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Decision making in crisis management: time criticality assessment

Keywords

cascade, domino, effect, model, CIP, MS&A, dynamic, decision making, time criticality

Abstract

The paper is proposing a methodology for assessing the time-criticality in the scope of a crisis resulting from a cascade of disruptions of a set of Critical Infrastructures (CIs). The success of the management of the crisis depends on taking the good decisions and undertaking the corresponding actions at the good timing. Identifying time intervals for decisions and actions requires dynamic models capable to assess the crisis time evolution. While, measuring the "criticality" of different time-intervals requires the use of suitable metrics.
Based on previous works for modelling cascade of CIs' disruptions incorporating CIs dependency and vulnerability, the authors propose a methodology to assess the time-criticality and propose the appropriate metrics.
The methodology is based on the idea of comparing the time-profile of a given cascade of disruptions between two configurations: unstressed and stressed CIs. The CIs become stressed under the action of a threat and combined with the dependency between CIs. The unstressed configuration represents a risk-noise.
Two metrics are proposed in order to carry on the comparison between the time-profiles of the stressed and the unstressed CIs. The proposed metrics are: the time to attend 90% of the asymptotic occurrence probability and the time to attend the most probable occurrence rate, describing the cascade likelihood.
An academic case is presented in order to demonstrate the applicability of the methodology and illustrate some interesting features.

1. Introduction

The paper is proposing a methodology for assessing the time-criticality in the scope of a crisis resulting from a cascade of disruptions of a set of Critical Infrastructures (CIs). The success of the management of the crisis depends on taking the good decisions and undertaking the corresponding actions at the good timing. Time and timing are then crucial, especially, if the cascade of disruptions may lead to hazardous consequences under the action of a given threat. The existing dependencies between CIs may even amplify the severity of the hazardous consequences of the cascade.
Identifying time intervals for decisions and actions requires dynamic models capable to assess the crisis time evolution. While, measuring the "criticality" of different time-intervals requires the use of suitable metrics.
Based on previous works for modelling cascade of CIs' disruptions incorporating CIs dependency and vulnerability, the authors propose a methodology to assess the time-criticality and propose the appropriate metrics.
The methodology is based on the idea of comparing the time-profile of a given cascade of disruptions between two configurations: unstressed and stressed CIs. The CIs become stressed under the action of a threat combined with the dependency between CIs. The unstressed configuration represents a risk-noise.
Two metrics are proposed in order to carry on the comparison between the time-profiles of the stressed and the unstressed CIs. The proposed metrics are: the time to attend 90% of the asymptotic occurrence probability and the time to attend the most probable occurrence rate, describing the cascade likelihood.

The paper is structured, after the above introduction, in three parties:

- A first part treating the concept of sequential events and their mathematical modelling. It is merely a succinct review limited to the restricted scope of interest of the paper, section 2.
- The second part, section 3, recall the basis of the mathematical models used here and borrowed from previous work. It describes the methodology and the corresponding basic models: the unstressed CIs and the stressed CI under the actions of threats and the CIs independencies. The cascade likelihood is equally recalled, measured with its: occurrence probability and occurrence rate. The concepts: "time to attend 90% of the asymptotic probability" and "time to attend the most probable occurrence rate" are introduced.
- In the third part, an academic case is presented in order to demonstrate the applicability of the methodology and illustrate some interesting features.

It may be helpful to underline that the main target of this work is to propose a methodology to measure the time criticality and its corresponding metrics.
Finally, although the paper focuses on disruptions following Stochastic Poisson Processes (SPP), the methodology is extendable to other stochastic processes. The focus on PSP was motivated by the avoidance of useless numerical complexity given our main target mentioned above.

2. Sequential events

A set of sequential disruptions is treated as a sequence of some ordered events. Many techniques have been developed in systems reliability and safety assessments to carry on "sequential failures analysis", such as: "Event trees" [1,2,3], "Fuzzy approaches" [4], "Dynamic Fault Trees" with "Priority Gates" [5,6,7] or without [8], Petri Net and Markov Chain Process [9] or "Monte-Carlo Simulation" [10]. The problem is also known as cascade modelling.
In cascade modelling, one would often like to determine the cascade likelihood using metrics such as: "occurrence probabilities" and/or probability distributions. Other probabilistic quantities can also be of interest, depending on the case. Other probabilistic quantities are, e.g., "mean time to….".
The paper uses a model that describes cascades of disruptions using an integral equation, [11,12]. The integral equation admits an analytical solution if the occurrence probability distribution functions (pdf) of the disruptions obey Stochastic Poisson Processes (SPP). The model takes into account the "vulnerability to the threat" and the "CIs' dependencies" using time constant factors called: "vulnerability stress factor" and "dependent stress factor", respectively, [11]. If the involved disruptions do not follow SPP, the integral equation will admit no analytical solution. Still, the integral equation can be determined using numerical techniques such as: Monte-Carlo simulation.
The above mentioned model is used in this paper in order to illustrate how to assess the criticality of time-intervals within the time-line of the decision making and actions undertaken processes.

3. Methodology description

The fundamental idea is to admit that CI disruptions are random and systemic when the CI is not under the action of a threat and is isolated from the other CIs. Under the stress resultant from a given threat actions and because the dependency of the CI on the other CIs, the occurrence of the individual disruptions increases.
The considered CIs are then all coherent in the sense of the reliability theory. In other words, the stressed CI can't show but higher disruption likelihood.
The model described in [11,12] is based on three basic hypotheses. The first hypothesis is that the vulnerability of a given CI under the action of a well-defined threat is described by a parameter acting on the systemic disruption rate. The second hypothesis is that the dependency of the disruptions between CIs can also be described using another parameter acting on the systemic disruption of the CIs under consideration. The third hypothesis is that both parameters are independent of each other. They can be time-dependent or constant. These parameters are called "vulnerability stress factor" and "dependency stress factor".
We recall that the sequence of distributions is defined by a given number of individual disruptions occurring in a well-defined chronological order.
There are two possible ways to phenomenologically express the likelihood as a function of the CI's stress level: either to use the disruption occurrence probability, or the disruption occurrence rate. Both lead to similar evaluations if they are well-calibrated. However, they are not similar in their formal mathematical expressions.
The model described in the paper integrates the stress of a CI in the disruption occurrence rate. First, one should treat the unstressed isolated CIs, as following.

3.1 Systemic disruptions

Although, we should integrate the stresses generated by the threats and the dependencies between the CIs, we should at first understand the disruptions of the unstressed CI's and their dynamics.

In the absence of identified threats, an isolated CI from all others CIs can still prove random disruptions initiated by different systemic mechanisms and expressed by different phenomenological failure modes. For this isolated CI, disruptions will be called "systemic". The systemic disruptions depends, then, on: the design of the CI, the materials used in its structures and the operational conditions of the CI.

Systemic disruptions occurrence is a stochastic phenomenon. Accordingly, the disruption occurrence can be described using an adequate: occurrence probability functions, occurrence probability density functions or disruption occurrence rates. We will be using occurrence rates in this paper.

The disruption occurrence rate of a given disruption mode in an isolated CI, $\lambda_i(o)$, will be called a systemic disruption occurrence rate. The isolated CI will be described as unstressed CI.

However, our interest is to describe how dependency and vulnerability may be affecting the systemic disruption rates when the CI becomes stressed as a result of the CIs dependencies (§3.2) and the threats actions (§3.3).

3.2 Dependency

In order to describe a cascade of disruptions, the dependencies between CIs should be considered. In the model given in [11], a disruption dependency matrix (D-D matrix) is established describing the existing dependencies between a given set of identified CIs. It is obvious that the set of considered CIs depends on the mode of the disruptions considered.

The dependency of the disruption occurrence of a given CI "i" on the disruption occurrence of another CI "j" is described by a factor ε_{ij} that is called the CI disruption occurrence dependency stress factor. The disruption occurrence rate $\lambda_i(j)$ of a given CI "i" given the disruption occurrence of the CI "j" is then described by:

$$\lambda_i(j) = \lambda_i(o)(1+\varepsilon_{ij})$$

Where, $\lambda_i(o)$ is the systemic disruption occurrence rate of the CI, "i", and ε_{ij} is its dependency stress factor regarding the disruption occurrence of the CI, "j".

A disruption dependency is "directional" if the disruption of the CI "j" impacts on the disruption of the CI "i" and the inverse is not true. In that case, one has $\varepsilon_{ij} > 0$ and $\varepsilon_{ji} = 0$. If the dependency is not directional, it is called "interdependency" rather than "dependency", given $(\varepsilon_{lk} > 0, \forall l, k)$ and, generally, $(\varepsilon_{ij} \neq \varepsilon_{ji})$.

If the CI, "i", is acted upon by the disruptions of other M CIs, its effective disruption rate $\lambda_i^{0,M}$ will, then, be given by:

$$\lambda_i^{0,M} = \lambda_i(o)\left[\prod_{j=1}^{M}(1+\varepsilon_{ij})\right]$$

In this model, the disruptions of many CIs act independently on the CI. We have not considered the possibility of a compound damage mechanisms. Considering independently the impact of each other disruption gives a conservative estimation of the effective disruption rate.

3.3 Vulnerability

The term "Vulnerability" is used here to describe the dependency between a well-defined threat and the occurrence of a disruption mode of a given CI. A CI does not react to all threats in the same manner. The stochastic disruption of the CI is dependent on the threat specifications. In the used model, from [11], a vulnerability matrix is established for each identified CI disruption mode and corresponds to a well-specified set of threats. It is obvious that the set of the involved threats depends on the location of the CI. The threat is generally specified by its: intensity, magnitude, likelihood, locality and dynamics.

The vulnerability of a given CI "i" to a well-defined threat "j" will be described using a vulnerability stress factor "υ_{ij}". The disruption occurrence rate $\lambda_i(j)$ of a given CI "i" under the action of the threat "j" will then be given by, [11]:

$$\lambda_i(j) = \lambda_i(o)(1+\upsilon_{ij}) \quad (1)$$

Where, $\lambda_i(o)$ is the systemic disruption occurrence rate of the CI, "i", and "υ_{ij}" is its vulnerability stress factor regarding the threat, "j". The stress factor "υ_{ij}" is a positive parameter and can generally be a time-function.

If the CI, "i", is acted upon by multiple N threats, its effective disruption rate $\lambda_i^{N,0}$ will, then, be given by:

$$\lambda_i^{N,0} = \lambda_i(o)\left[\prod_{j=1}^{N}(1+\upsilon_{ij})\right] \quad (2)$$

$\lambda_i^{N,0}$: is the effective disruption occurrence rate.

In the presented model, threats act on the same CI independently. We have not considered the possibility of a compound damage mechanisms. Considering independently the vulnerability of each threat gives a conservative estimation of the effective disruption rate. Modelling the CIs' dependencies will follow a similar logical scheme as it is shown in the following.

3.4 Multi-threat & multi-dependency

In a complex case, where there are multi-threat actions and many dependent CIs, the overall effective disruption rate $\lambda_i^{N,M}$ will be given by, [11]:

$$\lambda_i^{N,M} = \lambda_i(o)\left[\prod_{k=1}^{N}(1+\upsilon_{ik})\right]\left[\prod_{j=1}^{M}(1+\varepsilon_{ij})\right] \quad (5)$$

Where N refers to the number of the simultaneous acting threats and M refers to the number of the dependent CIs.

3.5 Disruption occurrence likelihood

Let T be a well-defined cascade of disruptions, occurs if and only if some discrete and independent disruptions e_i occur in a well-defined chronological order $[e_1 \to e_2 \to e_3 ... \to e_n]$. The cascade of disruptions T gives, consequently, place to a sever accident or a major crisis. The corresponding occurring instants of the elementary disruptions are $[\xi_1, \xi_2, \xi_3, ..., \xi_n]$, where $[t_1 < t_2 < t_3 < ... < t_n]$. Each of these instances $[t_1, t_2, t_3, ..., t_n]$ has its distribution probability density function (pdf). The first disruption event is e_1 and the last is e_n. The probability $p_n(t)$ that the major crisis T happens within the interval [0,t] is, then, given by, [13]:

$$p_n(t) = \int_0^t \rho_1(\xi_1)d\xi_1 * \int_{\xi_1}^t \rho_2(\xi_2)d\xi_2 * \ldots * \int_{\xi_{n-1}}^t \rho_n(\xi_n)d\xi_n \quad (6)$$

where, ρ_i are the probability density functions (pdf) characterizing the occurrence instances of events e_i. Whatever the type of these density probability functions, the integral in Eq.(6) can hopefully be solved in many cases.

It can, generally, be solved: numerically, e.g., using Monte-Carlo Simulation (MCS). However, there exists an analytical solution if ρ_i obeys a Poisson probability density function, [13].

The only focus of this paper is not the model itself, but the methodology using it in order to introduce the concept of time-criticality and to assess it in a decision making process.

Accordingly, we will consider the case of time-constant disruption occurrence rates and time-constant stress factors. It produces an exact analytical solution which is very interesting in illustrating the main target of our work and to avoid all numerical complexity relative to the integral. An exact solution of the integral equation of a given cascade of disruption has been developed and commented in [13]. It has the following form:

$$p_n(t) = \sum_{j=1}^{n} C_j^n * \left(1 - e^{-\left(\sum_{l=n-j+1}^{n}\lambda_l\right)t}\right)$$

Each event e_i is defined by a constant occurrence rate λ_i, $\{i \in [1,2,...,n]\}$ and the coefficient C_1^{i+1} is given by:

$$C_1^{i+1} = \sum_{j=1}^{i} C_j^i, \quad C_{j+1}^{i+1} = -\frac{\lambda_{i+1}}{\sum_{l=i-j+1}^{i+1}\lambda_l} C_j^i,$$

$$j = 1,2,...,i, \text{ and } i \in [1,2,...,n] \quad (8)$$

where, $C_1^1 = 1$.

The disruption occurrence rates λ_i, in Eq.(7)(8), are the systemic ones if there are neither threats nor dependency. Otherwise, the disruption occurrence rates λ_i, are the stressed disruption rates, as determined above.

4. A study-case

The considered situation is shortly described as following. An aging dam is located in a well-defined populated area. The dam is regulating the flow of a river thanks to a large retention lac behind.

If the water level in the lac reaches a well-defined alarm-level-1, a nearby water pumping station starts up automatically to evacuate the water excess to a small emergency retention area far from the lac, provisionally. That would allow to stabilize the water level below the alarm-level-1. If the water level could not be stabilized and reaches alarm-level-2, the risk of losing the dam's structure integrity becomes significant. Subsequently, the population in the area should be evacuated within 24-36 hours. The hypothetical crisis scenario examines the situation in case of an unexpected torrential rains combined with the flooding of the river. Given the threat "torrential rain and flood", the crisis scenario is composed of four basic "disruptions", as following:

- Disruption d_1: loss of electricity supply from the grid to the pumping station.
- Disruption d_2: loss of the water pumping station. That covers the loss of the emergency local electrical supply (a large diesel unit), the loss of automatic start up system and other systemic mechanical failure modes of the pumping unites.
- Disruption d_3: loss of the dam structure integrity. That covers all cracks with sizes larger than a critical value and/or the full collapse of the structure.
- Disruption d_4: failure to evacuate the population out of the disaster area. That covers: the failures of the population alert systems (media and SMS), the unavailability of emergency resources, the loss of accessibility to the evacuation meeting points and the loss of transportation capabilities. It includes systemic, humans and organizational failure modes.

We are interested in the period of time 24-36 hours from the moment when the water level behind the dam reaches alarm-level-2. This is the period necessary for the evacuation of the population from the exposed area. Starting from this instance when the threat became active.

We are considering a hypothetical major crisis occurs when four disruptions $[d_1, d_2, d_3, d_4]$ occur in the mentioned order. The systemic occurrence rates of the elementary disruptions are constant and having the following values: 10^{-4} /h, $5*10^{-3}$ /h, $2.5*10^{-2}$ /h, $1.25*10^{-1}$ /h, respectively. Thus, they are following SPPs.

The concerned CIs could be vulnerable to the given threat and, to different extends, can be dependent on each other.

Five stressed situations will then be examined and are mentioned below.

- Unstressed referential case (#0): no vulnerability to the threat and no dependencies. Disruptions may happen on a systemic random basis and in the given order $[d_1 \rightarrow d_2 \rightarrow d_3 \rightarrow d_4]$. Services supply losses occur in the order that produces the crisis just by the systemic occurrence of each event in the cascade. This possibility is a background noise and exists whether the threat is active or not and whether the CI's are dependent or not. The only way to decrease its likelihood is to redesign the whole CI systems.
- Stressed case (#1): only d_4 is vulnerable to the threat and there is no dependency. The vulnerability stress factor is equal to 1.5. The threat intensity is judged moderate.
- Stressed case (#2): all $[d_1, d_2, d_3, d_4]$ are equally vulnerable to the threat and there is no dependency. The vulnerability stress factors are all equal to 1.5. The threat intensity is judged moderate.
- Stressed case (#3): all $[d_1, d_2, d_3, d_4]$ are equally vulnerable to the threat and there is no interdependency. The vulnerability stress factors have uniformly increased and equal to 10. That may express high vulnerability of the CIs to that kind of threats or a threat with a very high intensity.
- Stressed case (#4): all $[d_1, d_2, d_3, d_4]$ are equally vulnerable to the threat. The vulnerability stress factors are all equal to 1.5. The threat intensity is judged moderate. Disruptions d_3 and d_4 show dependency on d_2 and their dependency stress factors are 0.8 and 0.4, respectively. Disruption d_4 is dependent on d_3 with a dependency stress factor equal to 0.4. [$\varepsilon_{32} = 0.8, \varepsilon_{42} = 0.4, \varepsilon_{43} = 0.4$]

The time profiles of the occurrence probability and of the occurrence rates are assessed over a period of time equal to 80 hours starting from the moment when the water level behind the dam attends the alarm-level-2. We use the time interval to reach 90% of the asymptotic occurrence probability as a characteristic figure. The 90% of the asymptotic occurrence probability will be called the reduced asymptotic probability (RAP) and the time to attend it is called TTA-ARP. Theoretically, the asymptotic values are attended when $t \rightarrow \infty$ which is not a practical measure in taking decisions.

Regarding the occurrence rates, we use the most probable value of the occurrence rate (MPR) as a

characteristic figure and the time to attend it will be referred to as TTA-MPR.
The criticality of time to decide and/or to react will be measured using a metric based on the cascade occurrence rate. While, the cascade occurrence probability itself will be used to measure the cascade occurrence likelihood.
The methodology we propose distinguishes clearly between:

- The cascade occurrence probability: is the matric to be used to measure the likelihood of the cascade,
- The cascade occurrence rate: that will be used to elaborate a metric to measure the criticality of the time to decide and/or to react.

The methodology uses two formal criteria to help in decision making and crisis management. The details are presented and commented in the following sections.

4.1 Unstressed reference case (#0)

The CIs are not vulnerable to the threat and the CIs' are not dependent. The likelihood of this cascade of disruptions is the following:

- The occurrence probability of the cascade is time dependent. It attends the ARP value of $3.15e-6$ after 46 hours, *Figure 1.*
- The occurrence rate of the cascade is also a time dependent function. It attends its MPR value $1.13e-7$ after 21 hours, *Figure 2.*

The systemic occurrence of this cascade of disruptions may result inacceptable consequences. Therefore the crisis managers would be interested in identifying the likelihood of the situation and its evolution with the time. Assessing this background-risk is useful in measuring the "time criticality" for deciding and acting during the crisis, as will be explained in the following.
Given that the most probable value of the cascade occurrence rate, the background risk-noise, is about 10^{-7} and occurs around 21 hours, one may propose the following classification based on three classes, T*able 1*:

- Class 3 – high: the occurrence arte is almost one decade around the most probable value of the noise risk [$>10^{-7}$]. This is the case between 4 hours and 60 hours from the start of the active phase of the threat.
- Class 2 – medium: the occurrence rate is one decade less than in class 1, $[10^{-8},10^{-7}]$. This is the case in two intervals: from 1h to 4 hours and from 60 hours to 85 hours.
- Class 1 – low: the occurrence rate is one decade below class 2, $[<10^{-8}]$. This is the case before 1 hour and after 85 hours, in the unstressed case (background-risk).

The unstressed case serves in establishing the scale of criticality to be used in assessing the stressed cases representing crisis situations. Four hypothetical crisis situations are presented in the following. They are synthetized in Table (1) and Table (2), as well.

4.2 Stressed case #1

The stressed case #1 consider the vulnerability υ of the four modes of disruption given above is taken in to account. The vulnerability stress factors are 0, 0, 0 and 1.5 for the disruption modes $[d_1, d_2, d_3, d_4]$ under the action of the considered threat, respectively. Only, the disruption mode $[d_4]$ is vulnerable to the given threat with a stress factor equal to 150%, i.e. it is occurrence rate is increased by factor 2.5 (1+1.5=2.5).
Accordingly, the stressed occurrence rates are, respectively: 10^{-4} /h, $5*10^{-3}$ /h, $2.5*10^{-2}$ /h, $3.125*10^{-1}$ /h. The likelihood of this cascade of disruptions is as following:

- The occurrence probability of the cascade is time dependent. It attends its ARP value of $2.8e-7$ after 20 hours, *Figure 1.*
- The occurrence rate of the cascade is also a time dependent function. It attends its MPR value $2.4e-8$ after 9 hours, *Figure 2.*

It is interesting to notice that although d_4 is vulnerable to the threat, the occurrence probability of the cascade is diminished by a decade and the crisis spends shorter time in the classes of high likelihood (red) and medium likelihood (yellow).
This situation can be considered in two contradicting manners, regarding the cascade likelihood:

- A comfortable manner because the cascade occurrence probability (likelihood) is lowered. So, this cascade of disruptions is less probable. Besides, the periods of time when the cascade attends its highest likelihood (high and medium) are short (20-4=16h).
- Another uncomfortable manner regarding these short hot periods. This hot period may not be long enough to decide and take the appropriate actions if the cascade occurs instead of its low likelihood.

It is worth underlining that the moderately stressed CIs resulted in a less likely crisis. But if the crisis occurs, time to decide and react is shorter.

4.3 Stressed case #2

All disruptions $[d_1, d_2, d_3, d_4]$ are equally vulnerable to the threat and have vulnerability stress factor equal to 1.5. They show the same vulnerability regarding the considered threat.
Accordingly, the stressed occurrence rates are $2.5e-4$ /h, $1.25e-2$ /h, $6.25*10^{-2}$ /h, $3.125*10^{-1}$ /h.
The likelihood estimations of this cascade of disruptions are as following:

- The occurrence probability of the cascade is time dependent. It attends its RAP value of $3.12e-6$ after 18 hours, Fig.(1).
- The occurrence rate of the cascade is also a time dependent function. It attends its MPR value of $2.8e-7$ after 8.5 hours, Fig.(2).

The interest of this case is that all occurrence rates increase by factor 1.5. In spite of this increase, likelihood of the cascade is almost similar to the unstressed case. The two differ in:

- The ARP is attended after 18.5h rather than 44h in the unstressed case.
- The MPR is attended at 8.5h rather than 21h in the unstressed case.

Case#2 has a similar likelihood to the unstressed case, but it has shorter times to decide and react. It has a faster dynamic.

4.4 Stressed case #3

All disruptions $[d_1, d_2, d_3, d_4]$ are equally vulnerable to the threat and have vulnerability stress factor equal to 10. That can be understood either as the threat is considered of higher intensity or the CIs have more vulnerability to the threat.

- The occurrence probability of the cascade is time dependent. It attends its RAP value of $3.12e-6$ after 4 hours, Fig.(1).
- The occurrence rate of the cascade is also a time dependent function. It attends its MPR value of $1.24e-6$ after 2 hours, Fig.(2).

Case #3 is close to case #0 but with significantly faster dynamics.

4.5 Stressed case #4

All disruptions $[d_1, d_2, d_3, d_4]$ are equally vulnerable to the threat and have vulnerability stress factor equal to 1.5. The threat is considered of moderate intensity similar to case #2. Dependencies between disruptions are considered. Disruptions d_3 and d_4 show dependency on d_2 and their dependency stress factors are 0.8 and 0.4, respectively. Disruption d_4 show dependency on d_3 with a dependency stress factor equal to 0.4. [$\varepsilon_{32} = 0.8, \varepsilon_{42} = 0.4, \varepsilon_{43} = 0.4$]

- The occurrence probability of the cascade is time dependent. It attends its RAP value of $8.32e-6$ after 17 hours, *Figure 1.*
- The occurrence rate of the cascade is also a time dependent function. It attends its MPR value of $8.00e-7$ after 7.8h, *Figure 2.*

The occurrence probability is higher than in case #0 (and all the other cases). Its dynamic behaviour is faster than in case #1 but of the same order as the three other cases.

5. Conclusions

Based on a dynamic model describing the cascade of disruptions, a methodology is proposed to measure the criticality of time to take decisions and actions in crises situations.
A methodology is proposed and can briefly be described as based on:

- The incorporation of the vulnerability and the dependency in the disruption occurrence rate.
- The systemic cascade, corresponding CIs are unstressed, is used as a referential to establish a criticality grid, *Table 1.*
- The dynamic of a cascade (stressed and unstressed) is characterized by its occurrence probability and its occurred rate and their time-evolution profiles.
- The occurrence probability is used to measure the cascade likelihood.
- The occurrence rate time-profile is a good measure of the cascade dynamic. It is used to measure the time-criticality regarding decision and action making.

Using exact dynamic models to assess cascade reveals some interesting effects:

- The likelihood of a given cascade does not necessarily increasing with the threat intensity, in spite of the individual increase of the likelihood of the events composing the cascade.
- That is why analyst should not focus on only one cascade scenario but on the set of all possible scenarios.
- Schematically, higher are the threat intensity and/or the CIs dependency, faster goes the dynamic of the cascade.

The methodology is limited by the inherent limits of the models it uses, [11, 12, 13], i.e.:

- The capability to describe CIs disruptions using the appropriate stochastic process, "probability distribution function, pdf".
- The degree of exactitude of the hypothesis that different dependencies between CIs are not interacting. It is too conservative.
- The degree of exactitude of the hypothesis that Multi-threats CIs are not interacting. It is too conservative.

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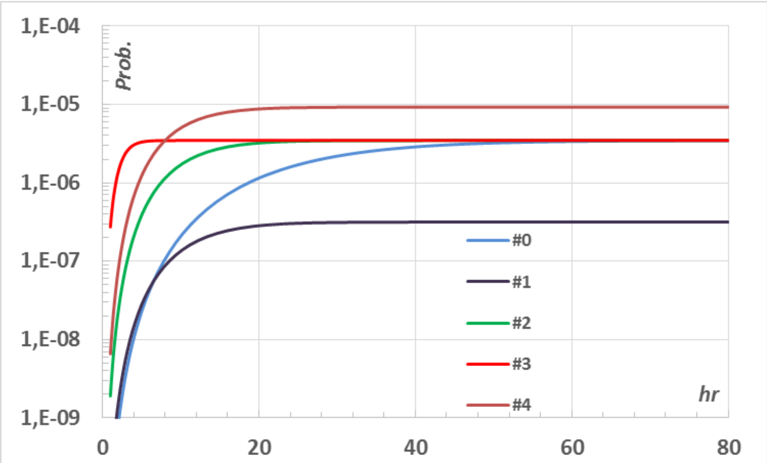

Figure 1: Time profile of the cascading occurrence probabilities

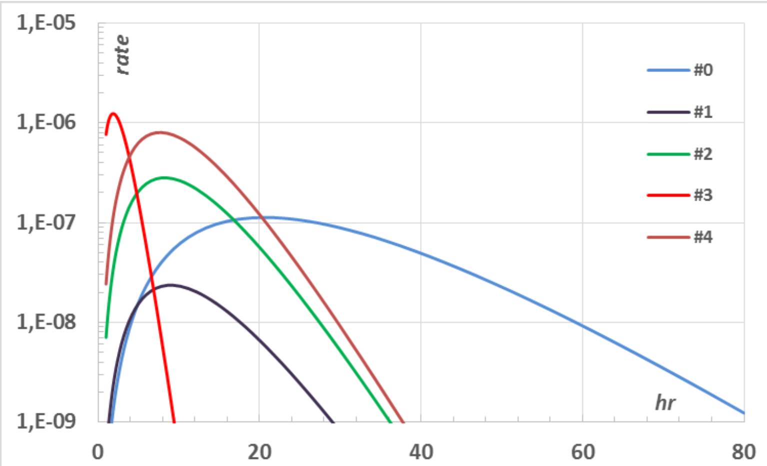

Figure 2. Time profile of the cascading occurrence rates

Table 1. The classification of the criticality according to the occurrence rate.

Cat.	Likelihood	Occurrence rate	Time interval (hours)				
			Case #0	Case #1	Case #2	Case #3	Case #4
1	Low	$\leq 10^{-8}$	0 - ~4 h 60 - ∞ h	0-~4 h 20- ∞ h	0 – 1 h 18 - ∞ h	1 – 5 h 8 - ∞ h	30 - ∞ h
2	Medium	$10^{-8} - 10^{-7}$	4-15h 27-60h	4-20 h	1 – 3 h 18 – 28 h	5 – 8 h	0 – 2 h 20 – 30 h
3	High	$\geq 10^{-7}$	15-27h	----	3 – 18 h	----	2-20h

Table 2. The occurrence probability and the occurrence rate characteristics.

	As. Prob.	RAP	TTA. RAP (h)	MPOR	TTA MPR (h)
Case #0	3.46e-6	3.11e-6	44	1.13e-7	20
Case #1	3.16e-7	2.84e-7	20	2.35e-8	9
Case #2	3.47e-6	3.12e-6	18.5	2.81e-7	8.5
Case #3	3.44e-6	3.12e-6	4	1.24e-6	2
Case #4	9.25e-6	8.32e-6	17	8.00e-7	7.8