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Preliminary reliability, risk and availability analysis and evaluation of bulk cargo transportation system in variable operation conditions

Keywords

system operation process, multi-state system, reliability, risk, availability

Abstract

In the paper, definitions and theoretical results on system operations process, multi-state system reliability, risk and availability modelling are illustrated by the example of their application to a bulk cargo transportation system operating in Gdynia Port Bulk Cargo Terminal. The bulk cargo transportation system is considered in varying in time operation conditions. The system reliability structure and its components reliability functions are changing in variable operation conditions. The system reliability structures are fixed with a high accuracy. Whereas, the input reliability characteristics of the bulk cargo transportation system components and the system operation process characteristics are not sufficiently exact because of the lack of statistical data. Anyway, the obtained evaluation may be a very useful example in simple and quick systems reliability characteristics evaluation, especially during the design and improving the transportation systems operating in ports.

1. Introduction

Taking into account the importance of the reliability and operating process effectiveness of technical systems it seems reasonable to expand the two-state approach to multi-state approach in their reliability analysis. The assumption that the systems are composed of multi-state components with reliability states degrading in time gives the possibility for more precise analysis and diagnosis of their reliability and operational processes' effectiveness. This assumption allows us to distinguish a system reliability critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operational process effectiveness. Then, an important system reliability characteristic is the time to the moment of exceeding the system reliability critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state reliability function that is a basic characteristic of the multi-state system. Determining the multi-state reliability function, the risk function and the availability of systems on the base of their components' reliability functions is then the main research problem. Modeling complicated systems operations' processes is difficult mainly because of large number of operations states and impossibility of precise describing of changes between these states. One of the useful approaches in modeling these complicated processes is applying the semi-markov processes Modeling of multi-state real technical systems' reliability and linking it with semi-markov model of these systems' operation processes is the main and practically important research problem of this paper. The paper is devoted to this research problem with reference to basic reliability structures of technical systems and particularly to reliability analysis of a port bulk cargo transportation system in variable operation conditions. This approach to system reliability investigation is based on the multi-state system reliability analysis and on semi-markov processes modeling.

2. The bulk cargo transportation system description

The Baltic Bulk Terminal Ltd. In Gdynia is designated for storage and reloading of bulk cargo such as different kinds of fertilisers i.e.: ammonium sulphate, but its main area of activity is to load bulk cargo on board the ships for export. The BBT is not equipped with any devices to enable the discharge of vessels.

There are two independent shipment systems:

1. The system of reloading rail wagons.
2. The system of loading vessels.

Cargo is brought to BBT by trains consisting of self discharging wagons which are discharged to a hopper and then by means of conveyors are transported into the one of four storage tanks (silos). Loading of fertilisers from storage tanks on board the ship is done by means of special reloading system which consists of several belt conveyors and one bucket conveyor which allows the transfer of bulk cargo in a vertical direction. Researched system is a system of belt conveyors, called later on the transport system.

In the conveyor reloading system we distinguish the following transportation subsystems:

S_1, S_2, S_3 – the belt conveyors.

In the conveyor loading system we distinguish the following transportation subsystems:

S_4 – the dosage conveyor,
S_5 – the horizontal conveyor,
S_6 – the horizontal conveyor,
S_7 – the sloping conveyors,
S_8 – the dosage conveyor with buffer,
S_9 – the loading system.

The scheme of this system is presented in *Figure 1*.

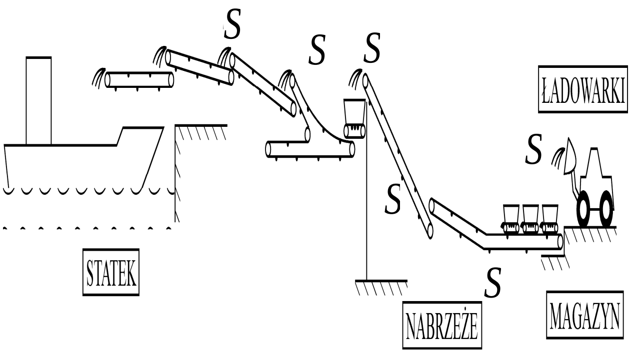

Figure 1. The scheme of port bulk cargo transportation system.

After discussion with experts, taking into account the reliability of the operation of the system, we distinguish the following four reliability states of its components:

- a reliability state 3 – ensuring the highest efficiency of the conveyor,
- a reliability state 2 – ensuring less efficient of the working conveyor by spilling cargo out of the belt caused by partial damage to some of the rollers or misalignment of the belt,
- a reliability state 1 – ensuring less efficiency of the working conveyor controlled directly by operator caused by i.e.: stretched or slightly damaged belt,
- a reliability state 0 – the conveyor unable to work which my be caused by i.e.: breakage of the belt, failure of rollers or elongated belt beyond adjustment range.

We preliminarily assume that the bulk cargo transportation system is composed of nine subsystems S_1, S_2, S_3 and S_4, S_5, S_6, S_7, S_8, S_9, having an essential influence on its reliability.

We mark the reliability functions of these subsystems respectively by the vectors

$$R_i(t\,,\cdot\,) = [R_i(t,0), R_i(t,1), R_i(t,2), R_i(t,3)],$$
$$t \in <0,\infty),\ i = 1,2,3,\ldots,9,$$

with the co-ordinates

$$R_i(t,u) = P(S_i(t) \geq u \mid S_i(0) = z) = P(T_i(u) > t)$$

defined for $t \in <0,\infty)$, $u = 0,1,2,3$, $i = 1,2,3,\ldots,9$, where $T_i(u)$, $i = 1,2,3,\ldots,9$, are independent random variables representing the lifetimes of subsystems S_i in the reliability state subset $\{u,u+1,...,3\}$.

Further, assuming that the system is in the reliability state subset $\{u,u+1,...,3\}$ if all its subsystems are in this subset of reliability states, we conclude that the system is a series system of subsystems S_1, S_2, S_3 and S_4, S_5, S_6, S_7, S_8, S_9 with a general scheme presented in *Figure 2.*

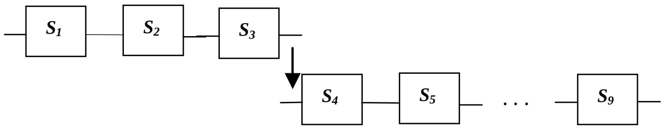

Figure 2. General scheme of transportation system structure

The bulk cargo transportation system consists nine subsystems S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9:

- the subsystem S_1 composed of 1 rubber belt, 2 drums, set of 121 bow rollers, set of 23 belt supporting rollers,

- the subsystem S_2 composed of 1 rubber belt, 2 drums, set of 44 bow rollers, set of 14 belt supporting rollers,
- the subsystem S_3 composed of 1 rubber belt, 2 drums, set of 185 bow rollers, set of 60 belt supporting rollers,
- the subsystem S_4 composed of three identical belt conveyors, each composed of 1 rubber belt, 2 drums, set of 12 bow rollers, set of 3 belt supporting rollers,
- the subsystem S_5 composed of 1 rubber belt, 2 drums, set of 125 bow rollers, set of 45 belt supporting rollers,
- the subsystem S_6 composed of 1 rubber belt, 2 drums, set of 65 bow rollers, set of 20 belt supporting rollers,
- the subsystem S_7 composed of 1 rubber belt, 2 drums, set of 12 bow rollers, set of 3 belt supporting rollers,
- the subsystem S_8 composed of 1 rubber belt, 2 drums, set of 162 bow rollers, set of 53 belt supporting rollers,
- the subsystem S_9 composed of 3 rubber belts, 6 drums, set of 64 bow rollers, set of 20 belt supporting rollers.

3. The bulk cargo transportation system operation process characteristics evaluation

Technical subsystems S_1, $S_2,\ldots,$ S_9, indicated in *Figure 1* are forming a general bulk cargo transportation system reliability structure presented in *Figure 2*. However, the bulk cargo transportation system reliability structure and the subsystems reliability depend on its changing in time operation states.

Taking into account the operation process of the considered system we distinguish the following as its three operation states:

- an operation state $z_1 -$ the discharging rail wagons to storage tanks or hall when subsystems S_1, S_2, S_3, are used, with the structure given in *Figure 3*.
- an operation state $z_2 -$ the loading of fertilizers from storage tanks or hall on board the ship is done by using S_4, S_5, S_6, S_7, S_8 , S_9, subsystems, with the structure given in *Figure 4*.
- an operation state $z_3 -$ the loading of fertilizers from rail wagons on board the ship is done by using S_1, S_2, S_3, S_6, S_7, S_8 , S_9 subsystems, with the structure given in *Figure 5*.

According to expert opinions in the operation process, $Z(t),\ t \geq 0$, we distinguished three operation states: z_1, z_2, z_3. On the basis of data coming from experts, the probabilities of transitions between the operation states are approximately given by

$$[p_{bl}]_{3x3} = \begin{bmatrix} 0 & 0.37 & 0.63 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Using the results given above and when the mean values of the conditional sojourn times M_{bl}, $b,l = 1,2,3$, $b \neq l$, are given, we can find the mean values of the unconditional sojourn times M_b, $b = 1,2,3$, in particular operation states z_b. Next, knowing the mean values of the unconditional sojourn times in particular operation states z_b and solving the system of equations

$$\begin{cases} [\pi_1, \pi_2, \pi_3] = [\pi_1, \pi_2, \pi_3][p_{bl}]_{3x3} \\ \pi_1 + \pi_2 + \pi_3 = 1, \end{cases}$$

$$\pi_1 = 0.5,\ \pi_2 = 0.185,\ \pi_3 = 0.315,$$

we obtain the limit values of the transient probabilities $p_b(t)$ at the operational states z_b ,

$$p_1 = 0.6679,\quad p_2 = 0.0945,\quad p_3 = 0.2376. \qquad (1)$$

4. The bulk cargo transportation system in variable operation conditions reliability, risk and availability evaluation

We assume as earlier that the bulk cargo transportation system is composed of $n = 9$ subsystems S_i, $i = 1,2,3,...,9$ and that the changes of the process $Z(t)$ of the bulk cargo transportation system operation states have an influence on the system subsystems S_i reliability and on the reliability structure as well. Thus, we denote the conditional reliability function of the bulk cargo transportation system subsystem S_i while the system is at the operational state z_b, $b = 1,2,3,$ by

$$R_i^{(b)}(t, \cdot)$$
$$= [1,\ R_i^{(b)}(t,1),\ R_i^{(b)}(t,2),\ \ldots,\ R_i^{(b)}(t,z)],$$

where for $t \in <0,\infty)$, $b = 1,2,3,$ $u = 1,2,...,z,$

$$R_i^{(b)}(t,u) = P(T_i^{(b)}(u) > t \,|\, Z(t) = z_b),$$

and the conditional reliability function of the bulk cargo transportation system while the system is at the operational state z_b, $b = 1,2,3,$ by

$$\boldsymbol{R}_{n_b}^{(b)}(t,\cdot)$$
$$= [1,\ \boldsymbol{R}_{n_b}^{(b)}(t,1),\ \boldsymbol{R}_{n_b}^{(b)}(t,2),..,\ \boldsymbol{R}_{n_b}^{(b)}(t,z)\ ],$$

where for $t \in <0,\infty)$, $b = 1,2,3,$ $n_b \in \{1,2,...,9\}$, $u = 1,2,...,z,$

$$\boldsymbol{R}_{n_b}^{(b)}(t,u),\ = P(T^{(b)}(u) > t | Z(t) = z_b).$$

We assume that the bulk cargo transportation system subsystems S_i, $i = 1,2,3,...,9,$ are its three-state components, i.e. $z = 3$, with the multi-state reliability functions

$$R_{ij}^{(b)}(t,\cdot) = [1,\ R_{ij}^{(b)}(t,1)\ , R_{ij}^{(b)}(t,2)\ , R_{ij}^{(b)}(t,3)\ ],$$

with exponential co-ordinates different in various operation states z_b, $b = 1,2,3,$

At system operational state z_1, system is composed of three series subsystems S_1, S_2, S_3.

The subsystem S_1 is a multi-state series system consists of $n = 147$ components and its reliability function [6] is given by

$$[\ \overline{\boldsymbol{R}}_{147}(t,\cdot)\ ]^{(1)} = [1,\ [\ \overline{\boldsymbol{R}}_{147}(t,1)]^{(1)},\ \overline{\boldsymbol{R}}_{147}(t,2)\ ]^{(1)},\ [\ \overline{\boldsymbol{R}}_{147}(t,3)\ ]^{(1)}],\ \ t \in <0,\infty), \quad (2)$$

where

$$[\ \overline{\boldsymbol{R}}_{147}(t,1)\ ]^{(1)} = \exp[-13.132t], \quad (3)$$

$$[\ \overline{\boldsymbol{R}}_{147}(t,2)\ ]^{(1)} = \exp[-16.569t], \quad (4)$$

$$[\ \overline{\boldsymbol{R}}_{147}(t,3)\ ]^{(1)} = \exp[-21.642t]. \quad (5)$$

The subsystem S_2 is a multi-state series system, consists of $n = 61$ components, its reliability function [6] is given by

$$[\ \overline{\boldsymbol{R}}_{61}(t,\cdot)\ ]^{(1)} = [1,\ [\ \overline{\boldsymbol{R}}_{61}(t,1)\ ]^{(1)},\ [\ \overline{\boldsymbol{R}}_{61}(t,2)\ ]^{(1)},\ [\ \overline{\boldsymbol{R}}_{61}(t,3)\ ]^{(1)}],\ \ t \in <0,\infty), \quad (6)$$

where

$$[\ \overline{\boldsymbol{R}}_{61}(t,1)\ ]^{(1)} = \exp[-5.204t], \quad (7)$$

$$[\ \overline{\boldsymbol{R}}_{61}(t,2)\ ]^{(1)} = \exp[-6.517t], \quad (8)$$

$$[\ \overline{\boldsymbol{R}}_{61}(t,3)\ ]^{(1)} = \exp[-8.406t]. \quad (9)$$

Thus the subsystem S_3 is a multi-state series system, consists of $n = 248$ components, its reliability function [6] is given by

$$[\ \overline{\boldsymbol{R}}_{248}(t,\cdot)\ ]^{(1)} = [1,\ [\ \overline{\boldsymbol{R}}_{248}(t,1)\ ]^{(1)},\ [\ \overline{\boldsymbol{R}}_{248}(t,2)\ ]^{(1)},\ [\ \overline{\boldsymbol{R}}_{248}(t,3)\ ]^{(1)}],\ \ t \in <0,\infty), \quad (10)$$

where

$$[\overline{\boldsymbol{R}}_{248}(t;1)]^{(1)} = \exp[-21.227t], \quad (11)$$

$$[\ \overline{\boldsymbol{R}}_{248}(t,2)\ ]^{(1)} = \exp[-26.577t], \quad (12)$$

$$[\ \overline{\boldsymbol{R}}_{248}(t,3)\ ]^{(1)} = \exp[-34.232t]. \quad (13)$$

The reliability function of the bulk cargo transportation system, at the operational state z_1, [6], is given by

$$[\ \overline{\boldsymbol{R}}\ (t,\cdot)]^{(1)}$$
$$= [1,\ [\ \overline{\boldsymbol{R}}(t;1)\ ]^{(1)},\ [\ \overline{\boldsymbol{R}}(t;2)\ ]^{(1)},\ \overline{\boldsymbol{R}}(t;3)\ ]^{(1)}], \quad (14)$$

where

$$[\ \overline{\boldsymbol{R}}(t;1)\ ]^{(1)} = \exp[-39.563t], \quad (15)$$

$$[\ \overline{\boldsymbol{R}}(t;2)\ ]^{(1)} = \exp[-49.663t], \quad (16)$$

$$[\ \overline{\boldsymbol{R}}(t;3)\ ]^{(1)} = \exp[-64.280t]. \quad (17)$$

The expected values and standard deviations of the bulk cargo transportation system conditional lifetimes in the reliability state subsets calculated from the above result given by (15)-(17), at the operational state z_1 are:

$$m^{(1)}(1) \cong 0.025,\ m^{(1)}(2) \cong 0.020,$$

$$m^{(1)}(3) \cong 0.016,\ \text{y}, \quad (18)$$

$$\sigma^{(1)}(1) \cong 0.025\ ,\ \sigma^{(1)}(2) \cong 0.020,$$

$$\sigma^{(1)}(3) \cong 0.016, \quad (19)$$

from (18)-(19) it follows the conditional lifetimes in the particular reliability states at the operational state z_1 are:

$$\overline{m}^{(1)}(1) \cong 0.005, \overline{m}^{(1)}(2) \cong 0.004,$$
$$\overline{m}^{(1)}(3) \cong 0.016.$$

At system operational state z_2, system is composed of one series-parallel subsystem S_4 and five series subsystems S_5, S_6, S_7, S_8, S_9.
The subsystem S_4 consists of k = 3 identical belt conveyors, each composed of l = 18 components. Thus the subsystem S_4 is a multi-state series-parallel system, its reliability function [6], is given by

$$[\boldsymbol{R}_{3,18}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}_{3,18}(t,1)]^{(2)}, [\boldsymbol{R}_{3,18}(t,2)]^{(2)},$$
$$[\boldsymbol{R}_{3,18}(t,3)]^{(2)}], \ t \in <0,\infty), \quad (20)$$

where

$$[\boldsymbol{R}_{3,18}(t,1)]^{(2)} = 1-[1-\exp[-2.751t]]^3$$
$$= \exp[-8.253t]$$
$$-3\exp[-5.502t]+3\exp[-2.751t], \quad (21)$$

$$[\boldsymbol{R}_{3,18}(t,2)]^{(2)} = 1-[1-\exp[-2.956t]]^3$$
$$= \exp[-8.868t]-3\exp[-5.912t]$$
$$+3\exp[-2.956t], \quad (22)$$

$$[\boldsymbol{R}_{3,18}(t,3)]^{(2)} = 1-[1-\exp[-3.276t]]^3$$
$$= \exp[-9.828t]-3\exp[-6.552t]$$
$$+3\exp[-3.276t]], \quad (23)$$

Thus the subsystem S_5 is a multi-state series system, its reliability function [6], is given by

$$[\overline{\boldsymbol{R}}_{173}(t,\cdot)]^{(2)} = [1, [\overline{\boldsymbol{R}}_{173}(t,1)]^{(2)}, [\overline{\boldsymbol{R}}_{173}(t,2)]^{(2)},$$
$$[\overline{\boldsymbol{R}}_{173}(t,3)]^{(2)}], \ t \in <0,\infty), \quad (24)$$

where

$$[\overline{\boldsymbol{R}}_{173}(t,1)]^{(2)} = \exp[-14.642t], \quad (25)$$

$$[\overline{\boldsymbol{R}}_{173}(t,2)]^{(2)} = \exp[-18.297t], \quad (26)$$

$$[\overline{\boldsymbol{R}}_{173}(t,3)]^{(2)} = \exp[-23.662t]. \quad (27)$$

Thus the subsystem S_6 is a multi-state series system, consists of n = 88 components, its reliability function [6], is given by

$$[\overline{\boldsymbol{R}}_{88}(t,\cdot)]^{(2)} = [1, [\overline{\boldsymbol{R}}_{88}(t,1)]^{(2)}, [\overline{\boldsymbol{R}}_{88}(t,2)]^{(2)},$$
$$[\overline{\boldsymbol{R}}_{88}(t,3)]^{(2)}], \ t \in <0,\infty), \quad (28)$$

where

$$[\overline{\boldsymbol{R}}_{88}(t,1)]^{(2)} = \exp[-7.547t], \quad (29)$$

$$[\overline{\boldsymbol{R}}_{88}(t,2)]^{(2)} = \exp[-9.457t], \quad (30)$$

$$[\overline{\boldsymbol{R}}_{88}(t,3)]^{(2)} = \exp[-12.272t]. \quad (31)$$

Thus the subsystem S_7 is a multi-state series system, consists of n = 18 components, its reliability function [6], is given by

$$[\overline{\boldsymbol{R}}_{18}(t,\cdot)]^{(2)} = [1, [\overline{\boldsymbol{R}}_{18}(t,1)]^{(2)}, [\overline{\boldsymbol{R}}_{18}(t,2)]^{(2)},$$
$$[\overline{\boldsymbol{R}}_{18}(t,3)]^{(2)}], \ t \in <0,\infty), \quad (32)$$

where

$$[\overline{\boldsymbol{R}}_{18}(t,1)]^{(2)} = \exp[-2.751t], \quad (33)$$

$$[\overline{\boldsymbol{R}}_{18}(t,2)]^{(2)} = \exp[-2.956t], \quad (34)$$

$$[\overline{\boldsymbol{R}}_{18}(t,3)]^{(2)} = \exp[-3.276t]. \quad (35)$$

Thus the subsystem S_8 is a multi-state series system, consists of n = 218 components, its reliability function [6], is given by

$$[\overline{\boldsymbol{R}}_{218}(t,\cdot)]^{(2)} = [1, [\overline{\boldsymbol{R}}_{218}(t,1)]^{(2)}, [\overline{\boldsymbol{R}}_{218}(t,2)]^{(2)},$$
$$[\overline{\boldsymbol{R}}_{218}(t,3)]^{(2)}], \ t \in <0,\infty), \quad (36)$$

where

$$[\overline{\boldsymbol{R}}_{218}(t,1)]^{(2)} = \exp[-18.639t], \quad (37)$$

$$[\overline{\boldsymbol{R}}_{218}(t,2)]^{(2)} = \exp[-23.333t], \quad (38)$$

$$[\overline{\boldsymbol{R}}_{218}(t,3)]^{(2)} = \exp[-30.226t]. \quad (39)$$

Thus the subsystem S_9 is a multi-state series system, consists of n = 93 components, its reliability function [6], is given by

$$[\,\overline{\boldsymbol{R}}_{93}(t,\cdot)\,]^{(2)} = [1, [\,\overline{\boldsymbol{R}}_{93}(t,1)\,]^{(2)}, [\,\overline{\boldsymbol{R}}_{93}(t,2)\,]^{(2)},$$

$$[\,\overline{\boldsymbol{R}}_{93}(t,3)\,]^{(2)}],\quad t \in <0,\infty), \qquad (40)$$

where

$$[\,\overline{\boldsymbol{R}}_{93}(t,1)\,]^{(2)} = \exp[-5.926t], \qquad (41)$$

$$[\,\overline{\boldsymbol{R}}_{93}(t,2)\,]^{(2)} = \exp[-8.063t], \qquad (42)$$

$$[\,\overline{\boldsymbol{R}}_{93}(t,3)\,]^{(2)} = \exp[-10.152t]. \qquad (43)$$

The reliability function of the bulk cargo transportation system, at the operational state z_2, [6], is given by

$$[\overline{\boldsymbol{R}}(t,\cdot)]^{(2)} = [1, [\,\overline{\boldsymbol{R}}(t;1)\,]^{(2)}, [\,\overline{\boldsymbol{R}}(t;2)\,]^{(2)},$$

$$[\,\overline{\boldsymbol{R}}(t;3)\,]^{(2)}],\ t \geq 0, \qquad (44)$$

where

$$[\,\overline{\boldsymbol{R}}(t;1)\,]^{(2)} = \exp[-57.758t]-3\exp[-55.007t]$$

$$+3\exp[-52.256t], \qquad (45)$$

$$[\,\overline{\boldsymbol{R}}(t;2)\,]^{(2)} = \exp[-70.974t]-3\exp[-68.018t]$$

$$+3\exp[-65.062t], \qquad (46)$$

$$[\,\overline{\boldsymbol{R}}(t;3)\,]^{(2)} = \exp[-89.416t]-3\exp[-86.140t]$$

$$+3\exp[-82.864t], \qquad (47)$$

The expected values and standard deviations of the bulk cargo transportation system lifetimes at the operational state z_2, in the safety state subsets calculated from the above result, according to (45)-(47), are:

$$m^{(2)}(1) \cong 0.020,\ m^{(2)}(2) \cong 0.016,$$

$$m^{(2)}(3) \cong 0.013, \qquad (48)$$

$$\sigma^{(2)}(1) \cong 0.020\ ,\ \sigma^{(2)}(2) \cong 0.016,$$

$$\sigma^{(2)}(3) \cong 0.013, \qquad (49)$$

and further, using (48), it follows the conditional lifetimes in the particular reliability states at the operational state z_2 are:

$$\overline{m}^{(2)}(1) \cong 0.004,\ \overline{m}^{(2)}(2) \cong 0.003,$$

$$\overline{m}^{(2)}(3) \cong 0.013.$$

At system operational state z_3, system is composed of seven non-homogenous series subsystems S_1, S_2, S_3, S_6, S_7, S_8, S_9.
The reliability function of the bulk cargo transportation system, at the operational state z_3, [6], is given by

$$[\overline{\boldsymbol{R}}(t,\cdot)]^{(3)} = [1, [\,\overline{\boldsymbol{R}}(t;1)\,]^{(3)}, [\,\overline{\boldsymbol{R}}(t;2)\,]^{(3)},$$

$$[\,\overline{\boldsymbol{R}}(t;3)\,]^{(3)}],\ t \geq 0, \qquad (50)$$

where

$$[\,\overline{\boldsymbol{R}}(t;1)\,]^{(3)} = \exp[-74.426t], \qquad (51)$$

$$[\,\overline{\boldsymbol{R}}(t;2)\,]^{(3)} = \exp[-93.472t], \qquad (52)$$

$$[\,\overline{\boldsymbol{R}}(t;3)\,]^{(3)} = \exp[-150.206t]. \qquad (53)$$

The expected values and standard deviations of the bulk cargo transportation system lifetimes at the operational state z_3, in the safety state subsets calculated from the above result, according to (51)-(53), are:

$$m^{(3)}(1) \cong 0.013,\ m^{(3)}(2) \cong 0.011,$$

$$m^{(3)}(3) \cong 0.007, \qquad (54)$$

$$\sigma^{(3)}(1) \cong 0.013\ ,\ \sigma^{(3)}(2) \cong 0.011,$$

$$\sigma^{(3)}(3) \cong 0.007, \qquad (55)$$

and further, using (54), it follows the conditional lifetimes in the particular reliability states at the operational state z_3 are:

$$\overline{m}^{(3)}(1) \cong 0.002,\ \overline{m}^{(2)}(2) \cong 0.004,$$

$$\overline{m}^{(3)}(3) \cong 0.007.$$

If the critical safety state is r = 2, then the system risk function, at the operational state z_2, according to (8), is given by

$$\boldsymbol{r}(t) = 1 - \exp[-93.472t] \text{ for } t \geq 0.$$

In the case when the system operation time is large enough, the unconditional reliability function of the bulk cargo transportation system is given by the vector

$$\boldsymbol{R_9}(t,\cdot) = [1, \boldsymbol{R_9}(t,1), \boldsymbol{R}_9(t,2), \boldsymbol{R_9}(t,3)], \quad t \geq 0, \tag{56}$$

the vector co-ordinates are given respectively by

$$\boldsymbol{R}_9(t,1) = p_1[\overline{\boldsymbol{R}}(t,1)]^{(1)} + p_2[\overline{\boldsymbol{R}}(t,1)]^{(2)} + p_3[\overline{\boldsymbol{R}}(t,1)]^{(3)}$$
$$= 0.6679\exp[-39.563t] + 0.0945 (\exp[-57.758t]-3\exp[-55.007t] +3\exp[-52.256t]) + 0.2376\exp[-74.426t] \text{ for } t \geq 0, \tag{57}$$

$$\boldsymbol{R}_9(t,2) = p_1[\overline{\boldsymbol{R}}(t,2)]^{(1)} + p_2[\overline{\boldsymbol{R}}(t,2)]^{(2)} + p_3[\overline{\boldsymbol{R}}(t,2)]^{(3)}$$
$$= 0.6679\exp[-49.663t] + 0.0945 (\exp[-70.974t]-3\exp[-68.018t] +3\exp[-65.062t]) + 0.2376\exp[-93.472t] \text{ for } t \geq 0, \tag{58}$$

$$\boldsymbol{R}_9(t,3) = p_1[\overline{\boldsymbol{R}}(t,3)]^{(1)} + p_2[\overline{\boldsymbol{R}}(t,3)]^{(2)} + p_3[\overline{\boldsymbol{R}}(t,3)]^{(3)}$$
$$= 0.6679\exp[-64.280t] + 0.0945 (\exp[-89.416t]-3\exp[-86.140t] +3\exp[-82.864t]) + 0.2376\exp[-150.206t] \text{ for } t \geq 0, \tag{59}$$

where $[\boldsymbol{R}(t,1)]^{(1)}$, $[\boldsymbol{R}(t,1)]^{(2)}$, $[\overline{\boldsymbol{R}}(t,1)]^{(3)}$ are given by (15), (45), (51) and $[\boldsymbol{R}(t,2)]^{(1)}$, $[\boldsymbol{R}(t,2)]^{(2)}$, $[\overline{\boldsymbol{R}}(t,2)]^{(3)}$ are given by (16), (46), (52) and $[\boldsymbol{R}(t,3)]^{(1)}$, $[\boldsymbol{R}(t,3)]^{(2)}$, $[\overline{\boldsymbol{R}}(t,3)]^{(3)}$ are given by (17), (47), (53).

The mean values of the system unconditional lifetimes in the reliability state subsets, after considering (17), (47), (81), and (57)-(59), respectively are

$$m(1) = 0.022, \; m(2) = 0.017, \; m(3) = 0.014.$$

The mean values of the system lifetimes in the particular reliability states are

$$\overline{m}(1) = 0.005, \; \overline{m}(2) = 0.003, \; \overline{m}(3) = 0.014,$$

If the critical safety state is r =2, then the system risk function, is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}_9(t,2) \quad t \geq 0,$$

$$\boldsymbol{r}(t) = 1 - [0.6679\exp[-49.663t] + 0.0945 (\exp[-70.974t]-3\exp[-68.018t] +3\exp[-65.062t]) + 0.2376\exp[-93.472t]].$$

Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, is $\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.00084$ years. If δ = 0.1 then $\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.00173$.

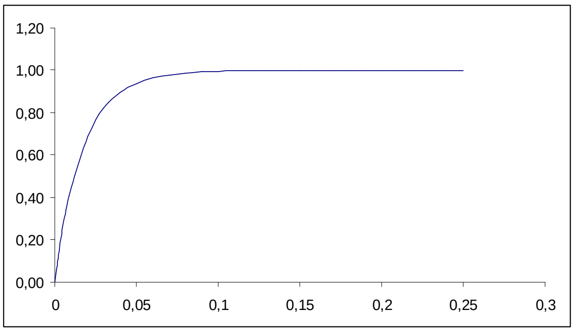

Figure 3. The graph of the port bulk cargo transportation system risk function $r(t)$

Further, assuming that the bulk cargo transportation system is repaired after its failure and that the time of the system renovation is ignored, we obtain the following results:

i) the distribution of the time $S_N(2)$ until the *Nth* exceeding of reliability critical state 2 of this system, for sufficiently large *N*, has approximately normal distribution $N(0.017N, 0.017\sqrt{N})$, i.e.,

$$F^{(N)}(t,2) = P(S_N(2) < t)$$

$$\cong F_{N(0,1)}(\frac{0.017N - t}{0.017\sqrt{N}}),\ t \in (-\infty, \infty),$$

ii) the expected value and the variance of the time $S_N(2)$ until the *Nth* exceeding the reliability critical state 1 of this system take respectively forms

$$E[S_N(2)] = 0.017N,\ D[S_N(2)] = 0.000289N,$$

iii) the distribution of the number $N(t,2)$ of exceeding the reliability critical state 2 of this system up to the moment $t, t \geq 0,$ for sufficiently large *t*, is approximately of the form

$$P(N(t,2) = N)$$

$$\cong F_{N(0,1)}(\frac{0.017N - t}{0.00222\sqrt{t}})$$

$$- F_{N(0,1)}(\frac{0.017(N+1) - t}{0.00222\sqrt{t}}),\ N = 0,1,2,...,$$

iv) the expected value and the variance of the number $N(t,2)$ of exceeding the reliability critical state 2 of this system at the moment $t, t \geq 0,$ for sufficiently large *t*, approximately take respectively forms

$$H(t,1) = 58.824t,\ D(t,1) = 58.824t.$$

Further, assuming that the bulk cargo transportation system is repaired after its failure and that the time of the system renovation is not ignored and it has the mean value $\mu_0(2) = 0.001,$ and the standard deviation $\sigma_0(2) = 0.001,$ we obtain the following results:

i) the distribution function of the time $\bar{\bar{S}}_N(2)$ until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution $N(0.018N, 0.01703\sqrt{N})$, i.e.,

$$\bar{\bar{F}}^{(N)}(t,2) =$$

$$P(\bar{\bar{S}}_N(2) < t) \cong F_{N(0,1)}(\frac{t - 0.018N)}{0.01703\sqrt{N}}),$$

$$t \in (-\infty, \infty),\ N = 1,2,...,$$

ii) the expected value and the variance of the time $\bar{\bar{S}}_N(2)$ until the *Nth* system's renovation take respectively forms

$$E[\bar{\bar{S}}_N(2)] \cong 0.018N,\ D[\bar{\bar{S}}_N(2)] \cong 0.00029N,$$

iii) the distribution function of the time $\bar{S}_N(2)$ until the *Nth* exceeding the reliability critical state 1 of this system takes form

$$\bar{F}^{(N)}(t,2) = P(\bar{S}_N(2) < t)$$

$$= F_{N(0,1)}(\frac{t - 0.018N + 0.001}{\sqrt{0.00029N - 0.000001}}),$$

$$t \in (-\infty, \infty),\ N = 1,2,...,$$

iv) the expected value and the variance of the time $\bar{S}_N(2)$ until the *Nth* exceeding the reliability critical state 2 of this system take respectively forms

$$E[\bar{S}_N(2)] \cong 0.017N + 0.001(N-1),$$

$$D[\bar{S}_N(2)] \cong 0.000289N + 0.000001(N-1),$$

v) the distribution of the number $\bar{\bar{N}}(t,2)$ of system's renovations up to the moment $t, t \geq 0,$ is of the form

$$P(\bar{\bar{N}}(t,2) = N)$$

$$\cong F_{N(0,1)}(\frac{0.018N - t)}{0.1269\sqrt{N}})$$

$$N = 1,2,...,$$

$$-_{N(0,1)}(\frac{0.018(N+1) - t)}{0.1269\sqrt{N}}),$$

vi) the expected value and the variance of the number $\bar{\bar{N}}(t,2)$ of system's renovations up to the moment $t, t \geq 0,$ take respectively forms

$$\overset{=}{H}(t,2) \cong 55.556t, \quad \overset{=}{D}(t,2) \cong 49.743t,$$

vii) the distribution of the number $\overline{N}(t,2)$ of exceeding the reliability critical state 2 of this system up to the moment $t, t \geq 0,$ is of the form

$$P(\overline{N}(t,2) = N) \cong F_{N(0,1)}\left(\frac{0.018N - t - 0.001}{0.1269\sqrt{t + 0.001}}\right)$$

$$- F_{N(0,1)}\left(\frac{0.018(N+1) - t - 0001}{0.1269\sqrt{t - 0.001}}\right), \ N = 1,2,\ldots,$$

viii) the expected value and the variance of the number $\overline{N}(t,2)$ of exceeding the reliability critical state 2 of this system up to the moment $t, t \geq 0,$ are respectively given by

$$\overline{H}(t,2) \cong \frac{t + 0.001}{0.018},$$

$$\overline{D}(t,1) \cong 49.743(t + 0.001),$$

ix) the availability coefficient of the system at the moment *t* is given by the formula

$$K(t,2) \cong 0.9444, \quad t \geq 0,$$

x) the availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0,$ is given by the formula

$$K(t,\tau,2) \cong 55.556 \int_{\tau}^{\infty} \boldsymbol{R}_3(t,2)dt, \quad t \geq 0, \quad \tau > 0.$$

5. Conclusion

In the paper the multi-state approach to the reliability, risk and availability analysis and evaluation of complex technical systems operating in variable operation conditions has been practically applied. Theoretical definitions and results have been illustrated by the example of their application in the reliability, risk and availability evaluation of a bulk cargo transportation system.

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