



Mathematical model of evaluation of telematics system reliability in internal transport on the production line

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ABSTRACT

In the modern economy we may observe a very rapid development of telematics systems. The authors of this paper present a mathematical model facilitating determination of telematics system's reliability in internal transport on a production line of a selected enterprise. Due to the fact that the transportation telematics system operates in the areas of determinist and indeterminist (uncertain, of incomplete fuzzy observation) environments, a model combining the Bayes classifier with fuzzy sets has been proposed. Moreover, in this paper the authors indicate a possibility of adjusting the model to other telematics systems used in a variety of sectors of the economy. As the telematics systems operate in determinist and indeterminist areas, and the system's reliability depends on all of its symptoms, therefore a combination of these two theories was essential to be able to describe a risk of their operation.

KEYWORDS: intelligent transport system, development of telematics systems

1. Introduction

Telematics systems in internal transport on the production line represent a huge challenge to their designers. First of all, such a system has to meet high requirements in respect of reliability. Each system error may lead to costly shutdowns of the whole production line. In production halls there occur various complex logistics processes. An efficiently operating telematics system affects timeliness of completion of individual operations and the company's profit. Efficiency and fluidity of transport of raw materials, intermediates and finished products in a production line is essential. In the event of mass production, quality and reliability of the above mentioned system are crucial. A telematics system in a production line is based on a number of installations, defined equipment and transportation systems. The whole structure is controlled by the use of IT technologies. A telematics system represents equipment i.e. crosses, lifts, belt conveyors, roll or chain conveyors, turntables, transport trucks. Each of these elements represents an important link, an object in the system's integrity. Damage to one of the

elements may stop the whole production line. It may also happen that damaging one element only decreases the system's efficiency. In particular, in automatic transport lines reliability is of crucial importance as in such systems the human factor is minimized. The operation of transport systems in production lines may rely on OPC DA and OPC UA standards which will facilitate full access to necessary information on the status of the equipment in real time. Elements of the telematics system in production lines include PLC Controllers, RTU modules, SCADA Systems, recorders, counters etc. Erroneous generation of information by one of the links affects quality of the remaining components of the system,. Therefore the authors present a model of evaluation of reliability in a local but also global aspect of the system. The model proposed here combines the Bayes model with a fuzzy function of resistance. Fuzzy sets are applied here for describing unprecise and incomplete data.

2. Mathematical description of a task of evaluation of the telematics system's reliability

In the first case it is to be assumed that there exist elements of the telematics system, such as sensors, detectors, drivers, controllers, which having got damaged would not be subject to repair, so readiness of a telematics object, probability of readiness (if the object is not repaired) is defined as [3]:

$$K_g(t)=R(t) \tag{1}$$

If Δt_u means operation time of given equipment (object) u of the telematics system, we subject to a subjective evaluation a percentage damage to the object. So let's assume the following form of non-damageability of the telematics object:

$$\begin{aligned} R_{k,PP}^{KT} &= R_k^{KT}(t, TP, \omega) = P_k^{KT}\{A(t, TP, \omega; 0 \leq \tau < t)\}, \\ R_{k,PW}^{KT} &= R_k^{KT}(L, \tilde{S}) = P_k^{KT}\{A(L, \tilde{S}; 0 \leq \tau < t)\}, \\ R_k^{KT} &= P_k^{KT}\{A(t, TP, \omega)\} \oplus \{A(L, \tilde{S})\}, \end{aligned} \tag{2}$$

where:

t – operating time of a structural part of a telematics object

L – degree of damage of a single object [0-100]

$\tilde{S} = [\tilde{S}_u, \tilde{S}_p]^T$ - fuzzy vector of resistance to operational conditions in the production line (the theory of fuzzy sets)

\tilde{S}_u - fuzzy value of a degree of faulty operation of the software

– decision making ability of the system [0-1]

\tilde{S}_p - number (size) of wrong impulses during operating time t

TP – temperature T of mechanical equipment P operating in the telematics system

ω - average velocity of all mobile components of the telematics system

k – subsequent 24-hours operation cycle of the telematics system or a stage after resetting of the production line

PP – operation parameters

PW – resistance parameters

$\dot{\cup}$ - orthogonal sum.

$P_k^{KT}\{A(t, L, S, TP, \omega)\}$ - means Bayes' probability function defining the condition that the telematics system is suitable for further use.

Through Bayes' probability function [1,2,7]

$$P(A) = \sum_{A_u \in A} p(A_u) = 1 \tag{3}$$

where:

$p(A_u)$ is the suitability function [0-1] of each object u- of this telematics system.

$p(A_u)$ means the function which meets the conditions:

$$\begin{aligned} \sum_{A_u \in 2^{\theta}} p(A_u) &= 1 \\ p(\phi) &= 0 \end{aligned} \tag{4}$$

Considering decompositions (distribution) of suitability of all objects of the telematics system it is possible to perform their aggregation to the combined decomposition (distribution) of the suitability of the whole telematics system thus obtaining a new base decomposition (distribution) m according to the rule [4,5]

$$p(C) = \frac{\sum_{A_1 \cap A_2 \cap \dots \cap A_u = C} p_1(A_1) \cdot p_2(A_2) \cdot \dots \cdot p_u(A_u)}{\sum_{A_1 \cap A_2 \cap \dots \cap A_u \neq \phi} (1 - p_1(A_1) \cdot p_2(A_2) \cdot \dots \cdot p_u(A_u))}. \tag{5}$$

The reliability system at the assumption of a fuzzy evaluation of resistance will be a rule system.

An example rule has the following form:

IF an object has operated in a given time interval Δt^n

And the values of operation parameters TP, ω and resistance parameters L, S occurred during this time **THEN** the object is not damaged.

$$A(\{L, \tilde{S}\} \oplus \{\omega, t, TP\}) \tag{6}$$

with an average value of the function of probability allocation $p_{ij}(\{L_j, \tilde{S}_j\})$ and $p_{ij}(\{TP_j, \omega_j, t_j\})$

where:

i – a number of trials

j – a number of objects

Thus, obtained rules of the function of mass from n-experts we combine with use of the Dempster's rule of combination (6). Then it is easy to determine the value of the conviction function (4), which in the context of the task presented in this manner will have the following form:

$$\begin{aligned} &P_k^{KT}\{A_{j_1 \dots j_n}^{KT}(t, TP, \omega)\} \oplus \{A_{j_1 \dots j_n}^{KT}(L, \tilde{S})\} \\ &= m_{j_1 \dots j_n}^{KT}\{A_{j_1 \dots j_n}^{KT}(t, TP, \omega)\} \oplus \{A_{j_1 \dots j_n}^{KT}(L, \tilde{S})\} \end{aligned} \tag{7}$$

where:

$P_{j_1 \dots j_n}^{KT}\{A_{j_1 \dots j_n}^{KT}(t, TP, \omega)\} \oplus \{A_{j_1 \dots j_n}^{KT}(L, \tilde{S})\}$ - is a mass function composed of independent decompositions (distribution) of reliability for any object of the telematics system in the production line.

Let's notice the fact that all values of the parameters in given time intervals are measured, calculated and remembered.

An example of a probabilistic classifier found in the Fig. 1-4:

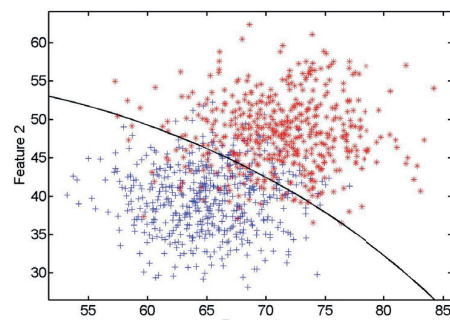


Fig. 1. Example 1 classification for simulation data [own study]

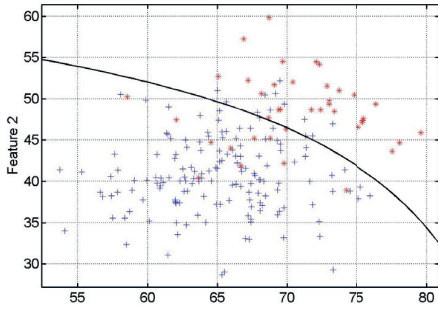


Fig. 2. Example 2 classification for simulation data [own study]

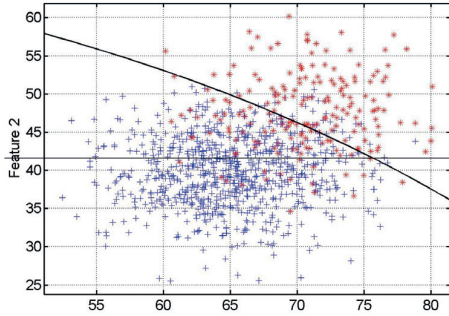


Fig. 3. Example 3 classification for simulation data [own study]

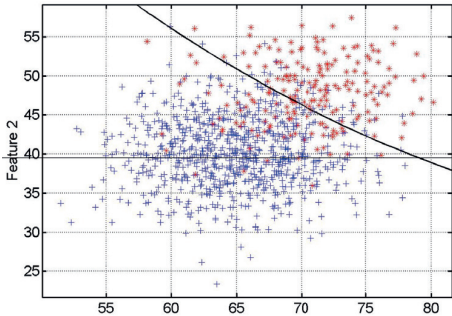


Fig. 4. Example 4 classification for simulation data [own study]

3. Practical example

Let's assume that we have five features describing the object and let us make them:

t, ω, \tilde{S}, L and TP .

The values of these variables in a given moment of time amount to: $t=4h, \tilde{S}=0.1\%, TP=50^\circ C, \omega=1200obr/min$. The rules will have the following form:

IF an object has operated in a given time interval for 4 hours and during this time the values of the operation parameters $TP=50^\circ C, \omega=1200obr/min$ and the resistance parameters $L=0.1\% \tilde{S}=0.1\%$ occurred

THEN the object is not damaged

$$A(\{L, \tilde{S}\} \oplus \{\omega, t, TP\}) \quad (8)$$

with an average value of the function of probability allocation $mp_{1j}(\{L_j, \tilde{S}_j\}) = 0.96$ and $p_{1j}(\{TP_j, \omega_j, t_j\}) = 0.99$ and $p_{2j}(\{L_j, \tilde{S}_j\}) = 0.99$ and $p_{2j}(\{TP_j, \omega_j, t_j\}) = 0.95$

Then, independently from each other, the allocation functions for the measured parameters have been assigned: object 1

$$p_1(\{\tilde{S}, L\}) = \begin{cases} A\{\tilde{S}, L\} & 0.96 \\ \neg A\{\tilde{S}, L\} & 0.04 \end{cases}$$

$$p_1(\{t, \omega, TP\}) = \begin{cases} A\{t, \omega, TP\} & 0.99 \\ \neg A\{t, \omega, TP\} & 0.01 \end{cases}$$

object 2 of the telematics system

$$p_2(\{\tilde{S}, L\}) = \begin{cases} A\{\tilde{S}, L\} & 0.99 \\ \neg A\{\tilde{S}, L\} & 0.01 \end{cases}$$

$$p_2(\{t, \omega, TP\}) = \begin{cases} A\{t, \omega, TP\} & 0.95 \\ \neg A\{t, \omega, TP\} & 0.05 \end{cases}$$

Having applied the Dempster's rule of combination we combine the above two decompositions (distribution) of reliability for the two objects

$p_3 = p_1 \oplus p_2$ and we obtain the combined decomposition of the form:

$$p_3(\{\tilde{S}, L\}) = \begin{cases} A\{\tilde{S}, L\} & 0.9995 \\ \neg A\{\tilde{S}, L\} & 0.0005 \end{cases}$$

$$p_3(\{t, \omega, TP\}) = \begin{cases} A\{t, \omega, TP\} & 0.9994 \\ \neg A\{t, \omega, TP\} & 0.0006 \end{cases}$$

The events $A\{\tilde{S}, TP\}$ and $A\{t\}$ prove the suitability of the telematics element for further use. So both of the elements prove for class A - the element is suitable for use. The events $\neg A$ prove unsuitability of the element for further use. Thus, we have divided the space of the events into two classes A and $\neg A$. In view of the above let's make an assumption $p_4(\{L, \tilde{S}, TP, \omega, t\}) = p_3(\{L, \tilde{S}, TP\} \oplus p_3(\{\omega, t\}))$, as a result of which we will obtain the new base decomposition (distribution):

$$p_3(\{L, \tilde{S}, TP, \omega, t\}) = \begin{cases} A\{L, \tilde{S}, TP, \omega, t\} & 0.999 \\ \neg A\{L, \tilde{S}, TP, \omega, t\} & 0.001 \end{cases}$$

Finally we obtain the function of conviction in the form:

$$P(A) = 0.999$$

$$P(\neg A) = 0.001$$

From the above it results that we have 99.9% certainty that the telematics system is suitable for further use.

The procedure described above enables us to change conviction into reliability of the telematics system in subsequent stages of operation of the equipment. At a certain point it may turn out that a given expert will not be able, based on the received parameters, to determine whether an element of the telematics system is suitable for further use or not suitable. Below we present a way of drawing conclusions in such a situation.

Let's assume, for simplification purposes, that we have only three features describing the object, and let's make them:

t, ω, \tilde{S} and TP .

The values of these parameters will be: $t=400h, S=1\%, TP=60^\circ C, \omega = 1500obr / min$ (let's make it a velocity of a rotating element in the production line) accordingly. The rules will be determined in the example below:

IF the object has operated in a given time interval of 40 hours and during this time interval there occurred the values of the operation parameters $TP = 60^\circ C, \omega = 1500obr / min$ and the resistance parameters $L = 0.3\% S = 0.4\%$

THEN the object is not damaged

$$A(\{L, \tilde{S}\} \oplus \{\omega, t, TP\}) \quad (9)$$

with an average value of the function of probability allocation $p_{1j}(A\{L_j, \tilde{S}_j\}, \neg A\{L_j, \tilde{S}_j\}) = 0.6$ and $p_{1j}(A\{TP_j, \omega_j, t_j\}, \neg A\{TP_j, \omega_j, t_j\}) = 0.7$ and $p_{2j}(\{L_j, \tilde{S}_j\}) = 0.9$ and $p_{2j}(\{TP_j, \omega_j, t_j\}) = 0.92$

Then, independently from each other, the allocation functions for the measured parameters have been assigned: object 1

$$p_1(\{\tilde{S}, L\}) = \begin{cases} A\{\tilde{S}, L\}, \neg A\{\tilde{S}, L\} & 0.6 \\ \Theta & 0.4 \end{cases}$$

$$p_1(\{t, \omega, TP\}) = \begin{cases} A\{t, \omega, TP\}, \neg A\{t, \omega, TP\} & 0.7 \\ \Theta & 0.3 \end{cases}$$

For object 2 of the telematics system

$$p_2(\{S, L\}) = \begin{cases} A\{\tilde{S}, L\} & 0.9 \\ \neg A\{\tilde{S}, L\} & 0.1 \end{cases}$$

$$p_2(\{t, \omega, TP\}) = \begin{cases} A\{t, \omega, TP\} & 0.92 \\ \neg A\{t, \omega, TP\} & 0.08 \end{cases}$$

Having applied Dempster's rule of combination opinions presented above:

$p_3 = p_1 \oplus p_2$ and we obtain the combined decomposition of the form:

$$p_3(\{\tilde{S}, L\}) = \begin{cases} A\{\tilde{S}, L\} & 0.9 \\ \neg A\{\tilde{S}, L\} & 0.1 \end{cases}$$

$$p_3(\{t, \omega, TP\}) = \begin{cases} A\{t, \omega, TP\} & 0.7 \\ \neg A\{t, \omega, TP\} & 0.3 \end{cases}$$

Due to the above, we make a composition $p_4(\{L, \tilde{S}, TP, \omega, t\}) = p_3(\{L, \tilde{S}, TP\} \oplus p_3(\{\omega, t\}))$, as a result of which we obtain a new base decomposition:

$$p_3(\{L, \tilde{S}, TP, \omega, t\}) = \begin{cases} A\{L, \tilde{S}, TP, \omega, t\} & 0.95 \\ \neg A\{L, \tilde{S}, TP, \omega, t\} & 0.05 \end{cases}$$

Finally, we obtain the conviction function in the form:

$$P(A) = 0.95$$

$$P(\neg A) = 0.05$$

It results from the above that we have 95% certainty that the telematics system is suitable for further use.

Values of the conviction function determined in the subsequent strokes enable us to determine the reliability characteristics. In Fig. 2, the characteristics performed based on the simulation of parameters selected at random [6] of the telematics system relying on exponential distribution have been presented. 96% efficiency of representation of the values obtained with the use of the above model with the generated data has been obtained.

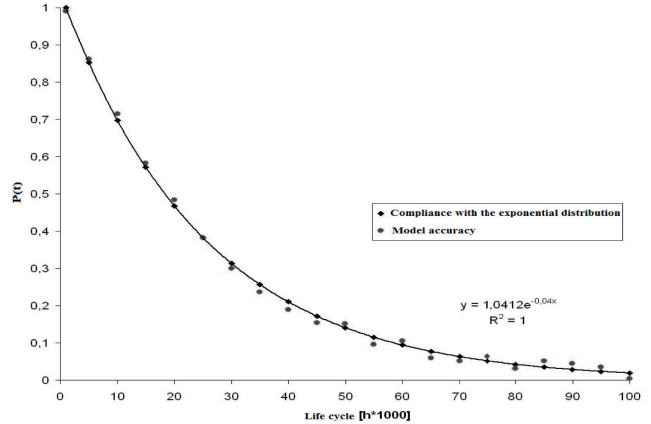


Fig. 5. Simulation results of the concluding algorithm based on the Bayes function of conviction [own study]

As one can see in Fig. 1, the area of generated data cover precisely the values of the function of conviction obtained by way of analysis.

If we are interested in the hypotheses belonging to a certain not empty subset

Δ of the whole space Θ , then representation Γ may be presented as follows [7]:

$$\Gamma_{\Delta}(\{\omega\}) = \Gamma(\{\omega\} \cap \Delta). \quad (10)$$

On this assumption, the measure P has the form:

$$P(H | \Delta) = (P(H \cup (\Theta - \Delta)) - P(\Theta - \Delta)) / (1 - P(\Theta - \Delta)). \quad (11)$$

Applying the formulas 9 and 10 we may, based on the possessed experts' rules determine e.g. the characteristics $y=P(S), y=P(L), y=P(TP), y=P(\tilde{S} | t), y=P(L|t)$.

In this work is presented a model of evaluation of reliability of the telematics system. This model enables us to comprise reliability of individual objects of the telematics system in one consistent base decomposition of reliability of the whole system. If we determine that the telematics system is not suitable for further use we assume the value of conviction Bel as a limit value for the given operating time. As a result of the problem presented in this manner we may obtain a base decomposition of e.g. the function of conviction from the operating time. An unquestionable advantage of the model is that the rule of combining reliability of individual elements of the system facilitates the creation of one base decomposition which will be sensitive to the weakest links of the system. IN other words, we will obtain a more accurate

decomposition of the function of conviction which will reflect in a more precise manner a base decomposition of conviction and a limit value of suitability of an object in given operating conditions e.g. in the area of time.

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