Kołowrocki Krzysztof

Soszyńska Joanna

Maritime University, Gdynia, Poland

Reliability, availability and safety of complex technical systems: modelling – identification – prediction – optimization

Keywords

reliability, safety, multistate system, operation process, complex system

Abstract

There is presented a practically well grounded approach concerned with the identification, evaluation, prediction and optimization of reliability, availability and safety of technical systems related to their operation processes. The main emphasis of this approach is on multi-state systems composed of ageing components and changing during the operation processes their structures and their components reliability and safety characteristics. There are proposed the convenient tools for analyzing these systems in the form of semi-markov modeling the systems' operation processes and multistate modeling the systems' reliability. There are described theoretical results of the proposed approach to reliability and safety analysis of multi-state systems with degrading components in their operation processes and the possibility of their practical applications to the reliability and safety analysis and optimization of the complex technical port and maritime transportation systems.

1. Introduction

Most real technical systems are very complex and it is difficult to analyze their reliability, availability and safety. Large numbers of components and subsystems and their operating complexity cause that the identification, evaluation, prediction and optimization of their reliability, availability and safety are complicated. The complexity of the systems' operation processes and their influence on changing in time the systems' structures and their components' reliability characteristics are very often met in real practice. We meet complex technical systems, for instance, in piping transportation of water, gas, oil and various chemical substances. Complex technical systems are also used in electrical energy distribution, in telecommunication, in rope transportation, in maritime transport and in shipyard and port transport systems using belt conveyers and elevators. Rope transportation systems like port elevators and ship-rope elevators used in shipyards during ship docking and undocking are model examples of such systems. Taking into account the importance of the safety and operating process effectiveness of such systems it seems reasonable to expand the two-state approach to multi-state approach [30], [34], [88]-[90] in their reliability and safety analysis. The assumption that the systems are composed of multi-state components with reliability states or safety states degrading in time without repair gives the possibility for more precise analysis of their reliability, safety and operational processes' effectiveness. This assumption allows us to distinguish a system reliability or safety critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operational process effectiveness. Then, an important system reliability or safety characteristic is the time to the moment of exceeding the system reliability or safety critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state reliability function and the system multi-state safety function that are basic characteristics of the multi-state system.

The convenient tools for analyzing these problems are semi-markov modeling [14], [58], [73] of the systems' operation processes and multistate approach [30], [34], [88]-[90] to the systems' reliability evaluation proposed in the paper.

The aim of the paper is to propose a complete approach to the reliability and safety identification, evaluation, prediction and optimization for as wide as possible a range of complex technical systems. Pointing out the possibility of this approach extensive and well founded practical application in the operating processes of these systems is also an important aspect of the paper.

The objective of this paper is to present recently developed mainly by the authors the general reliability, availability and safety analytical models of complex non-repairable and repairable multi-state technical systems related to their operation processes [35]-[38], [42]-[51], [72]-[80] and their practical applications to real industrial systems and processes [10], [16], [18], [26], [29], [32], [47], [48]-[49], [75], [80]. Integrated general models of complex industrial systems, linking their reliability, availability and safety models and their operation processes models and considering variable in different operation states their reliability and safety structures and their components reliability and safety parameters are considered. The common usage of the multistate system reliability and availability evaluation models [1]-[4], [6], [8], [17], [21]-[24], [27]-[34], [40], [42], [50], [55]-[57], [60], [62], [65]-[69], [86]-[89] and the semi-markov model [5], [7], [11], [13]-[14], [25], [58]-[59], [61], [63], [83] for the system operation processes modelling in order to construct the joint general system reliability and availability models related to their operation process is the proposed approach main idea. Joint models linking the reliability models of the considered typical multistate systems and their varying in time operation processes models are suggested to be applied in the reliability, availability and safety analysis of real complex technical systems. These joint reliability models of complex technical systems, together with linear programming [25] are proposed to reliability, availability and safety optimization [25], [53]-[54], [83]-[84], [90]-[91] and system operation cost analysis [41], [82], [90]-[91].

There are proposed the methods and tools useful in the statistical identifying the unknown parameters of the joint general system reliability and availability models related to their operation process [43]-[44]. There are presented statistical methods of determining unknown parameters of the semi-markov model of the complex technical system operation processes. There is suggested the chi-square goodness-of-fit test to be applied to verifying the distributions of the conditional system operation process sojourn times in the particular operation states. Moreover, there are presented the methods of estimating the unknown intensities of departure from the reliability state subsets of the exponential distribution of the component lifetimes of the multistate system in various operation states and the goodness-of-fit method s proposed to be applied to testing the hypotheses concerned with the exponential form of the multistate reliability function of the particular components of the system in variable operations conditions.

The proposed in the paper models and methods may be successfully applied, for instance, to reliability, availability and safety analysis, identification, prediction and optimization of the port and maritime transportation systems related to their varying in time their operation processes, their structures and their components reliability and safety characteristics.

2. Modeling complex technical systems operation processes

In analyzing the operation process of the complex technical system with the distinguished operation states $z_1, z_2, \ldots, z_\nu$, the semi-markov process may be used to construct its general probabilistic model [45]. To build this model the following parameters are defined:

- the vector of probabilities $[p_b(0)]_{1x\nu}$ of the system operation process initials operation states,
- the matrix of probabilities $[p_{bl}]_{\nu x\nu}$ of the system operation process transitions between the operation states,
- the matrix of conditional distribution functions $[H_{bl}(t)]_{\nu x\nu}$, of the system operation process conditional sojourn times θ_{bl} in the operation states.

To describe the system operation process conditional sojourn times in the particular operation states the uniform distribution, the triangle distribution, the double trapezium distribution, the quasi-trapezium distribution, the exponential distribution, the Weibull distribution, the normal distribution and the chimney distribution are suggested as suitable.

Under these definitions and assumptions, the following main operation process characteristics can be predicted: - the vector $[H_b(t)]_{1x\nu}$, of the unconditional distribution functions

$$H_b(t) = \sum_{l=1}^{\nu} p_{bl} H_{bl}(t), \;\; b = 1,2,...,\nu, \qquad (1)$$

of the sojourn times θ_b, $b = 1,2,...,v,$ of the system operation process at the operation states,

- the vector $[M_b]_{1xv}$, of the mean values

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl}, \; b = 1,2,...,v, \tag{2}$$

of the unconditional sojourn times θ_b, $b = 1,2,...,v,$

- the vector $[p_b]_{1xv}$ of the limit values of the transient probabilities

$$p_b = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \; b = 1,2,...,v, \tag{3}$$

at the particular operation states, where M_b, $b = 1,2,...,v,$ are given by (2), while the probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1, \end{cases} \tag{4}$$

- the vector $[\hat{M}_b]_{1xv}$ of the mean values

$$\hat{M}_b = E[\hat{\theta}_b] = p_b \theta, \; b = 1,2,...,v, \tag{5}$$

of the total sojourn times $\hat{\theta}_b$ in the particular operation states for sufficiently large operation time θ.

3. Modeling reliability, availability and safety of multistate systems with ageing components

In the systems' reliability and safety analysis it is practically reasonable to expand their two-state models to the multi-state models [34]. The multi-state series, parallel, "*m* out of *n*", consecutive "*m* out of *n*", series-parallel, parallel-series, series-"*m* out of *n*", "*m* out of *n*"-series, series-consecutive "*m* out of *n*" and consecutive "*m* out of *n*"-series systems with degrading components can be defined and their reliability functions can be determined. Having these definitions, the multi-state system risk function and other multi-state system reliability characteristics can be introduced and determined. The reliability functions of multi-state systems composed of components having exponential reliability functions can be given as well. Moreover, in an analogous way, a multi-state approach to defining the basic notions and analysis of systems' safety can be proposed.

Introducing the multi-state approach to reliability analysis of systems with ageing components we assume that:

- E_i, $i = 1,2,...,n$, are components of a system,
- all components and a system under consideration have the state set $\{0,1,...,z\}$, $z \geq 1$,
- the state indexes are ordered, the state 0 is the worst and the state z is the best, -
- $T_i(u)$, $i = 1,2,...,n$, are independent random variables representing the lifetimes of components E_i in the state subset $\{u,u+1,...,z\}$, while they were in the state z at the moment $t = 0$,
- $T(u)$ is a random variable representing the lifetime of a system in the state subset $\{u,u+1,...,z\}$ while it was in the state z at the moment $t = 0$,
- the system state degrades with time t without repair,
- $e_i(t)$ is a component E_i state at the moment t, $t \in <0,\infty)$, given that it was in the state z at the moment $t = 0$,
- $s(t)$ is a system state at the moment t, $t \in <0,\infty)$, given that it was in the state z at the moment $t = 0$.

The above assumptions mean that the states of the system with degrading components may be changed in time only from better to worse.

Under these assumptions, the multi-state system reliability characteristics, like ones presented below, may be introduced and determined.

Definition 1. A vector

$$R_i(t,\cdot) = [R_i(t,0), R_i(t,1), ..., R_i(t,z)], \; t \in <0,\infty),$$

where

$$R_i(t,u) = P(e_i(t) \geq u \mid e_i(0) = z) = P(T_i(u) > t),$$

$$t \in <0,\infty), \; u = 0,1,...,z, \; i = 1,2,...,n$$

is the probability that the component E_i is in the state subset $\{u, u+1,...,z\}$ at the moment t, $t \in <0,\infty)$, while it was in the state z at the moment $t = 0$, is called the multi-state reliability function of a component E_i.

Definition 2. A vector

$$\boldsymbol{R}_n(t,\cdot) = [\boldsymbol{R}_n(t,0), \boldsymbol{R}_n(t,1), ..., \boldsymbol{R}_n(t,z)], \; t \in <0,\infty),$$

where

$$\boldsymbol{R}_n(t,u) = P(s(t) \geq u \mid s(0) = z) = P(T(u) > t),$$

$t \in < 0, \infty), u = 0,1,...,z,$

is the probability that the system is in the state subset $\{u, u+1,..., z\}$ at the moment t, $t \in < 0, \infty)$, while it was in the state z at the moment $t = 0$, is called the multi-state reliability function of a system.

Definition 3. A probability

$$\boldsymbol{r}(t) = P(s(t) < r \mid s(0) = z) = P(T(r) \le t),$$

$$t \in < 0, \infty),$$

that the system is in the subset of states worse than the critical state r, $r \in \{1,...,z\}$ while it was in the state z at the moment $t = 0$ is called a risk function of the multi-state system or, in short, a risk.

4. Complex technical systems reliability, availability and safety evaluation and prediction

To construct the general reliability, availability and safety analytical models of complex non-repairable and repairable multi-state technical systems related to their operation processes, the linking their reliability, availability and safety models and their operation processes models and considering variable in different operation states their reliability and safety structures and their components reliability and safety parameters is practically very well justified [77].

Thus, we assume that the changes of the operation process states have an influence on the system multi-state components reliability and the system reliability structure, denoting the conditional reliability function of the system multi-state component E_i, $i = 1,2,...,n$, while the system is at the operation state z_b, $b = 1,2,...,v$, by

$$[R_i(t, \cdot)]^{(b)} = [1, [R_i(t, 1)]^{(b)}, ..., [R_i(t, z)]^{(b)}],$$

$$t \in < 0, \infty), \; b = 1,2,...,v.$$

To predict the complex technical system reliability and risk we determine the following characteristics:

- the conditional reliability functions of the system while the system is at the operational states z_b

$$[\boldsymbol{R}(t,\cdot)]^{(b)} = [1, [\boldsymbol{R}(t,1)]^{(b)}, ..., [\boldsymbol{R}(t, z)]^{(b)}, t \in < 0, \infty),$$

$$b = 1,2,...,\nu,$$

- the unconditional reliability function of the system

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), ..., \boldsymbol{R}(t, z)],$$

where

$$\boldsymbol{R}(t,u) \cong \sum_{b=1}^{\nu} p_b [\boldsymbol{R}(t,u)]^{(b)}, t \ge 0, b = 1,2,...,\nu, \quad (6)$$

- the mean values of the system unconditional lifetimes in the reliability state subsets $\{u, u+1,..., z\}$

$$\mu(u) \cong \sum_{b=1}^{\nu} p_b \mu_b(u), \; u = 1,2,..., z, \quad (7)$$

where

$$\mu_b(u) = \int_0^\infty [\boldsymbol{R}(t,u)]^{(b)} dt, \; u = 1,2,..., z, \quad (8)$$

are the mean values of the system conditional lifetimes in the reliability state subsets $\{u, u+1,..., z\}$ while the system is at the operation state z_b, $b = 1,2,...,v$,

- the variances of the system unconditional lifetimes in the reliability state subsets $\{u, u+1,..., z\}$

$$\sigma^2(u) = 2\int_0^\infty t \; \boldsymbol{R}(t,u) dt - [\mu(u)]^2, \; u = 1,2,..., z, \quad (9)$$

- the mean values of the system unconditional lifetimes in the particular reliability states

$$\bar{\mu}(u) = \mu(u) - \mu(u+1), \; u = 1,2,..., z-1,$$

$$\bar{\mu}(z) = \mu(z), \quad (10)$$

- the system risk function

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}(t,r), \; t \in < 0, \infty), \quad (11)$$

- the moment when the risk exceeds a permitted level δ

$$\tau = \boldsymbol{r}^{-1}(\delta), \quad (12)$$

where $\boldsymbol{r}^{-1}(t)$ is the inverse function of the risk function $\boldsymbol{r}(t)$.

Further, assuming that the reliability functions of the system components E_i, i = 1,2,...,n, in various operation states z_b, $b = 1,2,...,\nu$, are exponential reliability functions with the coordinates

$$[R_i(t,u)]^{(b)} = \exp[-[\lambda_i(u)]^{(b)} t], \; t \geq 0, \quad (13)$$

$$[\lambda_i(u)]^{(b)} > 0, \; u = 1,2,\ldots,z, \; b = 1,2,...,\nu,$$

it is possible to find the system unconditional multistate reliability functions for basic exponential complex technical multi-state systems like series, parallel, "m out of n", consecutive "m out of n", series-parallel, parallel-series, series"-m out of n", "m out of n"-series, series-consecutive "m out of n" and consecutive "m out of n"-series systems and other composed of them more complex systems. The exemplary results are:

i) for a series system

$$\overline{\boldsymbol{R}}_n(t,\cdot) = [1, \overline{\boldsymbol{R}}_n(t,1), ..., \overline{\boldsymbol{R}}_n(t,z)],$$

where

$$\overline{\boldsymbol{R}}_n(t,u) \cong \sum_{b=1}^{\nu} p_b \exp[-\sum_{i=1}^{n} [\lambda_i(u)]^{(b)} t], \quad (14)$$

$$t \geq 0, \; u = 1,2,...,z;$$

ii) for a parallel system

$$\boldsymbol{R}_n(t,\cdot) = [1, \boldsymbol{R}_n(t,1), ..., \boldsymbol{R}_n(t,z)],$$

where

$$\boldsymbol{R}_n(t,u) \cong 1 - \sum_{b=1}^{\nu} p_b \prod_{i=1}^{n} [1 - \exp[-[\lambda_i(u)]^{(b)} t]], \quad (15)$$

$$t \geq 0, \; u = 1,2,...,z.$$

For the considered exponential complex technical systems, it is possible to determine the mean values $\mu(u)$ and the standard deviations $\sigma(r)$ of the unconditional lifetimes of the system in the reliability state subsets $\{1,2,...,u\}$, $u = 1,2,...,z$, the mean values $\overline{\mu}(u)$ of the unconditional lifetimes of the system in the particular reliability states u, the system risk function $\boldsymbol{r}(t)$ and the moment τ when their risk exceeds a permitted level δ after substituting in (6)-(13) for $\boldsymbol{R}(t,u)$, $u = 1,2,...,z$, the coordinates of their unconditional reliability functions, like ones given for instance by (14) and (15) for series and parallel systems.

If we assume here that the considered systems are repairable in the sense that after exceeding the critical reliability state r is repaired and that the time of renovation is very small in comparison to their lifetimes in the reliability states subsets not worse than the critical one, then it is possible to obtain their renewal characteristics [46]. One of the basic characteristics of the renewable system is the expected value of the number $N(t,r)$ of exceeding the reliability critical state r of this system up to the moment t, $t \geq 0$, that for sufficiently large t, is given approximately by

$$H(t,r) = \frac{t}{\mu(r)}, \; r \in \{1,2,...,z\}. \quad (16)$$

If we assume here that the considered systems after exceeding the critical reliability state r are repaired and that the time of renovation is not very small in comparison to the their lifetimes in the reliability states subsets not worse than the critical one, then it is possible to obtain to obtain their renewal and ability characteristics [46]. One of the basic characteristic in this case is the expected value of the number $\overline{N}(t,r)$ of exceeding the reliability critical state r of this system up to the moment t, $t \geq 0$, that for sufficiently large t, is given approximately by

$$\bar{H}(t,r) \cong \frac{t + \mu_0(r)}{\mu(r) + \mu_o(r)}, \; r \in \{1,2,...,z\}, \quad (17)$$

where $\mu_o(r)$ is the mean value of the system renovation time.

5. Parameters of complex technical systems operation, reliability and safety models identification

There are proposed statistical methods of estimating the unknown parameters of the semi-markov model of the complex system operation process resulting in the following formulae [43]:

- for the vector $[p(0)]_{1x\nu}$ of the probabilities of the initial states

$$p_b(0) = \frac{n_b(0)}{n(0)} \text{ for } b = 1,2,...,\nu,$$

where $n_b(0)$ are the number of the realizations of the system operation process starting from the operation state z_b, $b = 1,2,...,\nu$, and $n(0)$ is the total number of all realizations of the system operation process starting at the initial moment t = 0,

- for the matrix $[p_{bl}]_{\nu x \nu}$ of the probabilities of the system operation process transitions from the operation state z_b to the operation state z_l

$$p_{bb} = 0 \quad \text{for } b = 1,2,...,\nu,$$

$$p_{bl} = \frac{n_{bl}}{n_b} \text{ for } b,l = 1,2,...,\nu,\ b \neq l\ ,$$

where n_{b1} $b,l = 1,2,...,\nu$, $b \neq l$, are the numbers of the system operation process transitions from the operation state z_b to the operation state z_l and n_b, $b = 1,2,...,\nu$, is the total number of the system operation process departures from the operation state z_b during the experiment time Θ,

- for the parameters of the suggested as suitable distributions of the conditional system operation process sojourn times in the particular operation states.

Moreover, in the proposed approach, testing the uniformity of statistical data sets coming from the complex systems operation processes and including the realizations of the system operation process conditional sojourn times in the operation states observed in different experiments are suggested. After that, there is suggested the chi-square goodness-of-fit test application to verifying the distributions of the system operation process conditional sojourn times in the particular operation states for distinguished as suitable distributions

There are also proposed the methods of estimating unknown parameters of the exponential distribution of the component lifetimes of the multistate system in the subsets of reliability states. These methods are considered for different kinds of the empirical investigations including the cases of small number of realizations and non-completed investigations.

It is suggested to assume that the coordinates of the vector of the system components conditional multistate reliability function are exponential reliability functions of the form given by (13), where $[\lambda(u)]^{(b)}$ is an unknown intensity of departure from this subset of the reliability states.

We want to estimate the value of this unknown intensity of departure from the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z$, on the basis of empirical data. The estimators of this unknown intensity of departure $[\lambda(u)]^{(b)}$, for various experimental conditions, are determined by maximum likelihood method in the following cases:

Case 1. The estimation on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Completed investigations, the same observation time on all experimental posts.

We assume that during the time $\tau^{(b)}$, $\tau^{(b)} > 0$, we have been observing the realizations of the component lifetime $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z$, at the operation state z_b, $b = 1,2,...,\nu$, on $n^{(b)}$ identical experimental posts. We assume that at the beginning of the experiment all components are new identical components staying at the best reliability state z and that during the fixed observation time $\tau^{(b)}$ all components have left the reliability states subset $\{1,2,..., z\}$, i.e. all observed components reached the worst reliability state 0. It means that the number $m^{(b)}(u)$ of components that have left the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z$, is equal to $n^{(b)}$, i.e. $m^{(b)}(u) = n^{(b)}$, $u = 1,2,..., z$. We mark by $t_i^{(b)}(u)$, $i = 1,2,..., m^{(b)}(u)$, $u = 1,2,..., z$, the moments of departures from the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z$, of the component on the $i-$th observational post.

In this case, the estimation of the unknown component intensity of departure from the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z$, is

$$[\hat{\lambda}(u)]^{(b)} = \frac{n^{(b)}}{\sum_{i=1}^{n^{(b)}} t_i^{(b)}(u)}\ .$$

Case 2. The estimation on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Non-completed investigations, the same observation time on all experimental posts.

We assume that during the time $\tau^{(b)}$, $\tau^{(b)} > 0$, we have been observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z$, at the operation state z_b, $b = 1,2,...,\nu$, on $n^{(b)}$ identical experimental posts. We assume that at the beginning of the experiment all components are new identical components staying at the best reliability state z and that during the fixed observation time $\tau^{(b)}$ not all components have left the reliability states subset $\{1,2,...,z\}$, i.e. $m^{(b)}$, $m^{(b)} < n^{(b)}$, observed components reached the worst reliability state 0. It means that the number $m^{(b)}(u)$ of components that have left the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ is less or equal to $n^{(b)}$, i.e. $m^{(b)}(u) \le n^{(b)}$, $u = 1,2,...,z$. We mark by $t_i^{(b)}(u)$, $i = 1,2,...,m^{(b)}(u)$, $u = 1,2,...,z,$ the moments of departures from the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ of the component on the $i-$th observational post.

In this case, the estimation of the unknown component intensity of departure from the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ is

$$[\hat{\lambda}(u)]^{(b)} = \frac{m^{(b)}(u)}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \tau^{(b)}[n^{(b)} - m^{(b)}(u)]}.$$

Assuming the observation time $\tau^{(b)}$ as the moment of departure from the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ of the components that have not left this reliability states subset we get so called pessimistic evaluation of the intensity of departure from the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ of the form

$$[\hat{\lambda}(u)]^{(b)} = \frac{n^{(b)}}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \tau^{(b)}[n^{(b)} - m^{(b)}(u)]}.$$

Case 3. The estimation on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Non-completed investigations, different observation times on particular experimental posts.

We assume that we have been observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ at the operation state z_b, $b = 1,2,...,\nu$, on $n^{(b)}$ identical experimental posts. We assume that the observation times on particular experimental posts are different and we mark by $\tau_i^{(b)}$, $\tau_i^{(b)} > 0,$ $i = 1,2,...,n^{(b)}$, the observation time respectively on the *i*-th experimental post. We assume that at the beginning of the experiment all components are new identical components staying at the best reliability state z and that during the fixed observation time $\tau^{(b)}$ not all components have left the reliability states subset $\{1,2,...,z\}$, i.e. $m^{(b)}$, $m^{(b)} < n^{(b)}$, observed components reached the worst reliability state 0. It means that the number $m^{(b)}(u)$ of components that have left the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ is less or equal to $n^{(b)}$, i.e. $m^{(b)}(u) \le n^{(b)}$, $u = 1,2,...,z$. We mark by $t_i^{(b)}(u)$, $i = 1,2,...,m^{(b)}(u)$, $u = 1,2,...,z,$ the moments of departures from the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ of the component on the $i-$th observational post.

In this case, the estimation of the unknown component intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ is

$$[\hat{\lambda}(u)]^{(b)} = \frac{m^{(b)}(u)}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \sum_{i=m^{(b)}(u)+1}^{n^{(b)}} \tau_i^{(b)}}.$$

Assuming the observation times $\tau_i^{(b)}$, $i = m^{(b)}(u), m^{(b)}(u)+1,...,n^{(b)}$, as the moment of departure from the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ of the components that have not left this reliability states subset we get so called a pessimistic evaluation of the intensity of departure from the reliability states subset of the form

$$[\hat{\lambda}(u)]^{(b)} = \frac{n^{(b)}}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \sum_{i=m^{(b)}(u)+1}^{n^{(b)}} \tau_i^{(b)}}.$$

Case 4. The estimation on the basis of the realizations of the component simple renewal flow (stream) on one experimental post.

We assume that during the time $\tau^{(b)}$, $\tau^{(b)} > 0,$ we have been observing the realizations of the component lifetime $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ at the operation state z_b, $b = 1,2,...,\nu$, on one experimental posts. We assume that at the moment when the component is leaving the reliability states subset $\{1,2,...,z\}$, i.e. the observed component reached the worst reliability state 0, it is replaced at once by the same new component staying at the reliability state z. It means that at the beginning all components are new identical components staying at the best reliability state z. We assume that during the fixed observation time $m^{(b)}$ components have left the reliability states subset $\{1,2,...,z\}$, i.e. $m^{(b)}$ observed components reached the worst reliability state 0. It means that the number $m^{(b)}(u)$ of components that have left the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ is equal either to $m^{(b)}$ or to $m^{(b)}+1$, i.e. $m^{(b)}(u)=m^{(b)}$ or $m^{(b)}(u)=m^{(b)}+1$, $u=1,2,...,z$. We mark by $t_i^{(b)}(u)$, $i=1,2,...,m^{(b)}(u)$, $u=1,2,...,z,$ the moments of departures from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the component on the $i-$th observational post.

In this case, the estimation of the unknown component intensity of departure from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ is

$$[\hat{\lambda}(u)]^{(b)}=\frac{m^{(b)}(u)}{\sum_{i=1}^{m^{(b)}(u)}t_i^{(b)}(u)+d^{(b)}(u)},$$

where

$$d^{(b)}(u)=\begin{cases}\tau^{(b)}-\sum_{i=1}^{m^{(b)}(u)}t_i^{(b)}(1)\ \textit{if}\ m^{(b)}(u)=m^{(b)}\\ 0\qquad \textit{if}\ m^{(b)}(u)=m^{(b)}+1.\end{cases}$$

In the case if $m^{(b)}(u)=m^{(b)}$, after assuming the observation time $\tau^{(b)}$ as the moment of departure from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the last component that has not left this reliability states subset we get so called a pessimistic evaluation of the intensity of departure from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the form

$$[\hat{\lambda}(u)]^{(b)}=\frac{m^{(b)}+1}{\sum_{i=1}^{m^{(b)}(u)}t_i^{(b)}(u)+d^{(b)}(u)}.$$

Case 5. The estimation on the basis of the realizations of the component simple renewal flows (streams) on several experimental posts – The same observation time on all experimental posts.

We assume that during the time $\tau^{(b)}$, $\tau^{(b)}>0,$ we have been observing the realizations of the component lifetime $T^{(b)}(u)$ in the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ at the operation state z_b, $b=1,2,...,\nu$, on $n^{(b)}$ experimental posts. We assume that, at each observation post, at the moment when the component is leaving the reliability states subset $\{1,2,...,z\}$, i.e. the observed component reached the worst reliability state 0, it is replaced at once by the same new component staying at the reliability state z. It means that, at each experiment post, at the beginning all components are new identical components staying at the best reliability state z. We assume that, at the j-th, $j=1,2,...,n^{(b)}$, experimental post, during the fixed observation time $m_j^{(b)}$ components have left the reliability states subset $\{1,2,...,z\}$, i.e. $m_j^{(b)}$ observed components reached the worst reliability state 0. It means that the number $m_j^{(b)}(u)$ of components that have left the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ is equal either to $m_j^{(b)}$ or to $m_j^{(b)}+1$, i.e. $m_j^{(b)}(u)=m_j^{(b)}$ or $m_j^{(b)}(u)=m_j^{(b)}+1$, $u=1,2,...,z$. We mark by $[t_i^{(b)}(u)]^{(j)}$, $i=1,2,...,m_j^{(b)}(u)$, the moments to the components departures from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ at the j-th, $j=1,2,...,n^{(b)}$, experimental post.

In this case, the estimation of the unknown component intensity of departure from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ is

$$[\hat{\lambda}(u)]^{(b)}=\frac{\sum_{j=1}^{n^{(b)}}m_j^{(b)}(u)}{\sum_{j=1}^{n^{(b)}}\sum_{i=1}^{m_j^{(b)}(u)}[t_i^{(b)}(u)]^{j}+\sum_{j=1}^{n^{(b)}}d_j^{(b)}(u)},$$

where for $j=1,2,...,n^{(b)}$

$$d_j^{(b)}(u)=\begin{cases}\tau^{(b)}-\sum_{i=1}^{m_j^{(b)}(u)}[t_i^{(b)}(1)]^{(j)}\ \textit{if}\ m_j^{(b)}(u)=m_j^{(b)}\\ 0\qquad \textit{if}\ m_j^{(b)}(u)=m_j^{(b)}+1.\end{cases}$$

In the case if there exist $j,\ j \in \{1,2,...,n^{(b)}\}$, such that $m_j^{(b)}(u) = m_j^{(b)}$, $u = 1,2,...,z$, assuming the observation time $\tau^{(b)}$ as the moment of departures from the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z$, of the last components on all experimental posts that have not left this reliability states subset we get so called pessimistic evaluation of the intensity of departure from the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z$, of the form

$$[\hat{\lambda}(u)]^{(b)} = \frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)} + n^{(b)}}{\sum_{j=1}^{n^{(b)}} \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(u)]^j + \sum_{j=1}^{n^{(b)}} d_j^{(b)}(u)}.$$

Case 6. The estimation on the basis of the realizations of the component simple renewal flows (streams) on several experimental posts – Different observation times on experimental posts.

We assume that we have been observing the realizations of the component lifetime $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z$, at the operation state z_b, $b = 1,2,...,\nu$, on $n^{(b)}$ experimental posts. We assume that the observation times on particular experimental posts are different and we mark by $\tau_j^{(b)}$, $\tau_j^{(b)} > 0$, $i = 1,2,...,n^{(b)}$, the observation time respectively on the *i*-th experimental post. We assume that, at each observation post, at the moment when the component is leaving the reliability states subset $\{1,2,...,z\}$, i.e. the observed component reached the worst reliability state 0, it is replaced at once by the same new component staying at the reliability state z. It means that, at each experiment post, at the beginning all components are new identical components staying at the best reliability state z. We assume that, at the j-th, $j = 1,2,...,n^{(b)}$, experimental post, during the fixed observation time $m_j^{(b)}$ components have left the reliability states subset $\{1,2,...,z\}$, i.e. $m_j^{(b)}$ observed components reached the worst reliability state 0. It means that the number $m_j^{(b)}(u)$ of components that have left the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z$, is equal either to $m_j^{(b)}$ or to $m_j^{(b)} + 1$, i.e. $m_j^{(b)}(u) = m_j^{(b)}$ or $m_j^{(b)}(u) = m_j^{(b)} + 1$, $u = 1,2,...,z$. We mark by $[t_i^{(b)}(u)]^{(j)}$, $i = 1,2,...,m_j^{(b)}(u)$, the moments to the components departures from the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z$, at the j-th, $j = 1,2,...,n^{(b)}$, experimental post.

In this case, the maximum likelihood evaluation of the unknown component intensity of departure from the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z$, is

$$[\hat{\lambda}(u)]^{(b)} = \frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)}(u)}{\sum_{j=1}^{n^{(b)}} \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(u)]^j + \sum_{j=1}^{n^{(b)}} \bar{d}_j^{(b)}(u)},$$

where for $j = 1,2,...,n^{(b)}$

$$\bar{d}_j^{(b)}(u) = \begin{cases} \tau_j^{(b)} - \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(1)]^{(j)} & if\ m_j^{(b)}(u) = m_j^{(b)} \\ 0 & if\ m_j^{(b)}(u) = m_j^{(b)} + 1. \end{cases}$$

In the case if there exist $j,\ j \in \{1,2,...,n^{(b)}\}$, such that $m_j^{(b)}(u) = m_j^{(b)}$, $u = 1,2,...,z$, assuming the observation times $\tau_j^{(b)}$, $j = 1,2,...,n^{(b)}$, as the moments of departures from the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z$, of the last components on experimental posts that have not left this reliability states subset we get so called a pessimistic evaluation of the intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z$, of the form

$$[\hat{\lambda}(u)]^{(b)} = \frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)} + n^{(b)}}{\sum_{j=1}^{n^{(b)}} \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(u)]^j + \sum_{j=1}^{n^{(b)}} \bar{d}_j^{(b)}(u)}.$$

There is also suggested in the proposed approach, the chi-square goodness-of-fit test application to verifying the hypotheses concerned with the exponential form of the multistate reliability function of the particular components of the system in variable operations conditions.

6. Complex technical systems operation, reliability, availability, safety optimization and cost analysis

The results of the joint general model of reliability of systems in variable operation conditions and linear programming are proposed to complex technical systems reliability, availability and risk optimization [46]. These theoretical tools application in finding the optimal values of limit transient probabilities of the system operation states maximizing the system lifetimes in the reliability or safety state subsets are very well founded in practice.

It is expressed in (6) that the system operation process has a significant influence on the system reliability. This influence is also clearly expressed in the equation (7) for the mean values of the system unconditional lifetimes in the reliability state subsets.

Thus, to improve the system reliability, if r, $r = 1,2,...,z,$ is the system critical reliability state, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,...,\nu$, of the transient probabilities p_b, $b = 1,2,...,\nu$, in the system operation states to maximize the mean value $\mu(r)$ of the unconditional system lifetime in the reliability state subset $\{r, r+1,...,z\}$, $r = 1,2,...,z,$ under the assumption that the mean values $\mu_b(r)$, $b = 1,2,...,\nu$, $r = 1,2,...,z,$ of the system conditional lifetimes in this reliability state subset are fixed. More exactly, we formulate the optimization problem as a linear programming model with the objective function of the following linear form

$$\mu(r) = \sum_{b=1}^{\nu} p_b \mu_b(r) \tag{18}$$

for a fixed $r \in \{1,2,...,z\}$ and with the following boundary constraints

$$\sum_{b=1}^{\nu} p_b = 1, \quad \breve{p}_b \le p_b \le \hat{p}_b, \quad b = 1,2,...,\nu,$$

where $\mu_b(r)$, $\mu_b(r) \ge 0$, $b = 1,2,...,\nu$, are fixed mean values of the system conditional lifetimes in the reliability state subset $\{r, r+1,...,z\}$ and $\breve{p}_b$, $0 \le \breve{p}_b \le 1$ and $\hat{p}_b$, $0 \le \hat{p}_b \le 1$, $\breve{p}_b \le \hat{p}_b$, $b = 1,2,...,\nu$, are lower and upper bounds of the unknown transient probabilities p_b, $b = 1,2,...,\nu$, respectively.

Now, we can obtain the optimal solution of the formulated linear programming problem [46], i.e. we can find the optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,...,\nu$, that maximize the system mean lifetime in the reliability state subset $\{r, r+1,...,z\}$, defined by the linear form (18) giving its maximum value in the following form

$$\dot{\mu}(r) = \sum_{b=1}^{\nu} \dot{p}_b \mu_b(r) \tag{19}$$

for a fixed critical reliability state $r \in \{1,2,...,z\}$.

From the above, replacing r by u, $u = 1,2,...,z,$ we obtain the corresponding optimal solutions for the mean values of the system unconditional lifetimes in the reliability state subsets $\{u, u+1,...,z\}$ of the form

$$\dot{\mu}(u) = \sum_{b=1}^{\nu} \dot{p}_b \mu_b(u) \ \text{ for } \ u = 1,2,...,z, \tag{20}$$

and by (9) the corresponding values of the variances of the system unconditional lifetimes in the reliability state subsets are

$$\dot{\sigma}^2(u) = 2\int_0^{\infty} t \ \dot{\boldsymbol{R}}_n(t,u)dt - [\dot{\mu}(u)]^2, \tag{21}$$

$$u = 1,2,...,z,$$

where $\dot{\mu}(u)$ is given by (20) and $\dot{\boldsymbol{R}}_n(t,u)$, according to (6), is the coordinate of the corresponding optimal unconditional multistate reliability function of the system

$$\dot{\boldsymbol{R}}_n(t,\cdot) = [1, \ \dot{\boldsymbol{R}}_n(t,1), ..., \ \dot{\boldsymbol{R}}_n(t,z)],$$

given by

$$\dot{\boldsymbol{R}}_n(t,u) \cong \sum_{b=1}^{\nu} \dot{p}_b [\boldsymbol{R}_n(t,u)]^{(b)}, \ t \ge 0, \tag{22}$$

$$u = 1,2,...,z,$$

and by (10) the optimal solutions for the mean values of the system unconditional lifetimes in the particular reliability states are of the form

$$\dot{\bar{\mu}}(u) = \dot{\mu}(u) - \dot{\mu}(u+1), \ u = 0,1,...,z-1,$$

$$\dot{\bar{\mu}}(z) = \dot{\mu}(z). \tag{23}$$

Moreover, considering (11) and (12), the corresponding optimal system risk function and the moment when the risk exceeds a permitted level δ, respectively are given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{R}}_n(t,r), \quad t \in (-\infty,\infty), \tag{24}$$

and

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta), \tag{25}$$

where $\dot{\boldsymbol{r}}^{-1}(t)$ is the inverse function of the risk function $\dot{\boldsymbol{r}}(t)$ given by (24).

Further, replacing $\mu(r)$ by $\dot{\mu}(r)$ in the expressions (16) and (17) for the repaired systems characteristics we may get their corresponding optimal values

$$\dot{H}(t,r) = \frac{t}{\dot{\mu}(r)}, \quad r \in \{1,2,...,z\}, \tag{26}$$

$$\dot{\bar{H}}(t,r) \cong \frac{t+\mu_0(r)}{\dot{\mu}(r)+\mu_o(r)}, \quad r \in \{1,2,...,z\}. \tag{27}$$

The way of cost analysis of complex technical systems in variable operation process is proposed and its application to the evaluation the cost before and after the system operation process optimization is suggested [41]. The methods of corrective and preventive maintenance policy maximizing availability and minimizing renovation cost of the complex technical systems in variable operation conditions are suggested in the proposed approach as well [90].

7. Modelling, identification and prediction of operation, reliability, availability and safety of port and maritime complex technical systems

The objective of this section is to express the very well grounded applications of the constructed general reliability, availability and safety analytical models of complex non-repairable and repairable multi-state technical systems related to their operation processes and the methods of these models unknown parameters identification to the evaluation and optimization of complex port transportation systems and technical systems of ships operating at sea waters. Presented particular statistical identification methods and selected cases of the constructed models are applied to the reliability, availability and safety parameters identification and characteristics evaluation and optimisation of the port oil pipeline transportation system and the maritime ferry technical system [48]-[49].

7.1. Port oil pipeline transportation system reliability and risk identification and prediction

The considered oil terminal is designated for the reception from ships, the storage and sending by carriages or cars the oil products. It is also designated for receiving from carriages or cars, the storage and loading the tankers with oil products such like petrol and oil. The considered terminal is composed of three parts *A*, *B* and *C*, linked by the piping transportation system with the pier.

The oil pipeline transportation system consists three subsystems S_1, S_2, S_3:

- the subsystem S_1 composed of two identical pipelines, each composed of 178 pipe segments of length 12m and two valves,
- the subsystem S_2 composed of two identical pipelines, each composed of 717 pipe segments of length 12m and to valves,
- the subsystem S_3 composed of two identical and one different pipelines, each composed of 360 pipe segments of either 10 m or 7,5 m length and two valves.

The subsystems S_1, S_2, S_3 are forming a general series port oil pipeline system reliability structure. However, the pipeline system reliability structure and the subsystems and components reliability depend on its changing in time operation states.

Taking into account the varying in time operation process of the considered system we distinguish the following as its eight operation states:

- an operation state z_1 − transport of one kind of medium from the terminal part B to part C using two out of three pipelines in subsystem S_3,
- an operation state z_2 − transport of one kind of medium from the terminal part C (from carriages) to part B using one out of three pipelines in subsystem S_3,
- an operation state z_3 − transport of one kind of medium from the terminal part B through part A to pier using one out of two pipelines in subsystem S_2 and one out of two pipelines in subsystem S_1,
- an operation state z_4 − transport of two kinds of medium from the pier through parts A and B to part C using one out of two pipelines in subsystem S_1, one out of two pipelines in subsystem S_2 and two out of three pipelines in subsystem S_3,
- an operation state z_5 − transport of one kind of medium from the pier through part A to B using

one out of two pipelines in subsystem S_1 and one out of two pipelines in subsystem S_2,

- an operation state z_6 – transport of one kind of medium from the terminal part B to C using two out of three pipelines in subsystem S_3, and simultaneously transport one kind of medium from the pier through part A to B using one out of two pipelines in parts S_1 and one out of two pipelines in subsystem S_2,
- an operation state z_7 – transport of one kind of medium from the terminal part B to C using one out of three pipelines in part S_3, and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines in part S_3.

On the basis of the statistical data coming from experts, the transient probabilities p_{bl} between the operation states z_b and z_l were evaluated. Their approximate values are given in the matrix below

$$[p_{bl}] =$$

$$\begin{bmatrix} 0 & 0.022 & 0.022 & 0 & 0.534 & 0.111 & 0.311 \\ 0.2 & 0 & 0 & 0 & 0 & 0 & 0.8 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0.488 & 0.023 & 0 & 0.023 & 0 & 0.233 & 0.233 \\ 0.095 & 0 & 0 & 0 & 0.667 & 0 & 0238 \\ 0.516 & 0.064 & 0 & 0 & 0.226 & 0.194 & 0 \end{bmatrix}$$

Unfortunately, because of lack of sufficient statistical data, it is not possible yet to determine the matrix of the conditional distribution functions $[H_{bl}(t)]_{7x7}$ of the lifetimes θ_{bl} for $b,l = 1,2,...,7,$ $b \neq l,$ and further consequently it is also not possible to determine the vector $[H_b(t)]_{1x8}$ of the unconditional distribution functions of the lifetimes θ_b of this operation process at the operation states z_b, $b = 1,2,...,7.$ On the basis of the statistical data coming from experiment it is possible to evaluate approximately the conditional mean values $M_{bl} = E[\theta_{bl}],$ $b,l = 1,2,...,7,$ $b \neq l,$ of sojourn times in the particular operation states and their approximate evolutions are as follows:

$$M_{12} = 1920,\ M_{13} = 480,\ M_{15} = 1999.4,$$

$$M_{16} = 1250,\ M_{17} = 1129.6,\ M_{21} = 9960,$$

$$M_{27} = 810,\ M_{31} = 575,\ M_{47} = 380,$$

$$M_{51} = 874.7,\ M_{52} = 480,\ M_{54} = 300,$$

$$M_{56} = 436.3,\ M_{57} = 1042.5,\ M_{61} = 325,$$

$$M_{65} = 510.7,\ M_{67} = 438,\ M_{71} = 874.1,$$

$$M_{72} = 510,\ M_{75} = 2585.7,\ M_{76} = 2380.$$

Hence, by (2), the unconditional mean sojourn times in the particular operation states are:

$$M_1 \cong 1610.52,\ M_2 \cong 2640,\ M_3 = 575,$$

$$M_4 = 380,\ M_5 \cong 789.35,\ M_6 \cong 475.76,$$

$$M_7 \cong 1529.76. \qquad (28)$$

The limit values of the transient probabilities at the operational states, according to (3)-(4) and (28), are:

$$p_1 = 0.389,\ p_2 = 0.062,\ p_3 = 0.003,$$

$$p_4 = 0.002,\ p_5 = 0.20,\ p_6 = 0.058,$$

$$p_7 = 0.286. \qquad (29)$$

After discussion with experts, taking into account the safety of the operation of the oil pipeline transportation system, in all operation states z_b, $b = 1,2,...7,$ we distinguish the following three reliability states $(z = 2)$ of the system and its components:

- a reliability state 2 – piping operation is fully safe,
- a reliability state 1 – piping operation is less safe and more dangerous because of the possibility of environment pollution,
- a reliability state 0 – piping is destroyed.

From the above, the oil piping transportation subsystems S_i, $i = 1,2,3,$ are composed of three-state components E_{ij}, i.e. z = 2, with the multi-state reliability functions

$$R_{ij}^{(b)}(t,\cdot) = [1,\ R_{ij}^{(b)}(t,1),\ R_{ij}^{(b)}(t,2)\ ],\ b = 1,2,...,7,$$

in different operation states z_b, $b = 1,2,...,7,$ with the co-ordinates $R_{ij}^{(b)}(t,1)$ and $R_{ij}^{(b)}(t,2)$ that by the arbitrary assumption are exponential.

At the system operational state z_1, the system is composed of the subsystem S_3, which is a series-"2 out of 3" system containing three series subsystems with the structure showed in *Figure 1*.

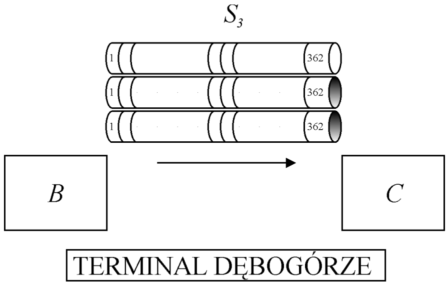

Figure 1. The scheme of port oil transportation system at operation state z_1

Thus, the subsystem S_3 is a multi-state series-"2 out of 3" system and its multi-state reliability function at the operational state z_1 is given by

$$[\boldsymbol{R}(t,\cdot)]^{(1)} = [1, [\boldsymbol{R}(t,1)]^{(1)}, [\boldsymbol{R}(t,2)]^{(1)}], \; t \geq 0,$$

where

$$[\boldsymbol{R}(t,1)]^{(1)} = [\boldsymbol{R}^{(2)}_{3,362}(t,1)]^{(1)} = \exp[-4.3019t]$$

$$+ 2\exp[-4.7375t] - 2\exp[-6.8885t]$$

$$[\boldsymbol{R}(t,2)]^{(1)} = [\boldsymbol{R}^{(2)}_{3,362}(t,2)]^{(1)} = \exp[-5.3668t]$$

$$+ 2\exp[-5.5544t] - 2\exp[-8.2378t] \quad (30)$$

At the system operational state z_2, the system is composed of a series-parallel subsystem S_3, which contains three pipelines with the structure showed in *Figure 2*.

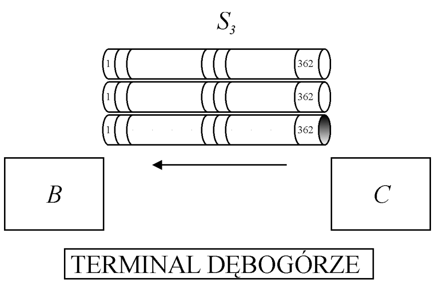

Figure 2. The scheme of port oil transportation system at operation state z_2

Thus, the subsystem S_3 is a multi-state series-parallel system and its multi-state reliability function at the operational state z_2 is given by

$$[\boldsymbol{R}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}(t,1)]^{(2)}, [\boldsymbol{R}(t,2)]^{(2)}], \; t \geq 0,$$

where

$$[\boldsymbol{R}(t,1)]^{(2)} = \boldsymbol{R}_{3,362}(t,1)]^{(2)}$$

$$= \exp[-2.5865t] + 2\exp[-2.15098t]$$

$$- 2\exp[-4.7375t] - \exp[-4.30196t]$$

$$+ \exp[-6.8884t],$$

$$[\boldsymbol{R}(t,2)]^{(2)} = \boldsymbol{R}_{3,362}(t,2)]^{(2)}$$

$$= \exp[-2.8710t] + 2\exp[-2.6834t]$$

$$- 2\exp[-5.5544t] - \exp[-5.3668t]$$

$$+ \exp[-8.4378t]. \quad (31)$$

Proceeding in an analogous way it is possible to determine the system conditional reliability function in the remaining operation states. Next, in the case when the operation time is large enough, the unconditional multi-state reliability function of the pipeline system is given by the vector

$$\boldsymbol{R}_3(t, \cdot) = [1, \boldsymbol{R}_3(t,1), \boldsymbol{R}_3(t,2)], \; t \geq 0,$$

where according to (6) and considering (29), the vector co-ordinates are given respectively by

$$\boldsymbol{R}_3(t,1) = 0.389 \cdot [\boldsymbol{R}(t,1)]^{(1)} + 0.062 \cdot [\boldsymbol{R}(t,1)]^{(2)}$$

$$+ 0.003 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(3)} + 0.002 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(4)}$$

$$+ 0.2 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(5)} + 0.058 \cdot [\overline{\boldsymbol{R}}(t,1)]^{(6)}$$

$$+ 0.286 \cdot [\boldsymbol{R}(t,1)]^{(7)} \text{ for } t \geq 0,$$

$$\boldsymbol{R}_3(t,2) = 0.389 \cdot [\boldsymbol{R}(t,2)]^{(1)} + 0.062 \cdot [\boldsymbol{R}(t,2)]^{(2)}$$

$$+ 0.003 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(3)} + 0.002 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(4)}$$

$$+ 0.2 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(5)} + 0.058 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(6)}$$

$$+ 0.286 \cdot [\boldsymbol{R}(t,2)]^{(7)}, \; t \geq 0, \quad (32)$$

where $[\boldsymbol{R}(t,1)]^{(1)}$, $[\boldsymbol{R}(t,1)]^{(2)}$, $[\boldsymbol{R}(t,2)]^{(1)}$, $[\boldsymbol{R}(t,2)]^{(2)}$ are given by (30) and (31) and

$[\overline{\boldsymbol{R}}(t,1)]^{(3)}$, $[\overline{\boldsymbol{R}}(t,1)]^{(4)}$, $[\overline{\boldsymbol{R}}(t,1)]^{(5)}$, $[\overline{\boldsymbol{R}}(t,1)]^{(6)}$, $[\boldsymbol{R}(t,1)]^{(7)}$, $[\boldsymbol{R}(t,2)]^{(1)}$, $[\boldsymbol{R}(t,2)]^{(2)}$, $[\overline{\boldsymbol{R}}(t,2)]^{(3)}$, $[\overline{\boldsymbol{R}}(t,2)]^{(4)}$, $[\overline{\boldsymbol{R}}(t,2)]^{(5)}$, $[\overline{\boldsymbol{R}}(t,2)]^{(6)}$, $[\overline{\boldsymbol{R}}(t,2)]^{(7)}$ are given by similar expressions that can be found.

Hence, the mean values and the standard deviations of the pipeline system unconditional lifetimes in the reliability state subsets, according to (7)-(9), respectively are:

$$\mu(1) = p_1\mu_1(1) + p_2\mu_2(1) + p_3\mu_3(1)$$

$$+ p_4\mu_4(1) + p_5\mu_5(1) + p_6\mu_6(1) + p_7\mu_7(1)$$

$$= 0.389 \cdot 0.364 + 0.062 \cdot 0.807 + 0.003 \cdot 0.307$$

$$+ 0.002 \cdot 0.079 + 0.2 \cdot 0.307 + 0.058 \cdot 0.079$$

$$+ 0.286 \cdot 0.364 \cong 0.363 \text{ years},$$

$$\sigma(1) \cong 0.308 \text{ years},$$

$$\mu(2) \cong 0.294 \text{ years}, \ \sigma(2) \cong 0.252 \text{ years}. \quad (33)$$

The mean values of the pipeline system lifetimes in the particular reliability states, by (10), are:

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 0.069,$$

$$\bar{\mu}(2) = \mu(2) = 0.294 \text{ years}.$$

If the critical reliability state is r =1, then the system risk function, according to (11), is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}_3(t,1) \text{ for } t \geq 0.$$

Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, from (12), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.066 \text{ years}.$$

7.2. Maritime ferry technical system safety and risk identification and prediction

We consider a passenger ro-ro ferry operating in Baltic Sea between Gdynia and Karlskrona ports on regular everyday line. In the ferry safety analysis we omit the protection and rescue subsystem and the social subsystem and we consider its strictly technical subsystems only.

We assume that the ferry is composed of five main technical subsystems S_1, S_2, S_3, S_4, S_5, having an essential influence on its safety. These subsystems are:

S_1 - a navigational subsystem,
S_2 - a propulsion and controlling subsystem,
S_3 - a loading and unloading subsystem,
S_4 - a hull subsystem,
S_5 - an anchoring and mooring subsystem.

The ferry technical system is the series system of subsystems S_1, S_2, S_3, S_4, S_5. However, the system safety structure and the subsystems and components safety depend on its changing in time operation states. Taking into account the operation process of the considered ferry technical system we distinguish the following as its eighteen operation states:

- an operation state z_1 – loading at Gdynia Port,
- an operation state z_2 – unmooring operations at Gdynia Port,
- an operation state z_3 – leaving Gdynia Port and navigation to “GD” buoy,
- an operation state z_4 – navigation at restricted waters from “GD” buoy to the end of Traffic Separation Scheme,
- an operation state z_5 – navigation at open waters from the end of Traffic Separation Scheme to “Angoring” buoy,
- an operation state z_6 – navigation at restricted waters from “Angoring” buoy to “Verko” Berth at Karlskrona,
- an operation state z_7 – mooring operations at Karlskrona Port,
- an operation state z_8 – unloading at Karlskrona Port,
- an operation state z_9 – loading at Karlskrona Port,
- an operation state z_{10} – unmooring operations at Karlskrona Port,
- an operation state z_{11} – ship turning at Karlskrona Port,
- an operation state z_{12} – leaving Karlskrona Port and navigation at restricted waters to “Angoring” buoy,
- an operation state z_{13} – navigation at open waters from “Angoring” buoy to the entering Traffic Separation Scheme,
- an operation state z_{14} – navigation at restricted waters from the entering Traffic Separation Scheme to “GD” buoy,
- an operation state z_{15} – navigation from “GD” buoy to turning area,

- an operation state z_{16} – ship turning at Gdynia Port,
- an operation state z_{17} – mooring operations at Gdynia Port,
- an operation state z_{18} – unloading at Gdynia Port.

The ferry operation process is very regular in the sense that the operation state changes are from the particular state z_b, $b = 1,2,...,17$, to the neighboring state z_{b+1}, $b = 1,2,...,17$, and from z_{18} to z_1 only. Therefore, the probabilities of transitions between the operation states are given by

$$[p_{bl}] = \begin{bmatrix} 0\,1\,0 \ldots 0\,0 \\ 0\,0\,1 \ldots 0\,0 \\ \ldots \\ 0\,0\,0 \ldots 0\,1 \\ 1\,0\,0 \ldots 0\,0 \end{bmatrix}.$$

On the basis of statistical data coming from experts the mean values of the conditional sojourn times in the operation states are:

$$M_{12} = 54.33,\ M_{23} = 2.57,\ M_{34} = 36.57,$$

$$M_{45} = 52.5,\ M_{56} = 525.95,\ M_{67} = 37.16,$$

$$M_{78} = 7.02,\ M_{89} = 21.43,\ M_{910} = 53.69,$$

$$M_{1011} = 2.93,\ M_{1112} = 4.38,\ M_{1213} = 23.86,$$

$$M_{1314} = 509.69,\ M_{1415} = 50.14,\ M_{1516} = 34.28,$$

$$M_{1617} = 4.52,\ M_{1718} = 5.62,\ M_{181} = 18.74.$$

Hence, by (2), the unconditional mean lifetimes in the operation states are (in minutes):

$$M_1 = 54.33,\ M_2 = 2.57,\ M_3 = 36.57,$$

$$M_4 = 52.5,\ M_5 = 525.95,\ M_6 = 37.16,$$

$$M_7 = 7.02,\ M_8 = 21.43,\ M_9 = 53.69,$$

$$M_{10} = 2.93,\ M_{11} = 4.38,\ M_{12} = 23.86,$$

$$M_{13} = 509.69,\ M_{14} = 50.14,\ M_{15} = 34.28,$$

$$M_{16} = 4.52,\ M_{17} = 5.62,\ M_{18} = 18.74. \qquad (34)$$

The limit values of the transient probabilities at the operational states z_b (the long term proportions p_b of transients at the operational states z_b), according to (3)-(4) and (34), are given by

$$p_1 = 0.037,\ p_2 = 0.002,\ p_3 = 0.025,$$

$$p_4 = 0.036,\ p_5 = 0.364,\ p_6 = 0.025,$$

$$p_7 = 0.005,\ p_8 = 0.014,\ p_9 = 0.037,$$

$$p_{10} = 0.002,\ p_{11} = 0.003,\ p_{12} = 0.017,$$

$$p_{13} = 0.354,\ p_{14} = 0.035,\ p_{15} = 0.024,$$

$$p_{16} = 0.003,\ p_{17} = 0.004,\ p_{18} = 0.013. \qquad (35)$$

We assume as earlier that the ferry technical system is composed of $n = 5$ subsystems S_i, $i = 1,2,...,5$, and that the changes of the process of ship operation states have an influence on the system subsystems S_i safety and on the system safety structure as well. The subsystems S_i, $i = 1,2,3,4,5$ are composed of five-state components, i.e. z = 4, with the multi-state safety functions

$$s_i^{(b)}(t,\cdot)$$

$$= [1,\ s_i^{(b)}(t,1),\ s_i^{(b)}(t,2),\ s_i^{(b)}(t,3),\ s_i^{(b)}(t,4)\,],$$

$$t \in <0,\infty),\ b = 1,2,...,18,\ u = 1,2,3,4,$$

with exponential co-ordinates different in various operation states z_b, $b = 1,2,...,18$.

On the basis of expert opinions, the ferry technical system safety structures and the ship components safety functions in different operation states are fixed. For instance, at the operation state z_1, i.e. at the loading state the ferry built of $n_1 = 2$ subsystems S_3 and S_4 forming a series structure shown in *Figure 3*.

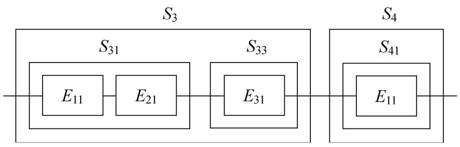

Figure 3. The scheme of the ferry structure at the operation state z_1

The conditional safety function of the ferry technical system while the ferry is at the operational state z_1 is given by

$$\bar{\boldsymbol{s}}_2^{(1)}(t,\ \cdot)$$

$$= [1,\ \bar{\boldsymbol{s}}_2^{(1)}(t,\ 1),\ \bar{\boldsymbol{s}}_2^{(1)}(t,\ 2),\ \bar{\boldsymbol{s}}_2^{(1)}(t,\ 3),\ \bar{\boldsymbol{s}}_2^{(1)}(t,\ 4)\ ],$$

where

$$\bar{\boldsymbol{s}}_2^{(1)}(t,\ 1)$$

$$= \exp[-0.433t]\ \exp[-0.05t] = \exp[-0.483t],$$

$$\bar{\boldsymbol{s}}_2^{(1)}(t,\ 2)$$

$$= \exp[-0.59t]\ \exp[-0.06t] = \exp[-0.65t]$$

$$\bar{\boldsymbol{s}}_2^{(1)}(t,\ 3)$$

$$= \exp[-0.695t]\ \exp[-0.065t] = \exp[-0.76t],$$

$$\bar{\boldsymbol{s}}_2^{(1)}(t,\ 4)$$

$$= \exp[-0.85t]\ \exp[-0.07t] = \exp[-0.92t]. \qquad (36)$$

At the operation states z_2, i.e. at the unmooring operations state the ferry technical system is built of subsystems S_1, S_2 and S_5 forming a parallel-series structure shown in *Figure 4*.

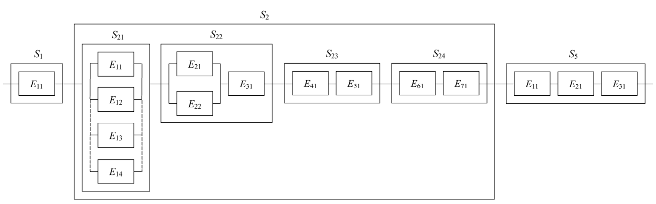

Figure 4. The scheme of the ferry structure at the operation state z_2

The conditional safety function of the ferry while the ferry is at the operational state z_2 is given by

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ \cdot)$$

$$=[1,\ \bar{\boldsymbol{s}}_3^{(2)}(t,\ 1),\ \bar{\boldsymbol{s}}_3^{(2)}(t,\ 2),\ \bar{\boldsymbol{s}}_3^{(2)}(t,\ 3),\ \bar{\boldsymbol{s}}_3^{(2)}(t,\ 4)\ ],$$

where

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 1) = 12\exp[-0.462t] + 8\exp[-0.561t]$$

$$-16\exp[-0.495t] - 3\exp[-0.594t],$$

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 2) = 12\exp[-0.54t] + 8\exp[-0.65t]$$

$$+ 6\exp[-0.62t] - 16\exp[-0.58t]$$

$$- 6\exp[-0.61t] - 3\exp[-0.69t],$$

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 3) = 12\exp[-0.62t] + 8\exp[-0.745t]$$

$$+ 6\exp[-0.72t] - 16\exp[-0.67t]$$

$$- 6\exp[-0.695t] - 3\exp[-0.795t],$$

$$\bar{\boldsymbol{s}}_3^{(2)}(t,\ 4) = 12\exp[-0.685t] + 8\exp[-0.82t]$$

$$+ 6\exp[-0.795t] - 16\exp[-0.74t]$$

$$- 6\exp[-0.765t] - 3\exp[-0.875t]. \qquad (37)$$

At the remaining operation states z_b, $b = 3,...,18$, after proceeding in an analogous way, we determine the system conditional safety functions in particular operation states.
In the case when the system operation time is large enough, the unconditional safety function of the ferry is given by the vector

$$\boldsymbol{s}_5(t,\ \cdot)$$

$$= [1,\ \boldsymbol{s}_5(t,1),\ \boldsymbol{s}_5(t,2),\ \boldsymbol{s}_5(t,3),\ \boldsymbol{s}_5(t,4)],\ t \geq 0,$$

where, according to (6) and after considering (35), its co-ordinates are as follows:

$$\boldsymbol{s}_5(t,u) = 0.037\cdot\bar{\boldsymbol{s}}_2^{(1)}(t,u) + 0.002\cdot\bar{\boldsymbol{s}}_3^{(2)}(t,u)$$

$$+ 0.025\cdot\bar{\boldsymbol{s}}_2^{(3)}(t,u) + 0.036\cdot\bar{\boldsymbol{s}}_3^{(4)}(t,u)$$

$$+ 0.364\cdot\bar{\boldsymbol{s}}_3^{(5)}(t,u) + 0.025\cdot\bar{\boldsymbol{s}}_3^{(6)}(t,u)$$

$$+ 0.005\cdot\bar{\boldsymbol{s}}_3^{(7)}(t,u) + 0.014\cdot\bar{\boldsymbol{s}}_2^{(8)}(t,u)$$

$$+ 0.037 \cdot \bar{s}_2^{(9)}(t,u) + 0.002 \cdot \bar{s}_3^{(10)}(t,u)$$

$$+ 0.003 \cdot \bar{s}_2^{(11)}(t,u) + 0.017 \cdot \bar{s}_3^{(12)}(t,u)$$

$$+ 0.354 \cdot \bar{s}_3^{(13)}(t,u) + 0.035 \cdot \bar{s}_3^{(14)}(t,u)$$

$$+ 0.024 \cdot \bar{s}_2^{(15)}(t,u) + 0.003 \cdot \bar{s}_2^{(16)}(t,u)$$

$$+ 0.004 \cdot \bar{s}_3^{(17)}(t,u) + 0.013 \cdot \bar{s}_2^{(18)}(t,u), \quad (38)$$

for $t \geq 0$, $u = 1,2,3,4$, where $s_2^{(1)}(t,u)$ and $s_3^{(2)}(t,u)$ are given by (36)-(37) and $s_{nb}^{(b)}(t,u)$ for $b = 3,4,...,18$, are given by similar expressions that can be found.

Thus, the mean values and standard deviations of the system unconditional lifetimes in the safety state subsets, according to (7)-(9) respectively are:

$$\mu(1) \cong 4.07, \; \sigma(1) \cong 4.1,$$

$$\mu(2) \cong 0.037 \cdot 1.54 + 0.002 \cdot 2.43 + 0.025 \cdot 3.9$$

$$+ 0.036 \cdot 3.80 + 0.364 \cdot 3.80 + 0.025 \cdot 3.24$$

$$+ 0.005 \cdot 2.43 + 0.014 \cdot 2.50 + 0.037 \cdot 2.50$$

$$+ 0.002 \cdot 2.43 + 0.003 \cdot 3.37 + 0.017 \cdot 3.80$$

$$+ 0.354 \cdot 3.80 + 0.035 \cdot 3.80 + 0.024 \cdot 3.90$$

$$+ 0.003 \cdot 3.37 + 0.004 \cdot 2.43 \cong 3.59,$$

$$\sigma(2) \cong 3.34,$$

$$\mu(3) \cong 3.19, \quad \sigma(3) \cong 3.65,$$

$$\mu(4) \cong 2.87, \quad \sigma(4) \cong 2.75. \quad (39)$$

The mean values of the system lifetimes in the particular safety states, by (10), are

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 0.48,$$

$$\bar{\mu}(2) = \mu(2) - \mu(3) = 0.4,$$

$$\bar{\mu}(3) = \mu(3) - \mu(4) = 0.32,$$

$$\bar{\mu}(4) = \mu(4) = 2.87.$$

If the critical safety state is r = 2, then the system risk function, according to (11) , is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{s}_5(t, 2).$$

Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, from (12), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.19 \text{ years}.$$

8. Optimization and cost analysis of operation, reliability, availability and safety of port and maritime complex technical systems

The results of the theoretical models of complex technical systems reliability, availability and safety optimization are applied to reliability and maintenance optimization of the port piping oil transportation system and safety and maintenance optimization of the technical system of the ferry operating at the sea and to their operation cost analysis [50], [90]. For these systems the optimal transient probabilities of the operation states maximizing the system lifetimes respectively in the reliability and safety state subsets improving the piping system reliability and the ferry technical system safety are determined. The cost analyses of these systems in variable operation conditions before and after the operation process optimization can be performed. The corrective and preventive maintenance policy maximizing availability and minimizing renovation cost of these systems can be performed as well.

8.1. Port oil pipeline transportation system reliability, risk and availability optimization

The objective function (19), in this case as the critical state is $r = 1$ and considering (33), takes the form

$$\mu(1) = p_1 \cdot 0.364 + p_2 \cdot 0.807 + p_3 \cdot 0.307$$

$$+ p_4 \cdot 0.079 + p_5 \cdot 0.307 + p_6 \cdot 0.079$$

$$+ p_7 \cdot 0.364. \quad (40)$$

On the basis of the lower $\breve{p}_b$ and upper $\hat{p}_b$ bounds of the unknown transient probabilities p_b, $b = 1,2,...,7$, coming from experts, we assume the following boundary constraints

$$\sum_{b=1}^{7} p_b = 1,$$

$0.21 \le p_1 \le 0.86,\ \ 0.01 \le p_2 \le 0.94,$

$0.02 \le p_3 \le 0.10,\ \ 0.06 \le p_4 \le 0.14,$

$0.05 \le p_5 \le 0.46,\ \ 0.001 \le p_6 \le 0.59,$

$0.05 \le p_7 \le 0.92.$

Finally, applying linear programming [46], we get the optimal transient probabilities

$$\dot{p}_1 = 0.21,\ \ \dot{p}_2 = 0.609,\ \ \dot{p}_3 = 0.02,\ \ \dot{p}_4 = 0.06,$$

$$\dot{p}_5 = 0.05,\ \dot{p}_6 = 0.001,\ \dot{p}_7 = 0.05, \qquad (41)$$

that maximize the system mean lifetime in the reliability state subset $\{1,2\}$ expressed by the linear form (40) giving, according to (19) and (41), its optimal value

$$\dot{\mu}(1) = 0.21 \cdot 0.364 + 0.609 \cdot 0.807$$

$$+ 0.02 \cdot 0.307 + 0.06 \cdot 0.079 + 0.05 \cdot 0.307$$

$$+ 0.001 \cdot 0.079 + 0.05 \cdot 0.364 = 0.61. \qquad (42)$$

Further, substituting the optimal solution (7) into the formula (20), we obtain the optimal solution for the mean value of the system unconditional lifetime in the reliability state subset $\{2\}$

$$\dot{\mu}(2) = 0.21 \cdot 0.304 + 0.609 \cdot 0.666$$

$$+ 0.02 \cdot 0.218 + 0.06 \cdot 0.058 + 0.05 \cdot 0.218$$

$$+ 0.001 \cdot 0.058 + 0.05 \cdot 0.304 = 0.50. \qquad (43)$$

Hence, according to (23), the optimal solutions for the mean values of the system unconditional lifetimes in the particular reliability states are

$$\dot{\bar{\mu}}(1) = \dot{\mu}(1) - \dot{\mu}(2) = 0.11,\ \dot{\bar{\mu}}(2) = \dot{\mu}(2) = 0.50.$$

Moreover, according to (22) and (32), the corresponding optimal unconditional multistate reliability function of the system is of the form

$$\dot{\bar{\boldsymbol{R}}}_3(t,\cdot) = [1,\ \dot{\bar{\boldsymbol{R}}}_3(t,1),\ \dot{\bar{\boldsymbol{R}}}_3(t,2)\,],\ t \ge 0,$$

with the coordinates given by

$$\dot{\bar{\boldsymbol{R}}}_3(t,1) = 0.21 \cdot [\bar{\boldsymbol{R}}(t,1)]^{(1)}$$

$$+ 0.609 \cdot [\bar{\boldsymbol{R}}(t,1)]^{(2)} + 0.02 \cdot [\bar{\boldsymbol{R}}(t,1)]^{(3)}$$

$$+ 0.06 \cdot [\bar{\boldsymbol{R}}(t,1)]^{(4)} + 0.05 \cdot [\bar{\boldsymbol{R}}(t,1)]^{(5)}$$

$$+ 0.001 \cdot [\bar{\boldsymbol{R}}(t,1)]^{(6)} + 0.05 \cdot [\bar{\boldsymbol{R}}(t,1)]^{(7)}\,],$$

$$\dot{\bar{\boldsymbol{R}}}_3(t,2) = 0.21 \cdot [\bar{\boldsymbol{R}}(t,2)]^{(1)}$$

$$+ 0.609 \cdot [\bar{\boldsymbol{R}}(t,2)]^{(2)} + 0.02 \cdot [\bar{\boldsymbol{R}}(t,2)]^{(3)}$$

$$+ 0.06 \cdot [\bar{\boldsymbol{R}}(t,2)]^{(4)} + 0.05 \cdot [\bar{\boldsymbol{R}}(t,2)]^{(5)}$$

$$+ 0.001 \cdot [\bar{\boldsymbol{R}}(t,2)]^{(6)} + 0.05 \cdot [\bar{\boldsymbol{R}}(t,2)]^{(7)}\,], \qquad (44)$$

where $[\bar{\boldsymbol{R}}(t,1)]^{(b)}$, $[\bar{\boldsymbol{R}}(t,2)]^{(b)}$, $b = 1,2,...,7$, are given in [47].

Further, according to (21) and (44), the corresponding optimal standard deviations of the system unconditional lifetime in the system reliability state subsets are

$$\dot{\sigma}(1) \cong 0.505,\ \dot{\sigma}(2) \cong 0.420.$$

If the critical safety state is $r = 1$, then the optimal system risk function, according to (24), is given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\bar{\boldsymbol{R}}}_3(t,1),\ t \ge 0,$$

where $\dot{\bar{\boldsymbol{R}}}_3(t,1)$ is given by (44).

Hence and considering (25), the moment when the optimal system risk function exceeds a permitted level $\delta = 0.05$, is

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta) \cong 0.1 \text{ years.}$$

8.2. Maritime ferry technical system safety and risk optimization

In this case, as the critical state is $r = 2$, then considering the expression for $\mu(2)$ in (39), the objective function (19), takes the form

$$\mu(2) = p_1 \cdot 1.54 + p_2 \cdot 2.43 + p_3 \cdot 3.90$$

$$+ p_4 \cdot 3.80 + p_5 \cdot 3.80 + p_6 \cdot 3.24$$

$$+ p_7 \cdot 2.43 + p_8 \cdot 2.50 + p_9 \cdot 2.50$$

$$+ p_{10} \cdot 2.43 + p_{11} \cdot 3.37 + p_{12} \cdot 3.80$$

$$+ p_{13} \cdot 3.80 + p_{14} \cdot 3.80 + p_{15} \cdot 3.90$$

$$+ p_{16} \cdot 3.37 + p_{17} \cdot 2.43 + p_{18} \cdot 1.54. \qquad (45)$$

On the basis of the lower $\breve{p}_b$ and upper $\hat{p}_b$ bounds of the unknown transient probabilities p_b, $b = 1,2,...,18$, coming from experts we assume the following bound constraints

$$\sum_{b=1}^{18} p_b = 1,$$

$$0.0006 \le p_1 \le 0.056, \; 0.001 \le p_2 \le 0.002,$$

$$0.018 \le p_3 \le 0.027, \; 0.027 \le p_4 \le 0.056,$$

$$0.286 \le p_5 \le 0.780, \; 0.018 \le p_6 \le 0.024,$$

$$0.002 \le p_7 \le 0.018, \; 0.001 \le p_8 \le 0.018,$$

$$0.001 \le p_9 \le 0.056, \; 0.001 \le p_{10} \le 0.003,$$

$$0.002 \le p_{11} \le 0.004, \; 0.013 \le p_{12} \le 0.024,$$

$$0.286 \le p_{13} \le 0.780, \; 0.025 \le p_{14} \le 0.043,$$

$$0.018 \le p_{15} \le 0.024, \; 0.002 \le p_{16} \le 0.004,$$

$$0.002 \le p_{17} \le 0.007, \; 0.001 \le p_{18} \le 0.018.$$

Finally, after applying linear programming [46], we get the optimal transient probabilities

$$\dot{p}_1 = 0.0006, \; \dot{p}_2 = 0.001, \; \dot{p}_3 = 0.027,$$

$$\dot{p}_4 = 0.056, \; \dot{p}_5 = 0.552, \; \dot{p}_6 = 0.018,$$

$$\dot{p}_7 = 0.002, \; \dot{p}_8 = 0.001, \; \dot{p}_9 = 0.001,$$

$$\dot{p}_{10} = 0.001, \; \dot{p}_{11} = 0.002, \; \dot{p}_{12} = 0.013,$$

$$\dot{p}_{13} = 0.286, \; \dot{p}_{14} = 0.025, \; \dot{p}_{15} = 0.024,$$

$$\dot{p}_{16} = 0.002, \; \dot{p}_{17} = 0.002, \; \dot{p}_{18} = 0.001, \qquad (46)$$

that maximize the system mean lifetime in the safety state subset $\{2,3,4\}$ expressed by the linear form (45) giving, according to (19) and (46), its optimal value

$$\dot{\mu}(2) \cong \dot{p}_1 \cdot 1.54 + \dot{p}_2 \cdot 2.43 + \dot{p}_3 \cdot 3.90$$

$$+ \dot{p}_4 \cdot 3.80 + \dot{p}_5 \cdot 3.80 + \dot{p}_6 \cdot 3.24$$

$$+ \dot{p}_7 \cdot 2.43 + \dot{p}_8 \cdot 2.50 + \dot{p}_9 \cdot 2.50$$

$$+ \dot{p}_{10} \cdot 2.43 + \dot{p}_{11} \cdot 3.37 + \dot{p}_{12} \cdot 3.80$$

$$+ \dot{p}_{13} \cdot 3.80 + \dot{p}_{14} \cdot 3.80 + \dot{p}_{15} \cdot 3.90$$

$$+ \dot{p}_{16} \cdot 3.37 + \dot{p}_{17} \cdot 2.43 + \dot{p}_{18} \cdot 1.54$$

$$= 3.83 \qquad (47)$$

Further, substituting the optimal solution (46) into the formulae (20), we obtain the optimal solution for the mean value of the system unconditional lifetime in the safety state subset $\{1,2,3,4\}$, $\{3,4\}$ and $\{4\}$ that respectively amounts:

$$\dot{\mu}(1) \cong 4.28, \; \dot{\mu}(3) \cong 3.41, \; \dot{\mu}(4) \cong 3.08. \qquad (48)$$

Hence, according to (23) and considering (47)-(48), the optimal solutions for the mean values of the system unconditional lifetimes in the particular safety states are

$$\dot{\bar{\mu}}(1) \cong 0.45, \; \dot{\bar{\mu}}(2) \cong 0.42, \; \dot{\bar{\mu}}(3) \cong 0.33,$$

$$\dot{\bar{\mu}}(1) \cong 3.08.$$

Moreover, according to (22) and (38), the corresponding optimal unconditional multistate safety function of the system is of the form

$$\dot{\boldsymbol{s}}_5(t, \cdot) =$$

$$[1, \; \dot{\boldsymbol{s}}_5(t,1), \dot{\boldsymbol{s}}_5(t,2), \dot{\boldsymbol{s}}_5(t,3), \dot{\boldsymbol{s}}_5(t,4)], \; t \ge 0,$$

where according to (22) and after considering the values of p_b given by (46), its co-ordinates are as follows:

$$\dot{\boldsymbol{s}}_5(t,u) \cong 0.0006 \cdot \boldsymbol{s}_2^{(1)}(t,u) + 0.001 \cdot \boldsymbol{s}_3^{(2)}(t,u)$$

$$+ 0.027 \cdot \boldsymbol{s}_2^{(3)}(t,u) + 0.056 \cdot \boldsymbol{s}_3^{(4)}(t,u)$$

$$+ 0.552 \cdot \boldsymbol{s}_3^{(5)}(t,u) + 0.018 \cdot \boldsymbol{s}_3^{(6)}(t,u)$$

$$+ 0.002 \cdot \boldsymbol{s}_3^{(7)}(t,u) + 0.001 \cdot \boldsymbol{s}_2^{(8)}(t,u)$$

$$+0.001\cdot \boldsymbol{s}_2^{(9)}(t,u) + 0.001\cdot\boldsymbol{s}_3^{(10)}(t,u)$$

$$+0.001\cdot\boldsymbol{s}_2^{(11)}(t,u) \; + 0.013\cdot\boldsymbol{s}_3^{(12)}(t,u)$$

$$+0.286\cdot\boldsymbol{s}_3^{(13)}(t,u) + 0.025\cdot \boldsymbol{s}_3^{(14)}(t,u)$$

$$+0.024\cdot\boldsymbol{s}_2^{(15)}(t,u) + 0.002\cdot\boldsymbol{s}_2^{(16)}(t,u)$$
$$+0.002\cdot\boldsymbol{s}_3^{(17)}(t,u) + 0.001\cdot\boldsymbol{s}_2^{(18)}(t,u) \quad (49)$$

for $t \ge 0$, $u = 1,2,3,4$, where $\boldsymbol{s}_{n_b}^{(b)}(t,u)$ for $b = 1,2,...,18$, are given in [48].

If the critical safety state is r = 2, then the system risk function, according to (24), is given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{s}}_5(t,2) \text{ for t} \ge 0,$$

where $\dot{\boldsymbol{s}}_5(t,2)$ is given by (49) for $u = 2$.

Hence, considering (25), the moment when the optimal system risk function exceeds a permitted level δ = 0.05, is

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta) \cong 0.25 \text{ years.}$$

8.3. Port oil pipeline transportation system preliminary cost analysis

First, we analyze the port pipeline system cost before its operation process optimization. The system is composed of $n = 2870$ components (pipe segments). According to the information coming from experts, the mean operation cost of a single basic component of the considered pipeline transportation system during the operation time θ = 1 year amounts

$$c_i(\theta) = 9.6 \text{ PLN}, \; i = 1,2,...,2870.$$

Thus, the total operation cost of the non-repaired pipeline transportation system during the operation time θ, $\theta \ge 0$, is given by

$$C(\theta) = 9.6 \cdot 2870 = 27552 \text{ PLN}, \; \theta \ge 0.$$

In the case when the pipeline transportation system is repaired after exceeding the critical reliability state $r = 1$ and its renovation time is ignored, according to the expert opinion, we assume that the cost of the system singular renovation is

$$c_{ig} = 88500\,PLN.$$

It can be fixed, using (16), that the mean value of the number of exceeding the critical reliability state during the operation time θ = 1 year is

$$H(\theta,1) = 2.755\theta = 2.755.$$

Thus, the total operation cost of the repaired pipeline transportation system with ignored its renovation time during the operation time θ = 1 year amounts [46]

$$C_{ig}(\theta) = 27552 + 2.755 \cdot 88500$$
$$= 27552 + 243817.5$$
$$= 271369.5 \text{ PLN.} \quad (50)$$

In the case when the pipeline transportation system is repaired after exceeding the critical reliability state $r = 1$ and its renovation time is not ignored and have distribution function with the mean value and the standard deviation respectively

$$\mu_0(r) = 0.005, \;\; \sigma_0(r) = 0.005,$$

according to the expert opinion, we assume that the cost of the system singular renovation is

$$c_{nig} = 88500\,PLN.$$

It can be fixed, applying (17), that the mean value of the number of exceeding the critical reliability state during the operation time θ = 1 year is

$$\overline{H}(t,1) \cong \frac{\theta + 0.005}{0.368} = \frac{1.005}{0.368} = 2.731.$$

Thus, the total operation cost of the renewed pipeline transportation system with ignored its renewal time during the operation time θ = 1 year amounts [46]

$$C_{ig}(\theta) = 27552 + 2.731 \cdot 88500$$
$$= 27552 + 241693.5$$
$$= 269245.5 \text{ PLN.} \quad (51)$$

After the optimization the operation process of the pipeline transportation system performed in Section 8.2, in the case when the system is repaired after exceeding the critical reliability state $r = 1$ and its renovation time is ignored, it can be fixed, using (26), that the mean value of the number of exceeding the critical reliability state during the operation time θ = 1 is

$$\dot{H}(\theta,1) = 1.639\theta = 1.639.$$

Thus, the total operation cost of the repaired pipeline transportation system with ignored its renovation time during the operation time θ = 1 year, after its operation process optimization, amounts [46]

$$C_{ig}(\theta) = 27552 + 1.639 \cdot 88500$$

$$= 27552 + 145051.5$$

$$= 172603.5 \text{ PLN.} \qquad (52)$$

After the optimization the operation process of the pipeline transportation system is repaired after exceeding the critical reliability state $r = 1$ and its renovation time is not ignored it can be fixed, applying (27), that the mean value of the number of exceeding the critical reliability state during the operation time θ = 1 year is

$$\dot{\bar{H}}(t,1) \cong \frac{\theta + 0.005}{0.615} = \frac{1.005}{0.615} = 1.634.$$

Thus, the total operation cost of the repaired pipeline transportation system with ignored its renewal time during the operation time θ = 1 year, after its operation process optimization, amounts [46]

$$C_{ig}(\theta) = 27552 + 1.634 \cdot 88500$$

$$= 27552 + 144609 = 172161 \text{ PLN.} \qquad (53)$$

Comparing the costs before the system operation process optimization given by (50) and (51) with the costs after the system operation process optimization given by (52) and (53) can justify the sensibility of this optimization action.

9. Conclusion

The joint model of reliability of complex technical systems in variable operation conditions linking a semi-markov modeling of the system operation processes with a multi-state approach to their reliability and safety analysis is proposed. The final results obtained from this joint model and a linear programming are suggested to be used to the complex technical systems reliability and safety optimization. It can be recognize that the proposed approach and theoretical tools may by very useful in reliability and safety identification, evaluation and optimization of a very wide class of real technical systems operating in varying conditions that have an influence on changing their reliability and safety structures and their components reliability and safety characteristics.

These tools practical application to the reliability and availability prediction and optimization of the oil piping transportation system operating in variable conditions in port and to the safety and risk evaluation and optimization of the ferry technical system operating in variable operation conditions at sea waters and the results achieved are very interesting for the reliability and safety practitioners from port and maritime transport industry and from other industrial sectors as well.

The pipeline transportation system is considered in the varying in time operation conditions. The system reliability structure and its components reliability functions are changing in variable operation conditions. The system reliability structures are fixed with a high accuracy. Whereas, the input reliability characteristics of the pipeline components and the system operation process characteristics are not sufficiently exact because of the lack of statistical data necessary for their estimation. The input characteristics of the ferry operation process are of high quality because of the very good statistical data necessary for their estimation. Whereas, the ferry technical system safety structures are fixed generally with not high accuracy in details concerned with the subsystems structures because of their complexity and concerned with the components safety characteristics because of the lack of statistical data necessary for their estimation. Therefore, the results of the proposed tools application may be considered as a preliminary illustration of their possibilities of using in practice. However, the obtained evaluation may be very useful examples in port and maritime technical transportation systems unknown parameters identification and characteristics prediction and optimization, especially during the design and when planning and improving its operation processes safety and effectiveness.

Acknowledgements

The paper describes partly the results of the work in the Poland-Singapore Joint Research Project titled “Safety and Reliability of Complex Industrial Systems and Processes” supported by grants from the Poland’s Ministry of Science and Higher

Education (MSHE grant No. 63/N-Singapore/2007/0) and the Agency for Science, Technology and Research of Singapore (A*STAR SERC grant No. 072 1340050).

References

[1] Amari, S.V. & Misra, R.B. (1997). Comment on: Dynamic reliability analysis of coherent multistate systems. *IEEE Transactions on Reliability* 46, 460-461.

[2] Aven, T. (1985). Reliability evaluation of multistate systems with multistate components. *IEEE Transactions on Reliability* 34, 473-479.

[3] Aven, T. & Jensen, U. (1999). *Stochastic Models in Reliability*. Springer-Verlag, New York.

[4] Aven, T. (1993). On performance measures for multistate monotone systems. *Reliability Engineering and System Safety* 41, 259-266.

[5] Barbu, V. & Limnios, N. (2006). Empirical estimation for discrete-time semi-Markov processes with applications in reliability. *Journal of Nonparametric Statistics*, Vol. 18, No. 7-8, 483-498.

[6] Barlow, R.E. & Wu, A.S. (1978). Coherent systems with multi-state components. *Mathematics of Operations Research* 4, 275-281.

[7] Black, M., Brint, A.T. & Brailsford, J.R. (2005). A semi-Markov approach for modelling asset deterioration. *Journal of the Operational Research Society*, Vol. 56, No. 11, 1241-1249.

[8] Brunelle, R. D. & Kapur, K. C. (1999). Review and classification of reliability measures for multistate and continuum models. *IEEE Transactions* 31, 1117-1180.

[9] Collet, J. (1996). Some remarks on rare-event approximation. *IEEE Transactions on Reliability* 45, 106-108.

[10] Dziula, P., Jurdziński, M., Kołowrocki, K. & Soszyńska, J. (2007). On multi-state safety analysis in shipping. *International Journal of Gnedenko e-Forum "Reliability: Theory & Application"*, Vol. 2, No 3-4, 40-53.

[11] Ferreira, F. & Pacheco, A. (2007). Comparison of level-crossing times for Markov and semi-Markov processes. *Statistics & Probability Letters*, Vol. 77, No. 2, 151-157.

[12] Gamiz, M.L. & Roman, Y. (2008). Non-parametric estimation of the availability in a general repairable. *Reliability Engineering & System Safety*, Vol. 93, No. 8, 1188-1196.

[13] Glynn, P.W. & Haas, P.J. (2006). Laws of large numbers and functional central limit theorems for generalized semi-Markov processes. *Stochastic Models*, Vol. 22, No. 2, 201-231.

[14] Grabski, F. (2002). *Semi-Markov Models of Systems Reliability and Operations*. Monograph. Analysis. Monograph. System Research Institute, Polish Academy of Science, (in Polish), Warsaw.

[15] Giudici, P. & Figini, S. (2009). *Applied data mining for business and industry*. John Wiley & Sons Ltd.

[16] Guze, S., Kołowrocki, K. & Soszyńska, J. (2008). Modeling environment and infrastructure influence on reliability and operation processes of port transportation systems. *Journal of Konbin*, Vol. 1, No 1(4), 25-38.

[17] Guze, S. & Kołowrocki, K. (2008). Reliability analysis of multi-state ageing consecutive „k out of n: F" systems. *International Journal of Materials & Structural Reliability*. Vol. 6. No. 1, 47-60.

[18] Habibullah, M. S., Lumanpauw, E., Kolowrocki, K., Soszynska, J. & Ming, N. G. (2009). A computional tool for general model of industrial systems operation processes. *Electronic Journal Reliability & Risk Analysis: Theory & Applications*, Vol. 2, No 4, 181-191.

[19] Helvacioglu, S. & Insel, M. (2008). Expert system applications in marine technologies. *Ocean Engineering*, Vol. 35, No. 11-12, 1067-1074.

[20] Hryniewicz, O. (1995). *Lifetime tests for imprecise data and fuzzy reliability requirements*. Reliability and Safety Analyses under Fuzziness. Onisawa T. and Kacprzyk J., Eds., Physica Verlag, Heidelberg, 169-182.

[21] Huang, J., Zuo, M.J. & Wu, Y. (2000). Generalized multi-state k-out-of-n:G systems. *IEEE Transactions on Reliability* 49, 105-111.

[22] Hudson, J.C. & Kapur, K.C. (1982). Reliability theory for multistate systems with multistate components. *Microelectronics and Reliability* 22, 1-7.

[23] Hudson, J.C. & Kapur, K.C. (1983). Reliability analysis of multistate systems with multistate components. *Transactions of Institute of Industrial Engineers* 15, 127-135.

[24] Hudson, J. & Kapur, K. (1985). Reliability bounds for multistate systems with multistate components. *Operations Research* 33, 735-744.

[25] Klabjan, D. & Adelman, D. (2006). Existence of optimal policies for semi-Markov decision processes using duality for infinite linear programming. *Siam Journal on Control and Optimization*, Vol. 44, No. 6, 2104-2122.

[26] Kołowrocki, K. (1998). On applications of asymptotic reliability functions to the reliability and risk evaluation of pipelines. *International*

Journal of Pressure Vessels and Piping 75, 545-558.

[27] Kołowrocki, K. (1999). On reliability and risk of large multi-state systems with degrading components. *Exploitation Problems of Machines*, 189-210.

[28] Kołowrocki, K. (2000). *On asymptotic approach to multi-state systems reliability evaluation.* Chapter 11. Recent Advances in Reliability Theory: Methodology, Practice and Inference. Limnios N. and Nikulin M. Eds., Birkhauser, Boston, 163-180.

[29] Kołowrocki, K. (2000). Asymptotic approach to reliability evaluation of piping and rope transportation systems. *Exploitation Problems of Machines* 2, 111-133.

[30] Kołowrocki, K. (2001). *Asymptotic Approach to System Reliability Analysis*. Monograph. System Research Institute, Polish Academy of Science, (in Polish), Warsaw.

[31] Kołowrocki, K. (2001). On limit reliability functions of large multi-state systems with ageing components. *Applied Mathematics and Computation* 121, 313-361.

[32] Kołowrocki, K. (2003). An asymptotic approach to reliability evaluation of large multi-state systems with applications to piping transportation systems. *International Journal of Pressure Vessels and Piping*, 80, 59-73.

[33] Kołowrocki, K. (2003). Asymptotic approach to reliability analysis of large systems with degrading components. *International Journal of Reliability, Quality and Safety Engineering*, Vol. 10, No 3, 249-288.

[34] Kołowrocki, K. (2004). *Reliability of Large Systems*. Elsevier.

[35] Kołowrocki, K. (2006). Reliability and risk evaluation of complex systems in their operation processes. *International Journal of Materials & Structural Reliability*, Vol. 4, No 2, 129-147.

[36] Kołowrocki, K. (2007). Reliability modelling of complex systems – Part 1. *International Journal of Gnedenko e-Forum "Reliability: Theory & Application"*, Vol. 2, No 3-4, 116-127.

[37] Kołowrocki, K. (2007). Reliability modelling of complex systems – Part 2. *International Journal of Gnedenko e-Forum "Reliability: Theory & Application"*, Vol. 2, No 3-4, 128-139.

[38] Kołowrocki, K. (2008). Reliability and risk analysis of large systems with ageing components. *Reliability Engineering and System Safety* 93, 1821-1829.

[39] Kołowrocki, K. (2008). Reliability of large systems, Section in Encyclopedia of Quantitative Risk Analysis and Assessment, John Wiley & Sons, Vol. 4, 1466-1471.

[40] Kołowrocki, K. (2008). Reliability and risk analysis of multi-state systems with degrading components. *International Journal of Materials & Structural Reliability*. Vol. 6. No. 2, 213-228.

[41] Kolowrocki K. (2008). Reliability, availability and cost analysis of large multi-state systems with ageing components. *Proc. International Conference ESREL 2008*, Valencia, Vol. 2, 1969-1983.

[42] Kołowrocki, K. & Soszyńska, J. (2006) Reliability and availability of complex systems. *Quality and Reliability Engineering International* Vol. 22, Issue 1, J. Wiley & Sons Ltd., 79-99.

[43] Kołowrocki K., Soszyńska J. (2009). Methods and algorithms for evaluating unknown parameters of operation processes of complex technical systems. *Proc. 3rd Summer Safety and Reliability Seminars – SSARS 2009*, Gdańsk-Sopot, Vol. 2, 211-222.

[44] Kołowrocki K., Soszyńska J. (2009). Methods and algorithms for evaluating unknown parameters of components reliability of complex technical systems. *Proc. 3rd Summer Safety and Reliability Seminars – SSARS 2009*, Gdańsk-Sopot, Vol. 2, 223-230.

[45] Kolowrocki, K. & Soszynska, J. (2009). Modelling environment and infrastructure influence on reliability and operation process of port oil transportation system. *Electronic Journal of Reliability & Risk Analysis: Theory & Applications*, Vol. 2, No 3, 2009, 131-142.

[46] Kolowrocki, K. & Soszynska, J. (2009). Reliability, risk and availability based optimization of complex technical systems operation processes. Part 1. Theoretical backgrounds. *Electronic Journal of Reliability & Risk Analysis: Theory & Applications*, Vol. 2, No 4, 141-152.

[47] Kolowrocki, K. & Soszynska, J. (2009). Reliability, risk and availability based optimization of complex technical systems operation processes. Part 2. Application in Port Transportation. *Electronic Journal of Reliability & Risk Analysis: Theory & Applications*, Vol. 2, No 4, 153-167.

[48] Kolowrocki, K. & Soszynska, J. (2009). Safety and risk evaluation of Stena Baltica ferry in variable operation conditions. *Electronic Journal of Reliability & Risk Analysis: Theory & Applications*, Vol. 2, No 4, 168-180.

[49] Kolowrocki, K. & Soszynska, J. (2010).

Reliability modeling of a port oil transportation system's operation processes. *International Journal of Performability Engineering*, Vol. 6, No. 1, 77-87.

[50] Kolowrocki, K. & Soszynska, J. (2010). Reliability and risk optimisation of complex industrial systems and processes with applications to maritime transport. *IEEE Transactions on Reliability*, (submitted).

[51] Kołowrocki, K., Soszyńska, J., Judziński, M. & Dziula, P. (2007). On multi-state safety analysis in shipping. *International Journal of Reliability, Quality and Safety Engineering. System Reliability and Safety*, Vol. 14, No 6, 547-567.

[52] Kossow, A. & Preuss, W. (1995). Reliability of linear consecutively-connected systems with multistate components. *IEEE Transactions on Reliability* 44, 518-522.

[53] Kuo, W. & Prasad, V.R. (2000). An annotated overview of system-reliability optimization, *IEEE Transactions on Reliability*, 49(2), 176-187.

[54] Kuo, W. & Zuo, M.J. (2003) *Optimal Reliability Modeling: Principles and Applications*. Hoboken: John Wiley & Sons, Inc.

[55] Levitin, G. & Lisnianski, A. (2000). Optimisation of imperfect preventive maintenance for multistate systems. *Reliability Engineering and System Safety* 67, 193-203.

[56] Levitin, G. & Lisnianski, A. (2000). Optimal replacement scheduling in multi-state series-parallel systems. *Quality and Reliability Engineering International* 16, 2000, 157-162.

[57] Levitin, G., Zhang, T. & Xie, M. (2006). State probability of a series-parallel repairable system with two-types of failure states. *International Journal of Systems Science*, 37(14), 1011 – 1020.

[58] Limnios, N. & Oprisan, G. (2001). *Semi-Markov Processes and Reliability*. Birkhauser, Boston.

[59] Limnios, N., Ouhbi, B. & Sadek, A. (2005). Empirical estimator of stationary distribution for semi-Markov processes. *Communications in Statistics-Theory and Methods*, Vol. 34, No. 4, 987-995.

[60] Lisnianski, A. & Levitin, G. (2003). *Multi-State System Reliability. Assessment, Optimisation and Applications.* World Scientific Publishing Co. Pte. Ltd., New Jersey, London, Singapore, Hong Kong.

[61] Macci, C. (2008). Large deviations for empirical estimators of the stationary distribution of a semi-Markov process with finite state space. *Communications in Statistics-Theory and Methods*, Vol. 37, No. 19, 3077-3089.

[62] Meng, F. (1993). Component-relevancy and characterisation in multi-state systems. *IEEE Transactions on Reliability* 42, 478-483.

[63] Mercier, S. (2008). Numerical bounds for semi-Markovian quantities and application to reliability. *Methodology and Computing in Applied Probability*, Vol. 10, No. 2, 179-198.

[64] Merrick, J.R.W. & van Dorp R. (2006). Speaking the truth in maritime risk assessment. Risk Analysis, Vol. 26, No. 1, 223-237.

[65] Natvig, B. (1982). Two suggestions of how to define a multi-state coherent system. *Adv. Applied Probability* 14, 434-455.

[66] Natvig, B. & Streller, A. (1984). The steady-state behaviour of multistate monotone systems. *J. Applied Probability* 21, 826-835.

[67] Natvig, B. (1984). Multi-state coherent systems. In: Encyclopaedia of Statistical Sciences, Wiley and Sons, New York.

[68] Ohio, F. & Nishida, T. (1984). On multistate coherent systems. *IEEE Transactions on Reliability* 33, 284-287.

[69] Pourret, O., Collet, J. & Bon, J-L. (1999). Evaluation of the unavailability of a multistate-component system using a binary model. *IEEE Transactions on Reliability* 64, 13-17.

[70] Rice, J. A. (2007). *Mathematical statistics and data analysis.* Duxbury. Thomson Brooks/Cole. University of California. Berkeley, 2007.

[71] Ross, S. M. (2007). *Introduction to probability models.* Academic Press, Elsevier, San Diego.

[72] Soszyńska, J. (2004). Reliability of large series system in variable operation conditions. *Joint Proceedings* 17, Gdynia Maritime University Press, Gdynia, 36-43.

[73] Soszyńska, J. (2004). Reliability of large parallel systems in variable operation conditions. *Faculty of Navigation Research Works* 16, Gdynia, 168-180.

[74] Soszyńska, J. (2006). Reliability of large series-parallel system in variable operation conditions. *International Journal of Automation and Computing*, Vol. 3, No 2, 199-206.

[75] Soszyńska, J. (2006). Reliability evaluation of a port oil transportation system in variable operation conditions. *International Journal of Pressure Vessels and Piping*, Vol. 83, Issue 4, 304-310.

[76] Soszyńska, J. (2006). Analiza bezpieczeństwa systemów wielostanowych w zmiennych warunkach eksploatacji. *Diagnostyka* 3(39), 25-34.

[77] Soszyńska, J. (2007). *Systems reliability analysis in variable operation conditions*. PhD

Thesis, Gdynia Maritime University-System Research Institute Warsaw, (in Polish).

[78] Soszyńska, J. (2007). Systems reliability analysis in variable operation conditions. *International Journal of Reliability, Quality and Safety Engineering. System Reliability and Safety,* Vol. 14, No 6, 617-634.

[79] Soszynska, J. (2009). Asymptotic approach to reliability evaluation of large "m out of l"-series system in variable operation conditions. *Journal of Reliability & Risk Analysis: Theory & Applications,* Vol. 2, 9-43.

[80] Soszyńska, J. (2010). Reliability and risk evaluation of a port oil pipeline transportation system in variable operation conditions. *International Journal of Pressure Vessels and Piping,* Vol. 87, No 2-3, 81-87.

[81] Stapleton, J. M. (2009). *Models for Probability and Statistical Inference*. John Wiley and Sons, New Jersey.

[82] Suich, R.C. & Patterson, R.L. (1991). k-out-of-n:G system: Some cost considerations. *IEEE Transactions on Reliability,* 40(3), 259-264.

[83] Tang, H., Yin, B.Q. & Xi, H.S. (2007). Error bounds of optimization algorithms for semi-Markov decision processes. *International Journal of Systems Science*, Vol. 38, No. 9, 725-736.

[84] Vercellis, S. (2009). *Data mining and optimization for decision making*. John Wiley & Sons Ltd.

[85] Wilson, A.G., Graves, T.L., Hamada, M.S. et al. (2006). Advances in data combination, analysis and collection for system reliability assessment. *Statistical Science,* Vol. 21, No. 4, 514-531.

[86] Xue, J. (1985). On multi-state system analysis. *IEEE Transactions on Reliability* 34, 329-337.

[87] Xue, J., Yang K. (1995). Dynamic reliability analysis of coherent multi-state systems. *IEEE Transactions on Reliability* 4, Vol. 44, 683-688.

[88] Xue, J. & Yang, K. (1995). Symmetric relations in multi-state systems. *IEEE Transactions on Reliability* 4, Vol. 44, 689-693.

[89] Yu, K., Koren, I. & Guo, Y. (1994). Generalized multistate monotone coherent systems. *IEEE Transactions on Reliability* 43, 242-250.

[90] Zio, E. (2006). *An introduction to the basics of reliability and risk analysis.* World Scientific Publishing Co. Pte. Ltd.

[91] Zio, E. (2009). *Computational methods for reliability and risk analysis.* World Scientific Publishing, Co. Pte. Ltd.