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MODELING THE SHAPE OF RAW MATERIALS ON THE EXAMPLE OF WALNUTS FRUIT (*JUGLANS REGIA L.*)[®]

Modelowanie kształtu surowców spożywczych na przykładzie owoców
 orzecha włoskiego (*Juglans regia L.*)[®]

Key words: modeling, common walnut, surface area, volume.

This article describes a direct method and an indirect method for acquiring information about the geometric parameters of common walnut fruit. The direct method involved measurements with the use of a caliper and geometric models (1D method). The indirect method was based on digital models constructed by 3D scanning (3D method). The aim of this study was to evaluate the accuracy of the above measurement methods in determining the surface area and volume of walnut fruit. The analysis of the two methods for determining the geometric parameters of walnuts revealed that the 3D method delivered more accurate results. In the 1D method, the surface area of walnut fruit can be determined with the use of a sphere (M1) and a spheroid (M4), and the volume of walnut fruit can be determined with the use of a sphere (M1), a spheroid (M4) and an ellipsoid (M5).

Symbols

A – total surface area (mm²),
 d_w – arithmetic mean diameter (mm),
 d_z – equivalent diameter (mm),
 L – length (mm),
 N – sample size,
 T – thickness (mm),
 V – volume (mm³),
 W – width (mm),
 1D – measurements involving the direct method,
 3D – measurements involving the 3D numerical model.

Słowa kluczowe: modelowanie, orzech włoski, pole powierzchni, objętość.

W artykule omówiono metodę bezpośrednią oraz metodę pośrednią pozyskiwania informacji o geometrycznych parametrach owoców orzecha włoskiego. Metoda pomiaru bezpośredniego, wykonana była z pomocą suwmiarki i modeli geometrycznych (metoda 1D). Metoda pośrednia oparta była na przestrzennych modelach numerycznych otrzymanych za pomocą skanowania 3D (metoda 3D). Celem pracy była ocena wyżej wymienionych metod pomiarowych w zakresie dokładności wyznaczania pola powierzchni i objętości owoców orzecha włoskiego. Z przeprowadzonych badań na owocach, wynika, że spośród zastosowanych dwóch metod wyznaczenia parametrów geometrycznych owoców najlepsze efekty uzyskano przy metodzie 3D. Do wyznaczenia pola powierzchni owoców metodą 1D można zastosować kulę (M1) i model elipsoidy obrotowej (M4). Wyznaczając objętość owoców orzecha włoskiego metodą 1D można wykorzystać kulę (M1), elipsoidę obrotową (M4) i elipsoidę (M5).

Wykaz ważniejszych oznaczeń

A – pole powierzchni całkowitej (mm²),
 d_w – arytmetyczna średnica zastępcza (mm),
 d_z – średnica zastępcza (mm),
 L – długość (mm),
 N – liczebność próby,
 T – grubość (mm),
 V – objętość (mm³),
 W – szerokość (mm),
 1D – pomiar metodą bezpośrednią,
 3D – pomiar metodą opartą o przestrzenny model numeryczny.

INTRODUCTION

Surface area and volume are used to design the processes of cleaning, dressing, peeling, coating and packaging of food raw materials and products [4, 10, 12, 13, 22]. Geometric parameters are usually determined with measuring devices that enable direct linear measurements, such as rulers, analog and digital calipers, and computer image analysis [5, 7, 11, 15]. All of the above methods support direct and rapid measurements of the studied object's linear dimensions, and they do not require the use of expensive devices. In most cases, a given object can be measured and the results can be recorded without any special preparation of the analyzed samples. The disadvantage of these methods is that the obtained results have numerous limitations, and they apply only to selected points on the sample. Additional calculations can also be performed using mathematical formulas from the literature [6, 8, 9].

The location of the points used to develop digital models of the analyzed samples is registered with 3D scanners. Non-contact 3D scanners register the distribution of points in the space of the analyzed object [17, 20]). A digital 3D model can be used to perform metrological analyses. The model's accuracy is determined by the scanner's resolution [18]. The digital model represents the shape of the analyzed sample, and it can be stored in a computer. This method can be applied to measure both small-sized and large-sized raw materials that are brittle and susceptible to damage [1, 2]. Two methods for acquiring information about the physical parameters of food raw materials are described in this article. The first method involves direct measurements with the use of a caliper (1D method), and the second method is based on digital models constructed by 3D scanning (3D method).

The aim of this study was to evaluate the accuracy of the above measurement methods in determining the basic geometric parameters of food raw materials on the examples of common walnut (*Juglans regia* L.) fruit. The relative error in the surface area and volume of walnuts determined with the analyzed methods was compared.

MATERIALS AND METHODS

The experimental material comprised common walnut (*Juglans regia* L.) fruit purchased in a grocery store of the PSS Społem retail chain in Olsztyn. Thirty walnuts without visible signs of damage were selected randomly for analysis. The walnuts were stored indoors, at a constant temperature of $20 \pm 1^\circ\text{C}$ and relative humidity of approximately 65%. The linear dimensions of walnuts were measured with a caliper with a resolution of 0.01 mm. The surface area and volume of walnuts were determined with the use of six geometric models composed of selected geometric shapes (Fig. 1).

The following mathematical formulas, describing surface area and volume, were derived for each geometric model:

– sphere (M1):

$$A_{M1} = \pi \cdot d_w^2 \quad (1)$$

$$V_{M1} = \frac{\pi \cdot d_w^3}{6} \quad (2)$$

– cylinder (M2):

$$A_{M2} = \pi \cdot d_z \cdot L + 2 \cdot \pi \cdot \left(\frac{d_z}{2}\right)^2 \quad (3)$$

$$V_{M2} = \frac{\pi \cdot d_z^2 \cdot L}{4} \quad (4)$$

– elliptic cylinder (M3):

$$A_{M3} \approx \pi \cdot L \cdot \left(\frac{3}{4} \cdot (W + T) - \sqrt{\frac{W \cdot T}{4}}\right) + 2 \cdot \pi \cdot \frac{W \cdot T}{4} \quad (5)$$

$$V_{M3} = \frac{\pi \cdot W \cdot T \cdot L}{4} \quad (6)$$

– spheroid (M4) when: $\frac{L}{2} > \frac{d_z}{2}$

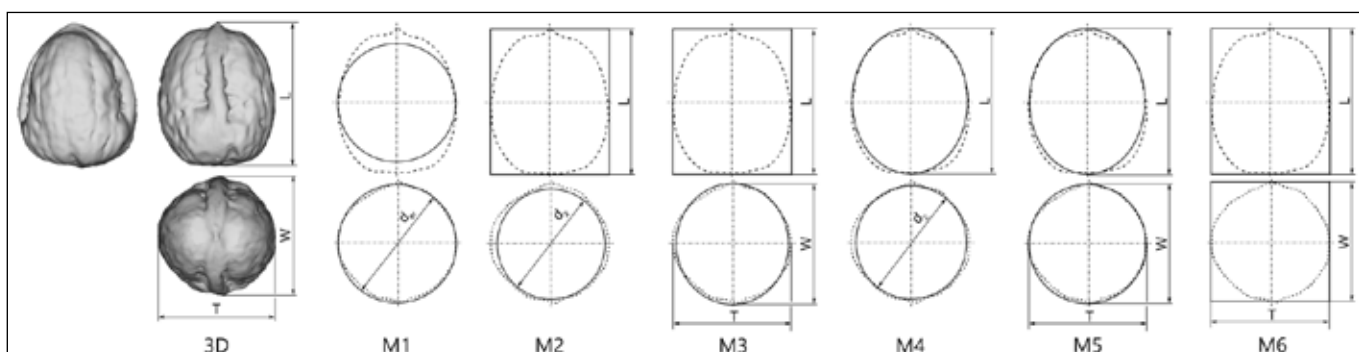


Fig. 1. Digital and geometric models of walnut fruit: 3D – digital model, M1 – sphere, M2 – cylinder, M3 – elliptic cylinder, M4 – spheroid, M5 – ellipsoid, M6 – cuboid; d_w – arithmetic mean diameter, d_z – mean diameter, L – length, W – width, T – thickness.

Rys. 1. Model numeryczny i modele geometryczne owoców orzecha włoskiego: 3D – model numeryczny, M1 – kula, M2 – walec, M3 – walec eliptyczny, M4 – elipsoida obrotowa, M5 – elipsoida, M6 – prostopadłościan; d_w – arytmetyczna średnica zastępcza, d_z – średnica zastępcza, L – długość, W – szerokość, T – grubość.

Source: The own study

Źródło: Badania własne

then:

$$A_{M4} = 2 \cdot \pi \cdot \left(\frac{d_z}{2}\right)^2 \cdot \left(1 + \frac{L}{\frac{d_z}{2} \cdot e} \cdot \arcsin(e)\right) = \frac{4 \cdot \pi \cdot d_z^2 + \pi \cdot L \cdot d_z \cdot e \cdot \arcsin(e)}{8} \quad (7)$$

where:

$$e = \sqrt{1 - \frac{d_z^2}{L^2}} \quad (8)$$

$$V_{M4} = \frac{\pi \cdot d_z^2 \cdot L}{6} \quad (9)$$

– ellipsoid (M5):

$$A_{M5} = 2 \cdot \pi \cdot \left[\left(\frac{L}{2}\right)^2 + \frac{\frac{T}{2} \cdot \left(\frac{L}{2}\right)^2}{\sqrt{\left(\frac{W}{2}\right)^2 - \left(\frac{L}{2}\right)^2}} \cdot F(\Theta, m) + \frac{T}{2} \cdot \sqrt{\left(\frac{W}{2}\right)^2 - \left(\frac{L}{2}\right)^2} \cdot E(\Theta, m) \right] \quad (10)$$

where:

$$m = \frac{\left(\frac{L}{2}\right)^2 \cdot \left(\left(\frac{T}{2}\right)^2 - \left(\frac{L}{2}\right)^2\right)}{\left(\frac{T}{2}\right)^2 \cdot \left(\left(\frac{W}{2}\right)^2 - \left(\frac{L}{2}\right)^2\right)} = \frac{L^2 \cdot T^2 - L^4}{T^2 \cdot W^2 - L^2 \cdot T^2} \quad (11)$$

$$\Theta = \arcsin\left(\sqrt{\frac{\sqrt{W^2 - L^2}}{|W|}}\right) \quad (12)$$

and $F(Q, m)$ and $E(Q, m)$ are incomplete elliptic integrals of the first and second kind [3]:

$$V_{M5} = \frac{\pi \cdot T \cdot W \cdot L}{6} \quad (13)$$

– cuboid (M6):

$$A_{M6} = 2 \cdot L \cdot W + 2 \cdot L \cdot T + 2 \cdot T \cdot W = 2 \cdot (L \cdot W + L \cdot T + T \cdot W) \quad (14)$$

$$V_{M6} = L \cdot W \cdot T \quad (15)$$

In models M1, M2 and M4, equivalent diameters were calculated with the following formulas:

$$d_w = \frac{L + W + T}{3} \quad (16)$$

$$d_z = \frac{W + T}{2} \quad (17)$$

Digital 3D models of walnut fruit were generated with the use of a NextEngine 3D laser scanner [16]. Scanning resolution was 248 points per mm², and the average scanning time to produce a complete model was approximately 30 minutes. The acquired series of 3D scans for constructing digital models were combined in the ScanStudio HD PRO program [16]. The MeshLab program was used to measure surface area and volume in the generated models [14]. The significance of differences between the average values of total surface area and volume of walnuts, measured with the 1D method and the 3D method, was compared. One-way analysis of variance (ANOVA) for multiple independent samples was conducted to identify homogeneous groups. Calculations were performed at a significance level of $\alpha = 0.05$, using STATISTICA v.13 PL software [3, 19, 21].

RESULTS

The surface area and volume of walnut fruit were measured based on digital models constructed by 3D scanning. The total surface area of walnuts determined with the 1D method and the 3D method is presented in Figure 2.

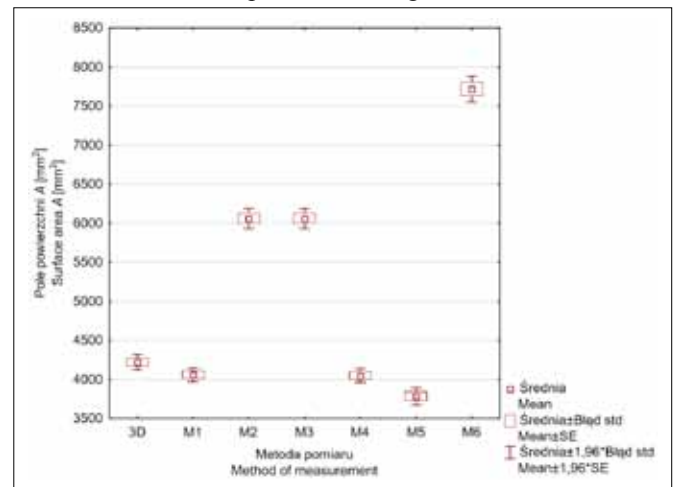


Fig. 2. Surface area of walnut fruit, determined by the 1D and 3D method.

Rys. 2. Pole powierzchni owoców orzecha włoskiego wyznaczone metodą 1D i 3D.

Source: The own study

Źródło: Badania własne

The results revealed that the analyzed parameters had a normal distribution. The significance of differences between the parameters of walnut fruit measured with the compared methods is presented in Table 1.

It was assumed that measurements of the surface area of walnut fruit, performed with the 3D method, were not burdened with error and that the obtained results can be used as a reference for the measurements performed with the 1D method. The relative error between the values obtained by direct measurement with the 1D method and those obtained using the 3D method was referred to as the “error of the method”. As shown

in Figure 3, the mean relative error was 3.77% and 3.98% when geometric models M1 and M4, respectively, were applied to measure the surface area of walnuts in the 1D method. Relative error was higher than 5% when the remaining geometric models were used in the 1D method.

Table 1. Results of the calculations to verify the significance of differences between the mean values of the area

Tabela 1. Wyniki obliczeń weryfikacji istotności różnic między średnimi wartościami pola powierzchni

Pole powierzchni *A* (ANOVA test) / Area *A* (ANOVA test)

$F(6, N=203)=601,47; p=0,000$

Prawdopodobieństwa porównań wielokrotnych /
Probability of multiple comparisons

Metoda pomiaru / Measurement method	Liczebność próby / Number of observations N	Współczynnik zmienności / Coefficient of variation	Odczylenie standardowe / Standard deviation	Średnia / Mean (mm ²)
3D	30	6,71	283,59	4220,96 ^a
M1	30	5,99	243,27	4059,40 ^a
M2	30	6,03	365,58	6061,35 ^b
M3	30	6,03	365,54	6061,20 ^b
M4	30	6,01	243,51	4050,72 ^a
M5	30	8,16	309,14	3784,39 ^c
M6	30	6,03	465,36	7717,09 ^d

Values in columns marked with identical letters do not differ significantly: a, b, c, d ($P \leq 0,05$)

Wartości w kolumnach z takimi samymi literami nie różnią się istotnie; a, b, c, d ($P \leq 0,05$)

Source: The own study

Źródło: Badania własne

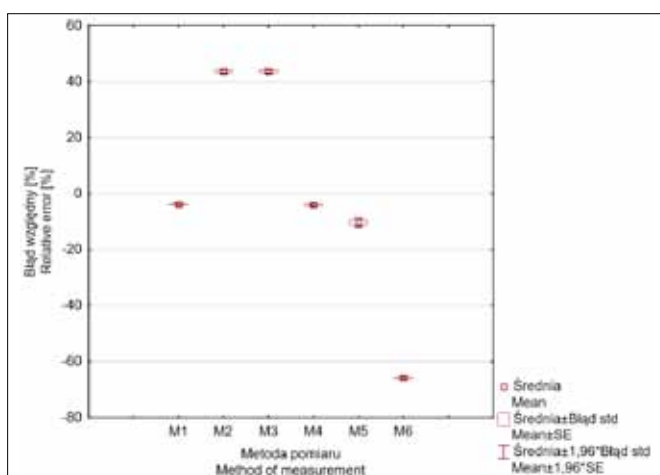


Fig. 3. Relative error in determining the surface area of walnut by the 1D method with the use of geometric models and the 3D method.

Rys. 3. Błąd względny wyznaczania pola powierzchni owoców orzecha włoskiego metodą 1D z wykorzystaniem modeli geometrycznych i metodą 3D.

Source: The own study

Źródło: Badania własne

The volume of walnut fruit determined with the analyzed measurement methods is presented in Figure 4.

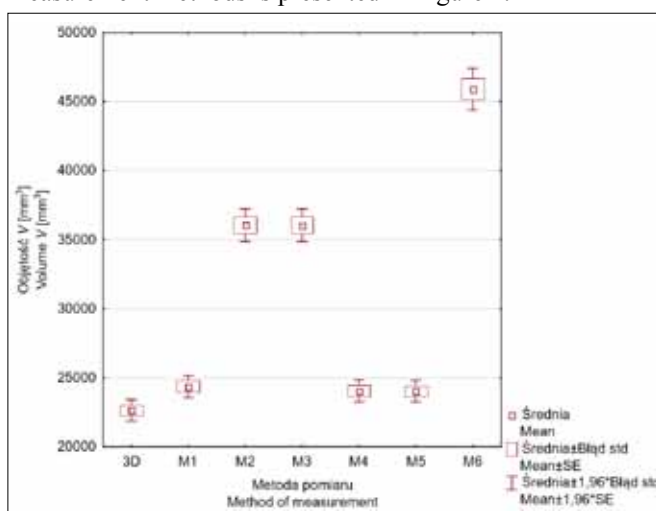


Fig. 4. Volume of walnut fruit, determined by the 1D and 3D method.

Rys. 4. Objętość owoców orzecha włoskiego wyznaczona metodą 1D i 3D.

Source: The own study

Źródło: Badania własne

The significance of differences between the average values of walnut volume was estimated by ANOVA. The results of detailed comparative analyses are presented in Table 2.

Table 2. Results of the calculations to verify the significance of differences between mean values of volume

Tabela 2. Wyniki obliczeń weryfikacji istotności różnic między średnimi wartościami objętości

Objętość *V* (ANOVA test) / Volume *V* (ANOVA test)

$F(6, N=203)=286,81; p=0,000$

Prawdopodobieństwa porównań wielokrotnych /
Probability of multiple comparisons

Metoda pomiaru / Measurement method	Liczebność próby / Number of observations N	Współczynnik zmienności / Coefficient of variation	Odczylenie standardowe / Standard deviation	Średnia / Mean (mm ³)
3D	30	9,50	2149,73	22626,00 ^a
M1	30	9,05	2205,97	24351,68 ^b
M2	30	9,17	3308,59	36052,72 ^c
M3	30	9,17	3306,57	36045,26 ^c
M4	30	9,17	2205,73	24035,14 ^{ab}
M5	30	9,17	2204,38	24030,17 ^{ab}
M6	30	9,17	4210,06	45894,25 ^d

Values in columns marked with identical letters do not differ significantly: a, b, c, d ($P \leq 0,05$)

Wartości w kolumnach z takimi samymi literami nie różnią się istotnie; a, b, c, d ($P \leq 0,05$)

Source: The own study

Źródło: Badania własne

It was assumed that measurements of the volume of walnut fruit, performed with the 3D method, were not burdened with error and that the obtained results can be used as a reference for the measurements performed with the 1D method. As shown in Figure 5, the mean relative error ranged from 6.25% to 7.69% when geometric models M1, M4 and M5 were applied to measure the volume of walnuts in the 1D method.

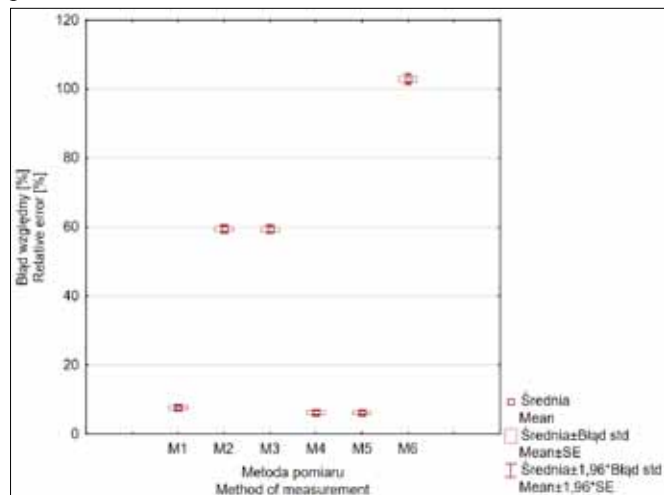


Fig. 5. Relative error in determining the volume of walnut by the 1D method with the use of geometric models and the 3D method.

Rys. 5. Błąd względny wyznaczania objętości owoców orzecha włoskiego metodą 1D z wykorzystaniem modeli geometrycznych oraz metodą 3D.

Source: The own study

Źródło: Badania własne

CONCLUSIONS

The analysis of the two methods for determining the geometric parameters of walnuts revealed that the 3D method delivered more accurate results. The geometric parameters

(linear dimensions, surface area, volume) of entire samples and their selected fragments can be determined based on digital 3D models. The surface area of walnuts was more accurately measured using digital 3D models (3D method) than the direct method involving geometric models (1D method). In the 1D method, the surface area of walnut fruit can be determined with the use of a sphere (M1) and a spheroid (M4), and the volume of walnut fruit can be determined with the use of a sphere (M1), a spheroid (M4) and an ellipsoid (M5). When geometric models M1 and M4 were applied to measure the surface area of walnuts in the 1D method, the mean relative error was 3.77% and 3.98%, respectively. When models M1, M4 and M5 were used to measure the volume of walnuts, the mean relative error ranged from 6.25% to 7.69%.

WNIOSKI

Z przeprowadzonych badań na owocach orzecha włoskiego, wynika, że spośród zastosowanych dwóch metod wyznaczenia parametrów geometrycznych owoców najlepsze efekty uzyskano przy metodzie 3D. Z przestrzennych modeli numerycznych można wyznaczyć parametry geometryczne (wymiary, powierzchnię, objętość) całych próbek jak i ich wybranych fragmentów. Pomiar pola powierzchni owoców z wykorzystaniem przestrzennych modeli numerycznych (metoda 3D) jest dokładniejszy niż pomiar pola powierzchni metodą bezpośrednią z wykorzystaniem modeli geometrycznych (metoda 1D). Do wyznaczenia pola powierzchni owoców metodą 1D można zastosować kulę (M1) i model elipsoidy obrotowej (M4). Wyznaczając objętość owoców orzecha włoskiego metodą 1D można wykorzystać kulę (M1), elipsoidę obrotową (M4) i elipsoidę (M5). Średni błąd względny pomiaru pola powierzchni stosując metodę pomiaru 1D i wykorzystując modele geometryczne M1 i M4 wyniósł 3,77% i 3,98%. Wykorzystując do wyznaczania objętości owoców modele M1, M4 i M5 popelnia się błąd względny pomiaru od 6,25% do 7,69%.

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