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GEOMETRIC TASKS DIFFICULTY FROM ANOTHER VIEW

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Abstract. Geometric knowledge and skills are necessary prerequisite for successful engineering study in many branches. Unfortunately, acquiring those is difficult for many even since the very beginning in primary and secondary schools and it needs to be supported. It is obvious that different students need different amounts of time to solve the same task. We raised a question whether or not the speed at which a student solves different tasks is a characteristic property of the individual student. We have analysed the data obtained during the eight years of operating GeoTest – an online system for the assignment of construction geometry tasks and automatic evaluation of the correctness of their solutions. The answer is that a student resolves some tasks quickly and some slowly (compared to another students).

Keywords: geometric constructions, data evaluation, task difficulty, observations

1 Introduction

Geometric knowledge and skills are necessary prerequisite for successful engineering study in many branches. Unfortunately, acquiring those is difficult for many even since the very beginning in primary and secondary schools. *Yet, too many students – and even some math teachers – end up saying that they "hate math" … They have somehow missed the intellectual math experience – and this may limit their lifelong interest in science, engineering, and technology* [7].

Students usually learn some partial and easily algorithmizable procedures successfully. However, especially construction plane geometry tasks and tasks requiring deriving or justification (which benefit, for example, kinematics and projection methods) are difficult to algorithmize and thus difficult for students. These tasks and problems are both illustrative and requiring the student to be aware of (and answer) questions such as: "Does this result satisfy given conditions and constraints in general?" and "What procedure leads to creating / finding the required elements?". These are tasks that, according to Van Hiele [8] criteria for evaluating geometric knowledge, belong to Level 2 (Abstraction) and 3 (Deduction) and according to the Five phases of the learning process they require a *Free orientation* (the fourth one) degree.

With GeoTest [3] we wanted to help develop the geometrical abilities of pupils and students. Its use was reported in [5]. At the same time, however, we have obtained data from the operation that indicate interesting phenomena. We will describe one of these observations.

Obviously, some students are more talented, proficient and faster in solving tasks than others.

Similarly, on the other hand: some tasks are easier for students than others, although they require a comparable, similar range of knowledge to be solved.

The phenomena indicating the difficulty of individual tasks that can be observed in GeoTest are as follows:

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- Which of the tasks teachers choose – this aspect is too much influenced by the teacher's personal preferences.
- The number of tries (submits) the student needs to successfully complete a task – this is influenced by such factors as inattention and distraction while solving, rather than geometry knowledge and skills of the student – until the student thinks he is able to solve the task, he does not submit the solution at all.
- Student’s time spent on solving the task until its first successful solution.

We found not only that this time varies for the student for different tasks (which certainly relates to the complexity of the task), but it turned out that the order of the student in the group of classmates in solving the task is not the same for different tasks.

2 Research question

Different students need different amounts of time to solve the same task [1]. There is nothing surprising about this, since every student has his/her own individual knowledge, skills and abilities, and every teacher knows both fast and slow students.

When looking at troubles students have acquiring geometric skills and knowledge, we wonder if significant differences can be traced between various geometric topics for a student or whether a student is simply “high achiever” or “low achiever” in general.

GeoTest system [3] is a web application for assigning, solving and automatic evaluation [4] of construction tasks. From eight years of its usage we collected a data from about 200 teachers; 10,000 students; 25,000 assigned tasks and 600,000 submitted answers. The data are stored in an SQL database.

GeoTest allows us to measure the time needed for each individual student to solve a specific task, including the time spent on faulty constructions and blind alleys.

Having such information, we can try to answer the following question:

Is a high or low speed a general characteristic of the student, or does each student solve some tasks quickly and other ones slowly?

Or more specifically:

If a student is quicker or slower at resolving a particular task compared to the other students, can it be deduced, based on this information, how quick, compared to such other students, the student is going to be at resolving another task?

3 Method

Unlike in the case of controlled experiments, it is difficult to create conditions under which it would make sense to compare solving times. We are thus, at least, trying to eliminate the influence of the school, teacher and age.

We will look for the answer to our research question using two approaches; the solution to this problem will be described for each of the approaches separately.

4 Approach 1:

To answer this question, we selected three tasks that were frequently assigned by teachers and the groups of students to which these tasks were assigned. From them, we selected groups of students of the same age, taught by the same teachers. In total, there were 121 students (groups of 29, 67 and 25 students, respectively) taught by three teachers. If the student did not resolve the task, the relevant column is missing in the graph.

In Figure 1, we mixed all the students, and in Figure 2, we separated the groups.

Figure 1 displays the time (vertical axis) that the individual students (horizontal axis) needed to solve three different tasks, namely the tasks to construct:

110 – a parallelogram, 120 – a triangle, and 130 – a rhombus.

On the left side, the students are sorted by the time needed to solve task 110, while on the right side, they are sorted by the time needed to solve task 130.

Figure 1 suggests no observable relationship between the times needed to solve these three tasks.

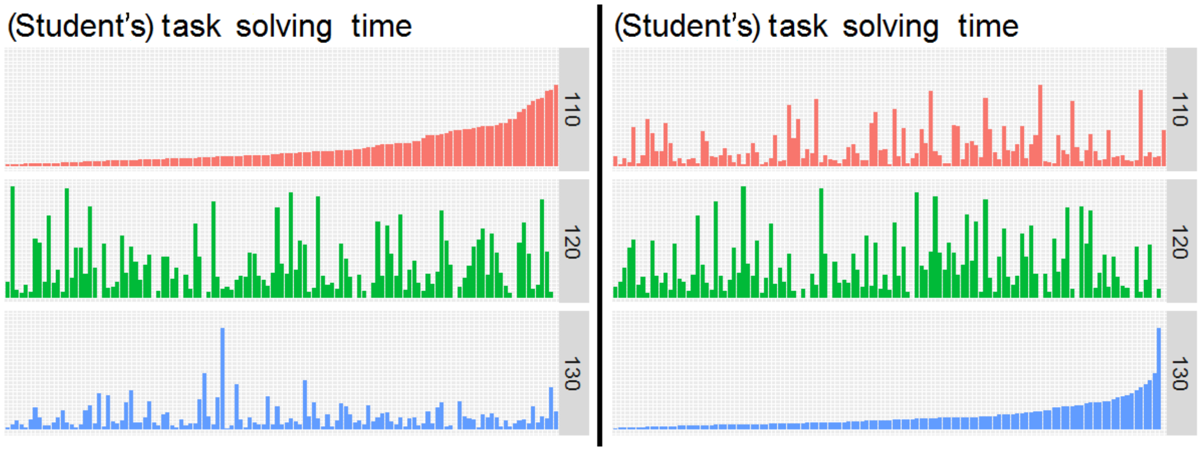

Figure 1: On the left side, the students are sorted by the time needed to solve task 110, while on the right side, they are sorted by the time needed to solve task 130

No observable relationship could be seen in similar graphs created for the individual groups of students (Figure 2) either

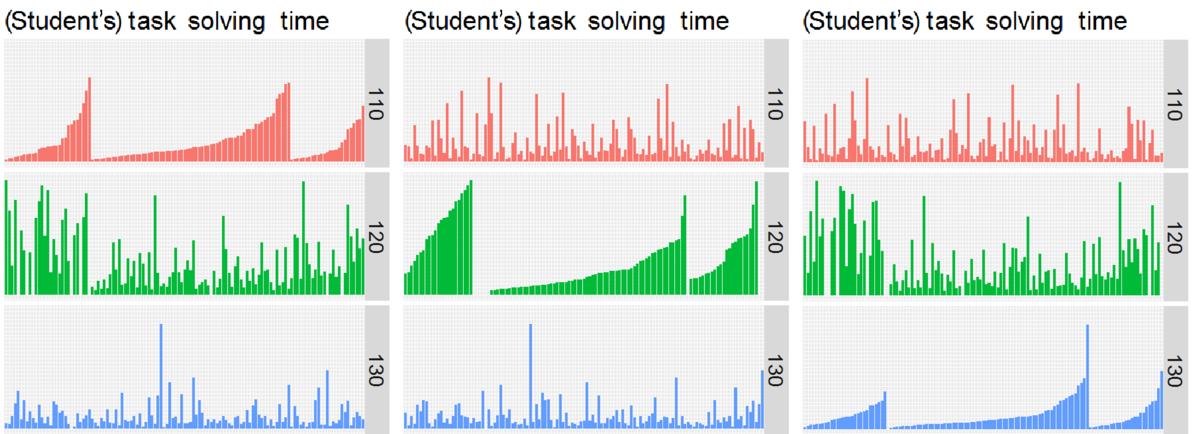

Figure 2: The data are the same as in the graph in Figure 1, but the students are arranged by group at first (three different groups of students)

A Spearman's correlation was run to determine the relationship between the vectors of students' solving times for these three tasks (N=121, 95% CI) with following results (Table 1):

Table 1: Table of the correlation of students'solving time between the tasks 110, 120, 130

Tasks pair	Spearman's ρ	p-value
110–120	0.07427	0.4182
110–130	0.12214	0.182
120–130	0.16995	0.06237

These show that only a weak positive monotonic correlation (Spearman's $\rho=0.17$, $p=0.06$) between tasks 120 and 130 can be observed.

5 Approach 2:

As part of the second observation, we monitored the consistency of the rating "this task is more difficult than that one" provided by two different student groups.

For this observation, we selected two groups of 26 and 30 students. To eliminate as many other possible sources influencing their speed as possible, the groups were selected in such a manner as to ensure that both groups included students from the same school and of the same age and taught by the same teacher in order to make sure that they had been taught the same things.

We created a vector of the times needed by the students to complete each task and evaluated the correlations (for a larger number of tasks this time). Where the student had not resolved the task, the missing time related to the unresolved task was replaced by the maximum time that the students of the given group needed to resolve the task.

In Figure 3, we can see the correlations in the form of heatmaps.

It can be seen that, for instance, for the tasks t90054 and t90074 (or t90054 and t90084) in Group1 (left), the colour is red, which means that the correlation is positive. This means that the students who were quick at resolving the first task were also quick at resolving the second task.

As regards the same pair of tasks in the second group (right), the colour is blue, which shows that the correlation coefficient is negative. Thus, an opposite relationship applies to the same tasks in another group, i.e. who was quick at resolving the first task was slow at resolving the second task.

While the first approach suggests that speed is not determined only by the particular student, the second approach suggests that it is also not determined exclusively by the particular task. This issue deserves a more detailed examination using a controlled experiment.

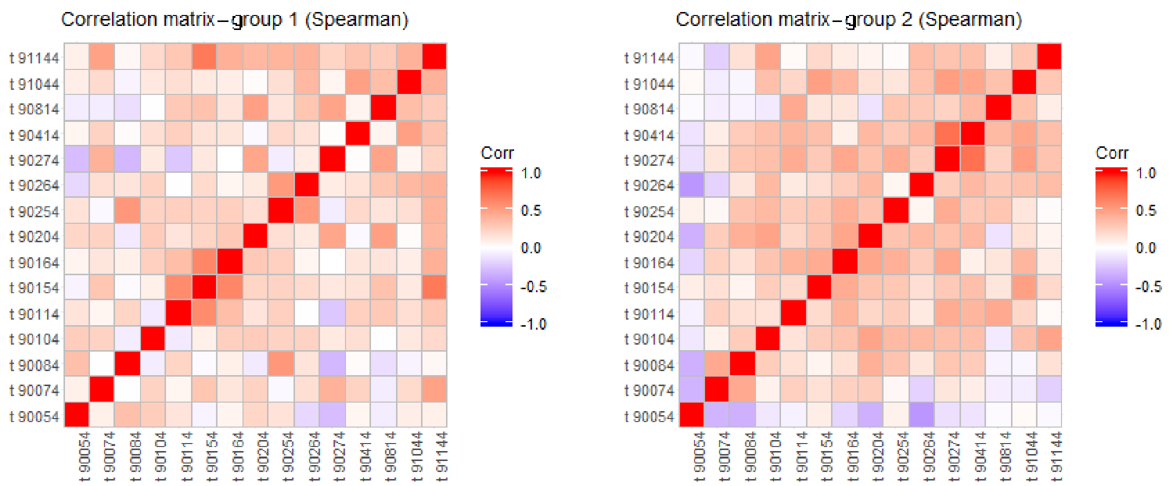

Figure 3: Heatmaps of the correlation matrix showing the correlation between the tasks, a comparison of two groups of students of the same age and taught by the same teacher. The tasks are arranged in the same order in both heatmaps

6 Answer to Research Question:

The conclusion may be surprising: The speed achieved by a particular student is not so straightforwardly determined by a comparison in the sense that "student A is faster than student B". Some students can solve a particular task quickly and another task slowly, or *vice versa*.

7 Discussion

We have analysed the data obtained during the eight years of operating GeoTest – an online system for the assignment of construction geometry tasks and automatic evaluation of the correctness of their solutions.

We raised a question whether or not the speed at which a student solves tasks is a property of the individual student. It seems that there is no dependence between the times spent on resolving various tasks by the same student. We supported this statement by calculating the correlation of individual students' solution times between two groups of students (from the same school and taught by the same teacher). The answer is that a student resolves some tasks quickly and some slowly (compared to another student).

The data obtained is not the result of a controlled experiment, but it is a result of a long-term use of the GeoTest system by teachers and students in schools. Observation is thus influenced by many factors that we are not able to fully understand. From the data used to find the answer to our question, we tried to eliminate some subjective influences (personality and teacher preferences, student age and school type) by selecting the data. However, the obtained results cannot be presented as a sufficiently verified. This should be supported by a controlled experiment.

However, we consider the results interesting and worthy of consideration for further research, especially since there are not enough opportunities to identify the type of tasks that are more difficult for a student than others during school lessons.

8 Conclusion

We have shown that if a student is quick at resolving a particular task, this does not mean that such a student is going to be (relatively) quick at resolving another one.

The answers have been obtained by analysing the data from the GeoTest system. GeoTest has been designed in an attempt to find a tool that could, in an attractive way, encourage geometry teaching, especially the teaching of plane geometry and spatial geometry construction tasks, thereby deepening the understanding of geometric rules.

Using a computer to solve geometric problems affects the student's success. For some, the need to use software is an obstacle in solving the given problem while others are relieved of their clumsiness in drawing.

For many students, the use of software changes the task solving strategy by allowing them to explore more (even wrong) ideas [6]. But, moreover, GeoTest (using GeoGebra tool) moves them away from a strictly visual assessment of the correctness of their solution to the solution that satisfies all given geometric relations. *The key to understanding dynamic geometry is not the memorization of terminology, procedures, propositions, or proofs. It is dependencies.* [7].

We add another perspective to these observations.

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TRUDNOŚĆ ZADAŃ GEOMETRYCZNYCH Z INNEGO PUNKTU WIDZENIA

Wiedza i umiejętności geometryczne są niezbędnym warunkiem udanego studiowania inżynierii w wielu branżach. Niestety, zdobycie ich jest trudne dla wielu i od początku, już w szkole podstawowej i średniej, należy je wspierać. Oczywistym jest, że różni uczniowie potrzebują różnej ilości czasu, aby rozwiązać to samo zadanie. W pracy autorzy postawili sobie pytanie, czy szybkość, z jaką uczeń rozwiązuje różne zadania, jest charakterystyczną właściwością każdego ucznia. Przeanalizowano dane uzyskane w ciągu ośmiu lat funkcjonowania GeoTest - internetowego systemu do przydzielania zadań geometrii konstrukcji i automatycznej oceny poprawności ich rozwiązań. Odpowiedź jest taka: uczeń rozwiązuje niektóre zadania szybko, a niektóre powoli (w porównaniu z innymi uczniami).