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THE FERRO-OILS VISCOSITY DEPENDED SIMULTANEOUSLY ON THE TEMPERATURE AND MAGNETIC OIL PARTICLES CONCENTRATION $\eta = \eta(T,\varphi)$ – PART II

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Abstract

The purpose of this article is to determine and describe the viscosity characteristics of the ferro-oils at context of temperature changes depend on different concentrations of magnetic particles. These characteristics were determined for the following test conditions: atmospheric pressure, constant shear rate and absent of an external magnetic field. In Part I of the paper there was determined and described four mathematical-physical models of fittings of the viscosity characteristics: exponential, fourth-degree polynomial and two homographic models – second- and third-degree. In present Part II of the paper above mentioned models have been developed. That has been done their depending on parameter of concentrations of magnetic particles φ in ferro-oil. There have been also undertaken comparison of the accuracy of estimation of reached results by selected mathematical models as well as these results have been rated according to their compliance with theoretical assumptions. Tests were performed for selected concentrations of magnetic particles in ferro-oil ranging: 1%, 2%, 4%, 6% and 8%. Tested ferro-oil was product from Unterensingen FerroTec Company (Germany) which is a mixture of colloidal mineral engine oil LongLife Gold Pennzoil class SAE 15W-40 with addition of Fe_3O_4 magnetic particles and a surfactant. Analysis of the results, identifying and fitting of characteristics has been made by means of software StatSoft STATISTICA ver. 9.1. The obtained characteristics going to be used by author in his future work to build mathematical models of viscosity changes of ferro-oils in the aspect of changes of parameters such as temperature, pressure, shear rate and the value of the external magnetic field. The determined parameters of physical properties will be used in analytical and numerical studies of flow and operating parameters of slide journal bearings lubricated with ferro-oils.

Keywords: *ferro-oil, dynamic viscosity, magnetic particles concentration*

1. Introduction

The present article is a continuation and development of research issues taken in the first part. In the preceding publications, there have been proposed and described four selected mathematical models of matches of the rheological characteristics of changes in viscosity of ferro-oil due to temperature. These characteristics were obtained by experimental testing carried out a rotational rheometer Haake Mars III with the tribological-geometric configuration of a “plate-cone” type. The proposed models had different mathematical constitution namely: exponential, polynomial of fourth-degree and homographic second- and third-degree functions. They have been described in dependence on practical possibilities of its applications and needs connected with numerical calculations.

In this, the second part of this paper, the first three of the four previously proposed models have been subjected to further analysis. The author has set itself the aim of finding dependence above-mentioned models on the essential parameter, which for the ferro-oil is the concentration of magnetic particles φ contained therein. Such as developed models have been re-examined, that also have been rated their accuracy of estimating the actual waveform of physical phenomena and eventually their susceptibility to further analytical and numerical studies.

2. The theoretical foundations

The analysis of the solutions of posed problem, which can be found in the specialist literature, does not give a satisfactory answer. On the one hand we have a number of papers that propose the rheological models for fluids of properties both Newtonian and non-Newtonian containing inclusions particles of varying concentrations but magnetically neutral. Among these models are the most popular: Einstein's, Batchelor's, Brinkman's, Phen-Thien's and Phom Moon's models. Their valuable description and comparison can be found, among others, in [2, 5, 8, 10]. In the authors' principle, these models can be apply to liquids contaminated with solid particles, usually dust. Unfortunately, this assumption also constitutes the weakness of these models, as in the case if we tried to adapt them for the purpose of describing ferrofluids. First, they do not describe the dynamic change of properties of the fluid under the influence of an external magnetic field. Secondly, they assume that the theoretical sizes of contaminant particles are usually several orders of magnitude greater than those are, which magnetic part of ferrofluids are. All in all these disadvantages make up the physical inadequacy of the models relative to the rheological phenomena occurring in the actual ferrofluid. In some works, the authors have attempted to use these models or to adapt them for the purposes of describing the phenomenon of magnetoviscosity of a ferrofluid [4, 5, 13]. One of such examples is a Synder's model the following form:

$$\eta_{nf} = \eta_{bf} \cdot (1 + \varphi \cdot 1.25)^{6.365}, \tag{1}$$

where:
η_{nf} – dynamic viscosity of the ferrofluid [Pa·s],
η_{bf} – dynamic viscosity of the base fluid [Pa·s],
φ – concentration of the magnetic particles [–].

In [13], there was done a comparison of the both results obtained by Sunder's model (1) and above mentioned other models: Einstein's, Batchelor's and Brinkman's with the results obtained experimentally. The results were far from a satisfactory – relative error of the estimation was between 15% and 40% depending on the model. Significantly better matching results were achieved by the authors in [4] using the relationship of the Herschey-Bulkley.

$$\tau = \tau_0 + k \cdot \gamma^n, \quad \text{for } \eta = k \cdot \gamma^{n-1}, \tag{2}$$

where:
τ – shear stress [Pa],
τ_0 – yield stress [Pa],
η – dynamic viscosity [Pa·s],
k – coefficient depending on concentration of particles [Pa·s^n],
n – coefficient of proportionality [–],
γ – shear rate [s^{-1}].

On the other hand, the rheological properties of ferrofluids are estimated and determined by using the methods and models of statistical physics. These calculations are based on relationships resulting from Faraday's law [3, 6, 9, 10, 12]. For individual magnetic particles it is determined their individual magnetic moment. These moments make up the total magnetization vector of ferrofluid. However, as indicated in his work Zubarev [14], the experimental results differ dramatically from theoretical calculations. Magnetoviscosity amplitudes of the concentrated ferrofluids are generally higher by 100% to 200%, and the increase in shear rate leads to a surprisingly rapid drop in the magnetoviscosity, which do not provide theoretical calculations.

The reason for this state of affairs lies in the strong simplifying assumptions adopted in the theoretical considerations. It is assumed that the magnetic particles in the ferrofluid are perfectly spherical, uniform in shape and size, well dispersed in a base fluid and do not interact with each other magnetically. In real ferrofluids, even those prepared commercially, there can be observed a strong dispersion of sizes and shapes of particles as well as their coagulation phenomena. Influence of the type, particle size and dispersion of the size of the ferrofluid is the foundation of many modern testing, e.g. [1, 7, 11]. Similarly, there are the effects of ellipticity of the shape, particle aggregation and the mutual interaction of magnetic [15, 16] during the investigations. There are no the general mathematical-physical model, which take into account all the mentioned above nuances at this moment.

In this context, studies leading to the creation of their own, even an empirical models, binding viscosity of the ferro-oil with the concentration of the magnetic particles in the aspect of changes in the basic operating parameters such as temperature $\eta = \eta(\varphi, T)$, pressure $\eta = \eta(\varphi, p)$ or the value of the external magnetic field induction $\eta = \eta(\varphi, B)$, seems to be absolutely reasonable.

3. Results of modelling and discussion

In this research work, the experimental designation of the viscosity characteristics of ferro-oil with few concentrations of magnetic particles: 1%, 2%, 4%, 6% and 8% due to the temperature has been done. Tested oil was a product made by FerroTec in Unterensingen. The average diameter of the magnetic particles was 10 nm and the volume content of the surfactant was 15%.

Tests were performed on HAAKE MARS III rheometer for the constant shear rate 100 1/s and temperature range up to 120ºC every 10K. There was used the configuration of rheometer chamber with the tribological configuration of "cone-plate" type and with Peltier's system of heating in the tests. Fig. 1 presents experimentally obtained results.

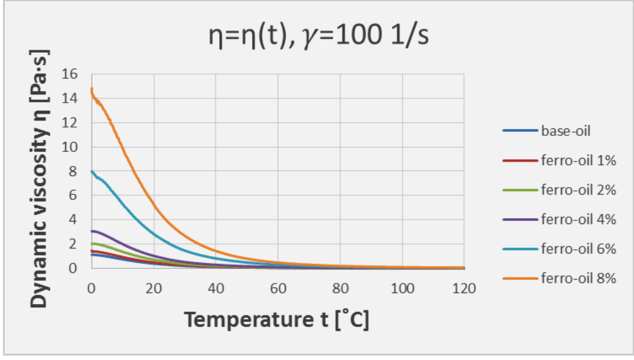

Fig. 1. Experimentally obtained results of the viscosity changes due to the temperature for base-oil and 1%, 2%, 4%, 6% and 8% ferro-oil

In the first part of this work, it has been proposed four models of matches of mathematical functions for the obtained characteristics. These were models: exponential (3), the fourth-degree polynomial (4), and two homographic models – the second- (5) and third degree (6) of the following form:

$$\eta_T = \eta_0 \exp[\delta_{T1} \cdot (T - T_0)], \tag{3}$$

$$\eta_T = \delta_{T2} \Delta T^4 + a_2 \Delta T^3 + b_2 \Delta T^2 + c_2 \Delta T + \eta_0, \tag{4}$$

$$\eta_T = \delta_{T3} \cdot \frac{1}{(T - T_0)^2 + a_3} + b_3, \tag{5}$$

$$\eta_T = \delta_{T4} \cdot \frac{1}{(T - T_0)^3 + a_4} + b_4, \tag{6}$$

where the parameters occurring in equations (3)-(6) are described in the form:

η_T – dynamic viscosity depended on the temperature [Pa·s],
η_0 – initial dynamic viscosity for $T_0 = 273.15$K and $p_0 = p_{at}$ [Pa·s],
δ_{T1} – major temperature coefficient for Model 1 [K^{-1}],

T – temperature [K],
T_0 – reference temperature 273.15 [K],
δ_{T2} – major temperature coefficient for Model 2 [Pa·s/K^4],
a_2 – 1st minor coefficient for Model 2 [Pa·s/K^3],
b_2 – 2nd minor coefficient for Model 2 [Pa·s/K^2],
c_2 – 3rd minor coefficient for Model 2 [Pa·s/K],
η_0 – initial viscosity displacement coefficient [Pa·s],
$\Delta T = (T - T_0)$ – reduced temperature [K],
δ_{T3} – major temperature coefficient for Model 3 [Pa·s·K^2],
a_3 – coefficient of the peak scale for Model 3 [K^2],
b_3 – coefficient of the viscosity displacement for Model 3 [Pa·s],
δ_{T4} – major temperature coefficient for Model 4 [Pa·s·K^3],
a_4 – coefficient of the peak scale for Model 4 [K^3],
b_4 – coefficient of the viscosity displacement for Model 4 [Pa·s].

The first three of these models have been subjected to further analysis leading to find relations of dynamic viscosity of the ferro-oil depending on concentration of the magnetic particles contained therein. Analyses of the results identify and matching characteristics were calculated using StatSoft STATISTICA 9.1 software. It has been applied Gauss-Newton's nonlinear estimation with regression made the least squares method adopted for the maximum number of iterations equal to 250 and the ratio of $1 \cdot 10^{-6}$ convergence criterion. The confidence level has been equal $p = 0.95$.

There has been done the analysis of the parameters of matching aiming to create a relation between them and concentration of the magnetic particles in the ferro-oil for Model 1. The initial parameter η_0 has been matched by using exponential relation and the temperature parameter δ_{T1} by linear one, what can be observed in Fig. 2.

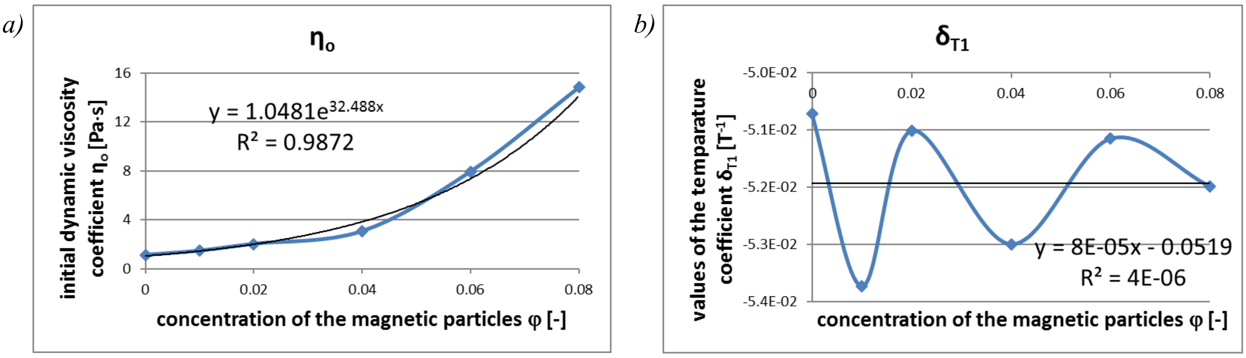

Fig. 2. Model 1: a) initial dynamic viscosity versus concentration of magnetic particles, b) temperature coefficient versus concentration of magnetic particles

Based on the foregoing dependences (1)-(6) it has been received the following relationship binding the ferro-oils dynamic viscosity simultaneously with the concentration of magnetic particles and temperature:

$$\eta_T = \eta_0 \cdot \exp(a_1 \cdot \varphi) \cdot \exp[k_1 \cdot (T - T_0)] = \eta_0 \cdot \exp[a_1 \cdot \varphi + k_1 \cdot (T - T_0)], \tag{7}$$

for:

$$a_1 = 32.488,\ k_1 = -0.05193.$$

Now in Fig. 3 are presented dependences between dynamic viscosity and temperature for sample of two chosen extreme concentrations of magnetic oil particles $\varphi = 0\%$ and $\varphi = 8\%$.

Also for Model 2, it has been made illustration of dependences between parameters δ_{T2}, a_2, b_2 or c_2 and magnetic particles concentration φ in ferro-oil. Fig. 4 presents waveforms of changes in above-mentioned parameters relative to concentration of magnetic particles.

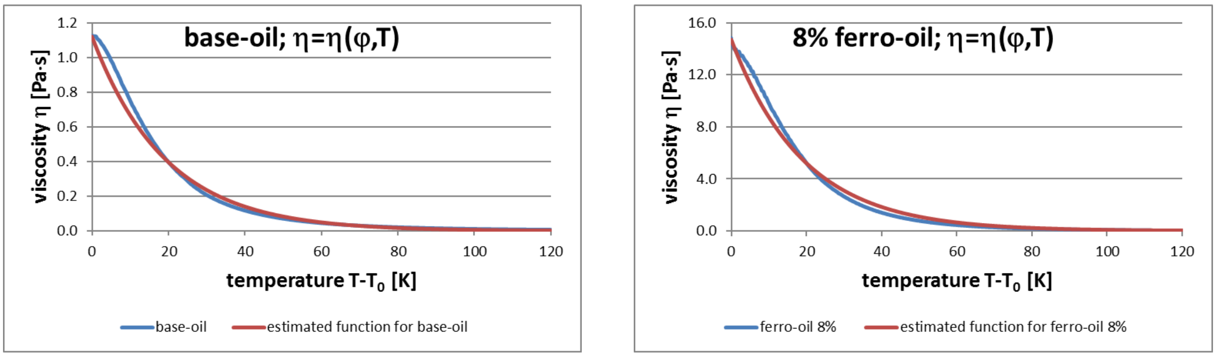

Fig. 3. Model 1: fitted functions for the results of the viscosity changes due to the temperature for base-oil and 8% ferro-oil

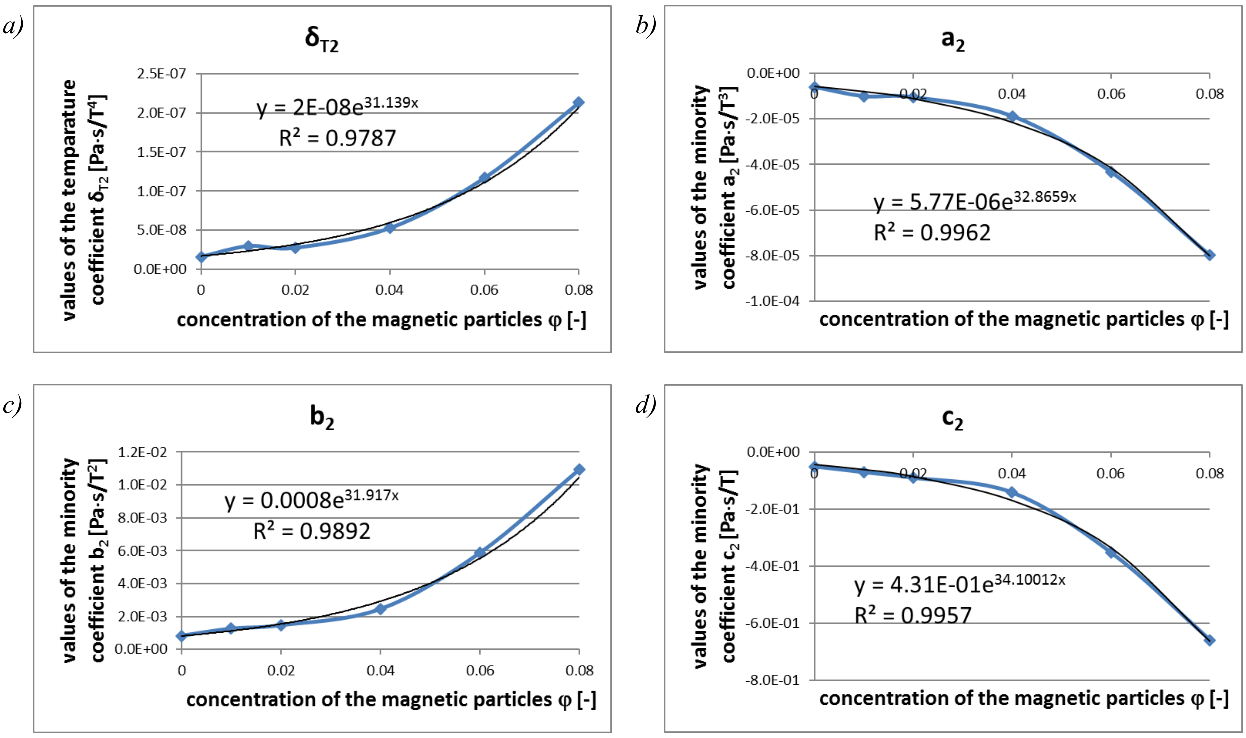

Fig. 4. Model 2: a) temperature coefficient versus concentration of magnetic particles, b) 1st minor coefficient versus concentration of magnetic particles, c) 2nd minor coefficient versus concentration of magnetic particles, d) 3rd minor coefficient versus concentration of magnetic particles

The change of each one of these parameters has been adjusted with exponential relation and finally, below shown relationship has been obtained:

$$\eta_T = \eta_0 + [a_{21}\Delta T^4 + a_{22}\Delta T^3 + a_{23}\Delta T^2 + a_{24}\Delta T]\cdot \exp(k_2 \cdot \varphi)\,, \tag{8}$$

for:

$$k_2 = 33.158, \quad a_{21} = 0.164\cdot 10^{-7}, \quad a_{22} = -0.58\cdot 10^{-5}, \quad a_{23} = 0.751\cdot 10^{-3}, \quad a_{24} = -0.04317\,,$$

where:

k_2 – the generalized aspect ratio coefficient.

The obtained mathematical model has proven to be exceptionally sensitive to the inaccuracy of parameter substitution and thus the results of estimation differed significantly from the values obtained empirically, which illustrate two selected examples in Fig. 5.

As the last homographic function model of two forms: the second degree and third was subjected to the analysis. The following can be viewed the analysis of matching parameters aimed at the relation between them and the concentration of magnetic particles in ferro-oil.

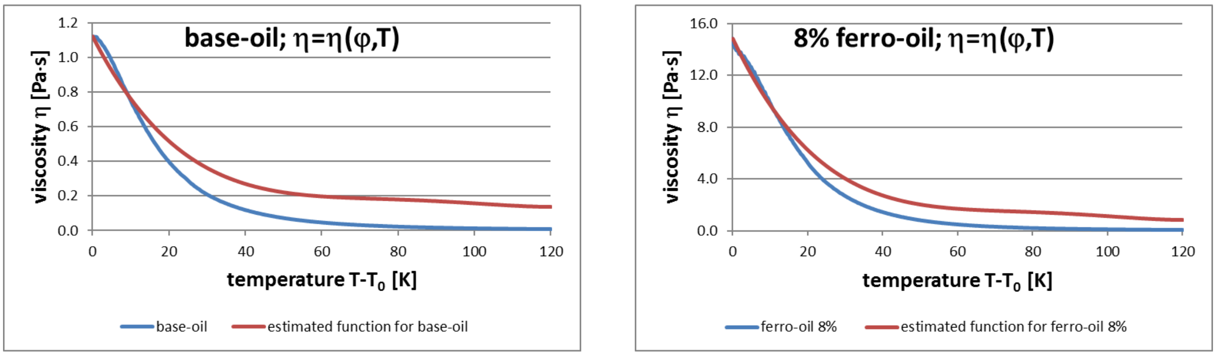

Fig. 5. Model 2: fitted functions for the results of the viscosity changes due to the temperature for base-oil and 8% ferro-oil

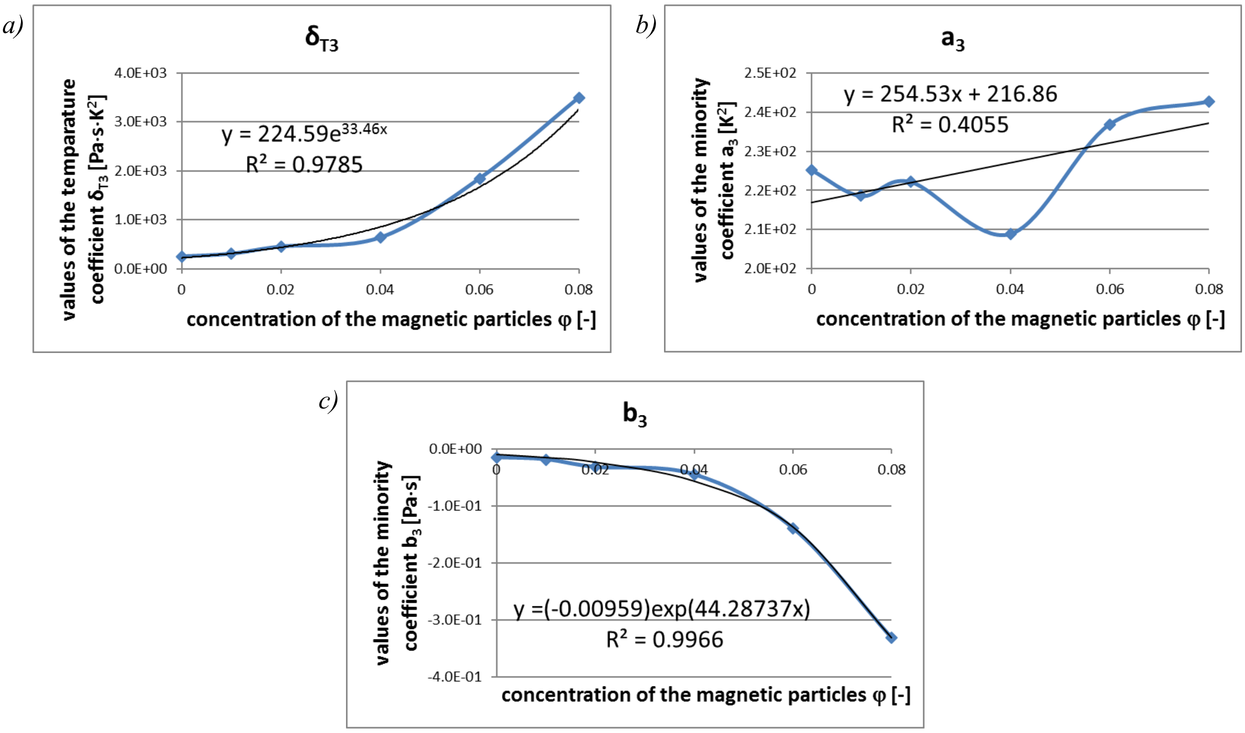

Fig. 6. Model 3: a) temperature coefficient versus concentration of magnetic particles, b) peak scale coefficient versus concentration of magnetic particles, c) coefficient of the viscosity displacement versus concentration of magnetic particles

Parameters of initial and displacement have been adjusted by using the exponential function whereas the parameter scale − by using a linear function. Based on the matches it has been obtained from the following relationship binding the dynamic viscosity ferro-oils with the concentration of magnetic particles:

$$\eta_T = \delta_{T3,\varphi} \cdot \exp(k_3 \cdot \varphi) \cdot \left(\frac{1}{(T - T_0)^2 + a_{31} \cdot \varphi + a_{32}} + b_{31} \cdot \exp(b_{32} \cdot \varphi) \right), \tag{9}$$

for:

$$\delta_{T3,\varphi} = 252.3881, \quad k_3 = 32.76, \quad a_{31} = 349.2, \quad a_{32} = 215, \quad b_{31} = -0.00959, \quad b_{32} = 44.28747.$$

Due to the fairly extensive mathematical form of the obtained relation, it has been made an analysis, which allowed for simplification. The reduced form of this relationship presents the following relation:

$$\eta_T = \delta_{T3,\varphi} \cdot \exp(k_3 \cdot \varphi) \cdot \left(\frac{1}{(T - T_0)^2 + a_{32R}} \right), \tag{9}$$

for:

$$\delta_{T3,\varphi} = 252.3881, \quad k_3 = 32.76, \quad a_{32R} = 225.7683.$$

There are presented below matches estimated with these models (the general and reduced one) for sample of two extreme chosen concentrations of magnetic particles $\varphi = 0\%$ and $\varphi = 8\%$.

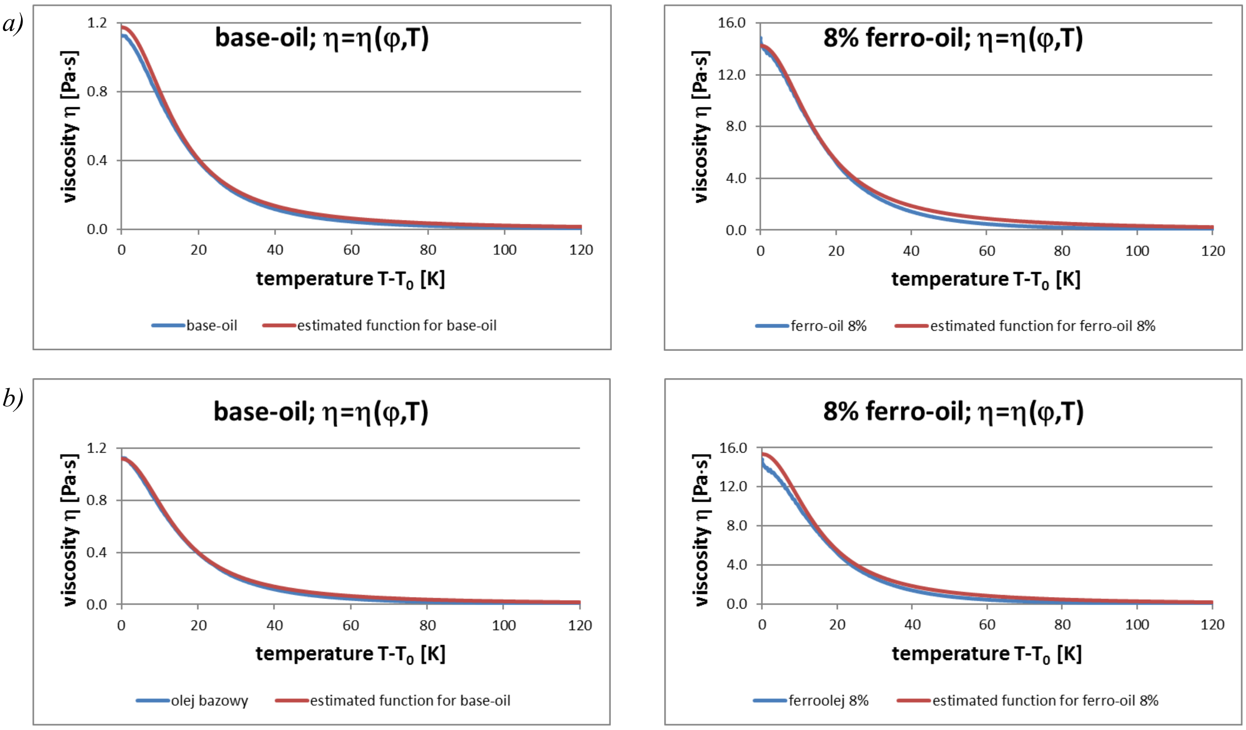

Fig. 7. Model 3 – fitted functions for the results of the viscosity changes due to the temperature for base-oil and 8% ferro-oil, for a) general form of model and b) reduced one

3. Observations and conclusions

Among the analysed models – Model 1 exponential and hyperbolic Model 3 – appear to match the actual waveforms of changes in viscosity of ferro-oil in terms of temperature depending on the concentration of magnetic particles with a satisfactory degree of accuracy. The polynomial Model 2 has a relatively major the absolute and relative errors, especially in the areas of the temperature stabilization of the viscosity – above temperature 20°C. According to the author's opinion, it is not possible to use it in further research analytical and numerical, so because of these errors and due to its purely mathematical origins.

The Model 1 described by the relationship (7) as well as Model 3 described by simplified relationship (10), seem to be perspective in terms of the possibility of their further use in research, because of its relatively simple and elegant mathematical construction, strong physical references and high precision of matching of actual characteristics. For both cases, the relative errors were at the similar level as they were reached by other authors in [4] using Herschey-Bulkley model.

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