

## NEW APPROACH TO MODELING GEOMETRIC ERRORS IN PRECISION MANUFACTURING MACHINES

Paweł Majda, Grzegorz Szwengier, Robert Jastrzębski

### Summary

This paper presents a new approach to modeling geometric errors in machine tools with linear rolling guideways. Modeling was performed using a finite element method. The results of the modeling were a visualization of deformations of a virtual milling machine with a cross table and with known characteristics of guide geometric errors. The simulation results show that the machine table is subject to a complex state of strain. This implies that the Abbe error during milling machine measurements can assume values dependent on the location of metrological device elements on the machine table. The calculation results were verified experimentally using real milling machines. The predictions of the computer simulation were confirmed.

**Keywords:** geometric errors, machine tool, volumetric error, FEM

### Nowe podejście do modelowania błędów geometrycznych precyzyjnych maszyn technologicznych

### Streszczenie

W artykule przedstawiono nowe podejście do modelowania błędów geometrycznych obrabiarek wyposażonych w liniowe toczne połączenia prowadnicowe. Modelowanie przeprowadzono z użyciem metody elementów skończonych. Efektem modelowania jest wizualizacja odkształceń frezarki poziomej ze stołem krzyżowym, która ma znane charakterystyki błędów geometrycznych połączeń prowadnicowych. Analiza wyników procesu symulacji wykazała, że stół takiej obrabiarki podlega złożonemu stanowi odkształcenia. Ma to istotne znaczenie praktyczne, ponieważ podczas wykonywania pomiarów błędów geometrycznych obrabiarek – błąd Abbego może przyjmować różne wartości w zależności od usytuowania przyrządów pomiarowych na stole frezarki. Wyniki obliczeń zweryfikowano doświadczalnie na rzeczywistych frezarkach. Potwierdzono przewidywania symulacji komputerowej.

**Słowa kluczowe:** błędy geometryczne, obrabiarka, błąd przestrzennego pozycjonowania, MES

## 1. Introduction

Guideway geometric errors are one of the causes of undesirable differences between nominal and real positions and orientation of a tool in relation to the

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Address: Prof. Grzegorz SZWENGIER, Paweł MAJDA, PhD DSc Eng., Szczecin University of Technology, Faculty of Mechanical Engineering, Institute of Manufacturing Engineering, Piastów 19, 70-310 Szczecin, e-mail: Pawel.Majda@ps.pl, Grzegorz.Szwen-gier@zut.edu.pl; Robert JASTRZĘBSKI, MSc Eng., SciLab Measurements Robert Jastrzębski, ul. Zacisze 53, 05-800 Pruszków, e-mail: rjastrzebski@scilab.pl

workpiece. These differences, in the entire workspace of a machine are known as volumetric errors – VE. The knowledge of VE is used in off-line software compensation of machine tool errors, thus increasing positioning accuracy. The efficiency of such a compensation is especially effective for large machine tools [1] (large machine tool tables are bigger than 1200x600 mm, medium sized machine tool tables are smaller than 1200x600 mm [2, 3]). One of the characteristic properties of large machine tools is their kinematic structure, usually a portal/gantry type, where a tool path runs in many axes against a fixed workpiece. In this case, regardless of the measurement method and determination of VE, the fixed table on which the workpiece lies is also a measurement datum for machine tool errors. VE is determined in relation to this datum. A similar procedure is applied for medium-sized machine tools. The fact that medium-sized machine tools usually have a movable table as a measurement datum should also be taken into account.

In direct measurement [4] of machine tool errors, e.g. by laser interferometer or electronic level, the elements of the measuring system are located on a movable table. In indirect measurements [4] the situation is similar, for example in the method proposed by Yang, Kim, Park and Lee [5], in which a ball bar test is used, with one end situated in different positions on the table. The methodology of measurements using a ball plate artefact, used by Bringmann and Knapp [6], and other material artefacts (Woody, Smith, Hocken and Miller [1]; Choi, Minb and Lee [7]), also requires the location of measuring system elements on the table. Effective tools for mapping VE using a lasertracing system (developed under the supervision of Schwenke [8, 9]) also require the location of the measuring equipment on the table. The common characteristic of all these mentioned methods is the measurement of geometric errors against the datum of the table. Questions arise whether (i) a correctly determined measurement datum exists and (ii) whether this situation may generate errors in the determination of VE.

Apart from the methodology of measurements, the accuracy of the software compensation of machine tool errors is very much affected by the manner and accuracy of VE modeling. It is widely accepted that the component of VE resulting from geometric errors in the axis can be modeled using a geometric transformation of solids treated as rigid bodies [10, 11]. The most commonly used is the homogeneous transformation matrix [6, 12, 13] or the Denavit-Hartenberg notation [14-16]. It has been shown analytically and confirmed experimentally that typical movable milling machine tables may be subject to deformation when changing position. So questions arise whether (i) the rigid body assumption is an acceptable simplification in VE modeling for milling machines with movable tables and (ii) whether the results of measurements of kinematic error characteristics in the axis are a reliable input parameters for the VE model? The range of analytical and experimental studies presented in this paper should help answer these questions.

Chapter 2 presents the main contribution of this paper. It describes, in general terms, a new approach to the modeling of machine tools with rolling guideway geometric errors. A virtual machine tool was created to examine and verify the assumptions used in VE determination. The evaluation of the obtained results identified the table of the machine tool to be the component which was subject to the most significant deformation resulting from the presence of geometric errors in the system.

Chapter 3 contains the results of measurements that were performed to verify the conclusions based on simulation studies.

Finally, conclusions and reflections on the error compensation of machine tools with non-rigid bodies were addressed in Chapter 4.

## **2. Modeling machine tools taking into account guideway geometric errors**

Machine modeling taking into account guideway geometric errors was performed using the finite element method (FEM) [17]. Using this method, it is difficult to achieve - during the model discretization stage – the assumption of the representation of the real geometry of the object. This problem is of particular importance because the calculations are performed for the variable structure of the machine. The variable structure denotes solutions for a variety of positions of the machine body elements, i.e. for different mutual positions of the tool relative to the workpiece (T-W). In such a case it is required that the calculation model is characterized by a reasonable number of degrees of freedom – it is desirable to keep it as low as possible. The realization of this assumption allows solution of the computational model in a short time. However, in model simplifications, one should bear in mind the basic criterion of reliable representation/mapping of the object's stiffness.

Simplification of the geometry of the machine tools and guide connection components is a stage which can be implemented independently of the other stages of modeling. The next stage is creating a FEM model of the machine; at this stage still not taking into account guideway geometric errors. The input data include: material constants, simplified geometry of the machine tool, geometry of guideway components, boundary conditions and configuration of the kinematic structure, i.e. the relative position of T-W. The result is, inter alia, the spatial distribution and geometric data for contact elements that model rolling elements. Such elements are geometrically non-linear (working exclusively on compressive strength) and physically non-linear (non-linearity of force-displacement resulting from the Hertz theory) [18-20]. In addition, the input parameter for the calculation is also the preload or gap in contact elements.

The next stage includes the analytical description of shape and/or relative arrangement of guide rails. In this study it was assumed that errors in these

components are defined using a function where the length of the guide is the argument of the function. The authors of this article suggest mapping guide rail geometric errors through the designation of preload and/or gap in contact elements with values resulting from the adopted geometric error function. The input data for the calculation include: the preload of the rolling elements, arrangement of contact elements (depending on the location of the machine body elements) and the functions describing the geometric errors of individual guideways. It should be emphasized that any change in the configuration of body elements will change the arrangement of contact elements. After each such change one should regenerate the calculation parameters for the contact elements. This enables analytical examination of the tested machine for various T-W configurations, including guideway geometric errors. Details of guideway geometric error modeling according to the above-mentioned concept are presented in [21] and [22].

It should be emphasized that even without defining additional excitations (such as the concentrated force, gravity and thermal deformation, etc.), the model of the machine, in the form described above, is subject to kinematic excitations resulting from the introduction of the guideway geometric errors into the model. The results of the calculation include nodal displacements, strains and stresses of the analyzed structure.

The algorithm for modeling machines, including geometric errors of rolling guides, is illustrated in Fig. 1.

Geometric errors for the calculations were assumed a priori. In this approach, a machine tool was modeled with arbitrarily adopted guideway geometric errors. Therefore, the calculation results can not be directly compared with those presented in the final experimental part. This approach can be considered justified if the conclusions drawn from the results of computer simulations are used to assist and guide experimental research, and not to describe the behavior of a particular machine.

Computational parameters for guideway blocks were adopted for a dynamic load of 22.8 kN, and a preload at 8% of dynamic load. The machine tool was modeled using a beam element. Figure 2a presents the discrete model of the machine and Fig. 2b shows the visualization of guide shapes adopted for error calculations. Fig. 2a also shows vertical, perfectly rigid and weightless rods, which are placed evenly and fixed to the machine table. These rods are intended to serve as markers that will help graphically evaluate the effects of deformation of the machine table after the solution of model equations. The locations of these markers in the real object is replaced by the workpiece or – in an examination of geometric accuracy of the machine tool – by the elements of the measuring apparatus. Therefore, by calculating the relative displacement of the tool-modelling node and the corresponding nodes belonging to the markers can one can directly determine the measure of spatial positioning error, i.e. VE.

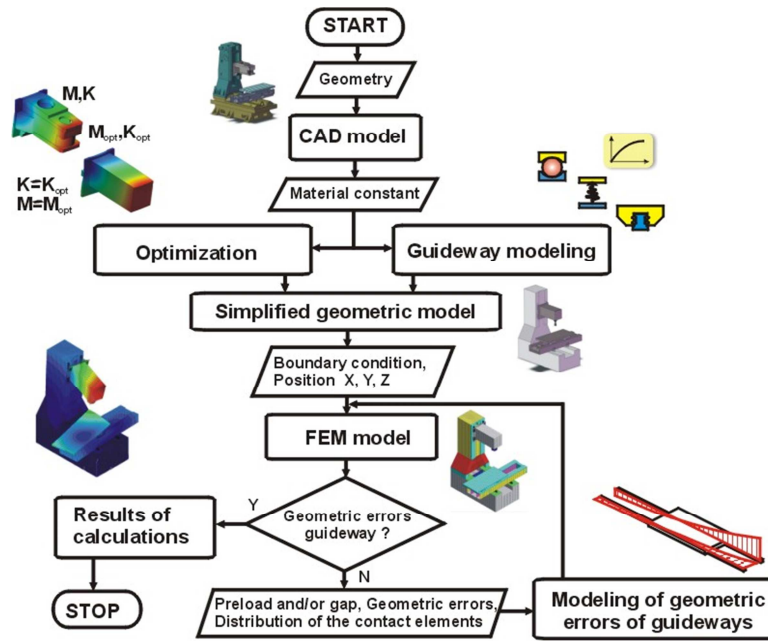


Fig. 1. Diagram of machine tool modeling taking into account guideway geometric errors

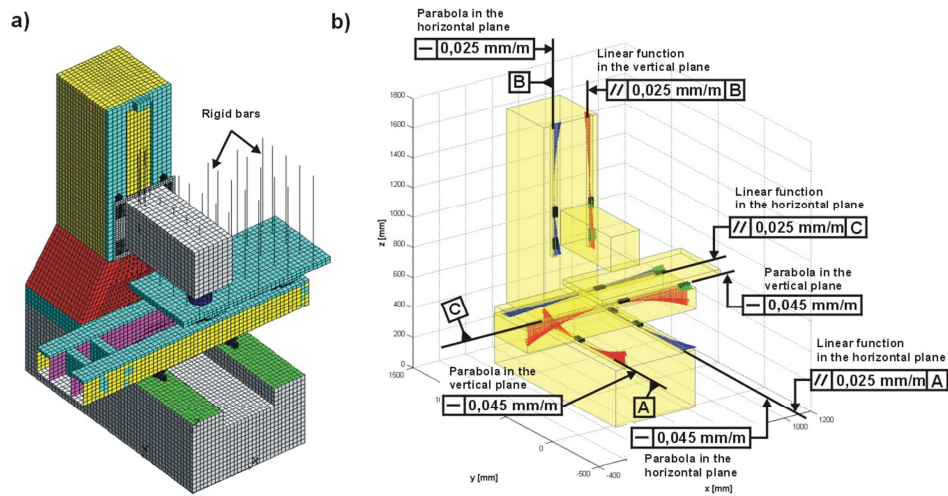


Fig. 2. FEM model of the machine tool (a), visualization of the guideway geometric errors adopted for calculations (b)

Figure 3 shows an example of visualization of the resultant displacement, as a result of FEM modeling of machine tools including guideway geometric errors. The presented method of modeling allows performing calculations for different configurations of the mutual position of machine body elements. Therefore, Figure 3 shows the coordinates X, Y and Z of the position of the tool relative to the machine coordinate system for which the computations were performed. Visual assessment of results presented in Fig. 3 shows that guideway geometric errors generate VE.

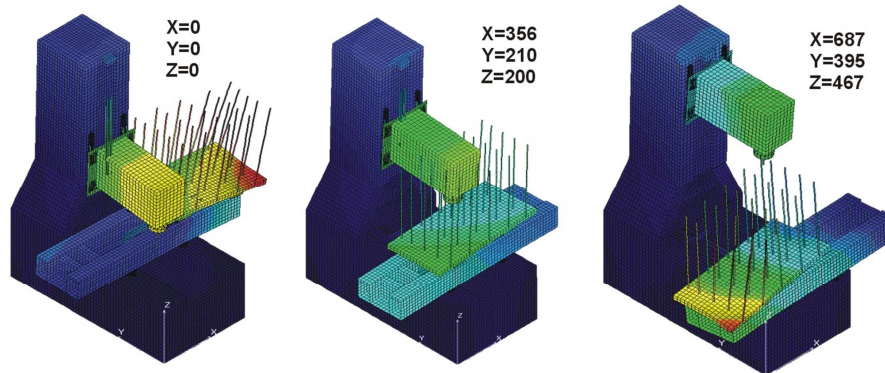


Fig. 3. Exemplary visualization of calculation results for the displacement area of different orientations of the machine body elements, taking into account guideway geometric errors

Moreover, they can also cause significant deformations of machine components. This is particularly evident when assessing the position of markers that are intended to reflect the movement of the workpiece (or elements of the measuring apparatus) situated on the machine table. Markers in the current configuration of the machine are subject to displacement, which is understandable given the assumptions. But they also are subject to different rotations (in different directions), indicating a complex strain state of the table. If the phenomenon can occur in real objects, a very important question appears: what measurement base should be adopted in experimental measurement of effects connected with the geometric accuracy of machines with movable tables?

All known (at least to the authors of this paper) methods of determining VE require positioning of measuring equipment on the machine table. If the table is to be subject to a complex strain state, it means that the resulting measurements (kinematic error characteristics) will depend on the location of the measuring apparatus on the table. Are the effects of deformability significant in experimental measurements and do they have any effect on the angular errors of pitch and roll? These issues will be discussed further below.

### 3. Experimental verification of computer simulations

One of the most interesting conclusions based on the results of simulation studies was a demonstration of significant deviation from the assumption of the perfect rigidity of the machine components. This particularly concerns movable machine tables, which under the influence of kinematic excitations resulting from guideway geometric errors, may be subject to complex states of strain. Effects of deformation effects on the characteristics of the machine table kinematic errors were verified experimentally. In the absence of significant deformation/strain, characteristics of angular errors should be independent of the location of the measuring apparatus on the table.

For discussion not limited to one design solution, we examined three different axial milling machines with the following kinematic structures and types of guideways:

- Milling Machine # 1 – XYOZ kinematic structure, table size 400 x 900 mm, rolling guides,
- Milling Machine # 2 – XZOY kinematic structure, table size 500 x 1750 mm, slide guides,
- Milling Machine # 3 – XYOZ kinematic structure, table size 350 x 650 mm, rolling guides,

Among the studied milling machines, # 3 had the smallest spacing of the X-axis guides.

The examination concerned pitch and roll errors. Measurements were made by electronic spirit levels. Measuring spirit levels were placed at the corners of the table. A reference level was placed on a stationary element of the machine. This approach enabled measurement of the table rotation relative to the stationary element. Figures 4, 5 and 6 show the arrangement of the measuring instruments and measurement results. Error bars represent the intervals of uncertainty (coverage factor  $k = 2$ ) from budgeting repeatability (standard deviation of a single measurement in the series) and the accuracy of the indication level.

The above-mentioned results of measurements of pitch and roll errors for medium-sized machines with movable (translational) tables indicate the following:

- The results of measurements of angular errors in various places of the table confirmed the conclusions based on simulation studies, i.e. tables in this group of machines may be subject to a complex strain state resulting from guideway geometric errors as evidenced by the qualitatively and quantitatively varied characteristics of angular errors. This particularly concerns roll error, which depending on the measuring site on the machine table, changes not only its value but even the direction of increase – Fig. 4c and Fig. 5c.

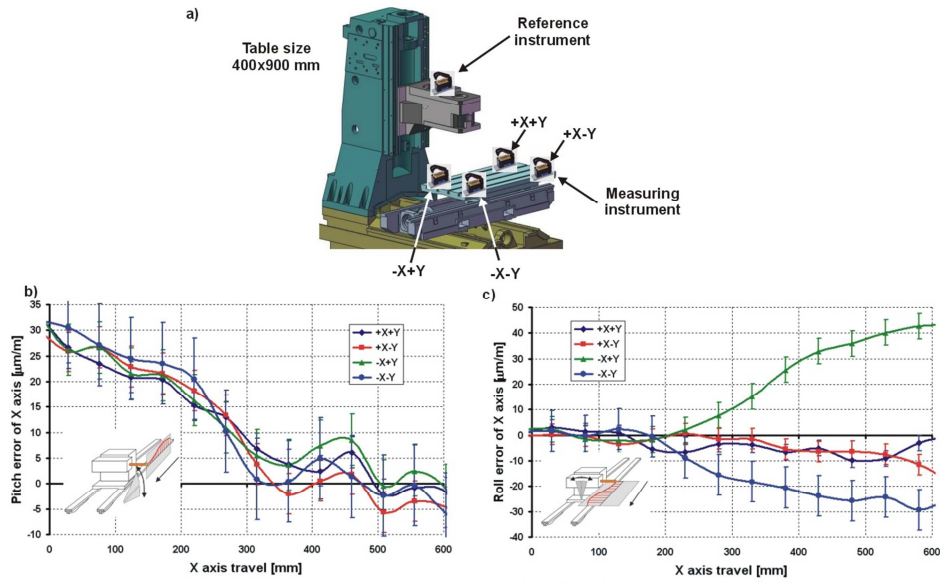


Fig. 4. The results of measurements of angular errors in the X axis of milling machine # 1:  
a) location of levels, b) pitch error, c) roll error

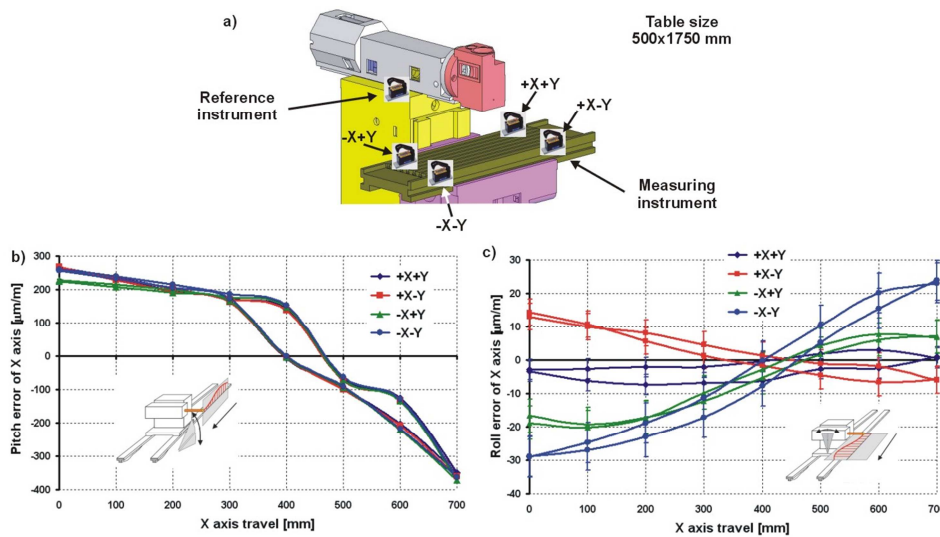


Fig. 5. The results of measurements of angular errors in the X axis of milling machine # 2:  
a) location of levels, b) pitch error, c) the roll error

• Comparing the results of examinations of sliding guideways (Fig. 5) and rolling (which had?) guideways (Fig. 4 and Fig. 6) we observed a significant



difference in the character of the measured angular errors. In machines with sliding guideways these errors had a visible hysteresis in their plot. Such effects were not observed for machines with rolling guideways. The effect of hysteresis was especially visible for pitch error in Fig. 5b. The value of this error was one order of magnitude higher than the values of all other errors examined in this study. This results from the construction of the machine in which the table was of a considerable size (mass). The displacement of the table is accompanied by the movement of the center of gravity and generation of a significant torque on the guides Z axis. This torque results in the deformation of guides in line with the direction of gravity, known as the “overhang of the table”. Lack of symmetry of this characteristics can be explained by the irregular weight distribution. On the right side of the table there is a cast iron mounting element of the drive.

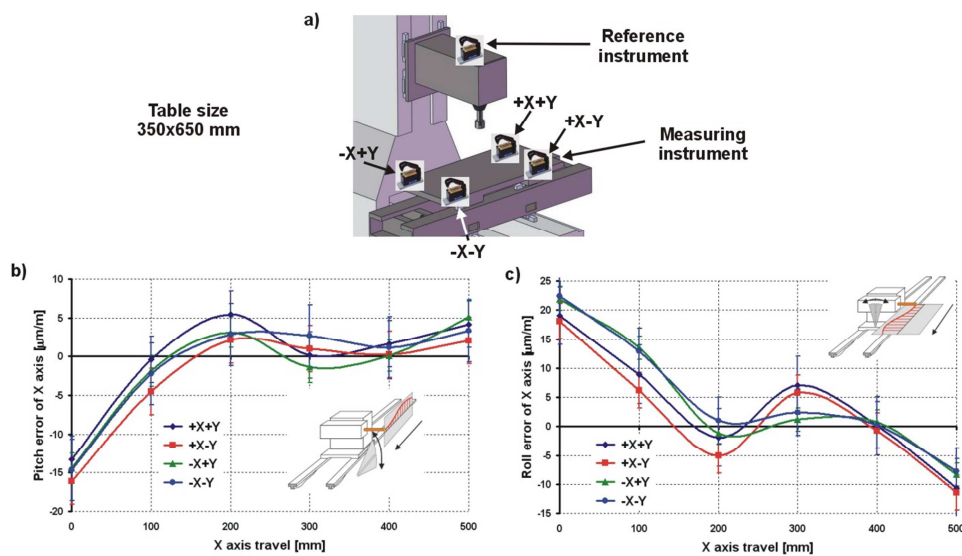


Fig. 6. The results of measurements of angular error in the X axis of milling machine # 3:  
a) location level, b) pitch error, c) the roll error

- The aforementioned effects of qualitative variation of angular error characteristics in the milling machine # 3, were not as pronounced (Fig. 6) as for the other machines. The values of its angular errors are comparable with the values obtained for the other machines. Milling machine # 3 had the smallest length and width of table, with a similar thickness. Therefore, the table of this machine is stiffer than in other machines, and thus better reproduces the kinematics of a rigid body – which explains the plot of the angular errors presented in Fig. 6.

#### 4. Conclusion

This paper presented FEM modeling of serial kinematic structures (such as machine tools and/or measuring machines), taking into account rolling guideway geometric errors. It helped clearly identify the cause of the variation of angular error characteristics of machine tools. The methodology may in the future take into account errors caused by thermal deformation and cutting forces. Simulations using this model enabled analytical examination of the effects of strain in the body elements of a typical mid-sized milling machine. Predictions based on simulations were experimentally confirmed, i.e. cross tables of real machines are subject to deformation due to the effect of guideway geometric errors.

The consequence of this phenomenon is the Abbe error [23], the value of which depends on the angle of rotation – which is equivalent with the dependence of its value on the pitch and roll errors examined in this paper. Thus Abbe errors occur during the measurement of errors of kinematic characteristics of the machine. Its value will be different in the corners and different in the central area of the table. Thus, any method of measuring errors of machine tools, which requires locating the elements of metrological devices on the table, will be affected by this phenomenon. This is of fundamental importance for the different methods of measuring VE. Always measure the VE in a specific configuration for the location of items metrological devices on the machine table. This means that the distribution of VE for a given configuration may be different from the distribution determined for other arrangements of metrological devices.

Significant deviation from the rigid body assumption is fundamental for the accuracy of VE modeling with regard to machine tools with cross tables. Commonly known VE modeling methods based on geometric rigid body transformations (homogeneous transformation matrix or the Denavit-Hartenberg notation) are inconsistent with the assumptions in the examined case – and so they are inaccurate.

The effects of the examined deformations of machine tables indicate that they may be more vulnerable (especially to bending in planes normal to the surface of the table) than guideways. Such a situation can be controlled by providing an adequate ratio between the stiffness of the table and guideways. Therefore, one should consider a recommendation for machine table designs. Such a recommendation should enable, taking into account the sustainability of the guideway, selection of the table's stiffness at which the guideways "will take into account the effects of deformations of their own geometric errors." This conclusion should be regarded as the practical effect of this present study.

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