = 0, \tag{2}$$

Where $|\overline{r}(t)|=|v(t)|$ represent the absolute value of a radius-vector $\overline{r}(t)$ of set $\Phi(t)$. The function (2) is differentiable in its domain of definition. In order to acquire the envelope $\Phi(t)$ of the set, let us combine the equation (2) with the following equation:

$$\Phi'(t): \langle \overline{P}_c - \overline{P}_{xyz}(t),\ \overline{P'}_{xyz}(t) \rangle_{R^4} + \overline{r}(t)\cdot\overline{r}'(t) = 0 \tag{3}$$

Expansion of equations (2) and (3) leads to the following system of equations:

$$\Phi(t): (x_1-x)^2 + (y_1-y)^2 + (z_1-z)^2 - r^2 = 0,$$
$$\Phi'(t): x'(x_1-x) + y'(y_1-y) + z'(z_1-z) - r'r = 0. \tag{4}$$

Let us consider the geometric sense of this equations. The first equation of the system (4) describes a one-parameter set of directed spheres $\vec{S}^2_{xyz}$ of radius $r=|v(t)|$ centered at the curve $\overline{P}_{xyz}(t) = \{x(t), y(t), z(t)\}$, which is acquired as a result of mapping of the curve $\overline{P}(t) \subset R^4 \rightarrow \overline{P}_{xyz}(t) \subset R^3_{xyz}$ on the basis of orthogonal projection on projection hyperplane R^3_{xyz}. Every sphere of the set is acquired as a result of cyclographic mapping of points of the curve $\overline{P}(t) \subset R^4$ on hyperplane R^3_{xyz}. Cyclographic mapping is performed by means of construction of α-hypercones with vertexes at points of the curve $\overline{P}(t)$ and axes perpendicular to projection hyperplane R^3_{xyz} with subsequent intersection of the acquired cones with hyperplane R^3_{xyz}. In this intersection the spheres S_{xyz} representing cyclographic images of points of the curve $\overline{P}(t)$ are generated. The second equation of the system (4) describes a non-linear bundle (T_1) of hyperplanes T_1 perpendicular to vector $\overline{P'}_{xyz}(t) = \{x'(t), y'(t), z'(t)\}$ tangent to the curve $\overline{P}_{xyz}(t)$ in its point $(x(t), y(t), z(t))$. At that, each plane T_1 (Fig.4) is parallel to the corresponding normal plane T of the line $\overline{P}_{xyz}(t)$ and is displaced from it, as it follows from the geometric sense of the second equation of the system (4), on distance

$$\rho_T = \frac{|\overline{r}\overline{r}'|}{\sqrt{\left(\overline{P'}_{xyz}(t)\right)^2}} \leq r. \tag{5}$$

From the equation (5) follows the necessary and sufficient condition of existence of real points of channel surface:

$$(x')^2+(y')^2+(z')^2 - r^2 \geq 0 \tag{6}$$

The intersection between the plane T_1 and a sphere S_{xyz} of the set of spheres, according to the condition (6), represents a circle λ. Then, as follows from the pattern depicted on figure 4, the radius r_1 of said circle equals

$$r_1(t) = r(t)\cdot\sqrt{1-\frac{\left(r'(t)\right)^2}{\left(\overline{P'}_{xyz}(t)\right)^2}} \tag{7}$$

It is obvious that the circle λ is a characteristic curve of an envelope Q_{xyz} and is defined as $\lambda=\Phi(t) \cap \Phi'(t) = T_1 \cap S_{xyz}$.

The geometric interpretation of the system of equations (4) in the hyperplane of projections (xyz) allows us to acquire the equation of envelope Q_{xyz} constituting a cyclographic image of the curve $\overline{P}(t) \subset R^4$. Let us consider the acquisition of this surface. Let us accept the following form of parametric equations of the curve $\overline{P}_{xyz}(t)$:

$$x = x(t),\ y = y(t),\ z = z(t),\ t \in G \subset R, \tag{8}$$

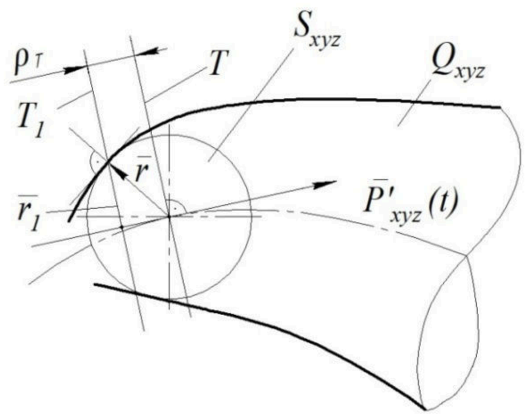

Figure 4: Channel surface formation pattern

Assuming that x, y and z are differentiable functions in the interval of definition of parameter t, it is always possible to shift from parameter t to internal parameter s_1 of the curve $\bar{P}_{xyz}(t)$:

$$ds_1 = |\bar{P}'_{xyz}(t)| \cdot dt = \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} \cdot dt \tag{9}$$

In every point (x,y,z) satisfying the condition $\bar{P}'_{xyz}(t) = \bar{i} \cdot x'(t) + \bar{j} \cdot y'(t) + \bar{k} \cdot z'(t) \neq 0$, i.e. in regular points of curve $\bar{P}'_{xyz}(t)$ that, additionally, do not constitute points of straightening ($\bar{P}'_{xyz}(t)$ and $\bar{P}''_{xyz}(t)$ are not collinear and, at that, $\bar{P}''_{xyz}(t) \neq 0$), the construction of Frenet trihedral with tangent unit vector $\bar{\tau}_1$, main normal unit vector $\bar{\nu}_1$, and binormal unit vector $\bar{\beta}_1$ is possible: $\bar{\tau}_1 = \dfrac{d\bar{P}_{xyz}}{ds_1} = \dot{\bar{P}}_{xyz}$; $\bar{\nu}_1 = \dfrac{\ddot{\bar{P}}_{xyz}}{|\ddot{\bar{P}}_{xyz}|}$; $\bar{\beta}_1 = [\bar{\tau}_1, \bar{\nu}_1]$.

Unit tangent vector $\bar{\tau}_1$ can take the following form:

$$\bar{\tau}_1 = \dot{\bar{P}}_{xyz} = \frac{d\bar{P}_{xyz}}{ds_1} = \bar{i} \cdot \frac{x'}{|\bar{P}'_{xyz}(t)|} + \bar{j} \cdot \frac{y'}{|\bar{P}'_{xyz}(t)|} + \bar{k} \cdot \frac{z'}{|\bar{P}'_{xyz}(t)|} = \bar{i} \cdot \tau_x + \bar{j} \cdot \tau_y + \bar{k} \cdot \tau_z \tag{10}$$

Main normal unit vector $\bar{\nu}_1$ can take the following form:

$$\bar{\nu}_1 = \bar{i} \cdot \frac{1}{k} \cdot \frac{d\tau_x}{ds_1} + \bar{j} \cdot \frac{1}{k} \cdot \frac{d\tau_y}{ds_1} + \bar{k} \cdot \frac{1}{k} \cdot \frac{d\tau_z}{ds_1} = \bar{i} \cdot \nu_x + \bar{j} \cdot \nu_y + \bar{k} \cdot \nu_z . \tag{11}$$

Where $k = |\ddot{\bar{P}}_{xyz}|$ represents curvature of curve $\bar{P}'_{xyz}(t)$ in the current point. Binormal unit vector $\bar{\beta}_1$ is of the following form:

$$\bar{\beta}_1 = \bar{i} \cdot \beta_x + \bar{j} \cdot \beta_y + \bar{k} \cdot \beta_z = [\bar{\tau}_1, \bar{\nu}_1] = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \tau_x & \tau_y & \tau_z \\ \nu_x & \nu_y & \nu_z \end{vmatrix} . \tag{12}$$

In the equation (11) the derivatives $\dfrac{d\tau_x}{ds_1}$, $\dfrac{d\tau_y}{ds_1}$ and $\dfrac{d\tau_z}{ds_1}$ can be acquired by the following technique: $\dfrac{d\tau_x}{ds_1} = \dfrac{d\tau_x}{dt} \dfrac{dt}{ds_1} = \tau'_x \cdot \dot{t}$; $\dfrac{d\tau_y}{ds_1} = \tau'_y \cdot \dot{t}$; $\dfrac{d\tau_z}{ds_1} = \tau'_z \cdot \dot{t}$.

Where $\dot{t} = \dfrac{1}{|\overline{P}'_{xyz}(t)|}$; derivatives τ'_x, τ'_y and τ'_z are acquired by derivation of the corresponding expressions τ'_x, τ'_y and τ'_z from the equation (10) with respect to t. Let us put down the equation of the circle λ of radius $r_1 = r_1(t)$ belonging to the plane T_1 in the coordinate system of moving Frenet trihedral:

$$x_\tau = -\rho_\tau = -\frac{v(t)v'(t)}{\sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}}; \tag{13}$$

$$y_v = r_1(t)\cos\varphi;\ z_\beta = r_1(t)\sin\varphi;\ 0 \le \varphi \le 2\pi,$$

where radius r_1 of the circle λ is acquired from the formula (7).

Let us put down formulas of transformations of the moving coordinate system $Ax_\tau y_v z_\beta$ to fixed coordinate system $Oxyz$ by use of the matrix of transformations $A^{-1} = \begin{vmatrix} \tau_x & v_x & \beta_x \\ \tau_y & v_y & \beta_y \\ \tau_z & v_z & \beta_z \end{vmatrix}$.

The matrix kind of the transformations has the form $D = A^{-1} \cdot B + C$,

$$B = \begin{bmatrix} x_\tau \\ y_v \\ z_v \end{bmatrix},\ C = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ D = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix},$$

where x, y, z represent coordinates of the current point of the curve $\overline{P}'_{xyz}(t)$.

It is possible to shift from the matrix kind of formulas of transformations to expanded coordinate kind:

$$\begin{aligned} x_1 &= x_\tau \cdot \tau_x + y_v \cdot v_x + z_\beta \cdot \beta_x + x, \\ y_1 &= x_\tau \cdot \tau_y + y_v \cdot v_y + z_\beta \cdot \beta_y + y, \\ z_1 &= x_\tau \cdot \tau_z + y_v \cdot v_z + z_\beta \cdot \beta_z + z. \end{aligned} \tag{14}$$

By means of combining (14) with the expressions of coordinates from the formulas (10), (11), (12), and (13), the expanded parametric equations of the envelope surface Q_{xyz} are acquired.

3 Results of experiment

3.1 Mathematical numerical model of the cyclographic image of the curve

Suppose we are given a parametric rational curve $\overline{P}(t) \subset R^4$ with the equations:

$$\overline{P}(t) = \{x(t), y(t), z(t), v(t)\} = \{t^4, 2t^3, 3t^2, 24t\},\ 1 \le t \le 3{,}5. \tag{15}$$

Its image $\overline{P}_{xyz}(t)$ in hyperplane R^3_{xyz} acquired by the use of orthogonal projection is defined by the equations

$$\overline{P}_{xyz}(t) = \{x(t), y(t), z(t)\} = \{t^4, 2t^3, 3t^2\}. \tag{16}$$

Tangent vector $\overline{P}'_{xyz}(t)$ of the image is of the following form:

$$\overline{P}'_{xyz}(t) = \{4t^4, 6t^3, 6t^2\}. \tag{17}$$

Derivative $\overline{P}''_{xyz}(t)$ is of the following form:

$$\overline{P}''_{xyz}(t) = \{12t^2, 12t, 6\}. \tag{18}$$

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Given (15) and corresponding equations (16), (17), and (18), it is required to acquire the cyclographic image of the curve $\overline{P}(t) \subset R^4$ in space R^3_{xyz}. Let us acquire unit vectors $\overline{\tau}_1$, $\overline{\nu}_1$, $\overline{\beta}_1$ of Frenet trihedral. Tangent unit vector of the curve $\overline{P}_{xyz}(t)$ is acquired as follows:

$$\overline{\tau}_1 = \frac{\overline{P}'_{xyz}(t)}{|\overline{P}'_{xyz}(t)|} = \left\{ \frac{2t^2}{\sqrt{4t^4+9t^2+9}}, \frac{3t}{\sqrt{4t^4+9t^2+9}}, \frac{3}{\sqrt{4t^4+9t^2+9}} \right\} \tag{19}$$

Binormal unit vector of the curve $\overline{P}_{xyz}(t)$ is acquired from the formula:

$$\overline{\beta}_1 = \frac{\left[\overline{P}'_{xyz}(t), \overline{P}''_{xyz}(t)\right]}{\left|\left[\overline{P}'_{xyz}(t), \overline{P}''_{xyz}(t)\right]\right|} \tag{20}$$

where the expressions in numerator and denominator are acquired the following way:

$$\left[\overline{P}'_{xyz}(t), \overline{P}''_{xyz}(t)\right] = \{-36t^2, 48t^2, -24t^2\}. \tag{21}$$

$$\left|\left[\overline{P}'_{xyz}(t), \overline{P}''_{xyz}(t)\right]\right| = 12t^2 \cdot \sqrt{4t^4+16t^2+9}. \tag{22}$$

By placing the expressions (21) and (22) into the equation (20), we acquire:

$$\overline{\beta}_1 = \left\{ -\frac{3}{\sqrt{4t^4+16t^2+9}}, \frac{4t}{\sqrt{4t^4+16t^2+9}}, -\frac{2t^2}{\sqrt{4t^4+16t^2+9}} \right\}. \tag{23}$$

Let us introduce the following designations: $\sqrt{4t^4+9t^2+9} = N$, $\sqrt{4t^4+16t^2+9} = M$.
Taking into the account these designations, let us acquire main normal unit vector $\overline{\nu}_1$ of the curve $\overline{P}_{xyz}(t)$:

$$\overline{\nu}_1 = [\overline{\beta}_1, \overline{\tau}_1] = \left\{ \frac{6t\cdot(2+t^2)}{M\cdot N}, \frac{9-4t^4}{M\cdot N}, \frac{t\cdot(9+8t^2)}{M\cdot N} \right\}. \tag{24}$$

The acquired equations (19), (23), and (24) define the projective components of unit vectors comprising the Frenet trihedral of the curve $\overline{P}_{xyz}(t)$. The equations (13) define the projective components of radius-vector of the curve $\overline{P}_{xyz}(t)$. By taking into account the expressions for these components in equation (14), we acquire the channel surface equation

$$\begin{aligned} x_1(t,\varphi) &= \frac{-72t\cdot L\cdot\sin\varphi}{M} - \frac{576t^3}{N^2} + \frac{144t^2\cdot(t^2+2)\cdot L\cdot\cos\varphi}{M\cdot N} + t^4, \\ y_1(t,\varphi) &= \frac{96t^2\cdot L\cdot\sin\varphi}{M} - \frac{864t^2}{N^2} + \frac{24t\cdot(-4t^4+9)\cdot L\cdot\cos\varphi}{M\cdot N} + 2t^3, \\ z_1(t,\varphi) &= \frac{-48t^2\cdot L\cdot\sin\varphi}{M} - \frac{864t}{N^2} - \frac{24t^2\cdot(8t^2+9)\cdot L\cdot\cos\varphi}{M\cdot N} + 3t^2, \\ & 1 \le t \le 3{,}5;\ 0 \le \varphi \le 2\pi, \end{aligned} \tag{25}$$

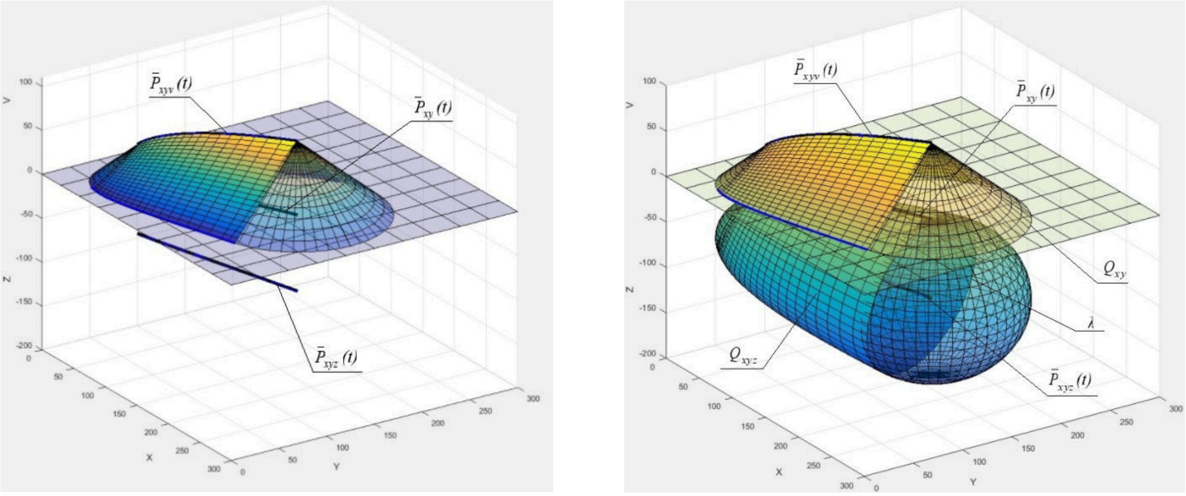
where

$$L = \sqrt{1 - 144\frac{1}{N\cdot t^2}}.$$

3.2 Results of computer implementation of cyclographic modeling

Figure 5 shows the sequence of execution of cyclographic modeling of spatial curve $\overline{P}(t) \subset R^4$ on Naumovich hyperdrawing by the computer algebra system MATLAB.

Figure 6 shows the result of cyclographic modeling of the curve $\overline{P}(t) \subset R^4$ on Naumovich hyperdrawing by using shaping functions: EQUATION CURVE, REVOLVE, LOFT in the CAD-system Autodesk Inventor.



a) b)

Figure 5: Cyclographic modeling of spatial curve on Naumovich hyperdrawing: a) formation of cyclographic images in the hyperplane projection (*xyv*); b) formation of cyclographic images in the hyperplane projection (*xyz*) and a full image of a cyclographic model of a curve

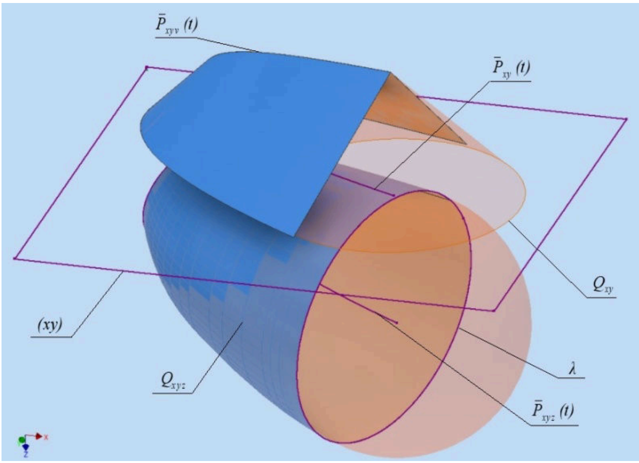


Figure 6: Cyclographic modeling of spatial curve on Naumovich hyperdrawing in the environment of the CAD-system Autodesk Inventor

4 Conclusions

An approach to cyclographic modeling of space R^4, based on the Naumovich hyperdrawing and the possibility of computer implementation of this hyperdrawing, allow us to achieve several positive effects:

1. The mathematical model of the cyclographic mapping of the space R^4 receives an accurate and complete geometrical interpretation when it is implemented on the Naumovich hyperdrawing.

2. Geometric interpretation allows us to visualize the process of sequential cyclographic modeling of objects of space R^4 in a virtual electronic 3D space. This is confirmed by the implementation of a cyclographic model of the curve of the space R^4 in the space R^3.

The performed experiments allow us to conclude that the mathematical description of the cyclographic model and its visualization are more compatible and complement each other in computer algebra programs than in CAD systems.

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SYMULACJA CYCLOGRAFII PRZESTRZENNEJ NA RYSUNKU 4D NAUMOWICZA

W niniejszej pracy przedstawiono możliwości zastosowania trójwymiarowego rysunku przestrzeni R^4, zaproponowanego przez N.V. Naumowicza w 1958 roku, w dziedzinie cyklicznego modelowania obiektów nieliniowych: krzywych i powierzchni przestrzeni R^4, których modele są szeroko stosowane w nowoczesnych systemach CAD/CAM i Computer Aided Geometric Design (CAGD). Możliwość zastosowania cyklografii przestrzennej na rysunku czterowymiarowym Naumowicza, jako obrazu cyklograficznego krzywej czterowymiarowej, pokazano na przykładzie powierzchni kanałowej.