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SIMULATION CALCULATIONS OF PARAMETERS FOR ELECTRICAL CIRCUITS AS AUTOMATION SYSTEMS BY USING MATHCAD AND PSPICE PROGRAMS

Abstract

Introduction and aims: The work shows the description of the analytical method of state variables (matrix notation) electrical circuit. As state variables assumed currents in the coils and the capacitor voltage. State variables describe the specific automation system. Purpose of these simulation calculations the automation system.

Material and methods: The material is a multi-dimensional automatic systems exposed to disturbance signals, which included analytical systems of differential equations of the first order. Interference signals created with trigonometric functions and procedures *rnd(x)* program *MathCAD* generating random numbers distributed natural. The method of analytical and numerical has been used in the paper.

Results: Simulations signal output of selected automation systems interference was determined on the basis of their analytical descriptions. The system of differential equations of the first order is solved numerically by difference equations in the program *MathCAD*. The results of simulation output signals of the automation system were verified in the program *PSpice*. Verification program *PSpice* was to create the equivalent circuit diagram of an electric circuit constituting the automation system and declare the appropriate analysis.

Conclusion: Using the calculation procedures of the program *MathCAD* can be made dynamic visualization of automation systems including interfering signals.

Keywords: Automatics systems, electrical circuits, calculations, *MathCAD*, *PSpice*.

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OBLICZENIA SYMULACYJNE PARAMETRÓW DLA OBWODÓW ELEKTRYCZNYCH JAKO UKŁADÓW AUTOMATYKI Z UŻYCIEM PROGRAMÓW MATHCAD I PSPICE

Streszczenie

Wstęp i cele: Praca pokazuje opis analityczny metodą zmiennych stanu (zapis macierzowy) obwodu elektrycznego. Jako zmienne stanu przyjęto prądy w cewkach i napięcie na kondensatorze. Zmienne stanu opisują określony układ automatyki. Cel to obliczenia symulacyjne danego układu automatyki.

Materiał i metody: Materiał to układy automatyki wielowymiarowe poddane działaniu sygnałów zakłócających, które ujęto analitycznie układami równań różniczkowych pierwszego rzędu. Sygnały zakłócające utworzono z funkcji trygonometrycznych i procedury *rnd(x)* programu *MathCAD* generującej liczby pseudolosowe o rozkładzie naturalnym. Zastosowano metodę analityczno-numeryczną.

Wyniki: Obliczenia symulacyjne sygnałów wyjściowe wybranych układów automatyki z zakłóceniami wyznaczono w oparciu o ich opisy analityczne. Układ równań różniczkowych pierwszego rzędu rozwiązano numerycznie metodą równań różnicowych w programie *MathCAD*. Uzyskane wyniki obliczeń symulacyjnych sygnałów wyjściowych wybranego układu automatyki zweryfikowano w programie *PSpice*. Weryfikacja w programie *PSpice* polegała na stworzeniu schematu zastępczego danego obwodu elektrycznego stanowiącego układ automatyki i zadeklarowania odpowiedniej analizy.

Wniosek: Za pomocą procedur obliczeniowych programu *MathCAD* można dokonywać wizualizacji dynamiki układów automatyki z uwzględnieniem sygnałów zakłócających.

Słowa kluczowe: Układy automatyki, obwody elektryczne, obliczenia, *MathCAD*, *PSpice*.

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1. Introduction

The electrical circuits are described, among others, by the systems of n first-order differential equations (a method state variable) with the constant time. The system of the n differential equations in vector - matrix notation has the following form:

$$\frac{d}{dt} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mm} \end{bmatrix} \cdot \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}. \quad (1)$$

Thus, in simplified terms:

$$\frac{d}{dt} \mathbf{x}(t) = \mathbf{A} \cdot \mathbf{x}(t) + \mathbf{B} \cdot \mathbf{u}(t), \quad (2)$$

where \mathbf{A} is the matrix of basic electrical circuit (state matrix - automation system), \mathbf{B} is the extortion matrix of electrical circuit (matrix inputs - automation system), \mathbf{x} is the state vector (state vector - automation system). Equation (2) engaging a description of the electrical circuit in the matrix form corresponds to the structure of the block diagram (Fig. 1). The block diagram included in the Figure 1 creates an automation system. In the automation systems $\dot{\mathbf{x}}(t)$ means a derivative of the state vector with respect to time.

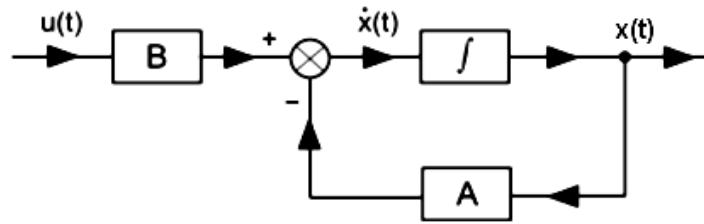


Fig. 1. The structure of the system described by equation (2)

2. Analytical description of selected automation systems from disturbance signals

The classic study of automation in the relationship: control input - output or input Interference - output does not capture the whole picture directly in the dynamics of the system. To get a comprehensive grasp of the dynamics of given system, it is described by state variables. This method allows to describe the state of the automation system at any time [4]. Below are shown and accordingly described some automation systems including disturbance signals.

2.1. Action of the disturbance signal at the input of the automation system

The dynamics of the continuous linear multidimensional stationary system (Fig. 2) in which there is impact signal noise to the input of create the state equation (3) and the output equation of the system (4).

$$\dot{\mathbf{x}}(t) = \mathbf{A} \cdot \mathbf{x}(t) + \mathbf{B} \cdot \mathbf{u}(t) + \mathbf{E} \cdot \mathbf{z}(t), \quad (3)$$

$$\mathbf{y}(t) = \mathbf{C} \cdot \mathbf{x}(t), \quad (4)$$

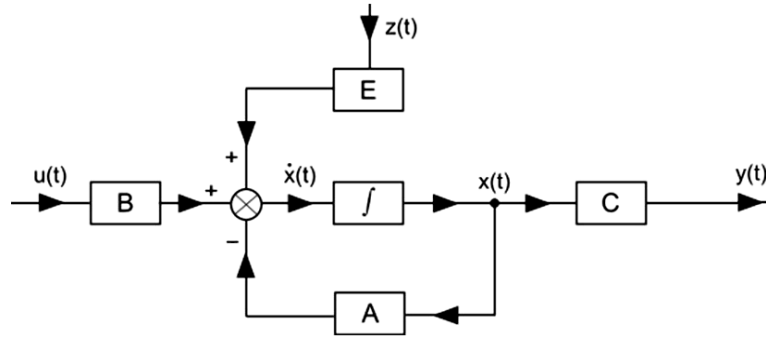


Fig. 2. Equivalent scheme of the automation system with disturbance signals at the input

where $\mathbf{x}(t) \in \mathfrak{R}^n$ - state vector, $\mathbf{u}(t) \in \mathfrak{R}^m$ - extortion vector, $\mathbf{y}(t) \in \mathfrak{R}^l$ - response vector, $\mathbf{z}(t) \in \mathfrak{R}^m$ - vector of the disturbance signals in the equations (3) and (4); \mathbf{A} - state matrix of dimension $n \times n$, \mathbf{B} - inputs matrix of dimension $n \times m$, \mathbf{C} - outputs matrix of dimension $l \times n$, \mathbf{E} - inputs matrix of disturbance signals of dimension $n \times m$ in equations (3) and (4).

2.2. Action disturbance signal on the output of automation systems

The linear continuous stationary and multivariate automation system (Fig. 3) in which there is some influence of signal noise to the system input describe the state equation (5) and the output equation (6).

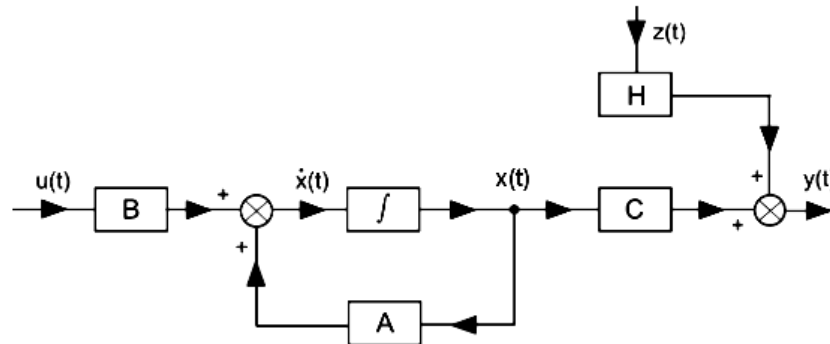


Fig. 3. Equivalent scheme of the automation system with disturbance signals at the output

$$\dot{\mathbf{x}}(t) = \mathbf{A} \cdot \mathbf{x}(t) + \mathbf{B} \cdot \mathbf{u}(t), \quad (5)$$

$$\mathbf{y}(t) = \mathbf{C} \cdot \mathbf{x}(t) + \mathbf{H} \cdot \mathbf{z}(t), \quad (6)$$

where $\mathbf{x}(t)$, $\mathbf{u}(t)$, $\mathbf{y}(t)$, $\mathbf{z}(t)$ - state vector, extortion vector, response vector and the disturbance signals in the equations (5) and (6); \mathbf{A} , \mathbf{B} , \mathbf{C} , $\mathbf{H}_{l \times m}$ - state matrix, inputs matrix, outputs matrix, and outputs the disturbance signals in the equations (5) and (6).

2.3. Effect of disturbance signals on the input and output of automation system

The dynamics of the linear continuous stationary and multidimensional system (Fig. 4), in which there is some influence from disturbance signals on the input and output of the system describes the following state equation (7) and output equation (8) of the system.

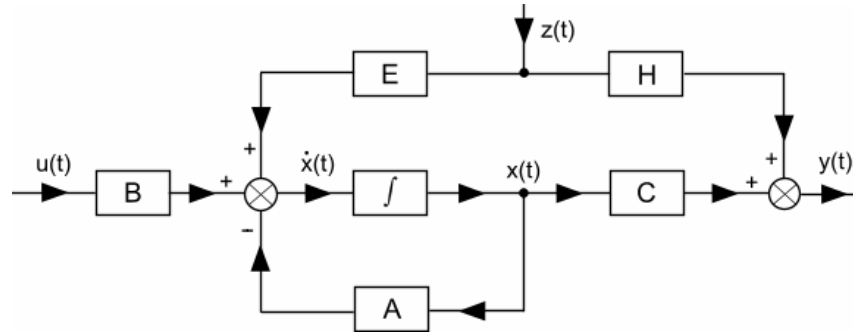


Fig. 4. Equivalent (block) scheme of the automation system with disturbance signals at the input and output

$$\dot{\mathbf{x}}(t) = \mathbf{A} \cdot \mathbf{x}(t) + \mathbf{B} \cdot \mathbf{u}(t) + \mathbf{E} \cdot \mathbf{z}(t), \quad (7)$$

$$\mathbf{y}(t) = \mathbf{C} \cdot \mathbf{x}(t) + \mathbf{H} \cdot \mathbf{z}(t), \quad (8)$$

where $\mathbf{x}(t)$, $\mathbf{u}(t)$, $\mathbf{y}(t)$, $\mathbf{z}(t)$ - state vector, extortion vector, response vector and the disturbance signals in the equations (7) and (8); \mathbf{A} , \mathbf{B} , \mathbf{C} , \mathbf{E} , \mathbf{H} - state matrix, inputs matrix, outputs matrix, and outputs the disturbance signals in the equations (7) and (8).

3. Simulation calculations of automation systems

3.1. Simulation of the output signals of the automation system created by the electrical circuit in the program *MathCAD*

For the calculation of simulation for output signals in the automation systems created by the electrical circuits has been used the electrical circuit, which some equivalent circuit created in the *PSpice* is shown at the Figure 5.

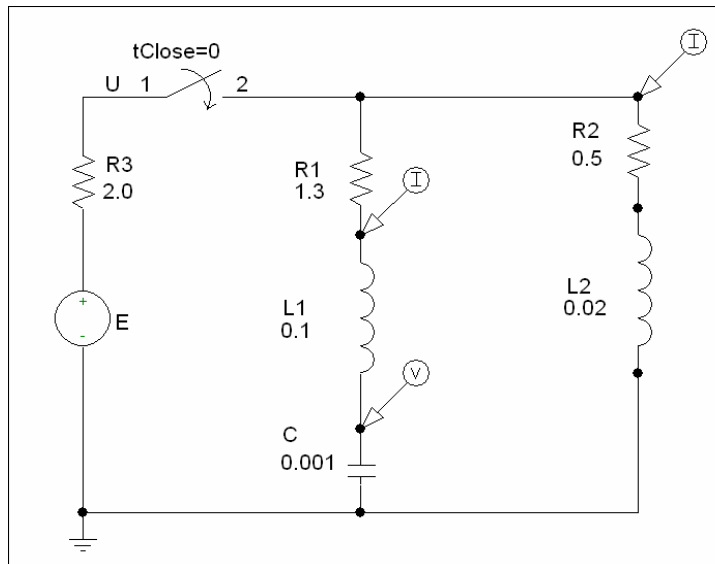


Fig. 5. Equivalent scheme of an electric circuit in the *PSpice* program

The electrical circuit placed on the Figure 5 has been described by using Kirchhoff's laws according to the method of the state variables. Kirchhoff's laws for the mesh and nodes have the following form:

$$E - R_3 \cdot i_3(t) - R_1 \cdot i_1(t) - L_1 \cdot \frac{di_1(t)}{dt} - u_c(t) = 0, \quad (9)$$

$$-R_2 \cdot i_2(t) - L_2 \cdot \frac{di_2(t)}{dt} + u_c(t) + L_1 \cdot \frac{di_1(t)}{dt} + R_1 \cdot i_1(t) = 0, \quad (10)$$

$$i_3(t) = i_1(t) + i_2(t), \quad (11)$$

$$\frac{du_c(t)}{dt} = \frac{1}{C} \cdot i_1(t). \quad (12)$$

As state variables accepted currents ($i_1(t)$, $i_2(t)$) in the inductors and the voltage $u_c(t)$ across the capacitor. In order to standardize some designations during the numerical calculations of the electrical circuits and automation systems the state variables were determined as follows: $x1(t) = i_1(t)$, $x2(t) = i_2(t)$ and $x3(t) = i_3(t)$. By introducing these designations for currents and voltages has been considered a system of three differential equations of the first order in the matrix form:

$$\begin{bmatrix} \frac{dx1(t)}{dt} \\ \frac{dx2(t)}{dt} \\ \frac{dx3(t)}{dt} \end{bmatrix} = \mathbf{A} \cdot \begin{bmatrix} x1(t) \\ x2(t) \\ x3(t) \end{bmatrix} + \mathbf{B} \cdot \begin{bmatrix} E \\ E \end{bmatrix}, \quad (13)$$

$$\mathbf{A} = \begin{bmatrix} -\frac{(R_1 + R_2)}{L_1} & -\frac{R_3}{L_1} & -\frac{1}{L_1} \\ -\frac{R_3}{L_2} & -\frac{(R_2 + R_3)}{L_2} & 0 \\ \frac{1}{C} & 0 & 0 \end{bmatrix}, \quad (14)$$

$$\mathbf{B} = \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \\ 0 & 0 \end{bmatrix}. \quad (15)$$

Accepting the equation

$$\frac{dx(t)}{dt} \approx \frac{x_{n+1} - x_n}{\Delta t} \quad (16)$$

and making the time discretization with the step Δt ($\Delta t = T_{\text{end}}/N$, $n = 0, 1, \dots, N$, $t_n = n \cdot \Delta t$) has been obtained by the following difference equations:

$$\begin{bmatrix} \frac{dx1(t)}{dt} \\ \frac{dx2(t)}{dt} \\ \frac{dx3(t)}{dt} \end{bmatrix} \approx \frac{1}{\Delta t} \begin{bmatrix} x1_{n+1} - x1_n \\ x2_{n+1} - x2_n \\ x3_{n+1} - x3_n \end{bmatrix}, \quad (17)$$

$$\begin{bmatrix} x1_{n+1} \\ x2_{n+1} \\ x3_{n+1} \end{bmatrix} = \begin{bmatrix} x1_n \\ x2_n \\ x3_n \end{bmatrix} + \mathbf{A} \cdot \begin{bmatrix} x1_n \\ x2_n \\ x3_n \end{bmatrix} \cdot \Delta t + \mathbf{B} \cdot \begin{bmatrix} E \\ E \end{bmatrix} \cdot \Delta t. \quad (18)$$

Simulation calculations based on the differential equation (2) were performed in *MathCAD* program.

Algorithm in MathCAD program:	Commentary:
$\begin{pmatrix} E \\ R_1 \\ R_2 \\ R_3 \\ L_1 \\ L_2 \\ C \end{pmatrix} := \begin{pmatrix} 10 \\ 1.3 \\ 0.5 \\ 2.0 \\ 0.1 \\ 0.02 \\ 0.001 \end{pmatrix}$	<p>Parameters of the electrical circuit</p>
$N := 10000 \quad n := 1 .. N \quad T_{\text{end}} := 0.7 \quad \Delta t := \frac{T_{\text{end}}}{N}$	<p>Discretization of time t, the time step Δt</p>
$A := \begin{bmatrix} -\frac{(R_1 + R_2)}{L_1} & -\frac{R_3}{L_1} & -\frac{1}{L_1} \\ -\frac{R_3}{L_2} & -\frac{(R_2 + R_3)}{L_2} & 0 \\ \frac{1}{C} & 0 & 0 \end{bmatrix}$ $B := \begin{pmatrix} \frac{E}{L_1} & 0 \\ 0 & \frac{E}{L_2} \\ 0 & 0 \end{pmatrix}$ $\begin{pmatrix} x1_{n+1} \\ x2_{n+1} \\ x3_{n+1} \end{pmatrix} := \begin{pmatrix} x1_n \\ x2_n \\ x3_n \end{pmatrix} + A \cdot \begin{pmatrix} x1_n \\ x2_n \\ x3_n \end{pmatrix} \cdot \Delta t + B \cdot \begin{pmatrix} E \\ E \end{pmatrix} \cdot \Delta t$ $\begin{pmatrix} x1_0 \\ x2_0 \\ x3_0 \end{pmatrix} := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	<p>A - state matrix</p> <p>B - input matrix</p> <p>The equation of state (2) in the numeric notation</p> <p>Warunki początkowe dla $t=0$</p>

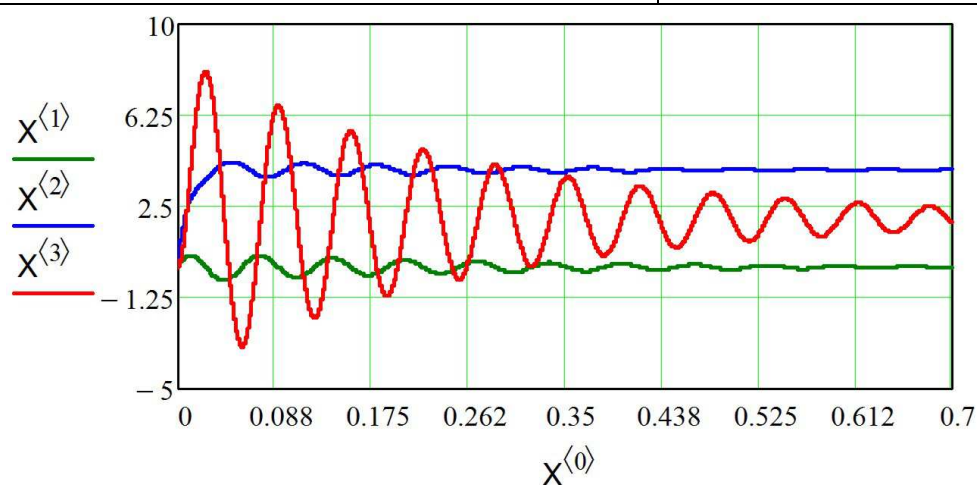


Fig. 6. Simulation of the output signals of the automation system created by the electrical circuit

3.2. Verification of the calculation of the state variables of an electric circuit constituting output signals in the automation program PSpice

In order to check the results of numerical calculations for state variables (x_1, x_2, x_3) of the electrical circuit (Fig. 5) in *MathCAD* program have been carried out some additional simulations in *PSpice* program. Simulation calculations in *PSpice* program were consisted in incorporation at the moment $t = 0$ (Sw tClose – switch off) of the DC voltage $E = 10$ V to the concerned electric circuit and to declare the appropriate analysis.

Due to join voltage to an electrical circuit at the moment $t = 0$ has occurred in the transient state. Transient states of the currents and voltage in the electrical circuit correspond to the state variable (x_1, x_2, x_3). Transient electrical circuit in *PSpice* program recognizes some procedure Transient Analysis and the parameters of the circuit switcher (Sw tClose). Transient Analysis - Print Step = 7 us, Final Time = 0.7 us, No – Print Deley = 0, Step Ceiling = 7 us, Sw tClose - tClose = 0, ttran = 0.01 ns, Rclosed = 0.0001, Ropen = 1 Meg. Simulation calculations in the *PSpice* program of the currents and voltages which are respectively state variables are given in the Figure 7. In both numerical programs, the *MathCAD* (Figure 6) and *PSpice* (Figure 7) have been obtained the same values of state variables.

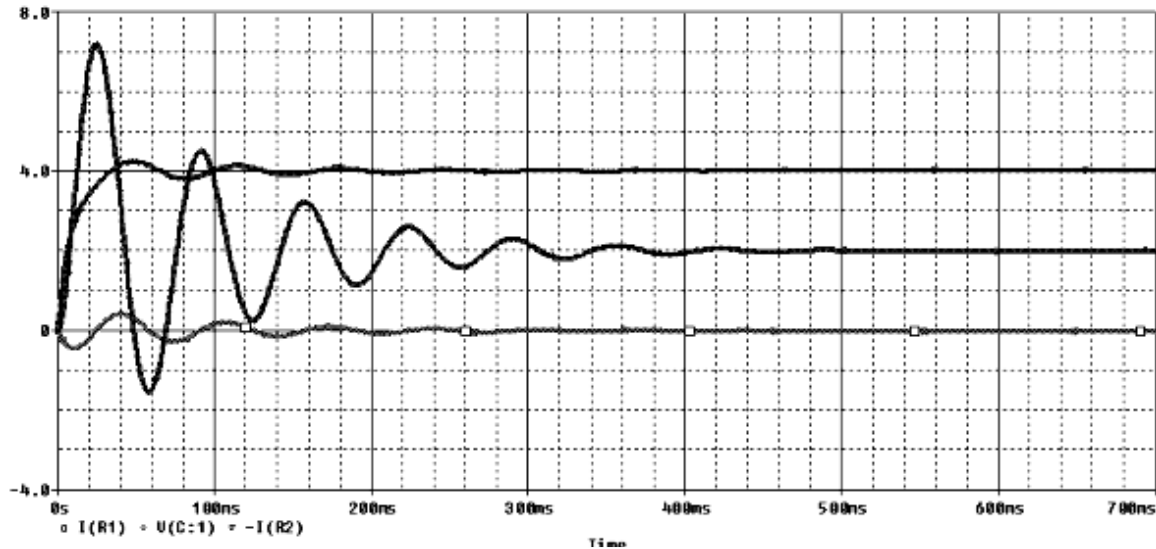


Fig. 7. Simulation of the output signals of the automation system created by the electrical circuit in the program *PSpice*

3.3. Simulation calculation of the output signals of the automation system with the influence of disturbance signals

In order to make some simulation calculations of the output signals of automation system, which was created with the electrical circuit and subjected the action of disturbance signals on the input and output have been respectively used the equations (7) and (8).

It have been created some disturbance signals from trigonometric functions and pseudorandom numbers distributed natural. Equations (7) and (8) in the numeric notation have been implemented in environment of *MathCAD* program.

Algorithm in MathCAD program:	Commentary:
$\begin{pmatrix} E & R_1 & R_2 & R_3 \\ L_1 & L_2 & C & f \end{pmatrix} := \begin{pmatrix} 10 & 1.3 & 0.5 & 2.0 \\ 0.1 & 0.02 & 0.001 & 50 \end{pmatrix}$	Parameters of the electrical circuit
$N := 10000 \quad n := 1 .. N \quad T_{\text{end}} := 0.7 \quad \Delta t := \frac{T_{\text{end}}}{N} \quad t_n := n \cdot \Delta t$	Discretization of time t , the time step Δt , time t on step n

$\mathbf{A} := \begin{pmatrix} -18 & -20 & -10 \\ -100 & -125 & 0 \\ 100 & 0 & 0 \end{pmatrix} \quad \mathbf{B} := \begin{pmatrix} 10 & 0 \\ 0 & 50 \\ 0 & 0 \end{pmatrix}$	<p><i>A</i> - state matrix <i>B</i> - input matrix</p>
$\mathbf{C} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \mathbf{E} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \mathbf{H} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	<p><i>C</i> - output matrix <i>E</i> - input matrix interference signals <i>H</i> - output matrix interference signals</p>
$\begin{pmatrix} z1_n \\ z2_n \\ z3_n \end{pmatrix} := \begin{bmatrix} 0.5 \cdot \sin(2 \cdot \pi \cdot f \cdot t_n) + 0.2 \cdot \cos(2 \cdot \pi \cdot f \cdot t_n) + (\text{mod}(0.7) - 0.2) \\ 0.2 \cdot \sin(2 \cdot \pi \cdot f \cdot t_n) + 0.5 \cdot \cos(2 \cdot \pi \cdot f \cdot t_n) + (\text{mod}(0.3) - 0.1) \\ 0.3 \cdot \sin(2 \cdot \pi \cdot f \cdot t_n) + 0.7 \cdot \cos(2 \cdot \pi \cdot f \cdot t_n) + (\text{mod}(0.5) - 0.3) \end{bmatrix}$	<p>Vector of interfering signals</p>
$\begin{pmatrix} x1_{n+1} \\ x2_{n+1} \\ x3_{n+1} \end{pmatrix} := \begin{pmatrix} x1_n \\ x2_n \\ x3_n \end{pmatrix} + \mathbf{A} \cdot \begin{pmatrix} x1_n \\ x2_n \\ x3_n \end{pmatrix} \cdot \Delta t + \mathbf{B} \cdot \begin{pmatrix} E \\ E \end{pmatrix} \cdot \Delta t + \mathbf{H} \cdot \begin{pmatrix} z1_n \\ z2_n \\ z3_n \end{pmatrix} \cdot \Delta t$	<p>The equation of state (7) in the numeric notation</p>
$\begin{pmatrix} x1_0 \\ x2_0 \\ x3_0 \end{pmatrix} := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} y1_n \\ y2_n \\ y3_n \end{pmatrix} := \mathbf{C} \cdot \begin{pmatrix} x1_n \\ x2_n \\ x3_n \end{pmatrix} + \mathbf{E} \cdot \begin{pmatrix} z1_n \\ z2_n \\ z3_n \end{pmatrix}$	<p>The initial conditions for $t = 0$ The equation of the output signals (8) in the numeric notation</p>

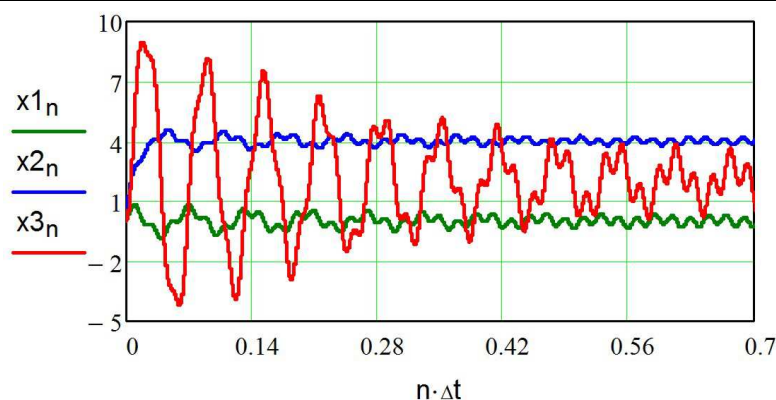


Fig. 8. Simulation of the output signals of the automation system created by the electrical circuit with the impact of interference signals on the input and output of the program *MathCAD*

4. Conclusions

- Using the electrical circuits can be modeled automation systems.
- Automation systems described by some differential equations of the first order can be analyzed by the difference equations in terms of calculation procedures of the program *MathCAD*.
- Using the calculation procedures of the program *MathCAD* can be made dynamic visualization of automation systems including interference signals.

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