Dynamic states of three-phase induction motors.
Selected problems

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The paper presents some problems arising during dynamic processes undergoing in three-phase induction motors, particularly in some asymmetric connection modes. Some characteristics illustrating the machine behaviour, particularly during start-up and at asymmetric connection modes, are presented with the use of basic differential equations describing operation of three-phase induction motors.

1. Introduction

The problems of dynamic states of electric machines arose relatively recently, as it was conditioned by development of modern computation equipment. The theory of electric machines including the operation modes at constant rotational speed and the problems of electromagnetically unsteady condition at constant rotational speed have been developed long time before. Nevertheless, the studies on dynamic state, i.e. on the behaviour of the machines under electromagnetically unsteady condition at varying rotational speed became feasible only when an appropriate computation equipment and computational software appeared. The dynamic state may be researched with the use of the circuit or field methods. The paper describes mainly the circuit method as the most suitable for analysis of the operational problems. In modern automated drive systems the dynamic states of electric machines occur nearly continuously and, therefore, knowledge of machine behaviour in these circumstances has a large practical meaning.

2. Basic equations of motion of an electromechanical transducer

Equation of motion of an electromechanical transducer under rotational motion, with assumption that the moment of inertia does not depend on time as well as flexibility in rotational motion and mechanical damping are neglected, may be written in the form (1):

$$M(t) = J(\frac{d^2}{dt^2}\upsilon(t)) + M_m \qquad (1)$$

where: J – total moment of inertia of the rotating masses; M_m – mechanical moment of the load; υ – the angle of deviation between the axes of the rotor and stator windings; ω – angular speed of the rotor; $\omega_s = 2 \cdot \pi \cdot f_s$ – angular speed of the rotating field, M(t) – electromagnetic moment.

Electromagnetic moment of the asynchronous electromechanical transducer obtained from the functional of energy transformation has a form (2):

$$\upsilon = \int \omega(t)dt \tag{2}$$

where: $i_s(t)$, $i_w(t)$ – the stator and rotor instantaneous currents, respectively; L_{sw} – coefficient of mutual inductance between particular phase windings of the stator and rotor; p – the number of pole pairs.

The electromagnetic moment may be expressed too with the help of the function of current matrix, depicted by the equation (3):

$$M(t) = \frac{p}{2}\left(\sum_{s=A}^{m} \sum_{w=n}^{n} i_s(t)i_1(t)\frac{d}{d\upsilon}L_{sw} \right) \tag{3}$$

where: $[i_s(t)]$, $[i_w(t)]$ – matrices of the stator and rotor instantaneous currents; $[L_{sw}]$ – the matrix of mutual stator – rotor inductances.

Such a way of formulation of the equations enables consideration of harmonic components of the machine magnetic field as well as harmonic components of the stator and rotor currents.

Matrices of coefficients of mutual inductance between symmetrically arranged stator and rotor windings, with consideration of harmonic components of the magnetic field may be written down in the form (4):

$$M(t) = p(i_A(t)i_B(t)i_C(t))\frac{d}{d\upsilon}\begin{pmatrix} L_{Aa} & L_{Ab} & L_{Ac} \\ L_{Ba} & L_{Bb} & L_{Bc} \\ L_{Ca} & L_{Cb} & L_{Cc} \end{pmatrix}\begin{pmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{pmatrix} \tag{4}$$

This part of the matrix only three elements of a single matrix column from the equation (3) are shown, that are denoted by L_{Aa}, L_{Ba}, and L_{Ca}. Taking into account large dimension of particular matrix elements only some selected matrix elements are presented.

For example, a following selected element of the matrix of A-phase stator current is presented:

$$i_A(t) = i_{A1\max}\cos(\omega t) + i_{A3\max}\cos(3\omega t) + i_{A5\max}\cos(5\omega t) + \ldots$$

Dynamic states of induction machines may be studied with the use of computation or measurement methods. Both methods are not simple, as they require good computation or measurement equipment. The paper presents some selected results of the dynamic states and stationary conditions resulting thereof. In order to carry out the numerical computation and to simulate various patterns,

inclusive of the time ones, for example the MATLAB software from Math Works may be used.

3. Basic characteristics of the dynamic states

As a result of various computations and computer simulations the dynamic states are depicted by time-patterns of the currents and moments, that are determined as time functions. What concerns the steady state, they are defined as the functions of rotational speed. Nevertheless, these characteristics are not determined in electromagnetically steady states, but in a dynamic process. Figure 1 shows such a characteristics determined during start-up of an unloaded motor supplied with symmetrical three-phase voltages, with symmetrical impedance of the rotor circuit.
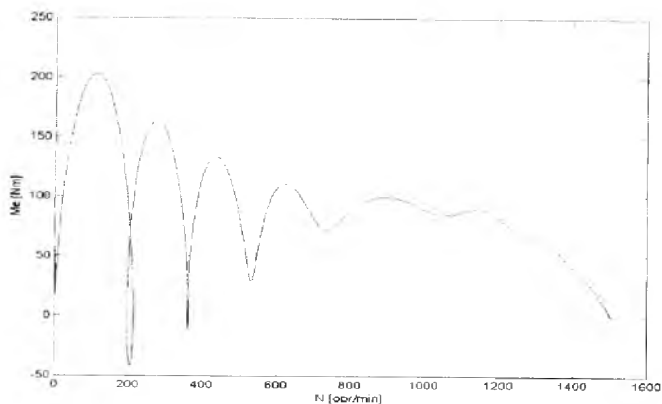

Fig. 1. Dynamic characteristics of an induction motor

It is easy to notice that the motor achieves the speed approaching the asynchronous speed. The pattern of dynamic moment has characteristic loops, being a result of speed oscillation during motor start-up. It may be easily found that such a characteristics presented as a function of rotational speed clearly deviates from the moment characteristics. The latter is determined too as a function of speed but in electromagnetically steady states and for various values of the speed. This characteristics is shown in Figure 5. Another form of the characteristics, however similar to the one from Fig. 1, is shown in Fig.2.

This characteristics is determined for the operational condition under which the voltage decays in one of the phases during the start-up. In result an asymmetric state occurs in the motor, due to asymmetry of the supply voltage. The pattern of the moment includes the loops characteristic for the dynamic state. Nevertheless, in the neighbourhood of the synchronous speed the motor does not achieve the steady operation point that is visible in Fig. 1, but instead its moment oscillates.
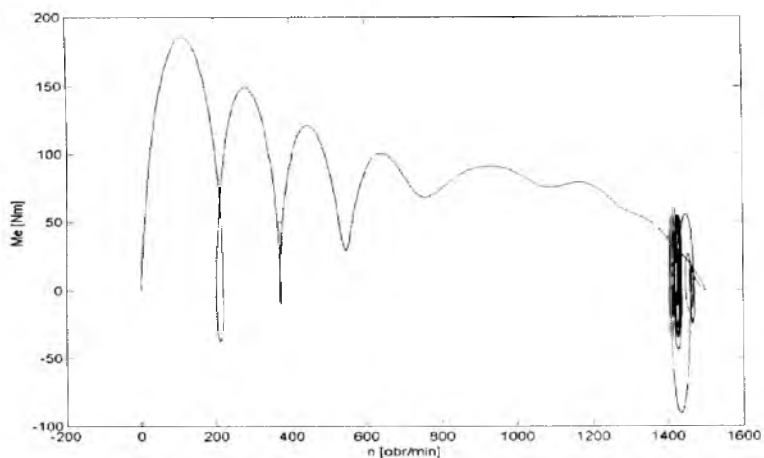
Fig. 2. Dynamic characteristics of the induction motor moment under condition of decay of one of the phases during the start-up

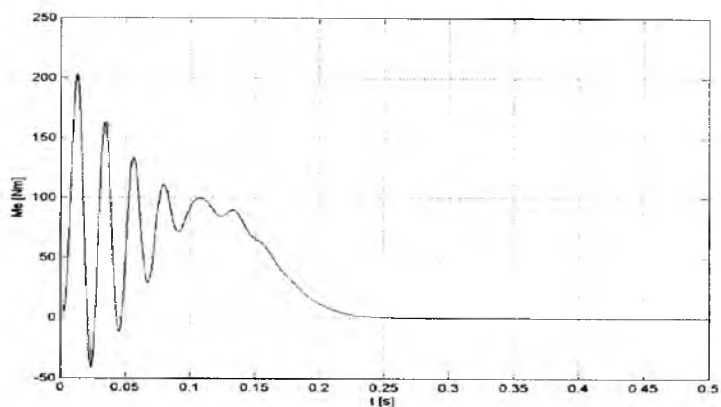Time pattern of the dynamic moment during start-up of unloaded motor is shown in Fig. 3.

Fig. 3. Time pattern of electromagnetic moment during start-up of an induction motor

The figure depicts moment oscillation within first two seconds of start-up of the induction motor. It is easy to notice that after several oscillations the characteristics approaches the steady one shown in Fig. 5. One can discern the maximum moment characteristic for an induction machine, that is considerably below the moment surge. The moment oscillation that, according to the electromagnetic parameters may take even negative value, is a reason of the loops occurring in the moment characteristics shown in Fig. 1. Negative values of the dynamic moment force the oscillation of the speed as a function of time, shown in Fig. 4.
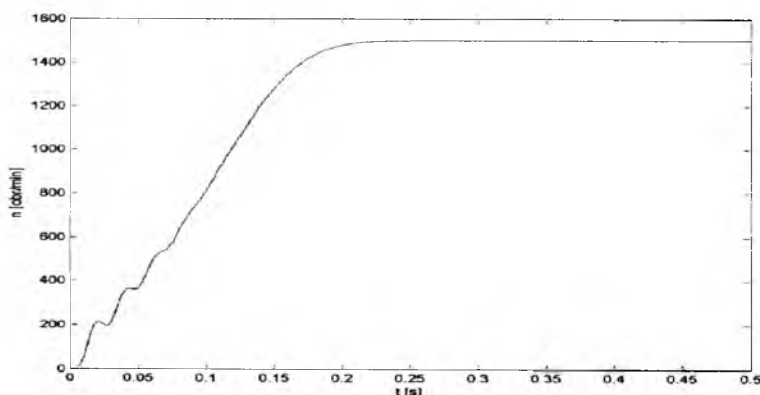
Fig. 4. Time pattern of rotational speed during start-up of an induction motor

4. Steady state at constant rotational speed

A particular case of dynamic (unsteady) state occurs when the steady state at constant rotational speed arises. Such an operational condition occurs in all known characteristics of the induction machines. For example, it is a case of the characteristics of moment vs. rotational speed (Fig.5) that is determined in electromagnetically steady states for various values of the rotational speed or the slip *s*. The slip *s* is calculated according to the relationship (5):

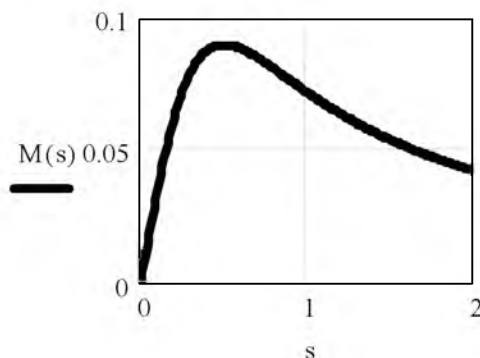$$s = \frac{\omega_s - \omega}{\omega_s} \tag{5}$$

Fig.5. Characteristics of the induction machine moment vs. the slip *s*

In case of such an operational condition the time functions of stator and rotor currents may be obtained for asymmetry of the three-phase supply voltages, or asymmetry of resistance of the external rotor circuit.

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The time function of stator current in the phase A may be formulated with the use of the method of symmetrical components in the form (6):

$$i_A(t) = i_{11m} \cdot \cos(\omega_z \cdot t - \varsigma_{11}) + i_{12m} \cdot \cos[(\omega_z + 2 \cdot \omega)t - \varsigma_{11}) +$$

$$+ i_{21m} \cdot \cos[(\omega_z - 2 \cdot \omega)t - \varsigma_{21})] + i_{22m} \cdot \cos(\omega_z \cdot t - \varsigma_{11}) \qquad (6)$$

The time function of rotor current is described by the equation (7):

$$i_a(t) = i_{341m} \cdot \cos[(\omega_z - \omega)t - \varsigma_{341})] + i_{342m} \cdot \cos[(\omega_z + 2 \cdot \omega)t - \varsigma_{342})] \qquad (7)$$

where: $\omega_s = 2 \cdot \pi \cdot f_s$ – pulsation of the mains voltage; f_s – frequency of the mains voltage; $\omega = 2 \cdot \pi \cdot f$ – pulsation corresponding to the assumed rotational speed.

The above equations give evidence that:
- In case of asymmetrical supply of a motor operating with asymmetrical resistance of external rotor circuit, in the stator and rotor currents the components of the following frequencies appear:
  a) in the stator current f_z, $(1 - 2s) \cdot f_z$, and $(3 - 2s) \cdot f_z$;
  b) in the rotor current sf_z, and $(2 - s) \cdot f_z$.
- In case of asymmetry of the motor supply with symmetrical resistance of the external rotor circuit the stator current components have a frequency f_z, while the frequencies of the rotor current components are equal to sf_z and $(2 - s) \cdot f_z$.
- In case of symmetrical supply of a motor operating with asymmetrical resistance of external rotor circuit the frequencies of the stator current components are equal to f_z and $(1 - 2s) \cdot f_z$, with equal frequency of both rotor current components, amounting to sf_z.

In the electric moment equation M(t) two constant components may be discerned: non-pulsating component M_{st} and pulsating components M(t) of the frequencies (8):

$$[2(\omega_z - \omega)t + \varsigma_1], \ [2(\omega_z + \omega)t + \varsigma_2], \ [2\omega_z t + \varsigma_1 + \varsigma_2] \text{ and } [2\omega t] \qquad (8)$$

In case of symmetry of the external rotor circuit and asymmetric supply the pulsating component of the moment has the frequency equal to $[2\omega_z t + \varsigma_1 + \varsigma_2]$. On the other hand, for symmetric supply and asymmetric external rotor circuit the frequency of pulsating component of the moment is equal to (9)

$$[2(\omega_z - \omega)t + \varsigma_1]. \qquad (9)$$

General form of the time function of the moment is given be the formula (10)

$$M(t) = M_1(t) - M_2(t) + M_3(t) + M_4(t) + M_5 - M_6 \qquad (10)$$

where:
– pulsating components

$$M_1(t) = cU_{1s}^2 (\cos(2\omega_z t - \omega t) - \varepsilon_1)$$

$$M_2(t) = dU_{2s}^2 (\cos(\omega_z - \omega)2t - \varepsilon_2)$$

$$M_3(t) = eU_{1s}U_{2s}(\cos(2\omega_z t) - \varepsilon_3)$$

$$M_4(t) = gU_{1s}U_{2s}(\cos(2\omega t) - \varepsilon_4)$$

– non-pulsating components

$$M_5 = KU_{1s}$$

$$M_6 = LU_{2s}^2$$

Substitution of the parameters of the considered machine into the formula describing the time function of the moment gives example pattern of the moment (Fig. 6). The pulsating components of the moment are noticeable in the moment pattern.
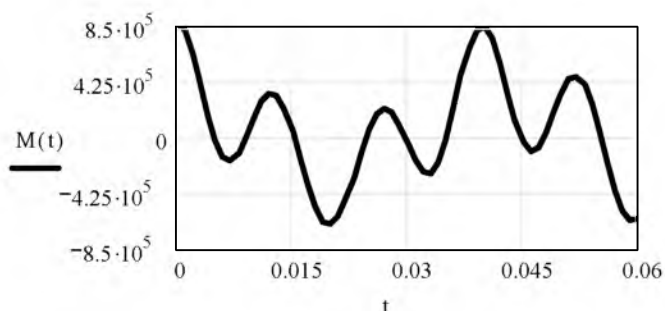

Fig. 6. Example pattern of the moment M(t)

5. Summary and conclusions

Studies on dynamic states of induction machines became feasible in result of implementation of modern computation and measurement equipment. Results of the studies enabled to assess complexity of the phenomena that accompany dynamical processes occurring in the induction machines. It was found, among others, that during the motor start-up a negative electromagnetic moment may arise, that is conducive to oscillation of the rotational speed. Computation programs developed for the dynamic states may be used in order to analyze the operational conditions of the machines in steady state.

References

[1]   Półrolniczak W. Praca dyplomowa magisterska. Politechnika Poznańska, Wydział Elektryczny. Poznań, 2010.
[2]   Stein Z. Zagadnienia stanów niesymetrycznych trójfazowych maszyn indukcyjnych. Wydawnictwa Uczelniane Politechniki Poznańskiej. Seria Rozprawy, nr 79. Poznań 1976.