

## ACOUSTIC TOMOGRAPHY OF A TURBULENT WATER JET

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*Measurement of parameters of flowing liquid is one of problems, which have many applications. In the present paper we describe linear and nonlinear acoustic tomography methods for reconstruction of parameters of a turbulent water jet with gas bubbles. Experiments on both techniques are described. In the linear acoustic method it was measured fluctuations of acoustic signal passed through the jet. In the nonlinear acoustic technique it was measured spectra of combination scattering signals in the jet. Experiments were done in a water tank with a submerged water jet. The results of the work demonstrate the possibility of the tomography of flowing bubble liquids.*

### INTRODUCTION

One of the most effective methods of diagnostics of turbulence media is the sounding method, which is used for investigation of the earth atmosphere, ionosphere, solar wind plasma and solar corona, and also plasma and hydrodynamics stream [1-6]. Perspective technique in this field proves to be interferometry method, when sounding signal passed through disturbed media is receives in several spatially separated points. Inhomogeneous media provoke phase, amplitude and frequency fluctuations of emission, propagating through them, and disturb the output signal of the instrument. The analysis of interferometer's response allows us to get information about propagation media. In this case, the projection length of the instrument baseline onto the wave front determines the maximum irregularity scale to which an interferometer is sensitive.

It is known that a bubble has prominent nonlinear properties. Nonlinear responses in scattered fields from a bubble are easily observed at the second or higher harmonics of the incident frequency, as well as at the subharmonics of the fundamental frequency and at the combination frequencies of the primary waves [7,8]. This opens up the possibility of using it for bubble detection and sizing. The advantage of nonlinear acoustic techniques are their high selectivity. Different nonlinear acoustical methods have been developed for bubble diagnostics [9-11]. The difference frequency technique is used in this paper for obtaining of the image of turbulent water jet with bubbles. For moving bubbles the specific Doppler frequency shift arises at the difference frequency, which can be used for bubble flow velocity measurement [12]. For low-density bubble populations such measurements can be done in the approximation of given primary waves. For dense bubble streams it is necessary to take into account attenuation of primary waves at bubbles inside the jet. The latter case is considered in this paper and called the difference frequency Doppler tomography method. In the present paper we use the difference frequency method for reconstruction of the flow parameters distribution across the submerged water jet filled with bubbles.

### 1. INTERFEROMETRIC METHOD

To adjust a procedure of that method and further application of it in natural media a special system was developed. Laboratory interferometry experiment on sounding disturbed water medium was carried out in the hydrodynamic tank in Nizhny Novgorod State University. The scheme of the experiment is shown in Fig.1. The transmitter, detectors and water pump nozzle developing turbulence stream are in the same plane at 45 cm depth from the water surface. The depth of the pool is 4.7 m.

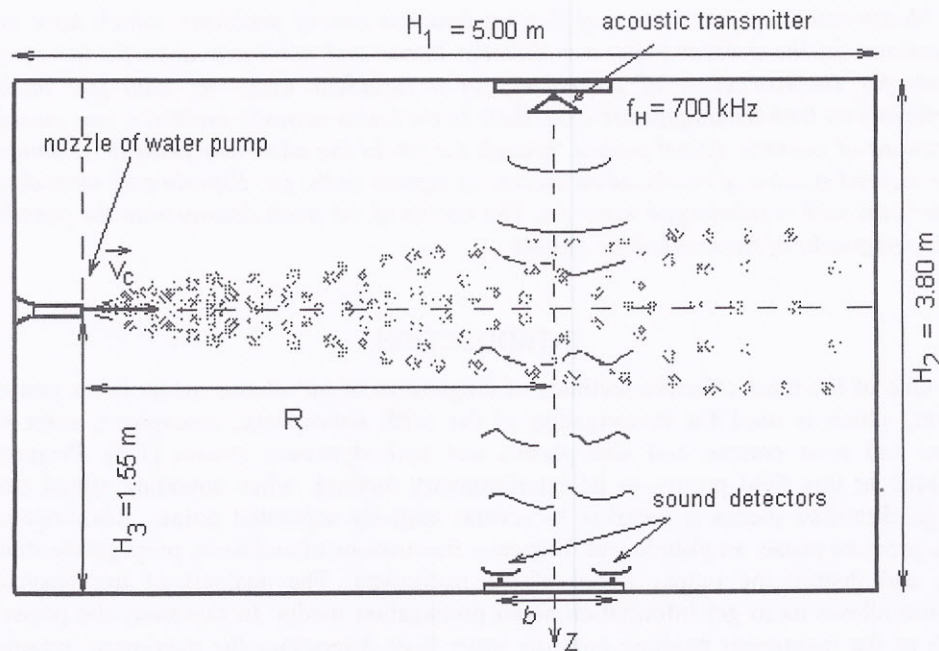


Fig.1. A scheme of the experiment.

Ultrasonic signal in the form of quasi-plane wave was propagated through submerged water jet from transmitter to two interferometer receiving antennas (see axis Z in Fig.1). The distance between detectors  $b$  – baseline of interferometer – was 15 cm, 24.5 cm, 34.5 cm. To exclude signal reflected from the water surface, sounding was made in pulse mode (signal frequency  $f=700$  kHz, frequency of sampling 60 Hz, impulse duration  $t=500$   $\mu$ s). Water stream was directed perpendicular to the propagation path of the emitted signal. Interferometer baseline was parallel to the water stream. Velocity of the water rejection from the nozzle  $V_c=23$  m/s, was decreased in inverse proportion to the distance between nozzle and the paths "transmitter – interferometer baseline". Clipper-limiter allowing to exclude the amplitude fluctuations of the received signal were installed in every receiving circuit.

The signals passed through the media by different paths and receiving in interferometer points were multiplied without time shift. The result of this multiplication was put into computer memory by means analog-digital converter and it proved to be discrete sample of the duration 180, 540, 1800 s with frequency of sampling  $f=60$  Hz.

This procedure is common for interferometer receiving and it has some advantages in comparison with traditional one point receiving as it allows to investigate the field fluctuations caused by turbulent medium only on two different propagation paths. In this case the influence of source self-radiation is excluded and it allows us to sound the medium not only by monochromatic but also wide band source signal. It proves to be very important when the later is raying by sound emission of natural sources. For example, when the solar corona and solar wind are investigated, signals of space radio sources are received; artificial and natural sound sources may be used to research turbulent processes in the ocean.

Autocorrelation function of the resulting interferometer signal was counted. Fourier-analysis of the signal sample was used, and field power spectrum of output signal were achieved.

The examples of the spectra taken in the course of experiments are shown in figure 2.

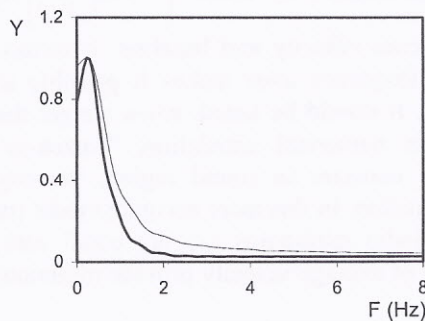


Fig.2. Examples of experimental spectra.

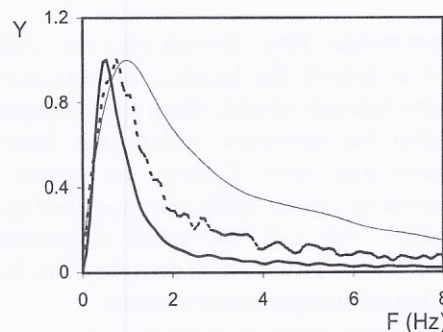


Fig.3. Interferometer response spectra for wide band signals.

Thick line shows interferometer response spectrum on monochromatic emission ( $f_H=700$  kHz) measured at baseline  $b=24.5$  cm and distance from the nozzle  $R=172$  cm. Duration of sample is  $t=1800$  s, frequency resolution is  $df=0.075$  Hz. Half width of the spectral line at the level of half amplitude  $\Delta f_0=0.41$  Hz. Thin line shows the spectrum taken at the same conditions, but at medium sounding by wideband signal ( $f_H=700$  kHz - centre

frequency of signal band,  $\Delta f=100$  kHz - signal bandwidth). Spectrum half-width is  $\Delta f_0=0.56$  Hz.

Figure 3 displays interferometer response spectrum on wideband signal measured on different bases, the sound paths distances from the nozzle  $R=140$  cm, sample duration  $t=540$  s, frequency resolution  $f_0=0.075$  Hz. You can see clearly the spectral line broadening when diminution baseline. When  $b=15$  cm (thin line in the Fig.3), spectral half-width  $\Delta f_0=1.80$  Hz, when  $b=24.5$  cm (dotted line) –  $\Delta f_0=0.70$  Hz, when  $b=34.5$  cm (thick line) –  $\Delta f_0=0.50$  Hz.

Observed data were compared with the results of early theoretical analysis [3]. Calculations were made in the approximation of geometrical optics. Spatial spectrum of the characters medium fluctuations (in our case such parameter was concentration of cavitation bubbles) was given by power low in certain interval of wave numbers  $[\kappa_0, \kappa_m]$  ( $\kappa_0=2\pi/L$ ,  $\kappa_m=2\pi/l_0$ ;  $L, l_0$  – outer and inner scales of irregularities respectively):

$$F = (\kappa_0^2 + \kappa^2)^{-\frac{p}{2}} \tag{1}$$

( $p$  – spectral index).

Our calculations demonstrated that measurements proved to be more informative at weak phase perturbation. In this case the expression of power spectrum coincides with the spectrum of difference phase fluctuations, obtained in similar task in work [6], where the experiment on sounding of surface layer of the atmosphere by sound monochromatic signals received in two point is described. At weak phase fluctuations interferometer power spectrum must be drop-down power low function. When stream velocity is directed along baseline, dependence spectrum from frequency  $\Omega$ , baseline  $b$  and stream velocity on axes  $V$  with accuracy of a constant factor, when  $\Omega \ll V\kappa_m$ , is described approximately by the expression:

$$Y(\Omega) = \left( 1 - \cos\left(\frac{\Omega b}{V}\right) \right) \left[ \left(\frac{\Omega}{V}\right)^2 + \kappa_0^2 \right]^{-\frac{(-p+1)}{2}} \tag{2}$$

It follows from (2) that there must be oscillations determined by factor  $\left[ 1 - \cos\left(\frac{\Omega b}{V}\right) \right]$  on spectrum wings. They depend only on relation of stream velocity and baseline. It means, when  $b$  is known the location of minimum on the frequency axes makes it possible to determine velocity stream along the propagation paths. It should be noted, when we got the expression for spectrum, which was further used in numerical simulation, “frozen-in” hypothesis was used. Velocity on stream axes was constant in sound region, velocity distribution on stream width was described by Gauss function. In this more accurate model (in comparison with (2)), the value of spectrum in periodic minimums are not equal zero. Besides, we should take into consideration fluctuations of average velocity provide minimum smoothing or disappearance of zeros.

Spectrum envelope in the frequency region  $\Omega \gg \kappa_0 V$  may be described by:

$$Y(\Omega) \approx [\Omega]^{-p+1}, \tag{3}$$

It follows that  $p$  – the index of the spatial spectrum of cavity bubbles concentration fluctuations – is determined by the form of experimental spectra.

At modeling medium characters (for example, outer and inner turbulence scale, the irregularities layer width and so on) were varied in such way that theoretical and experimental spectra were the same by their widths. After that observed data were investigated to determine the irregularities transfer velocity and spectral index  $p$ .

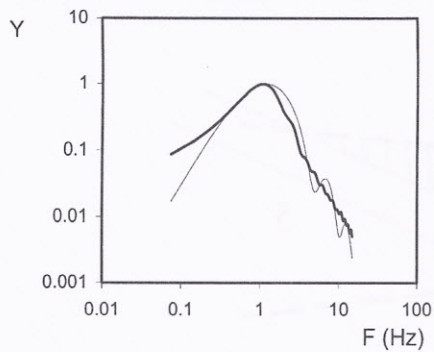


Fig.4. Experimental spectra (thick line), obtained at  $R=91$  cm,  $V=1.84$  m/s. Thin line - numerical simulation.

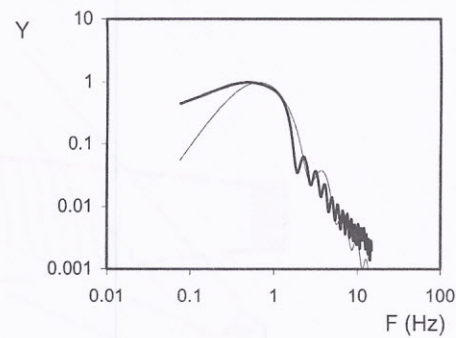


Fig.5. Experimental spectra (thick line), obtained at  $R = 172$  cm,  $V = 1.01$  m/s. Thin line - numerical simulation.

In the experiment we investigated weak phase fluctuations, when  $R$  was changed from 91 to 230 cm. Fig.4 and 5 show in logarithmic scale the examples of experimental spectra (thick line), when sounding the turbulent stream by monochromatic signals on two different distances from the nozzle to propagation paths:  $R=91$  cm, stream axial velocity in sounding region is equal  $V=1.84$  m/s (Fig.4) and  $R=172$  cm,  $V=1.01$  m/s (Fig.5). Sample duration  $t=540$  c, baseline  $b = 34.5$  cm. Thin line displays the results of numerical simulation.

Figs.4 and 5 represent good agreement of theoretical and practical data. Spectral index  $p$ , derived from graphs in most cases is equal to  $11/3$ , that corresponds to Kolmogorov spectrum. Oscillations are clearly seen on experimental spectral "wings". It is rather difficult to determine the velocity value by position of minimums at frequency axes. It is explained by the fact that average velocity fluctuations in experimental conditions could have been essential. Besides, relation was ambiguous, as detectors size were comparable with baseline (the detector diameter 6 cm). Nevertheless, the velocity values, counted by the first power spectrum minimum ( $\Omega_{\min}=V/b$ ) coincide with received values of stream velocity with the accuracy up to 50%.

## 2. NONLINEAR ACOUSTIC IMAGING OF A BUBBLE WATER JET

Experiments on obtaining of an image of the nonlinear acoustic scattering objects – a turbulent water jet with air bubbles - were done in the hydroacoustic water tank. The turbulent water jet was ejected from the cylindrical nozzle at the depth of about 0.5 m from the water surface. A scheme of the experiment is shown in Fig.6. A jet was insonified by two ultrasonic beams (1 and 2 in Fig.1) having frequencies 132 kHz and 195 kHz, respectively. Sound scatterers – bubbles are generated by hydrodynamic cavitation process near the nozzle and are taken by the jet down the flow. The difference frequency 63 kHz signal scattered from the jet is received by the focusing receiving system. The focusing system allow one to register the scattered wave near the focal point of the system. It consists of the hydrophone 3 and the spherical mirror 4 made of porous plastic material providing approximately the free-boundary condition at the mirror surface (about  $-1$  value of the reflection coefficient). The receiving system can be moved in horizontal and vertical plane automatically. The difference frequency signal after filtration amplification is input into computer.

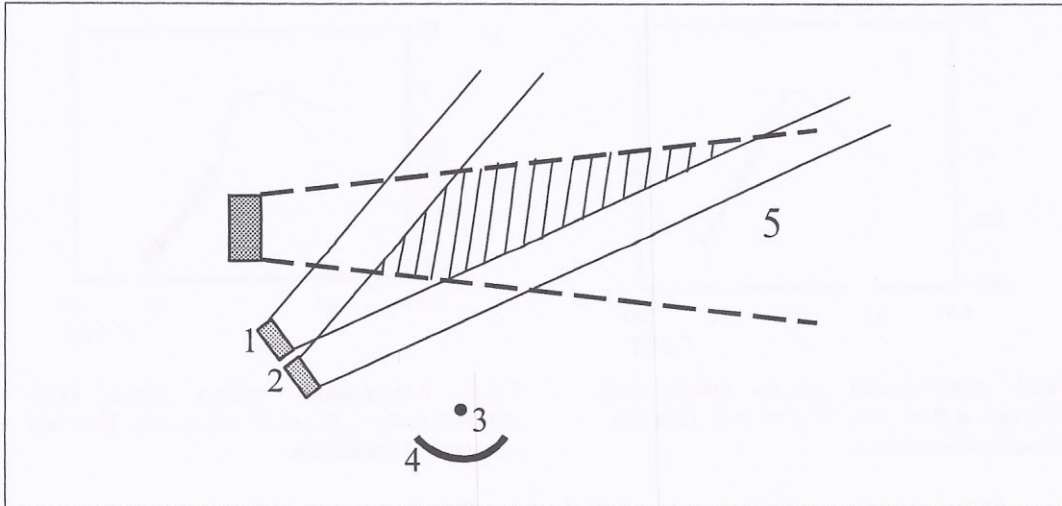


Fig.6. A scheme of experiment. 1 and 2 are primary acoustic wave transducers. 3 – hydrophone, 4 – spherical mirror, 5 – turbulent water jet with bubbles ejected by a nozzle.

Results of experiments are shown in Fig.7. It is a plane image of the jet with bubbles. X-coordinate corresponds to the jet axis, while Y-coordinate is perpendicular to the jet axis. From the figure 7 one can see the main features of the jet: the jet width increases down the flow and there are fluctuations of the bubble density in the jet due to the turbulence.

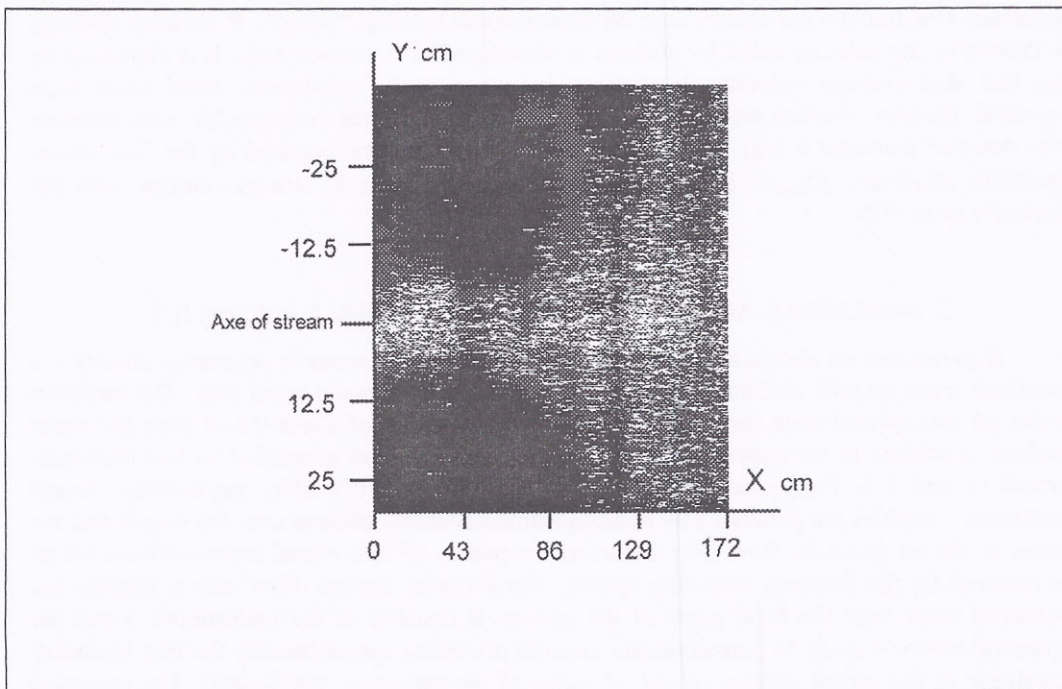


Fig.7. An image of the turbulent jet with gas bubbles obtained at the difference frequency 63 kHz. X-coordinate is along the jet axis, and Y – perpendicular to the jet.

## 3. NONLINEAR DOPPLER ACOUSTIC TOMOGRAPHY

A scheme of the experiment is shown in Fig.8. Two plane piezoceramic transducers insonify the jet and the scattered acoustic wave at the difference frequency is received by the receiving system. The frequency of transducers 1 and 2 were around 1 MHz, and the difference frequencies varied from 100 to 200 kHz.

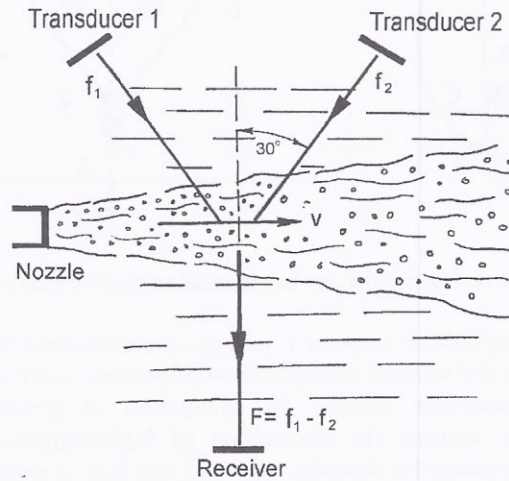


Fig.8. A scheme of the experiment.

In the experiment water flow was created by the centrifugal pump. The nozzle of 1 cm diameter ejected the jet with velocity of 25 m/s. Down the stream the jet diverges and the velocity decreases. The insonified part of the jet was at 60 cm distance from the nozzle. The mean diameter of the jet in the insonified zone was about 11 cm. Diameters of primary acoustic beams in the insonified zone are larger than the jet diameter. The nozzle, two transducers and the receiver were put at the depth of 41 cm from the water surface.

The receiving system represents a spherical hydrophone with the spherical reflector. The reflector of about 30 cm diameter has the focal distance 27.5 cm. The use of the reflector allows one to increase signal to noise ratio in the receiving acoustic signal. Heteronizing were used to transfer the receiving signal into low-frequency range for processing.

The experimentally measured spectra of acoustic signals at the difference frequency were compared to the numerical model for reconstruction of water flow parameters. The theoretical model was based on the approximation of the axially symmetric bubble distribution in the cross section of the jet. We supposed that bubbles move with the stream with the mean local flow velocity. It is taken into account the attenuation of primary acoustic waves inside the jet by resonance bubbles as well as the attenuation of the difference frequency scattered wave. Geometry of numerical and physical experiments is shown in figure 9. Since diameters of primary acoustic beams in the insonified zone are larger than the jet diameter, the primary waves can be considered as plane.

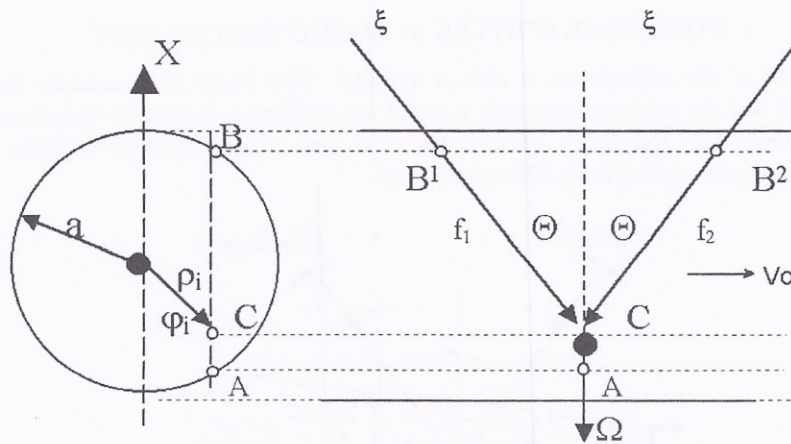


Fig.9. Geometry of the problem. Cross (left) and longitudinal (right) sections of the jet.

Elementary scattering volume in point C radiates the difference-frequency wave, which intensity is proportional to the squared multiplication of primary wave amplitudes and bubble concentrations in the scattering volume. In calculation of primary wave amplitudes insonifying the scattering volume the attenuation of high-frequency primary waves of frequencies  $f_2$  and  $f_1$  along geometry-acoustic rays  $B_1C$  and  $B_2C$  is accounted. Insonified part of the jet is considered as cylindrical volume filled in with microbubbles moving with the flow. The intensity of the difference-frequency-scattered signal generated from an elementary volume around the point C and the amplitude of the received signal is calculated.

Supposing that signals of combination scattering from elementary volumes are incoherent and taking into account the attenuation of primary waves and the difference-frequency wave inside the jet along the path CA, one can write the intensity of the difference-frequency signal at the receiver as follows:

$$\Delta W(\rho_i) \sim \exp(-\beta \rho_i) \cdot \int_0^{2\pi} \exp \left[ -2\sigma_f \int_B^C n(\xi) d\xi - \sigma_\Omega \int_C^A n(x) dx \right] d\varphi,$$

where  $\sigma_f, \sigma_\Omega$  - bubble extinction cross sections at the primary frequencies and the difference frequency, respectively,  $n(\xi), n(x)$  - distribution of bubble concentrations along the primary wave paths and the difference-frequency wave path. This expression allows one to calculate the intensity of spectral component of the difference-frequency signal in any frequency range. This intensity is the sum of intensities of nonlinear scattered signals from elementary volumes, which are at the same distance from the jet axis. Considering the flow velocity distribution across the jet dependent from the radial distance as

$$V = V_0 \exp(-\beta \rho^2),$$

one can obtain an expression for the frequency of the difference-frequency-scattered signal by the elementary volume as follows:

$$\Omega_i = 2f \cdot \sin\Theta \cdot \frac{V_0}{C_0} \cdot \exp(-\beta \rho_i^2).$$

Here  $V_0$  - flow velocity at the jet axis,  $\rho_i$  - radial distance from the jet axis to the elementary volume of scattering,  $\beta$  - scaled coefficient describing the effective jet radius,  $C_0$  - sound velocity. In paper [13] it was shown that the bubble distribution across the jet can be



described by the function:

$$n = n_0 \exp(-2\beta\rho^2) \sim V^2(\rho)$$

We used this model to calculate the form of spectral line. Free adjusting parameters of the model are the exponent power  $\beta$  of the flow velocity distribution (and bubble distribution) across the jet and the velocity  $V_0$  at the jet axis. Besides, values of the jet axis velocity and the bubble concentration were also varied in numerical simulations. Since the primary wave frequencies are closed to each other it was supposed that the primary waves have the same attenuation when propagate inside the jet.

In figure 10 it is shown normalized experimental spectrum of the difference frequency scattering (solid bold line) and theoretical spectra. Solid thin line in the figure 3 is the theoretical spectrum for the jet flow velocity  $V_0 = 3.2$  m/s and for the coefficient  $\beta = 0,2$  cm<sup>-2</sup>. Dashed line is the theoretical spectra for the jet flow velocity of 4M/c and for the same value of  $\beta$  as in previous case. Dotted line represents theoretical spectrum for  $\beta = 0,3$  cm<sup>-2</sup> and  $V_0 = 3.2$  m/s. In the given example the primary frequencies  $f_1$  and  $f_2$  were 1000 and 1100 kHz,

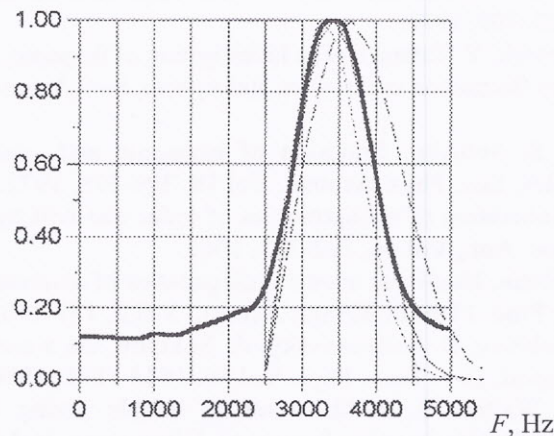


Fig.10. Experimental (solid bold line) and theoretically calculated spectra of the difference-frequency-scattered from the bubble jet acoustic signal.

respectively. The heterodyne frequency was 97.5 kHz. Consequently, the difference frequency  $\Omega = 100$  kHz corresponds to the frequency 2.5 kHz in figure 3. It is seen good agreement between experimental data and theoretical curve for  $V_0 = 3.2$  m/s,  $\beta = 0,2$  cm<sup>-2</sup>.

#### 4. CONCLUSION

Thus the experiments on insonifying of water turbulent stream by monochromatic sound wave with the use of interferometer receiving system have shown that output interferometer signal contains information on spatial spectrum of fluctuations of stream parameters. This information can be revealed that allows one to use this method for investigation of turbulent streams.

The conducted experiments developed model demonstrate also feasibility of the difference-frequency technique of imaging of the nonlinear scattering objects and the difference-frequency Doppler tomography technique for measurements of the spatial distribution of bubbles and velocity across the stream. The use of the difference-frequency

scattering for diagnostics of nonlinear targets is effective due to low level of reverberation at the difference frequency and due to high sensitivity of the nonlinear scattered wave spectrum to the variation of diagnostic parameters.

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