LINEAR BUCKLING OF MEDICAL ROBOT TOOL FOR SOFT TISSUE SURGERY

WYBOCZENIE LINOWE NARZĘDZIA ROBOTA MEDYCZNEGO DO CHIRURGII TKANKI MIĘKKIEJ

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ABSTRACT

Linear buckling is a physical phenomenon that can occur when an external force reaches a value equal to Euler's critical force. This paper gives a solution to an eigenvalue problem that describes the linear buckling. The main purpose of this article is to check when the linear buckling phenomenon will appear in the construction of medical robot tool. There are presented determined values of load factor coefficients, which are eigenvalue, and eigenvectors, which describe the shapes of deformation.

Keywords: linear buckling, eigenvalue problem, medical robot, finite element method

STRESZCZENIE

Wyboczenie liniowe jest zjawiskiem fizycznym, które może się pojawić w przypadku gdy przyłożona siła zewnętrzna będzie miała wartość równą sile krytycznej Eulera. W niniejszej pracy rozwiązano problem własny opisujący wyboczenie liniowe. Głównym celem artykułu jest sprawdzenie, kiedy pojawi się wyboczenie liniowe w narzędziu robota medycznego. Określono wartości współczynników obciążeniowych, które są wartościami własnymi, jak również wektory własne, które opisują kształty deformacji.

Słowa kluczowe: wyboczenie liniowe, zagadnienie własne, robot medyczny, metoda elementów skończonych

1. Introduction

Medical robotics is an important modern area in clinical medicine and biomechatronics, because it helps to ensure the optimal therapy. Surgical procedures with medical robots may be used for procedures on soft tissue or to service the artificial organs [1, 2, 3, 4].

One of the tools of medical robots is multibody effector with serial chain. Figure 1 shows an effector of the medical robot, called Robin Heart [5, 6].

Fig. 1. The effector of Robin Heart medical robot

The effector is attached to the main mechanism of the robot, which is called a constant point mechanism [7, 8]. This mechanism has three degrees of freedom for regional movement and the effector with five degrees of freedom manipulates the soft tissue. The main working space which results from first three degrees of freedom has a shape as shown in the picture below.

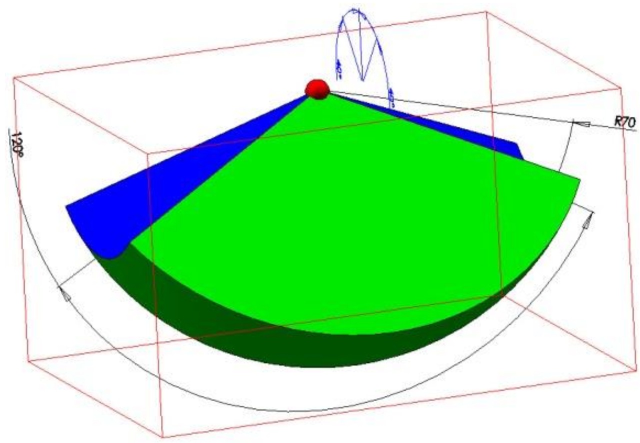

Fig. 2. Shape of the working space of a constant point mechanism

The mathematical formula that describes working space is:

$$|V| = 4\int_{0}^{\frac{\varphi_1}{2}} d\varphi_1 \int_{\varphi_2}^{\frac{\pi}{2}} sin(\varphi_2)d\varphi_2 \int_{0}^{\lambda} \lambda^2 d\lambda \tag{1}$$

From a standpoint of structural analysis, it is important to solve some problems, e.g. the deformation of robot in movement, or to give an answer to a question: ‘When does the effector yield to buckling phenomenon?’. It is also important to analyze modern biomechatronic devices due to the fields of mechanics. So far, there is a lack of elaborations about the elastic buckling of the medical robot effector.

In previous works with eigenvalue problems, authors calculated the linear buckling of a constant point mechanism in medical robot, Robin Heart [9]. Also, the natural frequencies of the tool for servicing artificial organs were calculated and the shapes of the vibration during resonance phenomenon were shown in the paper [10]. The next step in our work was to make a model of vectorial optimization, where two criteria were vital: natural frequency for increasing stiffness of the tool and mass that may minimize inertial forces [11]. The finite element method is used to calculate the elastic buckling problem because it gives a correct solution and the time of computations is not long when the geometrical model of the object is actually simplified. Important information about the problems of elastic buckling, which can be solved by using the finite element method, is announced in the work [12, 13]. Moreover, there is another work which describes the Lanczos method for solving eigenvalue problems [14].

2. Eigenvalue problem of linear buckling and the usage of the finite element method

The problem of linear buckling for the scheme from figure 3 can be outlined as:

$$EJ\frac{d^2y}{dz^2} + Fy = 0. \quad (2)$$

E – Young's modulus,
J – second area moment,
F – external force.
Transforming (2) and substituting (3) by (2), we obtain (4):

$$\frac{F}{EJ} = k^2 \quad (3)$$

$$\frac{d^2y}{dz^2} + k^2y = 0 \quad (4)$$

The general solution of equation (4) has the following pattern:

$$y = Acos(kz) + Bsin(kz) \quad (5)$$

If we take equation (3) and we know that:

$$kl = \pi, 2\pi, \dots, n\pi \quad (6)$$

n – natural number.

Then, we obtain Euler's critical force:

$$F_{kr} = k^2EJ = \frac{\pi^2EJ}{l^2} \quad (7)$$

The force that is given by (7), is the force which causes elastic buckling in thin rod of length l, in the case when some disturbances might occur in the rod.

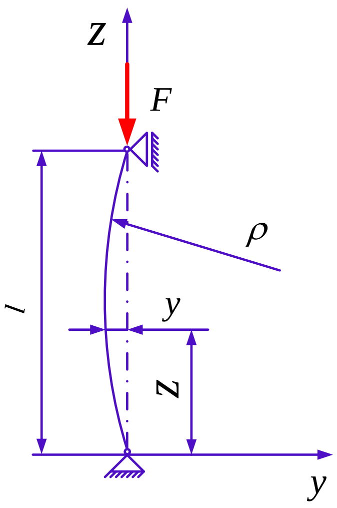

Fig. 3. Scheme of linear buckling

If to a thin rod will apply any force, then it is important to define the λ coefficient of load.
The value of critical Euler's load can be specified as:

$$F_{kr} = \lambda \cdot F \quad (8)$$

The load coefficient can be written as:

$$\lambda = \frac{F_{kr}}{F} \quad (9)$$

The linear buckling is determined by such conditions as:

$\lambda < 1$ – conditions of buckling (instability) (10)
$\lambda > 1$ – conditions of stability (11)

While using the finite element method, the load coefficient λ can be determined at the beginning of numerical analysis. The structural static equation for a little displacement, which describes the correlation between displacements and external forces.

$$[\boldsymbol{K}] \cdot \{\boldsymbol{u}\} = \{\boldsymbol{F}\} \tag{12}$$

$[\boldsymbol{K}]$ – stiffness matrix,
$\{\boldsymbol{u}\}$ – vector of nodal displacement,
$\{\boldsymbol{F}\}$ – vector of nodal forces.

Matrix $[\boldsymbol{K}]$ is obtained from the following equation:

$$[\boldsymbol{K}] = \int_V [\boldsymbol{B}]^T\,[\boldsymbol{D}][\boldsymbol{B}]dV \tag{13}$$

$[\boldsymbol{B}]$ – linear strain - displacement matrix $\{\boldsymbol{\varepsilon}\}$=**[B]**$\{\boldsymbol{d}\}$ (matrix of shape function),
$[\boldsymbol{D}]$ – constitutive matrix,
$\{\boldsymbol{\varepsilon}\}$ – vector of strain,
$\{\boldsymbol{d}\}$ – vector of displacement.

$$[\boldsymbol{K_G}] = \int_V [\boldsymbol{G}]^T\,[\boldsymbol{S}][\boldsymbol{G}]dV^2 \tag{14}$$

$[\boldsymbol{K_G}]$ – stress-stiffness matrix,
$[\boldsymbol{G}]$ – obtained from shape functions by appropriate differentiation,
$[\boldsymbol{S}]$ – initial stresses.

$$([\boldsymbol{K}] + [\boldsymbol{K_G}]) \cdot \{\boldsymbol{u}\} = \{\boldsymbol{F}\} \tag{15}$$

During the loss of stability for equal loads, there are other possible states of equilibrium.

$$([\boldsymbol{K}] + \boldsymbol{\lambda}[\boldsymbol{K_G}]) \cdot \{\boldsymbol{u}\} = \{\boldsymbol{F}\} \tag{16}$$

$$([\boldsymbol{K}] + \boldsymbol{\lambda}[\boldsymbol{K_G}]) \cdot \{\boldsymbol{u} + \boldsymbol{\delta u}\} = \{\boldsymbol{F}\} \tag{17}$$

After subtracting the equations (16) and (17), a symmetrical problem that defines the stability of the substitutional system is obtained to solve:

$$([\boldsymbol{K}] + \boldsymbol{\lambda}[\boldsymbol{K_G}]) \cdot \{\boldsymbol{\delta u}\} = \{\boldsymbol{0}\} \tag{18}$$

$\boldsymbol{\lambda}$ – eigenvalues, which are load coefficients,
$\boldsymbol{\delta u}$ – eigenvector, which is the shape of buckling.
The system of equation (18) is solved by using the Lanczos method for large symmetrical systems.

3. Model of the tool

The cleaning geometry operation was used with the purpose of simplifying the shape of the tool. This results in a significant shortening of computing time. The discrete model has 73 728 degrees of freedom to obtain an excellent accuracy of the numerical value in the linear buckling solution. Ten node tetra-hedral elements were used to discretized continuous model.

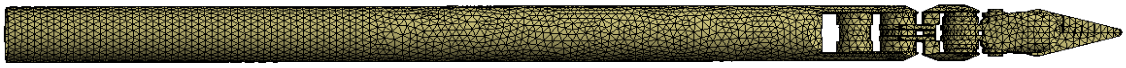

Fig. 4. Discrete model of the effector

Fig. 5. Boundary conditions of the model

The model was supported in plane A and 10 N force was applied in B, so that the tool can be compressed.

4. Results

Numerical calculations were carried out for three lengths of medical robot effector. The first length is 149.31 mm and four shapes of deformation are presented in figures 6 to 9. For lambda equal to 54.62, the effector is deformed in plane XY. In the second eigenvalue effector, it is deformed in orthogonal direction to XY plane. The third shape is similar to the deformation as in the first direction and the fourth is a like to the second shape of deformation.

Fig. 6. First shape of deformation for lambda equal to 54.62

Fig. 7. Second shape of deformation for lambda equal to 70.49

Fig. 8. Third shape of deformation for lambda equal to 146.34

Fig. 9. Fourth shape of deformation for lambda equal to 264.47

In figures 10–13, deformations of the 249.13 mm effector are shown.

Fig. 10. First shape of deformation for lambda equal to 22.751

Fig. 11. Second shape of deformation for lambda equal to 24.095

Fig. 12. Third shape of deformation for lambda equal to 111.34

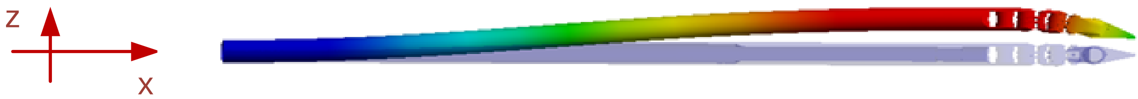

Fig. 13. Fourth shape of deformation for lambda equal to 177.97

In pictures 14 to 17, there are illustrated deformations of the effector’s length of 349.13 mm for the next load factors.

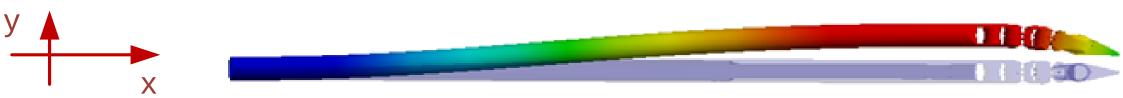

Fig. 14. First shape of deformation for lambda equal to 11.83

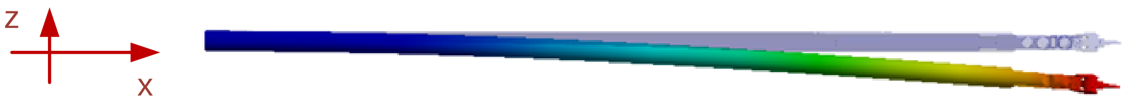

Fig. 15. Second shape of deformation for lambda equal to 12.08

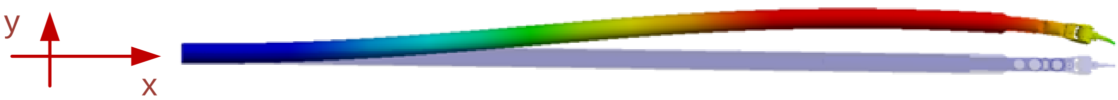

Fig. 16. Third shape of deformation for lambda equal to 86.335

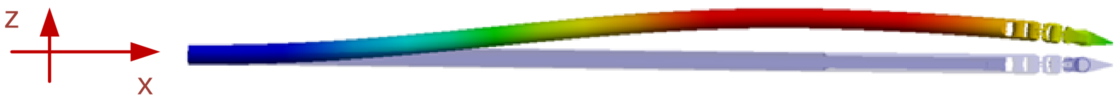

Fig. 17. Fourth shape of deformation for lambda equal to 109.08

Based on the results obtained from those three experiments with three lengths of the effector, it can be concluded that the load factor increases its value when the length of it increases as well.

5. Conclusions

Conducted studies are substantial for checking the buckling phenomenon, in order to use the medical robots safely during operations and before the usage to pass the control tests in medicine field.

The solution of physical model of the effector is obtained by using the finite element method. With the aim of solving the issue of linear buckling physical phenomenon, the discrete mesh model with correct numbers of degrees of freedom was created. For discretization of the geometric model, tetrahedral elements with ten nodes were used. After solving the issue with the model, the shapes of deformation due to the buckling physical phenomenon and coefficient of load were multiplied by the value of imposed force to acquire the Euler's critical force. Leaning on the numerical results, it can be deduced on which length and load the tool for soft tissue surgery starts to buckle. The outcomes showed that the effector of medical robot will not be in buckling state while working with the soft tissue. The significant aim of this work was to reduce the computation time of a discrete mesh model by using the cleaning geometry operation, which has simplified the shape of the tool.

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