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Modelling the accident probability in large-scale, maritime transportation system

Keywords

maritime transportation system, safety, collision probability, traffic simulation, semi-Markov model

Abstract

The probability of an accident in transportation systems can serve as a measure of these systems safety or risk, depending on the objective. Therefore numerous methods and models for risk evaluation, with respect to maritime, have been developed. However, these models are either too simplified, allowing relatively fast analysis but very often missing the substantial links among the model variables, or they are too slow for effective analysis, due to computational complexity, not necessarily being backed-up with the complexity of the model itself. Thereby, this paper introduces a novel method evaluating the probability of ship-ship collision in the maritime transportation system focusing on the open sea collisions, applying the queuing theory in the simulation model. The model allows relatively fast prediction as it focuses on the specific events (e.g. accidents), instead of simulating the whole traffic. To support this hypothesis a case study is presented focusing on a selected element of transportation system in operation.

1. Introduction

The probability of an accident in transportation systems can serve as a measure of these systems safety or risk, depending on the objective. When the former is in question, only the probability of an accident seems to be good enough. However when the risk is an objective, then firstly the proper definition should be adopted, see [1] and secondly the accident consequences should be accounted for as well. Nevertheless, the probability itself is an essentail factor in either of these two cases, thus its proper and effective modelling is of high importance.
A model for the probability evaluation should be complex enough to be able to capture the relations among variables allowing further analysis of accident consequences. On the other hand it should be computationaly effective to simulate numerous scenarios in reasonable time span. Otherwise such a model has rather weak potential to be used for effective and reliable process of risk management through the optimization of either the accident probability or its consequences. In the recent years numerous methods and models for risk evaluation, with respect to maritime, have been developed, for the literature review a reader is referred to [4], [8], [11]. However, most of existing models are either too simplified, allowing relatively fast analysis but very often they are missing substantial links among the model variables, or they are too slow for effective analysis, due to computational complexity, not necessarily backed-up with the complexity of the model itself.
Thereby, this paper introduces a novel method evaluating the probability of ship-ship collision in the maritime transportation system focusing on the open sea collisions, applying the queuing theory in the simulation model, see [13]. The model allows relatively fast prediction as it focuses on the selected discrete events, which can be defined arbitrary

depending on the criteria adopted, instead of simulating the whole traffic. In this paper the following events are considered, with the corresponding distances between ships, see Figure 1: collision alert (state 0, ships are within distance less than d1), high risk of collision (state 1, distance <d2), low risk of collision (state 2, distance < d3) and negligible risk of collision (state 3, distance <3nm). Thereby the presented model is able to predict not only the number of accidents but also the number of potentially dangerous situations, referred to as "near-misses", which might be even more relevant indicator for evaluation of the system safety than the number of accidents as in the reality the former occur more frequently than the latter, see [6], [10].
Finally a case study is presented with the use of the model, focusing on a selected waterways junction. The obtained results are presented and discussed.

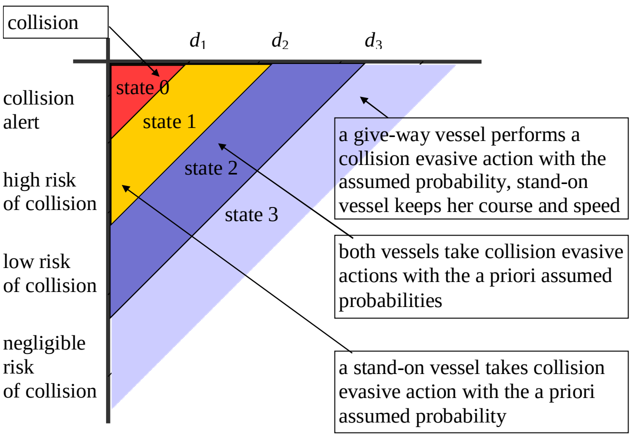

Figure 1. The discreet events adopted in the presented model.

2. Methods and models

This paper introduces a novel model evaluating the probability of failure occurrence in a transportation system, namely the ship-ship collision in the open sea. The model assumes maritime transportation system to be a discrete-events system, which can be described by two parameters: firstly by a time instant at which a discrete event takes place and secondly by the transition between states at this time instant. Basically the discrete-event models utilize three approaches as follows: event scheduling, process interaction or activity scanning. The first approach focuses on selected event, e.g. the time instants when system transition occurs, while the second focuses on processes, e.g. the flow of each entity through the system. Whereas in the third approach the conditions of all events defined in the model are scanned for each simulation run. For the detailed description of these methods a reader is referred to [14].

In this paper the discrete-event system is simulated using the second approached namely the event scheduling.

2.1. Maritime transportation system

Following the logic of the discrete-event systems the major elements of the analyzed transportation system need to be defined, namely: system infrastructure (waterways), system components (ships with their attributes), system components dynamics (traffic intensity and collision evasive manoeuvers), traffic events (state 0-3, see *Figure 1*) and the probabilities of occurrence of these traffic events.
The analyzed maritime transportation system refers to the selected part of the system operating under non-ice conditions in the Gulf of Finland between Helsinki and Tallinn.

System infrastructure
The analyzed transportation system consists of four waterways (North, South, East and West) with four potential collision areas, see *Figure 2*. The main flows are on the East-West lanes while two crossing flows referred to as collision flows are on North and South waterways.

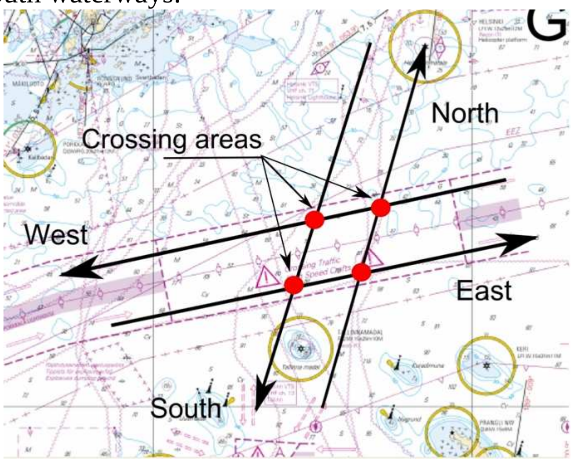

Figure 2. The maritime transportation system arrangement

Secondly, utilizing the system analysis formalism, the presented transportation system state can be defined as a three-dimensional vector consisting of independent components (X(t),O(t),Y(t)), where:

- X(t) – time between ships in the main flows,
- O(t) – time that the ship in the main flow spent in a collision area (the area bounded by the crossing areas),
- Y(t) – time interval between ships in the collision flows.

Then, the following definitions are given:

Definition 1. There is a collision situation if a ship cannot continue to move in an unimpeded manner and has to change the ship course or ship speed.
Definition 2. There is a collision threat if the time span between ships in collision area, measured with respect to a point of potential meeting, is smaller than the adopted admissible value:

$$X(t) \le Y(t) \le X(t) + O(t)\,. \qquad (1)$$

System components
In the analyzed system, six major ship types are considered as follows: a tanker, a container carrier, a passenger ship, a RoPax, a general cargo ship and a fast ferry. The ships attributes namely ship's dimensions, ship speed, ship course, mean velocity for different types of vessels at each waterway is adopted from the former study by [10].

System components dynamics
The system dynamics can be described in two-fold, firstly defining the dynamics of its component (traffic intensity) and secondly evaluating the ways how the system elements (ships) behave being in a collision situation.
Traffic intensity data for the analyzed transportation system comes from the previous work of [10] and concerns the traffic in the selected area of the Gulf of Finland, namely the waterways junction between Helsinki and Tallinn. The overall picture on the traffic intensity in the given area is depicted in *Figure 3*.

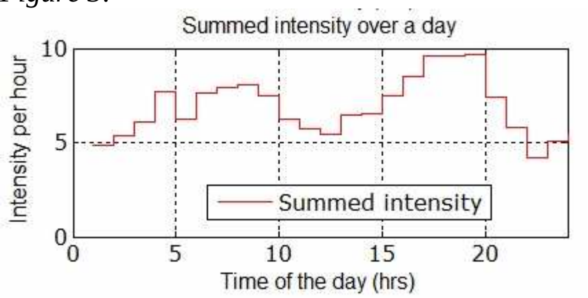

Figure 3. The intensity of the traffic streams in the analyzed transportation system.

Whereas, the collision evasive action is modeled, referring to the COLREG rule 17, where four actions, permitted or required, for each vessel given the collision course are defined as follows:

- at long range, both vessels are free to take any action however when a risk of collision is prominent, the give-way vessel is required to take proper action to achieve a safe passing distance and the stand-on vessel must keep her course and speed; we assume that the distance at which this stage commences is 3 nm, see [2];
- when it becomes apparent that the give-way vessel is not taking appropriate action, the stand-on vessel is permitted to take action to avoid collision by her maneuver alone; we assume that the outer limit of the third stage is 2 nm, see [2];
- when a collision cannot be avoided by the give-way vessel alone, the stand-on vessel must take such action.

In the model we a priori assume the following probabilities of taking evasive action by give-a-way and stand-on vessels, see Chauvin&Lardjane (2008):

- a give-a-way vessel: 0.94 for a ferry and 0.67 for other ship type;
- a stand-on vessel: 0.45 for a ferry and 0.32 for a cargo ship.

Obviously these numbers should be validated with the traffic observation, as they affect significantly the model output.

Traffic events
Four traffic events corresponding to four levels of risk are considered here. These depend on the mutual distance of vessels being on collision courses, see *Figure 1*. We assume that the waterway is divided into cells with a length of $CS=1/\rho_{cross}$ and the total crossing density ρ_{cross} is determined with the use of the following formulae:

$$\rho_{cross} = \max\{\rho_{EW-NS}\},$$

$$\rho_{EW-NS} = \sqrt{\rho_{EW}^{\ 2} + \rho_{NS}^{\ 2}},$$

$$\rho_{13} = \sqrt{\rho_1^{\ 2} + \rho_3^{\ 2}}\,, \qquad (2)$$

$$\rho_{EW} = \frac{LOA_{EW}}{T_{EW} \cdot V_{EW} + LOA_{EW}},$$

$$\rho_{NS} = \frac{LOA_{NS}}{T_{NS} \cdot V_{NS} + LOA_{NS}},$$

where LOA_{EW} – a length of a ship on the E-W waterway, LOA_{NS} a length of a ship on the N-S waterways, T_{EW}, T_{NS}, – the mean times between ships departing for the given waterways. The distance is measured in taxicab metric, according to the sequence of grid cells.

When defining the risk levels we assume that the collision happens (collision alert in *Figure 1*) if two encountering ships, being on collision courses are within a distance which is less than the adopted criterion called MDTC, see [10]. MDTC is the smallest distance between two ships being on collision courses, at which the collision evasive actions are still effective, however if the distance goes beyond this limit, the collision is inevitable. MDTC has been calculated for various ship types, and full range of relative bearings and collision angles, where the latter is defined as a difference between ships courses. In this paper we analyse the case, where the collision angle is close to 100 degree, and the corresponding MDTC values yield: 6*LOA* for all ship types excluding tankers and 8.5*LOA* for tankers. This values are dependent on the collision evasive manoeuvers taken, thus we consider here a situation when the evasive action is taken only by a give-a-way ship whereas stand-on ship follows her initial course and speed.

Then, to define the second risk level (high risk of collision in *Figure 1*) we assume a situation where two encountering ships being on collision courses are within distance of at least 1 nm. This distance will be considered in the simulation as a critical value between high and low risk of collision.

Furthermore we assume a distance of 3 *nm* as a safety distance corresponding to the negligible risk of collision, see *Figure 1*.

Then these risk levels have been expressed in the language that the model uses, in the following manner:

high risk of collision – both ships are entering a cell and a number of blank cells between two ships is equal at least d_1-2; one ship is entering a cell and second ship is in a cell or is leaving a cell and a number of blank cells between them is not less than d_1-1; both ships are in a cell or are leaving a cell and a number of blank cells between them is not less than d_1; if none of these conditions if fulfilled we define this situation as *a collision alert*; distance d_1 is determined from the equation:

$$d_1 = \left\lceil \frac{MDTC}{CS} \right\rceil \text{ [m]}, \tag{3}$$

Low risk of collision: both ships are entering a cell and a number of blank cells between these two ships is not less than d_2-2; one ship is entering a cell and a second ship is in a cell or is leaving a cell and a number of blank cells between these two ships is not less than d_2-1; both ships are in a cell or leaving a cell and a number of blank cells between them is equal not less than d_2; where the distance d_2 is as follows:

$$d_2 = \left\lceil \frac{1852}{CS} \right\rceil \text{ [m]}, \tag{4}$$

Negligible risk of collision: both ships are entering a cell and a number of blank cells between them is not less than d_3-2; one ship is entering a cell and another ship is in a cell or is leaving a cell and a number of blank cells between them is not less than d_3-1; both ships are in a cell or are leaving a cell and a number of blank cells between them is not less than d_3; where the distance d_3 is determined from the equation

$$d_3 = \left\lceil \frac{3 \cdot 1852}{CS} \right\rceil \text{ [m]}, \tag{5}$$

The presented model assumes varying the simulation time step (Δt), which corresponds to the speed of the fastest vessel in the analyzed cells. Thereby the Δt is determined as follows:

$$\Delta t = \frac{CS}{V_{\max}}, \tag{6}$$

where $V_{max}=max\{V_1, V_2, \dots V_n\}$ and V_n is a ship speed in knots on a given waterway and in the simulation it corresponds to the speed measured in cells pre time step.

Modeling the probability of a given traffic event

In order to obtain the probability of a given traffic event, defined in the previous section, we adopted the event scheduling approach. Thereby we take into consideration the $G_2/G_2/2$ queuing system with losses, a general arrival process, a general service process and a pair of double servers. The arrival process is considered as a semi-Markov stationary point process; see [13], [14]. Therefore it can be described by a transition matrix $[p_{ij}]$ and a matrix of conditional transition times distributions called Markov kernel $[F_{ij}]$, $i, j = 1,2,\dots, i \neq j$, where F_{ij} is a cumulative probability distribution of a holding time of a state i, if the next state will be j. The asymptotic probabilities $p_i(t)$ are given by formulas

$$\overline{p}_i = \lim_{t \to \infty} p_i(t) = \frac{\pi_i E[\theta_i]}{\sum_{j=1}^{m} \pi_j E[\theta_j]}, \quad \text{i = 1, 2, . . .,} \tag{7}$$

where π_i satisfy the system of equations

$$\begin{cases} [\pi_i] = [\pi_i][p_{ij}] \\ \sum_{j=1}^{m} \pi_j = 1, \end{cases} \tag{8}$$

and θ_i is a time of remaining at a state i. We assume that the embedded Markov chain is ergodic.

In the model the collision threat is tantamount to the failure appearance, where the failure is a loss of request. This means that the request is lost if the time between arriving requests is less than the residuary service time. Time instants of service starting are equal to the time instant of arriving at the collision area a ship belonging to a collision flow.
Then, we define the time between arriving requests at main router as T_α thereby its probabilistic distribution is given by a formula:

$$F_{T_\alpha}(t) = \sum_{i,j} p_i p_{ij} F_{ij}(t), \tag{9}$$

with the corresponding probability:

$$p = \int_0^\infty e^{-\mu t} dF_{T_\alpha}(t). \tag{10}$$

Whereas the probability of the event described by equation (1) is as follows:

$$\begin{aligned} &P(X(t) \le Y(t) \le X(t) + O(t)) = \\ &= P(Y(t) \le X(t) + O(t) / X(t) \\ &\le Y(t)) \cdot P(X(t) \le Y(t)) = \\ &= F_{Y-X/O}(0)[1 - F_{Y-X}(0)] \end{aligned} \tag{11}$$

Thereby the cumulative distribution functions of random variables X-Y and Y-X-O need to be determined.

2.2. Analysis of transportation system dynamics

As mentioned in previous chapter system dynamics is described in two-fold and one part was already addressed in section 2.1. However this chapter touches upon the second part, which means the statistical analysis of the dynamics of system's component, namely ships arrival intensities, see *Figure 3*. For this purpose we need to examine the analyzed flows and their nature. These has been performed with the use of the following statistical tests: the correlation test, (see *Table 1*), the randomness test (*Table 2*), the multiple range test (*Table 3*), test estimating the difference between two means (*Table 4*) and parametric tests (*Table 6*).
The correlation coefficients vary between -1 and 1 and they measure the strength of the linear relationship between two variables. The correlation is considered statistically significant, at the 95% confidence level, if the calculated P-value is less than 0.05. Otherwise such a pair of variables is not correlated. The results of the correlation of the flows intensities, expressed by Pearson moments are shown in *Table 1*.

Table 1. Test for correlation of the flows intensities.

	North Intensity	South Intensity	West Intensity
East Intensity	-0,2675	-0,5579	0,0532
P-Value	0,2063	*0,0046*	0,8049
North Intensity		0,3510	-0,0363
P-Value		0,0926	0,8662
South Intensity			0,0258
P-Value			0,9046

In the next step the statistical tests for series randomness are carried out. If a P-value for any test is greater than or equal to 0.05, we cannot reject the null hypothesis stating the series are random, assuming the 95% or even higher confidence level.

Table 2. Test for randomness

	Traffic intensities in the given flow			
	E	**W**	**N**	**S**
Runs above and below median				
Number of runs	9	9	9	7
Expected number of runs	13,0	13,0	13,0	13,0
Large sample test statistic	1,46	1,46	1,46	2,29
P-value	0,14	0,14	0,14	0,022
Runs up and down				
Number of runs	13	12	17	14
Expected number of runs	15,66	15,66	15,66	15,66
Large sample test statistic	1,09	1,59	0,41	0,59
P-value	0,27	0,11	0,67	0,56
Box-Pierce Test based on first 8 autocorrelations				
Large sample test statistic	7,98	11,56	10,61	14,56
P-value	0,44	0,17	0,22	0,07
H_0	+	+	+	-

Analysis of the stream intensity made for the hourly intervals, allows verification of the hypothesis for the Poisson point process input, see *Table 3*.

Table 3. Multiple Range Tests

	Count	*Mean*	*Homogeneous Groups*
South Intensity	24	1,21078	X
North Intensity	24	1,21487	X
East Intensity	24	2,2125	X
West Intensity	24	2,21458	X

While *Table 4* shows the pairs of variables being statistically different, assuming the 95% confidence level and these are marked with the asterisk. To discriminate among the means the Fisher's least significant difference (LSD) procedure is used, which is considered very detailed.

Table 4. The estimated difference between each pair of means

Contrast	*Sig.*	*Difference*	*+/- Limits*
East Intensity - North Intensity	*	0,997631	0,45746
East Intensity - South Intensity	*	1,00172	0,45746
East Intensity - West Intensity		-0,00208333	0,45746
North Intensity - South Intensity		0,00408497	0,45746

North Intensity - West Intensity	*	-0,999714	0,45746
South Intensity - West Intensity	*	-1,0038	0,45746

** denotes a statistically significant difference.*

In the following table the observed traffic flows intensities are depicted. Then these are analyzed with the use of non-parametric statistical tests and the results are gathered in *Table 6*.

Table 5. Non-parametrical tests for the analyzed flows

	Traffic intensities in the given flow			
	E	**W**	**N**	**S**
Sample mean	2,2125	2,2145	1,2148	1,2107
Sample median	2,0333	2,2416	1,0980	1,1470
Sample standard dev.	0,5218	0,5575	0,9511	1,0290

Thereby the presented analyzes demonstrate:

- a negative, statistically significant correlation, indicating a cyclic change of vessel traffic on SE direction, see *Table 1*.
- existence of two different groups overlapping with the main directions of SN and the EW, see *Tables 3, 4,6*.
- non constant intensity for the analyzed flows (variance test results), thus the flows are not are not Poisson flows.
- lack of randomness in the direction South in conjunction with the rejection of the hypothesis of variance equal to zero indicates the need for more research on the traffic changes as a function of time, *Table 2*.

Table 6. Non-parametrical tests for the analyzed flows

Hypothesis Tests for α = 0,05	**Traffic intensities in the given flow**			
	E	**W**	**N**	**S**
$\mathbf{H_0}$: mean = 2,0 $\mathbf{H_1}$: mean ≠ 2				
t-test	Do not reject $\mathbf{H_0}$			
Computed statistic	1,995	1,886	1,107	1,004
P-Value	0,058	0,072	0,280	0,326
$\mathbf{H_0}$: median = 2,0 $\mathbf{H_1}$: median ≠ 2				
sign test				
Number of values below hypothesized median	9	9	11	11
Number of values above hypothesized median	12	15	13	13
Large sample test statistic (continuity correction applied)	0,44	1,02	0,20	0,20
P-Value	0,66	0,31	0,84	0,84
signed rank test	Do not reject $\mathbf{H_0}$			
Average rank of values below hypothesized median	7,9	9,7	11,0	10,6
Average rank of values above hypothesized median	13,29	14,20	13,77	14,08
Large sample test statistic (continuity correction applied)	1,51	1,79	0,81	0,93
P-Value	0,13	0,07	0,42	0,35
$\mathbf{H_0}$: sigma = 0 $\mathbf{H_1}$: sigma > 0				
chi-squared test	Reject $\mathbf{H_0}$			
Computed statistic	6E20	7E20	2E21	2E21
P-Value	0,0	0,0	0,0	0,0

3. Results derived from a simulation model of maritime transportation system

The framework of the simulation model presented in previous chapters has been coded in Java language using SSJ V2.1.3 library with support of stochastic simulations, see [12], [9], [5], [3], [7]. Java-based simulation tool is very popular because of its object-oriented programming environment which effectively supports standardized components; see [3], [7]. As a result of the program the following data is obtained: the matrix of the system transitions' number between the states and the realizations of the conditional sojourn times at the state until the transition to the other state.
From these results we obtained the following: the matrix of probabilities of the system's transitions between the states and the vector of probabilities of the system being in the particular states during the simulation time. According to the adopted risk levels, the following states are considered:

- state 0 – collision alert;
- state 1 – high risk of collision;
- state 2 – low risk of collision;
- state 3 – negligible risk of collision.

The probability for a system being in a given state (i) is denoted by p_i, where i = 0,1,2,3.

In the simulation model the departure time of a ship can follow certain distribution. We can select among numerous alternatives both on the main and lateral waterways. The following distributions can be selected: deterministic, uniform, exponential, Erlang, normal, log-normal, Beta, gamma and triangular. Moreover it is also possible to simulate non-Poisson streams. In the case study, we assume as follows:

- the mean time between ships departing on the main waterways (E-W) equal to 2.2 h following an uniform distribution;
- the mean time between ships departing on the crossing waterways (N-S) equals 1.2 h with standard deviation 1 h.

- Various distributions for the distribution of the latter are adopted, and the differences are observed, see *Table 7*.

Following these, the probabilities for a transportation system being in a given state are derived.

Table 7. Obtained results showing influence of distribution adopted for flow intensity modeling.

Distribution \ Probability	p_0	p_1	p_2	p_3
Exponential	0.0026	0.0024	0.7704	0.2246
Erlang	0.0016	0.0022	0.6292	0.3670
Normal	0.0074	0.0847	0.5749	0.3330
Log-normal	0.0124	0.0016	0.6742	0.3118
Beta	0.0072	0.0788	0.6971	0.2169
Gamma	0.0020	0.0026	0.7321	0.2633

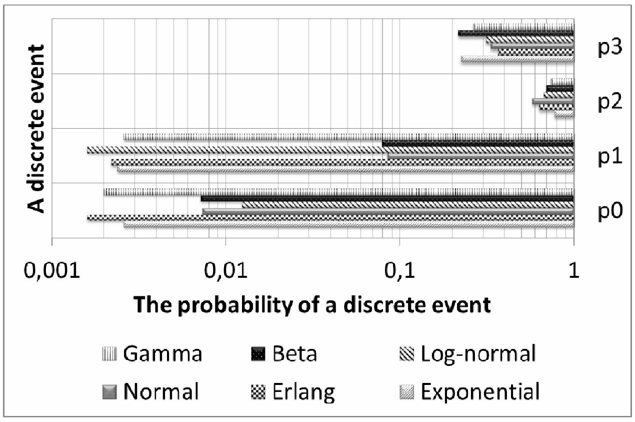

Figure 4. The case study results.

4. Conclusions

This paper introduces a novel method evaluating the probability of ship-ship collision in the maritime transportation system focusing on the open sea collisions, applying the queuing theory in the simulation model. The model allows relatively fast prediction as it focuses on the selected discrete events, which can be defined arbitrary depending on the criteria adopted, instead of simulating the whole traffic.

The advantage of presented approach to the commonly adopted traffic simulation models is the dynamic and linear division of the analyzed area into cells, which depends on the ship's speed. Moreover the model determines the probabilities for the transportation system being in numerous states, instead of calculating only the probability of collision events. Thereby, the obtained results can be validated with the available data on a number of events other that collision, for instance the near-misses which can be observed in a given transportation system. From the statistical view point this feature makes the presented model more reliable that the other models, as the validation can be performed with relatively high number of recorded events.

Although the model is promising, the drawbacks have to be reminded a swell. Firstly, the model is sensitive to the distribution of input data, see *Figure 4*. This is especially evident for the system being in a state p_1, where various distributions provide results which can differ by two orders of magnitude. In case of p_0 the results are more coherent, however significant differences can be observed as well. This leads to the conclusion, that this particular parameter must be properly defined a priori, otherwise the model results can hardly be reliable. Secondly, the effect of evasive manoeuver is an important factor, as discussed in Chapter 2.1, determining the transition of a system from one state to another.

Recapitulating, at this stage the presented model can serve as an efficient tool for predicting the transportation system safety, by predicting the probability of a system being in a given state; however the input must be selected carefully. Luckily the latter can be done quite detailed using the recorded data on traffic flow, derived from the AIS system.

Moreover the model is capable of optimizing transportation system safety given the criteria. Thus it can be effectively used in safety management or safety-based spatial planning of sea areas.

As the next stage of our research the extension of the presented model over larger sea area and more complex transportation system is anticipated.

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