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Integrated impact model on critical infrastructure safety related to its operation process and climate-weather change process

Keywords

Impact model, climate-weather process, operation process, safety, model, Critical infrastructure

Abstract

The main aim of the paper is to define the operation and climate change influence on the safety of a critical infrastructure considered as a complex system in its operating environment. As the result, a general safety analytical model of a complex technical system under the influence of the operation process related to climate-weather change process is proposed. Further, the conditional safety functions at the operation process related to climate-weather change process, the unconditional safety function and the risk function of the complex system at changing in time operation and climate-weather conditions are defined. Moreover, the mean lifetime up to the exceeding a critical safety state, the moment when the risk function value exceeds the acceptable safety level, the intensities of ageing of the critical infrastructure and its components and the coefficients of the operation and climate-weather impact on the critical infrastructure and its components intensities of ageing are proposed as the other significant safety indices for any critical infrastructure.

1. Introduction

The paper presents the operation and climate change influence on the safety of a critical infrastructure defined as a complex system in its operating environment that in the case of its degradation have significant destructive influence on all aspects of human activities like: the health, safety and security, economics and social conditions. A general safety analytical model of a complex technical system under the influence of the operation process related to climate-weather change process is proposed. It is the integrated model of complex technical system safety, linking its multistate safety model and the model of its operation process related to climate-weather change process at its operating area, considering variable at the different climate-weather states impacted by them the system safety structures and its components safety parameters.The conditional safety functions at the operation process related to climate-weather change process particular states, the unconditional safety function and the risk function of the complex system at changing in time operation and climate-weather conditions are defined. Other, practically significant, critical infrastructure safety indices introduced in the paper are its mean lifetime up to the exceeding a critical safety state, the moment when its risk function value exceeds the acceptable safety level, the intensities of ageing of the critical infrastructure and its components and the coefficients of the operation and climate-weather impact on the critical infrastructure and its components intensities of ageing. These safety indices are defined in general for any critical infrastructure and determined particularly for the port oil piping transportation system and the maritime ferry technical system considering varying in time

their safety structures and components safety parameters influenced by changing in time their operation and climate-weather conditions at their operating areas.
The time dependent interactions between the operation process related to the climate-weather change process states varying at the system operating area and the system safety structure and its components safety states changing are evident features of most real technical systems, including critical infrastructures as well. The common critical infrastructure safety and the operation process related to climate-weather change process at its operating area analysis is of great value in the industrial practice because of operation impacts and negative impacts of extreme weather hazards on the critical infrastructure safety. The convenient tools for analyzing this problem are the multistate critical infrastructures safety modelling [Kołowrocki, Soszyńska-Budny, 2011; Xue, 1985; Xue, Yang, 1995a-b] commonly used with the semi-Markov modeling [Ferreira, Pacheco, 2007; Glynn, Hass, 2006; Grabski, 2014; Kołowrocki 2005; Limnios, Oprisan, 2005; Mercier 2008] of the operation processes related to climate-weather change processes at their operating areas, leading to the construction the joint general safety models of the critical infrastructures under the influence of their operation processes related to climate-weather change processes at their operating areas.

2. System operation process related to climate-weather variable conditions

2.1. System operation process

We assume that the critical infrastructure during its operation process is taking $v, v \in N$, different operation states $z_1, z_2, \ldots, z_v$. Further, we define the critical infrastructure operation process $Z(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{z_1, z_2, \ldots, z_v\}$. Moreover, we assume that the critical infrastructure operation process $Z(t)$ is a semi-Markov process [Grabski, 2002], [Limnios, 2005], [Mercier, 2008], [Soszyńska, 2007], [Kołowrocki, Soszyńska-Budny, 2011] with the conditional sojourn times θ_{bl} at the operation states z_b when its next operation state is z_l, $b, l = 1,2,\ldots,v,\ b \neq l.$ Under these assumptions, the critical infrastructure operation process may be described by:

- the vector $[p_b(0)]_{1\times v}$ of the initial probabilities $p_b(0) = P(Z(0) = z_b)$, $b = 1,2,\ldots,v,$ of the critical infrastructure operation process $Z(t)$ staying at particular operation states at the moment $t = 0$;
- the matrix $[p_{bl}]_{v\times v}$ of probabilities p_{bl}, $b, l = 1,2,\ldots,v,$ of the critical infrastructure operation process $Z(t)$ transitions between the operation states z_b and z_l;
- the matrix $[H_{bl}(t)]_{v\times v}$ of conditional distribution functions $H_{bl}(t) = P(\theta_{bl} < t)$, $b, l = 1,2,\ldots,v,$ of the critical infrastructure operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states.

As the mean values of the conditional sojourn times θ_{bl} are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t) = \int_0^\infty t h_{bl}(t) dt,$$
$$b, l = 1,2,\ldots,v,\ b \neq l, \tag{1}$$

Thwn from the formula for total probability, it follows that the unconditional distribution functions of the sojourn times $\theta_{bl}, b, l = 1,2,\ldots,v,$ of the system operation process $Z(t)$ at the operation states z_b, $b = 1,2,\ldots,v,$ are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t),\ b = 1,2,\ldots,v. \tag{2}$$

Hence, the mean values $E[\theta_b]$ of the system operation process $Z(t)$ unconditional sojourn times θ_b, $b = 1,2,\ldots,v,$ at the operation states are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl}\ ,\ b = 1,2,\ldots,v, \tag{3}$$

where M_{bl} are defined by the formula (1) in a case of any distribution of sojourn times θ_{bl} and by the formulae (3.2)-(3.8) in the cases of particular defined respectively by (2.5)-(2.11) [EU-CIRCLE Report D2.1-GMU2, 2016], distributions of these sojourn times.
The limit values of the system operation process $Z(t)$ transient probabilities at the particular operation states

$$p_b(t) = P(Z(t) = z_b)\ ,\ t \in <0,+\infty),\ b = 1,2,\ldots,v,$$

are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \ b = 1,2,...,v, \quad (4)$$

where M_b, $b = 1,2,...,v$, are given by (3), while the steady probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \quad (5)$$

In the case of a periodic system operation process, the limit transient probabilities p_b, $b = 1,2,...,v$, at the operation states defined by (4), are the long term proportions of the system operation process $Z(t)$ sojourn times at the particular operation states z_b, $b = 1,2,...,v$.

Other interesting characteristics of the system operation process $Z(t)$ possible to obtain are its total sojourn times $\hat{\theta}_b$ at the particular operation states z_b, $b = 1,2,...,v$, during the fixed system opetation time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the system operation process total sojourn times $\hat{\theta}_b$ at the particular operation states z_b, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by

$$\hat{M}_b = E[\hat{\theta}_b] = p_b \theta, \ b = 1,2,...,v, \quad (6)$$

where p_b are given by (4).

2.2. Climate-weather change process

To model the climate-weather change process for the critical infrastructure operating area we assume that the climate-weather in this area is taking w, $w \in N$, different climate-weather states $c_1, c_2, \ldots, c_w$. Further, we define the climate-weather change process $C(t)$, $t \in <0, +\infty)$, with discrete operation states from the set $\{c_1, c_2, \ldots, c_w\}$. Assuming that the climate-weather change process $C(t)$ is a semi-Markov process it can be described by [Kołowrocki K., Soszyńska-Budny J., Torbicki M., Critical Infrastructure Operation Area Climate-Weather Change Process (C-WCP) Including Extreme Weather Hazards (EWH), C-WCP Model, 2016]:

- the vector $[q_b(0)]_{1Xw}$ of the initial probabilities $q_b(0) = P(C(0) = c_b)$, $b = 1,2,\ldots, w$, of the climate-weather change process $C(t)$ staying at particular climate-weather states c_b at the moment $t = 0$;
- the matrix $[q_{bl}]_{wxw}$ of the probabilities of transitions q_{bl}, b, $l = 1,2,\ldots, w$, $b \neq l$, of the climate-weather change process $C(t)$ from the climate-weather states c_b to c_l;
- the matrix $[C_{bl}(t)]_{wxw}$ of the conditional distribution functions $C_{bl}(t) = P(C_{bl} < t)$, b, $l = 1,2,\ldots, w$, of the conditional sojourn times C_{bl} at the climate-weather states c_b when its next climate-weather state is c_l, b, $l = 1,2,\ldots, w$, $b \neq l$.

Assuming that we have identified the above parameters of the climate-weather change process semi-Markov model, we can predict this process basic characteristics.

The mean values of the conditional sojourn times C_{bl}, are given by [Kołowrocki K., Soszyńska-Budny J., Torbicki M., Critical Infrastructure Operation Area Climate-Weather Change Process (C-WCP) Including Extreme Weather Hazards (EWH), C-WCP Model, 2016]

$$N_{bl} = E[C_{bl}] = \int_0^\infty t dC_{bl}(t) = \int_0^\infty t c_{bl}(t) dt,$$
$$b,l = 1,2,...,w. \quad (7)$$

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times C_b, $b = 1,2,...,w$, of the climate-weather change process $C(t)$ at the climate-weather states c_b $b = 1,2,...,w$, are given by [EU-CIRCLE Report D2.1-GMU3, 2016]

$$C_b(t) = \sum_{l=1}^{v} q_{bl} C_{bl}(t), \ b = 1,2,...,w, \quad (8)$$

Hence, the mean values $E[C_b]$ of the climate-weather change process $C(t)$ unconditional sojourn times C_b, $b = 1,2,...,w$, at the climate-weather states are given by

$$N_b = E[C_b] = \sum_{l=1}^{v} q_{bl} N_{bl}, \ b = 1,2,...,w, \quad (9)$$

where N_{bl} are defined by the formula (7) in a case of any distribution of sojourn times C_{bl} and by the formulae (3.2)-(3.8) given in [EU-CIRCLE Report D2.1-GMU3, 2016] in the cases of particular defined respectively by (2.5)-(2.11) in [EU-CIRCLE Report D2.1-GMU2, 2016], distributions of these sojourn times.

The limit values of the climate-weather change process $C(t)$ transient probabilities at the particular operation states

$$q_b(t) = P(C(t) = c_b),$$

$t \in <0,+\infty),\ b = 1,2,...,w,$ (10)

are given by [Kołowrocki, Soszyńska-Budny, 2011], [EU-CIRCLE Report D2.1-GMU3, 2016]

$$q_b = \lim_{t\to\infty} q_b(t) = \frac{\pi_b N_b}{\sum_{l=1}^{v} \pi_l N_l},\ b = 1,2,...,w, \qquad (11)$$

where N_b, $b = 1,2,...,w$, are given by (9), while the steady probabilities π_b of the vector $[\pi_b]_{1xw}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][q_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \qquad (12)$$

In the case of a periodic climate-weather change process, the limit transient probabilities q_b, $b = 1,2,...,w$, at the climate-weather states defined by (11), are the long term proportions of the climate-weather change process $C(t)$ sojourn times at the particular climate-weather states C_b, $b = 1,2,...,w$.

Other interesting characteristics of the system climate-weather change process $C(t)$ possible to obtain are its total sojourn times $\hat{C}_b$ at the particular climate-weather states c_b, $b = 1,2,...,w$, during the fixed time. It is well known, [Kołowrocki, Soszyńska-Budny, 2011], [EU-CIRCLE Report D2.1-GMU3, 2016] that the climate-weather change process total sojourn times $\hat{C}_b$ at the particular climate-weather states c_b for sufficiently large time C have approximately normal distributions with the expected value given by

$$\hat{N}[\hat{C}_b] = q_b C,\ b = 1,2,...,w, \qquad (13)$$

where q_b are given by (11).

2.3. Critical infrastructure operation process related to climate-weather change

Assuming that we have identified the unknown parameters of the critical infrastructure operation process related to climate-weather change $ZC(t)$, $t \in <0,+\infty)$, that can take $vw, v, w \in N$, different operation states $zc_{11}, zc_{12}, \ldots, zc_{vw}$, described by :

- the vector $[pq_{ij}(0)]_{1xvw}$ of initial probabilities of the critical infrastructure operation process related to climate-weather change $ZC(t)$ staying at the initial moment $t = 0$ at the operation and climate-weather states zc_{ij}, $i = 1,2,...,v \in N$, $j = 1,2,...,w$;
- the matrix $[pq_{ij\,kl}]_{vwxvw}$ of the probabilities of transitions of the critical infrastructure operation process related to climate-weather change $ZC(t)$ between the operation states zc_{ij} and zc_{kl}, $i = 1,2,...,v$, $j = 1,2,...,w$, $k = 1,2,...,v$, $l = 1,2,...,w$;
- the matrix $[HC_{ij\,kl}(t)]_{vwxvw}$ of the matrix of conditional distribution functions of the critical infrastructure operation process related to climate-weather change $ZC(t)$ conditional sojourn times $\theta C_{ij\,kl}$, $i = 1,2,...,v$, $j = 1,2,...,w$, $k = 1,2,...,v$, $l = 1,2,...,w$, at the operation state zc_{ik}, $i = 1,2,...,v$, $k = 1,2,...,v$, when the next operation state is zc_{jl}, $j = 1,2,...,w$, $l = 1,2,...,w$,

we can predict this process basic characteristics.

2.3.1. Critical infrastructure operation process related to climate-weather change characteristics - independent critical infrastructure operation process and climate-weather change process

The mean values of the conditional sojourn times $\theta C_{ij\,kl}$, $i = 1,2,...,v$, $j = 1,2,...,w$, $k = 1,2,...,v$, $l = 1,2,...,w$, at the operation state zc_{ij}, $i = 1,2,...,v$, $k = 1,2,...,v$, when the next operation state is zc_{kl}, $j = 1,2,...,w$, $l = 1,2,...,w$, are defined by [Kołowrocki, Soszyńska-Budny, 2011]

$$MN_{ij\,kl} = E[\theta C_{ij\,kl}] = \int_0^\infty t dHC_{ij\,kl}(t)dt = \int_0^\infty thc_{ij\,kl}(t)dt,$$
$$i = 1,2,...,v,\ j = 1,2,...,w,$$
$$k = 1,2,...,v,\ l = 1,2,...,w. \qquad (14)$$

In the case when the processess $Z(t)$ and $C(t)$ are independent, the expessions (14) tasks the form

$$MN_{ij\,kl} = E[\theta C_{ij\,kl}] = \int_0^\infty t[h_{ik}(t)C_{jl}(t) + H_{ik}(t)c_{jl}(t)]dt,$$
$$i = 1,2,...,v,\ j = 1,2,...,w,$$
$$k = 1,2,...,v,\ l = 1,2,...,w. \qquad (15)$$

Since from the formula for total probability, it follows that the unconditional distribution functions of the conditional sojourn times θC_{ij}, of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states state zc_{ij}, $i = 1,2,...,v$, $j = 1,2,...,w$, are given by

$$HC_{ij}(t) = \sum_{k=1}^{v}\sum_{l=1}^{w} p_{ij\,kl} HC_{ij\,kl}(t),\ t \in < 0,+\infty),$$
$$i = 1,2,...,v,\ \ j = 1,2,...,w, \tag{16}$$

In the case when the processess $Z(t)$ and $C(t)$ are independent, according to (6.11) [EU-CIRCLE Report D2.1-GMU2, 2016] and (6.18) [EU-CIRCLE Report D2.1-GMU3, 2016] the expessions (16) tasks the form

$$HC_{ij}(t) = \sum_{k=1}^{v}\sum_{l=1}^{w} p_{ik} q_{jl} H_{ik}(t) C_{jl}(t),\ \ t \in < 0,+\infty),$$
$$i = 1,2,...,v,\ \ j = 1,2,...,w, \tag{17}$$

From (16) it follows that the mean values $E[\theta C_{ij}]$ of the unconditional distribution functions of the conditional sojourn times θC_{ij}, of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states zc_{ij}, $i = 1,2,...,v$, $j = 1,2,...,w$, are given by

$$MN_{ij} = E[\theta C_{ij}] = \sum_{k=1}^{v}\sum_{l=1}^{w} p_{ij\,kl} MN_{ij\,kl},\ i = 1,2,...,v,$$
$$j = 1,2,...,w, \tag{18}$$

where $MN_{ij\,kl}$ are given by the formula (14).

In the case when the processess $Z(t)$ and $C(t)$ are independent, considering (17) and (6.18) [EU-CIRCLE Report D2.1-GMU3, 2016] the expession (18) tasks the form

$$MN_{ij} = E[\theta C_{ij}] = \sum_{k=1}^{v}\sum_{l=1}^{w} p_{ik} q_{jl} MN_{ij\,kl},\ \ i = 1,2,...,v,$$
$$j = 1,2,...,w, \tag{19}$$

where $MN_{ij\,kl}$ are given by the formula (15).

The transient probabilities of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states zc_{ij}, $i = 1,2,...,v$, $j = 1,2,...,w$, can be defined by

$$pq_{ij}(t) = P(ZC(t) = zc_{ij}),\ t \in < 0,+\infty),$$
$$i = 1,2,...,v,\ \ j = 1,2,...,w. \tag{20}$$

In the case when the processess $Z(t)$ and $C(t)$ are independent the expession (20) for the transient probabilities can be expressed in the following way

$$pq_{ij}(t) = P(ZC(t) = zc_{ij}) = P(Z(t) = z_i \cap C(t) = c_j)$$
$$= P(Z(t) = z_i) \cdot P(C(t) = c_j) = p_i(t) \cdot q_j(t),$$
$$t \in < 0,+\infty),\ i = 1,2,...,v,\ \ j = 1,2,...,w, \tag{21}$$

where

$$p_i(t) = P(Z(t) = z_i),\ t \in < 0,+\infty),\ i = 1,2,...,v, \tag{22}$$

are the transient probabilities of the operation process $Z(t)$ defined in Chapter 2 and

$$q_j(t) = P(C(t) = c_j),\ t \in < 0,+\infty),$$
$$j = 1,2,...,w, \tag{23}$$

are the transient probabilities of the climate-weather change process $C(t)$ defined in Chapter 4.

The limit values of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states zc_{ij}, $i = 1,2,...,v$, $j = 1,2,...,w$, can be found from [Kołowrocki, Soszyńska-Budny, 2011]

$$pq_{ij} = \lim_{t\to\infty} \frac{\pi_{ij} MN_{ij}}{\sum_{i=1}^{v}\sum_{j=1}^{w} \pi_{ij} MN_{ij}},\ i = 1,2,...,v,\ \ j = 1,2,...,w, \tag{24}$$

where MN_{ij}, $i = 1,2,...,v$, $j = 1,2,...,w$, are given by (19), while the steady probabilities π_{ij}, $i = 1,2,...,v$, $j = 1,2,...,w$, of the vector $[\pi_{ij}]_{1xvw}$ satisfy the system of equations

$$\begin{cases} [\pi_{ij}][pq_{ij\,kl}] = [\pi_{ij}] \\ \sum_{i-=1}^{v}\sum_{j=1}^{w} \pi_{ij} = 1, \end{cases} \tag{25}$$

where $pq_{ij\,kl}$, $i = 1,2,...,v$, $j = 1,2,...,w$, $k = 1,2,...,v$, $l = 1,2,...,w$, are given by (20).

In the case of a periodic system operation process, the limit transient probabilities pq_{ij}, $i = 1,2,...,v$, $j = 1,2,...,w$, at the operation states given by (24), are the long term proportions of the critical infrastructure operation process $ZC_{ij}(t)$ sojourn times at the particular operation states zc_{ij}, $i = 1,2,...,v$, $j = 1,2,...,w$.

Other interesting characteristics of the critical infrastructure operation process $ZC_{ij}(t)$ possible to obtain are its total sojourn times $\hat{\theta}\hat{C}_{ij}$, $i = 1,2,...,v$, $j = 1,2,...,w$, at the particular operation states zc_{ij}, $i = 1,2,...,v$, $j = 1,2,...,w$, during the fixed system opetation time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the system operation process total sojourn times $\hat{\theta}\hat{C}_{ij}$, at the particular operation states zc_{ij}, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by

$$\hat{M}\hat{N}_{ij} = E[\hat{\theta}\hat{C}_{ij}] = pq_{ij}\theta, \ i = 1,2,...,\nu, \ j = 1,2,...,w, \quad (26)$$

where pq_{ij}, $i = 1,2,...,\nu$, $j = 1,2,...,w$, are given by (24).

2.3.2. Critical infrastructure operation process related to climate-weather change characteristics - dependent critical infrastructure operation process and climate-weather change process

The mean values of the conditional sojourn times $\theta C_{ij\,kl}$, $i = 1,2,...,\nu$, $j = 1,2,...,w$, $k = 1,2,...,\nu$, $l = 1,2,...,w$, at the operation state zc_{ik}, $i = 1,2,...,\nu$, $k = 1,2,...,\nu$, when the next operation state is zc_{jl}, $j = 1,2,...,w$, $l = 1,2,...,w$, are defined by [Kołowrocki, Soszyńska-Budny, 2011]

$$MN_{ij\,kl} = E[\theta C_{ij\,kl}] = \int_0^\infty t dHC_{ij\,kl}(t)dt = \int_0^\infty t hc_{ij\,kl}(t)dt, \ i = 1,2,...,\nu, \ j = 1,2,...,w, \ k = 1,2,...,\nu, \ l = 1,2,...,w. \quad (27)$$

Since from the formula for total probability, it follows that the unconditional distribution functions of the conditional sojourn times θC_{ij}, of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states state zc_{ij}, $i = 1,2,...,\nu$, $j = 1,2,...,w$, are given by

$$HC_{ij}(t) = \sum_{k=1}^{\nu}\sum_{l=1}^{w} p_{ij\,kl} HC_{ij\,kl}(t), \ t \in <0,+\infty), \ i = 1,2,...,\nu, \ j = 1,2,...,w, \quad (28)$$

Hence, the mean values $E[\theta C_{ij}]$ of the unconditional distribution functions of the conditional sojourn times θC_{ij}, of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states zc_{ij}, $i = 1,2,...,\nu$, $j = 1,2,...,w$, are given by

$$MN_{ij} = E[\theta C_{ij}] = \sum_{k=1}^{\nu}\sum_{l=1}^{w} p_{ij\,kl} MN_{ij\,kl}, \ i = 1,2,...,\nu, \ j = 1,2,...,w, \quad (29)$$

where $MN_{ij\,kl}$ are defined by the formula (27).

The transient probabilities of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states zc_{ij}, $i = 1,2,...,\nu$, $j = 1,2,...,w$, can be defined by

$$pq_{ij}(t) = P(ZC(t) = zc_{ij}), \ t \in <0,+\infty), \ i = 1,2,...,\nu, \ j = 1,2,...,w. \quad (30)$$

In the case when the processess $Z(t)$ and $C(t)$ are dependent the transient probabilities can be expresed either by

$$pq_{ij}(t) = P(ZC(t) = zc_{ij}) = P(Z(t) = z_i \cap C(t) = c_j) = P(Z(t) = z_i) \cdot P(C(t) = c_j \mid Z(t) = z_i) = p_i(t) \cdot q_{j/i}(t), \ t \in <0,+\infty), \ i = 1,2,...,\nu, \ j = 1,2,...,w, \quad (31)$$

where

$$p_i(t) = P(Z(t) = z_i), \ t \in <0,+\infty), \ i = 1,2,...,\nu, \quad (32)$$

are transient probabilities of the operation process $Z(t)$ defined in Chapter 2 and

$$q_{j/i}(t) = P(C(t) = c_j \mid Z(t) = z_i), \ t \in <0,+\infty), \ i = 1,2,...,\nu, \ j = 1,2,...,w, \quad (33)$$

are conditional transient probabilities of the climate-weather change process $C(t)$ defined in Chapter 4 in case they are not conditional
or by

$$pq_{ij}(t) = P(ZC(t) = zc_{ij}) = P(Z(t) = z_i \cap C(t) = c_j) = P(C(t) = C_j) \cdot P(Z(t) = z_i \mid C(t) = c_j) = q_j(t) \cdot p_{i/j}(t), \ t \in <0,+\infty), \ i = 1,2,...,\nu, \ j = 1,2,...,w, \quad (34)$$

where

$$q_j(t) = P(C(t) = c_j),\ t \in <0,+\infty),\ \ j = 1,2,\dots,w, \quad (35)$$

are transient probabilities of the operation process $C(t)$ defined in Chapter 4 and

$$p_{i/j}(t) = P(Z(t) = z_i \mid C(t) = c_j),$$
$$t \in <0,+\infty),\ i = 1,2,\dots,v,\ \ j = 1,2,\dots,w, \quad (36)$$

are conditional transient probabilities of the climate-weather change process $Z(t)$ defined in Chapter 2 in case they are not conditional.

The limit values of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states zc_{ij}, $i = 1,2,\dots,v$, $j = 1,2,\dots,w$, can be found from [Kołowrocki, Soszyńska-Budny, 2011]

$$pq_{ij} = \lim_{t \to \infty} \frac{\pi_{ij} MN_{ij}}{\sum_{i=1}^{v}\sum_{j=1}^{w} \pi_{ij} MN_{ij}},\ i = 1,2,\dots,v,\ \ j = 1,2,\dots,w, \quad (37)$$

where MN_{ij}, $i = 1,2,\dots,v$, $j = 1,2,\dots,w$, are given by (29), while the steady probabilities π_{ij}, $i = 1,2,\dots,v$, $j = 1,2,\dots,w$, of the vector $[\pi_{ij}]_{1xvw}$ satisfy the system of equations

$$\begin{cases} [\pi_{ij}][pq_{ij\,kl}] = [\pi_{ij}] \\ \sum_{i-=1}^{v}\sum_{j=1}^{w} \pi_{ij} = 1. \end{cases} \quad (38)$$

In the case of a periodic system operation process, the limit transient probabilities pq_{ij}, $i = 1,2,\dots,v$, $j = 1,2,\dots,w$, at the operation states given by (37), are the long term proportions of the critical infrastructure operation process $ZC_{ij}(t)$ sojourn times at the particular operation states zc_{ij}, $i = 1,2,\dots,v$, $j = 1,2,\dots,w$.

Other interesting characteristics of the critical infrastructure operation process $ZC_{ij}(t)$ possible to obtain are its total sojourn times $\hat{\theta}\hat{C}_{ij}$, $i = 1,2,\dots,v$, $j = 1,2,\dots,w$, at the particular operation states zc_{ij}, $i = 1,2,\dots,v$, $j = 1,2,\dots,w$, during the fixed system opetation time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the system operation process total sojourn times $\hat{\theta}\hat{C}_{ij}$, at the particular operation states zc_{ij}, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by

$$\hat{M}\hat{N}_{ij} = E[\hat{\theta}\hat{C}_{ij}] = pq_{ij}\theta,$$
$$i = 1,2,\dots,v,\ \ j = 1,2,\dots,w, \quad (39)$$

where pq_{ij}, $i = 1,2,\dots,v$, $j = 1,2,\dots,w$, are given by (37).

3. Safety of multistate systems at variable operation conditions related to climate-weather change

We assume that the changes of the operation process related to climate-weather change $ZC(t)$, $t \in <0,+\infty)$, states $zc_{11}, zc_{12}, \dots, zc_{vw}$, have an influence on the system multistate components E_i, $i = 1,2,\dots,n$, safety. Consequently, we denote the system multistate component E_i, $i = 1,2,\dots,n$, conditional lifetime in the safety state subset $\{u, u+1,\dots,z\}$ while the operation process related to climate-weather change $ZC(t)$ is at the state zc_{bl}, $b = 1,2,\dots,v$, $l = 1,2,\dots,w$, by $T'''^{(bl)}_i(u)$ and its conditional safety function by the vector

$$[S'''_i(t,\cdot)]^{(bl)} = [1,\ [S'''_i(t,1)]^{(bl)}, \dots,$$
$$[S'''_i(t,z)]^{(bl)}],\ t \in <0,\infty),$$
$$b = 1,2,\dots,v,\ \ l = 1,2,\dots,w,\ \ i = 1,2,\dots,n,$$
$$(40)$$

with the coordinates defined by

$$[S'''_i(t,u)]^{(bl)} = P(T'''^{(bl)}_i(u) > t \mid ZC(t) = zc_{bl})$$
$$(41)$$

for $t \in <0,\infty)$, $u = 1,2,\dots,z$, $b = 1,2,\dots,v$, $l = 1,2,\dots,w$.

The safety function $[S'''_i(t,u)]^{(bl)}$ is the conditional probability that the component E_i lifetime $T''^{(b)}_i(u)$ in the safety state subset $\{u, u+1,\dots,z\}$ is greater than t, while the climate-weather change process $ZC(t)$ at the system operating area is at the state zc_{bl}, $b = 1,2,\dots,v$, $l = 1,2,\dots,w$.

In the case, the system components E_i $i = 1,2,\dots,n$, at the climate-weather change process $ZC(t)$ at the system operating area states zc_{bl}, $b = 1,2,\dots,v$, $l = 1,2,\dots,w$, have the exponential safety functions, the coordinates of the vector (40) are given by

$$[S'''_i(t,u)]^{(bl)} = P(T'''^{(bl)}_i(u) > t \mid ZC(t) = zc_{bl})$$
$$= \exp[-[\lambda'''_i(u)]^{(bl)} t],\ t \in <0,\infty),$$

$b = 1,2,...,\nu,\ l = 1,2,...,w,\ i = 1,2,...,n.$ (42)

Existing in (3.3) the intensities of ageing of the system components E_i $i = 1,2,...,n,$ (the intensities of the system components E_i $i = 1,2,...,n,$ departure from the safety state subset $\{u,u+1,...,z\}$) at the operation process related to climate-weather change process $ZC(t)$ states $zc_{bl}, b = 1,2,...,\nu,$ $l = 1,2,...,w,$ i.e. the coordinates of the vector

$$[\lambda'''_i(\cdot)]^{(bl)} = [0, [\lambda'''_i(1)]^{(bl)}, \ldots, [\lambda'''_i(z)]^{(bl)}],$$
$$t \in <0,+\infty),\ b = 1,2,...,\nu,\ l = 1,2,...,w,$$
$$i = 1,2,...,n, \quad (43)$$

are given by

$$[\lambda'''_i(u)]^{(bl)} = \rho'''^{(bl)}_i(u) \cdot \lambda_i(u),\ u = 1,2,...,z,$$
$$b = 1,2,...,\nu,\ l = 1,2,...,w,\ i = 1,2,...,n, \quad (44)$$

where $\lambda_i(u)$ are the intensities of ageing of the system components E_i, $i = 1,2,...,n,$ (the intensities of the system components E_i, $i = 1,2,...,n,$ departure from the safety state subset $\{u,u+1,...,z\}$) without operation and climate-weather change impact, i.e. the coordinate of the vector

$$\lambda_i(\cdot) = [0, \lambda_i(1), \ldots, [\lambda_i(z)],\ i = 1,2,...,n, \quad (45)$$

and

$$[\rho'''_i(u)]^{(bl)},\ u = 1,2,...,z,\ b = 1,2,...,\nu,$$
$$l = 1,2,...,w,\ i = 1,2,...,n, \quad (46)$$

are the coefficients of operation and climate-weather impact on the system components E_i, $i = 1,2,...,n$, intensities of ageing (the coefficients of operation and climate-weather impact on critical infrastructure component E, $i = 1,2,...,n$, intensities of departure from the safety state subset $\{u,u+1,...,z\}$) at the operation process related to climate-weather change process states $zc_{bl}, b = 1,2,...,\nu,$ $l = 1,2,...,w,$ i.e. the coordinate of the vector

$$[\rho'''_i(\cdot)]^{(bl)} = [0, [\rho'''_i(1)]^{(bl)}, \ldots, [\rho'''_i(z)]^{(bl)}],$$
$$b = 1,2,...,\nu,\ l = 1,2,...,w,\ i = 1,2,...,n. \quad (47)$$

The system component safety function (40), the system components intensities of ageing (43) and the coefficients of the climate-weather impact on the system components intensities of ageing (47) are main system component safety indices.

Similarly, we denote the system conditional lifetime in the safety state subset $\{u,u+1,...,z\}$ while the operation process related to climate-weather change process $ZC(t)$ at the system operating area is at the state $zc_{bl}, b = 1,2,...,\nu,$ $l = 1,2,...,w,$ by $T'''^{(bl)}(u)$ and the conditional safety function of the system by the vector

$$[\boldsymbol{S'''}(t,\cdot)]^{(bl)} = [1, [\boldsymbol{S'''}(t,1)]^{(bl)}, ...,$$
$$[\boldsymbol{S'''}(t,z)]^{(bl)}], \quad (48)$$

with the coordinates defined by

$$[\boldsymbol{S'''}(t,u)]^{(bl)} = P(T'''^{(bl)}(u) > t | ZC(t) = c_b) \quad (49)$$

for $t \in <0,\infty),$ $u = 1,2,...,z,$ $b = 1,2,...,\nu,$ $l = 1,2,...,w.$

Further, we denote the system unconditional lifetime in the safety state subset $\{u,u+1,...,z\}$ by $T'''(u)$ and the unconditional safety function of the system by the vector

$$\boldsymbol{S'''}(t,\cdot) = [1, \boldsymbol{S'''}(t,1), ..., \boldsymbol{S'''}(t,z)], \quad (50)$$

with the coordinates defined by

$$\boldsymbol{S'''}(t,u) = P(T'''(u) > t) \quad (51)$$

for $t \in <0,\infty),$ $u = 1,2,...,z.$

In the case when the system operation time θC_{ij} is large enough, the coordinates (51) of the unconditional safety function of the system defined by (50) are given by

$$\boldsymbol{S'''}(t,u) \cong \sum_{b=1}^{\nu}\sum_{l=1}^{w} pq_{bl}[\boldsymbol{S'''}(t,u)]^{(bl)}$$
$$\text{for } t \ge 0,\ u = 1,2,...,z, \quad (52)$$

where $[\boldsymbol{S'''}(t,u)]^{(bl)},$ $u = 1,2,...,z,$ $b = 1,2,...,\nu,$ $l = 1,2,...,w,$ are the coordinates of the system conditional safety functions defined by (48)-(49) and $pq_{bl},$ $b = 1,2,...,\nu,$ $l = 1,2,...,w,$ are the operation process related to climate-weather change process $ZC(t)$ at the system operating area limit transient probabilities at the state $zc_{bl}, b = 1,2,...,\nu,$ $l = 1,2,...,w,$ given by (37).

The exemplary graph of a five-state (z = 4) critical infrastructure safety function

$$\boldsymbol{S'''}(t,\cdot) = [1,\ \boldsymbol{S'''}(t,1),\ \boldsymbol{S'''}(t,2),\ \boldsymbol{S'''}(t,3),\ \boldsymbol{S'''}(t,4)],\ t \in <0,\infty),$$

is shown in *Figure 1*.

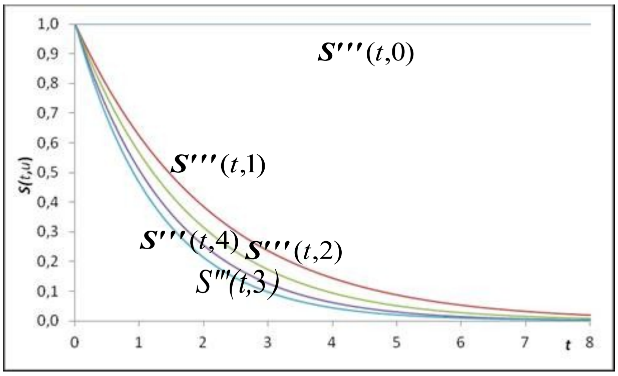

Figure 1. The graphs of a five-state critical infrastructure safety function $\boldsymbol{S'''}(t,\cdot)$ coordinates

The mean value of the system unconditional lifetime $T'''(u)$ in the safety state subset $\{u, u+1,...,z\}$ is given by [Soszyńska, 2010; Kołowrocki, Soszyńska-Budny, 2011], [EU-CIRCLE Report D2.1-GMU3, 2016]

$$\mu'''(u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}\mu'''_{bl}(u),\ u = 1,2,...,z, \tag{53}$$

where $\mu'''_{bl}(u)$ are the mean values of the system conditional lifetimes $T'''^{(bl)}(u)$ in the safety state subset $\{u, u+1,...,z\}$ at the operation process related to climate-weather change process $ZC(t)$ at the system operating area state zc_{bl}, $b = 1,2,...,v$, $l = 1,2,..., w$, given by

$$\mu'''_{bl}(u) = \int_0^\infty [\boldsymbol{S'''}(t,u)]^{(bl)} dt,\ u = 1,2,...,z, \tag{54}$$

$[\boldsymbol{S'''}(t,u)]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,..., v$, $l = 1,2,..., w$, are defined by (48)-(49) and q_b are given by (37). Whereas, the variance of the system unconditional lifetime $T'''(u)$ is given by

$$\sigma'''^2(u) = 2\int_0^\infty t\ \boldsymbol{S'''}(t,u)dt - [\mu'''(u)]^2,$$
$$u = 1,2,...,z, \tag{55}$$

where $\boldsymbol{S'''}(t,u)$, $u = 1,2,...,z$, are given by (51)-(52) and $\mu'''(u)$, $u = 0,1,...,z$, are given by (53)-(54).

Hence, according to (1.19) [Kołowrocki, Soszyńska-Budny, 2011], we get the following formulae for the mean values of the unconditional lifetimes of the system in particular safety states

$$\bar{\mu}'''(u) = \mu'''(u) - \mu'''(u+1),\ u = 0,1,...,z-1,$$
$$\bar{\mu}'''(z) = \mu'''(z), \tag{56}$$

where $\mu'''(u)$, $u = 0,1,...,z$, are given by (53).

Moreover, according to (1.20)-(1.21) [Kołowrocki, Soszyńska-Budny, 2011], if r is the system critical safety state, then the system risk function

$$\boldsymbol{r'''}(t) = P(S'''(t) < r \mid S'''(0) = z) = P(T'''(r) \le t),$$
$$t \in <0,\infty), \tag{57}$$

defined as a probability that the system is in the subset of safety states worse than the critical safety state r, $r \in \{1,...,z\}$ while it was in the safety state z at the moment t = 0 [Kołowrocki, 2014], [Kołowrocki, Soszyńska-Budny, 2011] is given by

$$\boldsymbol{r'''}(t) = 1 - \boldsymbol{S'''}(t,r),\ t \in <0,\infty), \tag{58}$$

where $\boldsymbol{S'''}(t,r)$ is the coordinate of the system unconditional safety function given by (52) for $u = r$.

The graph of the system risk function presented in *Figure 2* is called the fragility curve of the system.

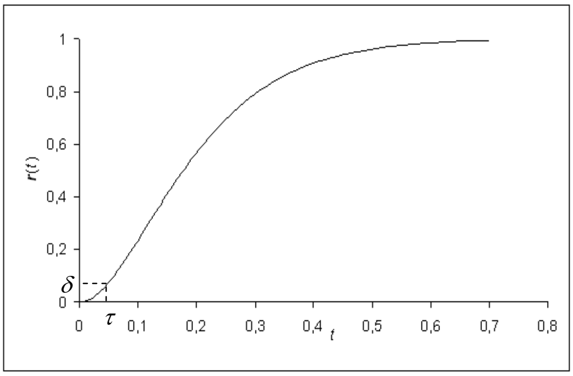

Figure 2. The graph (The fragility curve) of a system risk function $\boldsymbol{r'''}(t)$

The system safety function, the system risk function and the system fragility curve are main system safety indices. Other practically useful system safety indices are:

- the mean value of the unconditional system lifetime $T'''(r)$ up to the exceeding the critical safety state r given by

$$\mu'''(r) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}\mu'''_{bl}(r), \tag{59}$$

where $\mu'''_{bl}(r)$ are the mean values of the system conditional lifetimes $T'''^{(b)}(r)$ in the safety state subset $\{r, r+1,...,z\}$ at the climate-weather change process $ZC(t)$ at the system operating area state zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, given by

$$\mu'''_{bl}(r) = \int_0^{\infty}[\boldsymbol{S'''}(t,r)]^{(bl)}dt, \ \ b = 1,2,...,v,$$
$$l = 1,2,...,w, \tag{60}$$

$[\boldsymbol{S'''}(t,r)]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,..., v$, $l = 1,2,..., w$, are defined by (48)-(49) and pq_{bl} are given by (37);
- the standard deviation of the system lifetime $T(r)$ up to the exceeding the critical safety state r given by

$$\sigma'''(r) = \sqrt{n'''(r) - [\mu'''(r)]^2}\ , \tag{61}$$

where

$$n'''(r) = 2\int_0^{\infty} t\,\boldsymbol{S'''}(t,r)dt, \tag{62}$$

where $\boldsymbol{S'''}(t,r)$ is given by (51)-(52) and $\mu'''(r)$ is given by (59) for $u = r$.

- the moment τ the system risk function exceeds a permitted level δ given by

$$\tau = \boldsymbol{r'''}^{-1}(\delta), \tag{63}$$

and illustrated in Figure 2, where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r'''}(t)$ given by (58).

Other critical infrastructure safety indices are:
- the intensities of ageing of the critical infrastructure (the intensities of critical infrastructure departure from the safety state subset $\{u, u+1,...,z\}$) related to the operation and climate-weather change impact, i.e. the coordinates of the vector

$$\boldsymbol{\lambda'''}(t,\cdot) = [0, \ \boldsymbol{\lambda'''}(t,1), \ldots, \boldsymbol{\lambda'''}(t,z)\ ],$$
$$t \in <0,+\infty), \tag{64}$$

where

$$\boldsymbol{\lambda'''}(t,u) = \frac{-\dfrac{d\boldsymbol{S'''}(t,u)}{dt}}{\boldsymbol{S'''}(t,u)},$$
$$t \in <0,+\infty), \ \ u = 1,2,...,z; \tag{65}$$

- the coefficients of the operation and climate-weather impact on the critical infrastructure intensities of ageing (the coefficients of the operation and climate-weatcher impact on critical infrastructure intensities of departure from the safety state subset $\{u, u+1,...,z\}$), i.e. the coordinates of the vector

$$\boldsymbol{\rho'''}(t,\cdot) = [0, \ \boldsymbol{\rho'''}(t,1), \ldots, \boldsymbol{\rho'''}(t,z)\,],$$
$$t \in <0,+\infty), \tag{66}$$

where

$$\boldsymbol{\lambda'''}(t,u) = \boldsymbol{\rho'''}(t,u)\cdot \boldsymbol{\lambda}(t,u), \ \ t \in <0,+\infty),$$
$$u = 1,2,...,z. \tag{67}$$

and $\boldsymbol{\lambda}(t,u)$ are the intensities of ageing of the critical infrastructure (the intensities of the critical infrastructure departure from the safety state subset $\{u, u+1,...,z\}$) without of operation and climate-weather impact, i.e. the coordinate of the vector

$$\boldsymbol{\lambda}(t,\cdot) = [0, \ \boldsymbol{\lambda}(t,1), \ldots, \boldsymbol{\lambda}(t,z)\,], \ t \in <0,+\infty). \tag{68}$$

In the case, the critical infrastructure have the exponential safety functions, i.e.

$$\boldsymbol{S'''}(t,\cdot) = [0, \ \boldsymbol{S'''}(t,1), ..., \boldsymbol{S'''}(t,z)\ ],$$
$$t \in <0,+\infty), \tag{69}$$

where

$$\boldsymbol{S'''}(t,r) = \exp[-\boldsymbol{\lambda'''}(u)t], \ \ t \in <0,+\infty),$$
$$\boldsymbol{\lambda'''}(u) \geq 0, \ \ u = 1,2,\ldots,z, \tag{70}$$

the critical infrastructure safety indices defined by (64)-(68) take forms:
- the intensities of ageing of the critical infrastructure (the intensities of critical infrastructure departure from the safety state subset $\{u, u+1,...,z\}$) related to operation and climate-weather change impact, i.e. the coordinates of the vector

$$\boldsymbol{\lambda}'''(\cdot) = [0, \boldsymbol{\lambda}'''(1), ..., \boldsymbol{\lambda}'''(z)], \quad (71)$$

- the coefficients of the operation and climate-weather impact on the critical infrastructure intensities of ageing (the coefficients of the operation and climate-weather impact on critical infrastructure intensities of departure from the safety state subset $\{u, u+1, ..., z\}$), i.e. the coordinate of the vector

$$\boldsymbol{\rho}'''(\cdot) = [0, \boldsymbol{\rho}'''(1), ..., \boldsymbol{\rho}'''(z)], \quad (72)$$

where

$$\boldsymbol{\lambda}''(u) = \boldsymbol{\rho}''(u) \cdot \boldsymbol{\lambda}(u), \; u = 1,2,...,z. \quad (73)$$

and $\boldsymbol{\lambda}(u)$ are the intensities of ageing of the critical infrastructure (the intensities of the critical infrastructure departure from the safety state subset $\{u, u+1, ..., z\}$) without of operation and climate-weather impact, i.e. the coordinate of the vector

$$\boldsymbol{\lambda}(\cdot) = [0, \boldsymbol{\lambda}(1), ..., \boldsymbol{\lambda}(z)]. \quad (74)$$

4. Safety of multistate exponential systems at operation variable conditions related climate-weather change

We assume that the system components at the operation process related to climate-weather change process $ZC(t)$ at the system operating area states have the exponential safety functions. This assumption and the results given in Chapter 1 [Kołowrocki, Soszyńska-Budny, 2011] yield the following results formulated in the form of the following proposition.

Proposition 1
If components of the multi-state system at the operation process related to climate-weather change process $ZC(t)$ at the system operating area states $zc_{bl}, b = 1,2,..., v, \quad l = 1,2,..., w,$ have the exponential safety functions given by

$$[S'''_i(t,\cdot)]^{(bl)} = [1, [S'''_i(t,1)]^{(bl)}, ..., [S'''_i(t,z)]^{(bl)}],$$
$$t \in <0,\infty), \; b = 1,2,..., v, \; l = 1,2,..., w,$$
$$i = 1,2,...,n, \quad (75)$$

with the coordinates

$$[S'''_i(t,u)]^{(bl)} = P(T'''^{(bl)}_i(u) > t | ZC(t) = zc_{bl})$$
$$= \exp[-[\lambda'''_i(u)]^{(bl)} t], \; t \in <0,\infty),$$
$$b = 1,2,..., v, \; l = 1,2,..., w, \; i = 1,2,...,n, \quad (76)$$

and the intensities of ageing of the system components E_i, $i = 1,2,...,n,$ (the intensities of the system components E_i, $i = 1,2,...,n,$ departure from the safety state subset $\{u, u+1, ..., z\}$) related to operation and climate-weather change impact, existing in (4.2), are given by

$$[\lambda'''_i(u)]^{(bl)} = \rho'''^{(bl)}_i(u) \cdot \lambda_i(u), \; u = 1,2,...,z,$$
$$b = 1,2,..., v, \; l = 1,2,..., w, \; i = 1,2,...,n, \quad (77)$$

where $\lambda_i(u)$ are the intensities of ageing of the system components E_i, $i = 1,2,...,n,$ (the intensities of the system components E_i, $i = 1,2,...,n,$ departure from the safety state subset $\{u, u+1, ..., z\}$) without operation and climate-weather change impact and

$$[\rho'''_i(u)]^{(bl)}, \; u = 1,2,...,z, \; b = 1,2,..., v,$$
$$l = 1,2,..., w, \; i = 1,2,...,n, \quad (78)$$

are the coefficients of the operation and climate-weather impact on the system components E_i, i = 1,2,...,n, intensities E_i, $i = 1,2,...,n,$ of ageing (the coefficients of operation and climate-weather impact on critical infrastructure component E, i = 1,2,...,n, intensities of departure from the safety state subset $\{u, u+1, ..., z\}$) without operation and climate-weather change impact, in the case of series, parallel, "m out of n", consecutive "m out of n: F" systems and respectively by

$$[S'''_{ij}(t,\cdot)]^{(bl)} = [1, [S'''_{ij}(t,1)]^{(bl)}, ..., [S'''_{ij}(t,z)]^{(bl)}],$$
$$t \in <0,\infty), \; b = 1,2,..., v, \; l = 1,2,..., w,$$
$$i = 1,2,...,k, \; j = 1,2,...,l_i, \quad (79)$$

with the coordinates

$$[S'''_{ij}(t,u)]^{(bl)} = P(T'''^{(bl)}_{ij}(u) > t | ZC(t) = zc_b)$$
$$= \exp[-[\lambda'''_{ij}(u)]^{(bl)} t], \; t \in <0,\infty), \; b = 1,2,..., v,$$
$$l = 1,2,..., w, \; i = 1,2,...,k, \; j = 1,2,...,l_i, \quad (80)$$

and the intensities of ageing of the system components E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i, (the intensities of the system components E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i, departure from the safety state subset $\{u, u+1, ..., z\}$) related to operation and climate-weather change impact, existing in (4.6), are given by

$$[\lambda'''_{ij}(u)]^{(bl)} = \rho'''^{(bl)}_{ij}(u) \cdot \lambda_{ij}(u), \; u = 1,2,...,z,$$

$b = 1,2,\ldots, v,\ l = 1,2,\ldots, w,\ i = 1,2,\ldots,k,$
$j = 1,2,\ldots,l_i,$ (81)

where $\lambda_{ij}(u)$ are the intensities of ageing of the system components E_{ij}, $i = 1,2,\ldots,k, j = 1,2,\ldots,l_i$, (the intensities of the system components E_{ij}, $i = 1,2,\ldots,k$, $j = 1,2,\ldots,l_i$, departure from the safety state subset $\{u,u+1,\ldots,z\}$) without operation and climate-weather change impact and

$$[\rho'''_{ij}(u)]^{(bl)},\ u = 1,2,\ldots,z,\ b = 1,2,\ldots,v,$$
$$l = 1,2,\ldots, w,\ i = 1,2,\ldots,k, j = 1,2,\ldots,l_i, \quad (82)$$

are the coefficients of the operation and climate-weather impact on the system components E_{ij}, $i = 1,2,\ldots,k$, $j = 1,2,\ldots,l_i$, intensities E_{ij}, $i = 1,2,\ldots,k$, $j = 1,2,\ldots,l_i$, of ageing (the coefficients of operation and climate-weather impact on critical infrastructure component E, $i = 1,2,\ldots,n$, intensities of departure from the safety state subset $\{u,u+1,\ldots,z\}$) without operation and climate-weather change impact,
in the case of series-parallel, parallel-series, series-"*m* out of *k*", "m_i out of l_i"-series, series-consecutive "*m* out of *k*: F" and consecutive "m_i out of l_i: F"-series systems and the system operation time θ is large enough, then its multistate unconditional safety function is given by the vector:

i) for a series system

$$\boldsymbol{S'''}(t,\cdot) = [1,\ \boldsymbol{S'''}(t,1), \ldots, \boldsymbol{S'''}(t,z)]$$
$$\text{for } t \ge 0, \quad (83)$$

where

$$\boldsymbol{S'''}(t,u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \exp[-\sum_{i=1}^{n}[\lambda'''_i(u)]^{(bl)} t]$$
$$\text{for } t \ge 0,\ u = 1,2,\ldots,z, \quad (84)$$

ii) for a parallel system

$$\boldsymbol{S'''}(t,\cdot) = [1, \boldsymbol{S'''}(t,1), \ldots, \boldsymbol{S'''}(t,z)]$$
$$\text{for } t \ge 0, \quad (85)$$

where

$$\boldsymbol{S'''}(t,u) \cong 1 - \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \prod_{i=1}^{n}[1-\exp[-[\lambda'''_i(u)]^{(bl)} t]]$$
$$\text{for } t \ge 0,\ u = 1,2,\ldots,z, \quad (86)$$

iii) for a "*m* out of *n*" system

$$\boldsymbol{S'''}(t,\cdot) = [1, \boldsymbol{S'''}(t,1), \ldots, \boldsymbol{S'''}(t,z)]$$
$$\text{for } t \ge 0, \quad (87)$$

where

$$\boldsymbol{S'''}(t,u)$$
$$\cong 1 - \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{\substack{r_1,r_2,\ldots,r_n=0 \\ r_1+r_2+\ldots+r_n \le m-1}}^{1} \prod_{i=1}^{n} \exp[-r_i[\lambda'''_i(u)]^{(bl)} t]$$
$$[1-\exp[-[\lambda'''_i(u)]^{(bl)} t]]^{1-n}$$
$$\text{for } t \ge 0,\ u = 1,2,\ldots,z, \quad (88)$$

or

$$\boldsymbol{S'''}(t,\cdot) = [1,\ \boldsymbol{S'''}(t,1), \ldots, \boldsymbol{S'''}(t,z)]$$
$$\text{for } t \ge 0, \quad (89)$$

where

$$\boldsymbol{S'''}(t,u)$$
$$\cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{\substack{r_1,r_2,\ldots,r_n=0 \\ r_1+r_2+\ldots+r_n \le \bar{m}}}^{1} \prod_{i=1}^{n}[1-\exp[-[\lambda'''_i(u)]^{(bl)} t]]^{r_i}$$
$$\exp[-(1-r_i)[\lambda'''_i(u)]^{(bl)} t]$$
$$\text{for } t \ge 0,\ u = 1,2,\ldots,z, \quad (90)$$

and $\bar{m} = n - m$;

iv) for a consecutive "*m* out of *n*: F" system

$$\boldsymbol{CS'''}(t,\cdot) = [1,\ \boldsymbol{CS'''}(t,1), \ldots, \boldsymbol{CS'''}(t,z)]$$
$$\text{for } t \ge 0, \quad (91)$$

where

$$\boldsymbol{CS'''}(t,u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \prod_{i=1}^{k}[\boldsymbol{CS'''}_n^m(t,u)]^{(bl)}$$
$$\text{for } t \ge 0,\ u = 1,2,\ldots,z, \quad (92)$$

and $[\boldsymbol{CS'''}(t,u)]^{(bl)}$, $t \ge 0$, $i = 1, 2,\ldots, k$, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, are given by

$$[\boldsymbol{CS'''}(t,u)]^{(bl)} \cong \begin{cases} 1 & \text{for } n<m, \\ 1-\sum_{b=1}^{v}\sum_{l=1}^{w} pq_b \prod_{i=1}^{n}[1-\exp[-[\lambda'''_i(u)]^{(bl)}t]] & \text{for } n=m, \\ \sum_{b=1}^{v}\sum_{l=1}^{w} pq_b [\exp[-[\lambda'''_n(u)]^{(bl)}t][\boldsymbol{CS'''}^{m}_{n-1}(t,u)]^{(bl)} \\ +\sum_{i=1}^{m-1}\exp[-[\lambda'''_{n-i}(u)]^{(bl)}t][\boldsymbol{CS'''}_{n-i-1}(t,u)]^{(bl)} \\ \prod_{j=n-i+1}^{n}[1-\exp[-[\lambda'''_j(u)]^{(bl)}t]] & \text{for } n>m, \end{cases}$$

for $t \ge 0$, $u = 1,2,\ldots,z,$ (93)

v) for a series-parallel system

$$\boldsymbol{S'''}(t,\cdot) = [1,\ \boldsymbol{S'''}(t,1),\ \ldots,\ \boldsymbol{S'''}(t,z)]$$
for $t \ge 0,$ (94)

where

$$\boldsymbol{S'''}(t,u) \cong 1-\sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \prod_{i=1}^{k}[1-\exp[-\sum_{j=1}^{l_i}[\lambda'''_{ij}(u)]^{(bl)}t]]$$
for $t \ge 0,\ u = 1,2,\ldots,z,$ (95)

vi) for a parallel-series system

$$\boldsymbol{S'''}(t,\cdot) = [1,\ \boldsymbol{S'''}(t,1),\ \ldots,\ \boldsymbol{S'''}(t,z)]$$
for $t \ge 0,$ (96)

where

$$\boldsymbol{S'''}(t,u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \prod_{i=1}^{k}[1-\prod_{j=1}^{l_i}[1-\exp[-[\lambda'''_{ij}(u)]^{(bl)}t]]]$$
for $t \ge 0,\ u = 1,2,\ldots,z,$ (97)

vii) for a series-"m out of k" system

$$\boldsymbol{S'''}(t,\cdot) = [1,\ \boldsymbol{S'''}(t,1),\ \ldots,\ \boldsymbol{S'''}(t,z)]$$
for $t \ge 0,$ (98)

where

$$\boldsymbol{S'''}(t,u) \cong$$

$$1-\sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{\substack{r_1,r_2,\ldots,r_k=0 \\ r_1+r_2+\ldots+r_k \le m-1}}^{1} \prod_{i=1}^{k}[\prod_{j=1}^{l_i}\exp[-[\lambda'''_{ij}(u)]^{(bl)}t]]^{r_i}$$

$$\cdot[1-\prod_{j=1}^{l_i}\exp[-[\lambda'''_{ij}(u)]^{(bl)}t]]^{1-r_i}$$
for $t \ge 0,\ u = 1,2,\ldots,z,$ (99)

or

$$\boldsymbol{S'''}(t,\cdot) = [1,\ \boldsymbol{S'''}(t,1),\ \ldots,\ \boldsymbol{S'''}(t,z)]$$
for $t \ge 0,$ (100)

where

$$\boldsymbol{S'''}(t,u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \sum_{\substack{r_1,r_2,\ldots,r_k=0 \\ r_1+r_2+\ldots+r_k \le \bar{m}}}^{1} \prod_{i=1}^{k}[1-[\prod_{i=1}^{l_i}\exp[-[\lambda'''_{ij}(u)]^{(bl)}t]]^{r_i}$$

$$\cdot[\prod_{j=1}^{l_i}\exp[-[\lambda'''_{ij}(u)]^{(bl)}t]]^{1-r_i} \text{ for } t \ge 0,$$
$\bar{m} = k-m,\ u = 1,2,\ldots,z;$ (101)

viii) for a "m_i out of l_i"-series system

$$\boldsymbol{S'''}(t,\cdot) = [1,\ \boldsymbol{S'''}(t,1),\ \ldots,\ \boldsymbol{S'''}(t,z)]$$
for $t \ge 0,$ (102)

where

$$\boldsymbol{S'''}(t,u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_b \prod_{i=1}^{k}[1-\sum_{\substack{r_1,r_2,\ldots,r_{l_i}=0 \\ r_1+r_2+\ldots+r_{l_i} \le m_i-1}}^{1} \prod_{j=1}^{l_i}\exp[-r_j[\lambda'''_{ij}(u)]^{(bl)}t]]$$

$$\cdot[1-\exp[-[\lambda'''_{ij}(u)]^{(bl)}t]]^{1-r_j}]$$
for $t \ge 0,\ u = 1,2,\ldots,z,$ (103)

or

$$\boldsymbol{S'''}(t,\cdot) = [1,\ \boldsymbol{S'''}(t,1),\ \ldots,\ \boldsymbol{S'''}(t,z)]$$
for $t \ge 0,$ (104)

where

$$\boldsymbol{S'''}(t,u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl} \prod_{i=1}^{k}[\sum_{\substack{r_1,r_2,\ldots,r_{l_i}=0 \\ r_1+r_2+\ldots+r_{l_i} \le \bar{m}_i}}^{1} \prod_{j=1}^{l_i}[1-\exp[-[\lambda''_{ij}(u)]^{(bl)}t]]^{r_j}$$

$$\cdot\exp[-(1-r_j)[\lambda'''_{ij}(u)]^{(bl)}t]] \text{ for } t \ge 0,$$
$\bar{m}_i = l_i - m_i,\ i = 1,2,\ldots,k,\ u = 1,2,\ldots,z;$ (105)

ix) for a series-consecutive "m out of k: F" system

$$\boldsymbol{CS'''}(t,\cdot) = [1,\ \boldsymbol{CS'''}(t,1),\ ...,\ \boldsymbol{CS'''}(t,z)]$$
for $t \ge 0,$ (106)

where

$$\boldsymbol{CS'''}(t,u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}[\ \boldsymbol{CS'''}(t,u)]^{(bl)} \quad \text{for} \quad t \ge 0,$$
$u = 1,2,...,z,$ (107)

and $[\boldsymbol{CS'''}(t,u)]^{(bl)}$, $t \ge 0,\ b = 1,2,...,v,\ l = 1,2,...,w,$ are given by

$$[\boldsymbol{CS'''}(t,u)]^{(bl)} = \begin{cases} 1 & \text{for } k < m, \\ 1 - \prod_{i=1}^{k}[1 - \exp[-\sum_{j=1}^{l_i}[\lambda'''_{ij}(u)]^{(bl)} t]] & \text{for } k = m, \\ \exp[-\sum_{j=1}^{l_k}[\lambda'''_{kj}(u)]^{(bl)} t][\boldsymbol{CS'''}_{k-1;l_1,l_2,...l_k}(t,u)]^{(bl)} & \\ + \sum_{j=1}^{m-1}[\exp[-\sum_{v=1}^{l_{k-j}}[\lambda'''_{k-jv}(u)]^{(bl)} t]] & \\ \cdot[\boldsymbol{CS'''}_{k-j-1;l_1,l_2,...l_k}(t,u)]^{(bl)} & \\ \cdot \prod_{i=k-j+1}^{k}[1 - \exp[-\sum_{v=1}^{l_i}[\lambda'''_{iv}(u)]^{(bl)} t]] & \text{for } k > m, \end{cases}$$
for $t \ge 0,\ u = 1,2,...,z,$ (108)

x) for a consecutive "m_i out of l_i: F"-series system

$$\boldsymbol{CS'''}(t,\cdot) = [1,\ \boldsymbol{CS'''}(t,1),\ ...,\ \boldsymbol{CS'''}(t,z)]$$
for $t \ge 0,$ (109)

where

$$\boldsymbol{CS'''}(t,u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} pq_{bl}\prod_{i=1}^{k}[\boldsymbol{CS'''}^{m}_{i,l_i}(t,u)]^{(bl)}$$
for $t \ge 0,\ u = 1,2,...,z,$ (110)

and $[\boldsymbol{CS'''}^{m}_{i,l_i}(t,u)]^{(bl)}$, $t \ge 0,\ i = 1, 2,\dots, k,$ $b = 1,2,...,v,\ l = 1,2,...,w,$ are given by

$$[\boldsymbol{CS'''}^{m}_{i,l_i}(t,u)]^{(bl)} = \begin{cases} 1 & \text{for } l_i < m_i, \\ 1 - \prod_{j=1}^{l_i}[1 - \exp[-[\lambda'''_{ij}(u)]^{(bl)} t]] & \text{for } l_i = m_i, \\ \exp[-[\lambda'''_{il_i}(u)]^{(bl)} t][\boldsymbol{CS'''}^{m_i}_{i,l_i-1}(t,u)]^{(bl)} & \\ + \sum_{j=1}^{m_i-1}\exp[-[\lambda'''_{il_i-j}(u)]^{(bl)} t][\boldsymbol{CS'''}^{m_i}_{i,l_i-j-1}(t,u)]^{(bl)} & \\ \cdot \prod_{v=l_i-j+1}^{l_i}[1 - \exp[-[\lambda'''_{iv}(u)]^{(bl)} t]] & \text{for } l_i > m_i, \end{cases}$$
for $t \ge 0,\ u = 1,2,...,z,$ (111)

Remark 1

The formulae for the safety functions stated in *Proposition 1* are valid for the considered systems under the assumption that they do not change their structures at different operation process related to climate-weather change process *ZC*(*t*) at the system operating area states $zc_{bl},\ b = 1,2,...,v,\ \ l = 1,2,...,w.$ This limitation can be simply omitted by the replacement in these formulae the system's structure shape constant parameters $n,\ m,\ k,\ m_i,\ l_i,$ respectively by their changing at different operation states $zc_{bl},\ b = 1,2,...,v,\ \ l = 1,2,...,w,$ equivalent structure shape parameters $n^{(bl)},\ m^{(bl)},\ k^{(bl)},\ m_i^{(bl)},$ $l_i^{(bl)},\ b = 1,2,...,v,\ l = 1,2,...,w.$

For the exponential complex technical systems, considered in *Proposition 1*, we determine the mean values $\mu'''(u)$ and the standard deviations $\sigma'''(u)$ of the unconditional lifetimes of the system in the safety state subsets $\{u, u+1,...,z\},\ u = 1,2,...,z,$ the mean values $\bar{\mu}'''(u)$ of the unconditional lifetimes of the system in the particular safety states $u,\ u = 1,2,...,z,$ the system risk function $\boldsymbol{r'''}(t)$ and the moment τ'' when the system risk function exceeds a permitted level δ respectively defined by (53)-(63), after substituting for $\boldsymbol{S'''}(t,u),\ u = 1,2,...,z,$ the coordinates of the unconditional safety functions given respectively by (83)-(111).

5. Conclusions

The integrated general model of complex systems' safety, linking their safety models and their operation processes models and considering variable at different operation states their safety structures and their components safety parameters is constructed. The material given in this report delivers the procedures and algorithms that allow to find the main an practically important safety characteristics of the complex technical systems at the variable operation conditions. Next the results are applied to the safety evaluation of the port oil piping transportation system and the maritime ferry technical system. The predicted safety characteristics of these exemplary critical infrastructures operating at the variable conditions are different from those determined for this system operating at constant conditions [Kołowrocki, Soszyńska-Budny, 2011]. This fact justifies the sensibility of considering real systems at the variable operation conditions that is appearing out in a natural way from practice. This approach, upon the sufficient accuracy of the critical infrastructures' operation processes and the critical infrastructures' components safety parameters identification, makes their safety prediction much more precise.

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