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ELECTROMAGNETIC TORQUE RIPPLE WAVEFORM CALCULATION OF A FIVE-PHASE PENTACLE CONNECTED IM UNDER ONE PHASE FAILURE

OBLICZENIA PRZEBIEGÓW TĘTNIEŃ MOMENTU ELEKTROMAGNETYCZNEGO W SILNIKU INDUKCYJNYM Z PIĘCIOFAZOWYM UZWOJENIEM W UKŁADZIE PENTAGRAMU, W WARUNKACH USZKODZENIA JEDNEJ Z FAZ

Abstract: The presented paper deals with the five-phase induction motor (IM) with pentacle connected stator winging, which is working under one phase supply failure. A computation of the motor electromagnetic quantities is made using the space vector theory in the complex plane. Assuming, the motor is supplied by a PWM controlled inverter high modulation frequency, only the first stator voltage harmonics are taken in consideration. On the base of the measured parameters of the IM the trajectories of the stator and rotor current space vectors were investigated. Of these, the motor electromagnetic torque ripple waveform for four-phase supply working mode is derived.

Streszczenie: Przedstawiony artykuł dotyczy pięciofazowego silnika indukcyjnego (IM) z połączonym pentaklem skrzydłem stojana, który działa w przypadku awarii zasilania jednofazowego. Obliczanie wielkości elektromagnetycznych silnika odbywa się za pomocą teorii wektora przestrzeni w płaszczyźnie zespolonej. Zakładając, że silnik jest zasilany przez wysoką częstotliwość modulacji sterowaną PWM, brane są pod uwagę tylko pierwsze harmoniczne napięcia stojana. Na podstawie zmierzonych parametrów IM badano trajektorie wektorów przestrzennych prądu stojana i wirnika. Spośród nich wyprowadzono przebieg tętnienia momentu elektromagnetycznego silnika dla trybu pracy z zasilaniem czterofazowym.

Keywords: *induction motor, torque ripple, space phasor, one phase failure, complex plane*
Słowa kluczowe: *silnik indukcyjny, tętnienie momentu obrotowego, wskaznik przestrzenny, awaria jednej fazy, płaszczyzna zespolona*

1. Introduction

Five-phase asynchronous motors have been recently used in various applications of drives technology. Compared to a three-phase asynchronous motors have several advantages. They have a lower phase current for the same output motor power. This is very advantageous for the high power asynchronous motor in terms of semiconductor supply device. Five phase motors have a smoother run, comparing a three-phase one. This makes it an ideal drives for a residential home elevators. Their great advantage is, that they are able continue to work even in one phase supply failure. This failure often occurs with frequency-controlled drives at the current overload of one of the semiconductor components of the inverter. This feature can be a great advantage for the elevator drive of the high buildings or hospitals, where the shutdown of the elevator can cause considerable problems.

Stator winding of the five phase motors can be connected in three ways:

- star connection,
- pentagon connection,
- pentacle (pentagram) connection.

Each of the connections has its advantages and disadvantages. In the event of one phase supply failure, it is most advantageous a star connection. On the other side, pentacle connection gives the highest phase voltage, for the same DC inverter input supply.

Supply of the five phase induction motors is in the majority cases provided by a voltage source inverter (VSI). A number of pulse width modulation techniques (PWM) are available to control VSI output voltage. The space vector PWM control technique become the most popular because of the easy of digital implementation and better DC utilization compared to the ramp comparison sinusoidal PWM method.

2. Five-phase VSI modeling

The five-phase motors needs for their operation a semiconductor converter, which is able to create a five phase supply voltages. Power circuit topology of a five-phase inverter is shown in Fig.1. Inverter consists of a five transistor legs.

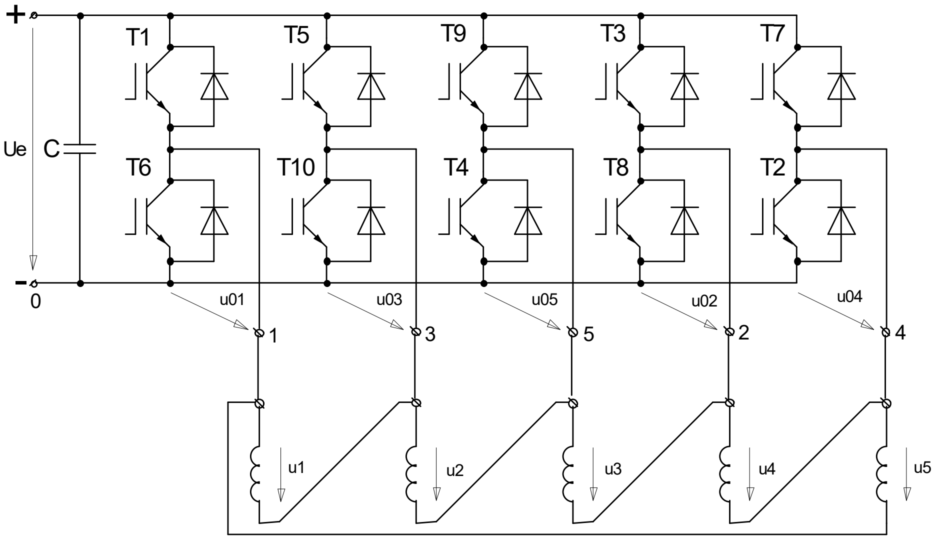

Fig.1. Supply inverter of the pentacle connected five phase induction machine

Each of the invertor legs consists of the two IGBT transistors with anti-parallel connected free-wheeling diodes used to ensure a negative current path. The inverters output terminals are numbered (1, 2, …,5).

Assume that the inverters DC supply voltage stay constant for any inverter supply currents.

Next, suppose that inverters output voltage is controlled by a PWM with very high modulation frequency (f_m >>10 kHz). On the base of this assumption we can suppose the inverter output voltages to be ideal sinusoidal.

The transistors of the first inverters leg (voltage between a transistors node and negative rail of the DC supply voltage) is controlled to obtain output voltage of the form

$$\mathbf{u}_{01} = \frac{U_e}{2} + r\frac{U_e}{2} cos\,\omega t = \frac{U_e}{2} + r\frac{U_e}{2} e^{j\omega t} \tag{1}$$

where, r - is a voltage control coefficient $r \in \langle 0,1 \rangle$, $\omega = 2\pi f$ - is angular frequency;

The voltages of the other legs are shifted by shifting factor $\mathbf{a} = e^{j\frac{2\pi}{5}}$.

$$\begin{aligned} &\mathbf{u}_{02} = \mathbf{a}\,\mathbf{u}_{01}; \qquad &&\mathbf{u}_{03} = \mathbf{a}^2\,\mathbf{u}_{01}; \\ &\mathbf{u}_{04} = \mathbf{a}^3\,\mathbf{u}_{01}; \qquad &&\mathbf{u}_{05} = \mathbf{a}^4\,\mathbf{u}_{01}; \end{aligned} \tag{2}$$

The motor phase voltages are given by a difference between two legs voltages as seen in Fig.1.

$$\begin{aligned} \mathbf{u}_1 &= \mathbf{u}_{01} - \mathbf{u}_{03} = \left(1 - \mathbf{a}^2\right) r\frac{U_e}{2} e^{j\omega t} \\ \mathbf{u}_2 &= \mathbf{u}_{03} - \mathbf{u}_{05} = \left(\mathbf{a}^2 - \mathbf{a}^4\right) r\frac{U_e}{2} e^{j\omega t} \\ \mathbf{u}_3 &= \mathbf{u}_{05} - \mathbf{u}_{02} = \left(\mathbf{a}^4 - \mathbf{a}\right) r\frac{U_e}{2} e^{j\omega t} \\ \mathbf{u}_4 &= \mathbf{u}_{02} - \mathbf{u}_{04} = \left(\mathbf{a} - \mathbf{a}^3\right) r\frac{U_e}{2} e^{j\omega t} \\ \mathbf{u}_5 &= \mathbf{u}_{04} - \mathbf{u}_{01} = \left(\mathbf{a}^3 - 1\right) r\frac{U_e}{2} e^{j\omega t} \end{aligned} \tag{3}$$

Fig.2 depicts the phase voltages formed in pentacle stator winding connection.

To simplify the calculation of the AC motor quantities, it is very advantageously to employ space vectors. The space vector very simplified the analyzing multi-phase electric systems.

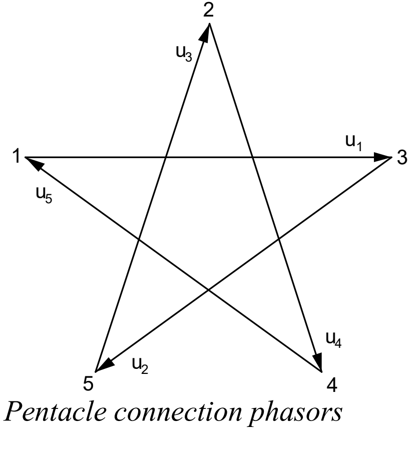

Fig.2. Pentacle connection phasors

Space vector of the five phase pentacle connected voltage system is defined by the relationship

$$\underline{\mathbf{u}}_s = \frac{2}{5}\Big[Re(\mathbf{u}_1) + \mathbf{a}_1\, Re(\mathbf{u}_2) + \mathbf{a}_1^2\, Re(\mathbf{u}_3) + K \quad \mathbf{a}_1^3\, Re(\mathbf{u}_4) + \mathbf{a}_1^4\, Re(\mathbf{u}_5)\Big] \qquad (4)$$

taking $\mathbf{a}_1 = e^{j\frac{4\pi}{5}}$ is a space shifting factor.

The pentacle connection forms the voltages of the second order.

Fig.3. show the space vector trajectory of a five phase voltage system. The trajectory was calculated for inverter DC input voltage $U_e = 350V$.

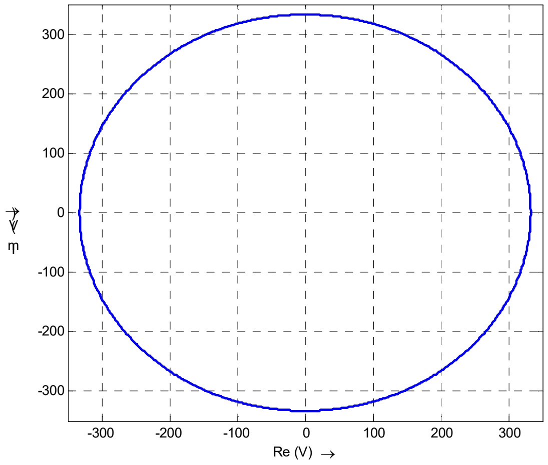

Fig.3. Voltage space vector trajectory

3. Current space vector calculation

For the stator and rotor current space vector count, there is advantageously used a classical equivalent circuit of induction machine shown in Fig.4.

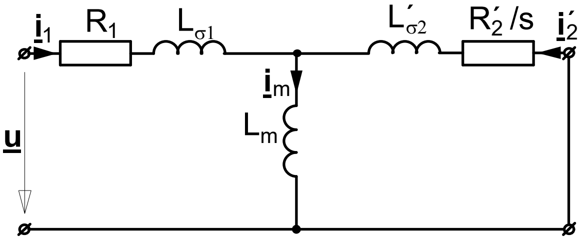

Fig.4. Equivalent circuit of IM

For next calculation will be used the following measured parameters of a five phase two-pole IM.

$P_n = 3kW; U_n = 5\times 230V / 50Hz;$
$n_n = 2910 ot / min;\ p = 1;$
$R_1 = 3{,}778\Omega;\ R'_2 = 2{,}498\Omega;$
$L_m = 0{,}436H;\ L_{1\sigma} = 6{,}83mH;\ L'_{2\sigma} = 11{,}88mH;$

From the equivalent circuit (Fig.4), for the stator current space vector following equation is valid

$$\underline{\mathbf{i}}_1 = \frac{\underline{\mathbf{u}}}{R_1 + \left[j\omega L_{1\sigma} + \dfrac{j\omega L_m\left(R'_2/s + j\omega L'_{2\sigma}\right)}{R'_2/s_2 + j\omega\left(L_m + L'_{2\sigma}\right)} \right]} \qquad (5)$$

where, $s = \dfrac{\omega - \omega_m}{\omega}$ is a slip of IM

The rotor current space vector can by determined using the Thevenin theorem. Fig.5 depicts Thevenin simplified equivalent circuit of IM.

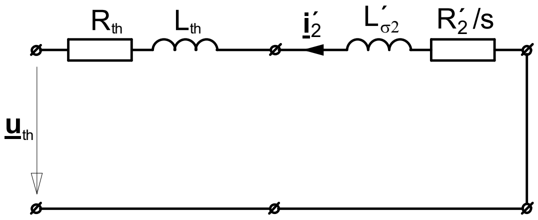

Fig. 5. Thevenin equivalent circuit of IM

For the Thevenin voltages follow

$$U_{th} = U_m \frac{\omega L_m}{\sqrt{R_1^2 + \omega\left(L_{1\sigma} + L_m\right)^2}} \qquad (6)$$

Having L_m ? $L_{1\sigma}$ and $\omega\left(L_m + L_{1\sigma}\right)$? R_1, then

$$R_{th} \approx R_1 \frac{L_m}{L_m + L_{1\sigma}}; \quad L_{th} \approx L_{1\sigma}$$

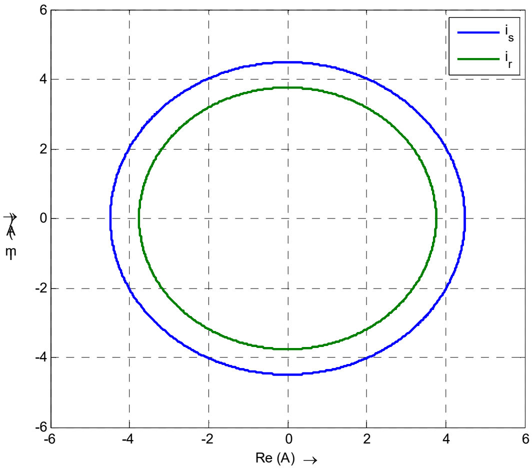

Fig.6. Stator and rotor current space vector trajectories

Then for rotor current space vector resulting equation

$$\underline{\mathbf{i}}_2 = -\frac{k_{th}\underline{\mathbf{u}}}{R_2'/s + R_{th} + j\omega\left(L_{th} + L_{2\sigma}'\right)} \tag{7}$$

Fig.6 shows the stator and rotor current space vector trajectories. The trajectories were calculated on the base of equations (6) and (7). Calculation was made for supply frequency of $f = 50\,Hz$ and rotor speed $n = 2910\,rev/min$ ($s = 0{,}03$).

Electromagnetic torque can be calculated on the base of following equation

$$M_{em} = \frac{5}{2} p L_m \, Im\left(\underline{\mathbf{i}}_1 \underline{\mathbf{i}}_2^*\right) \tag{8}$$

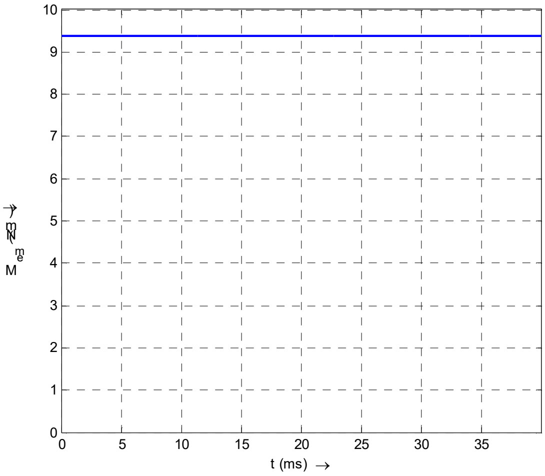

Fig. 7. Electromagnetic torque waveform

4. One phase failure operation

Assume failure of the 2-th phase (T3 and T8). In this case on the phases „3” and „4” we have voltage witch apparent between terminals 5-4, as depicts on the Fig.8.

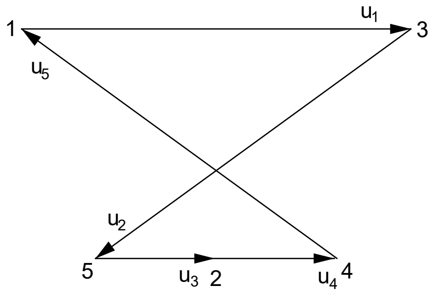

Fig. 8. Failed pentacle phasors

Phasor of the voltage between terminals 5-4 can by calculated as a different between leg voltages $\mathbf{u}_{04}$ and $\mathbf{u}_{05}$ as follow.

$$\mathbf{u}_{54} = \mathbf{u}_{05} - \mathbf{u}_{04} = \left(\mathbf{a}^4 - \mathbf{a}^3\right)\mathbf{u}_{01} \tag{9}$$

Assuming symmetry of the windings of each phase, that voltage is evenly distributed between the phases „3” and „4”

$$\mathbf{u}_3 = \mathbf{u}_4 = \frac{1}{2}\left(\mathbf{a}^4 - \mathbf{a}^3\right) r U_e e^{j\omega t} \tag{10}$$

On the base of equation (4) we can calculate once again the voltage space vector.

Fig. 9 depicts the trajectory of the voltage space vector of four phase operation. Trajectory has an elliptical form.

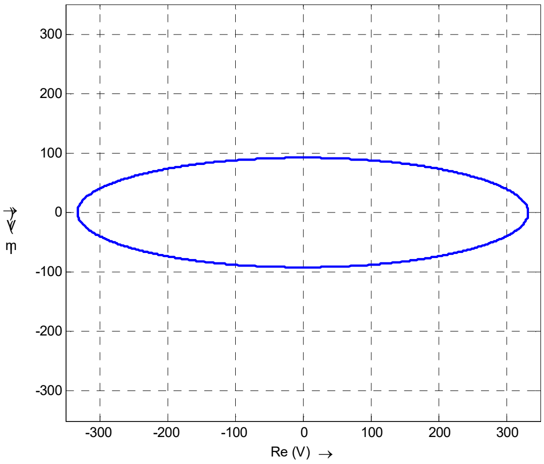

Fig. 8. Voltage space vector trajectory.

On the base of equation (5) and (7) the trajectories of the currents space vector were calculated. Fig. 9 shows calculated stator and rotor current space vector trajectories. Calculation was made for supply frequency of $f = 50\,Hz$ and rotor speed $n = 2886\,rev/min$ ($s = 0{,}038$).

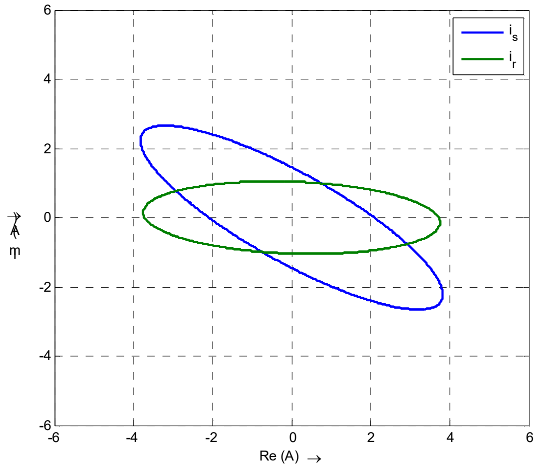

Fig. 9. Stator and rotor current space vector trajectories

Electromagnetic torque was calculated on the base of equation (8). Calculated waveform is depicted in Fig.10. The electromagnetic waveform is strong pulse with second harmonic of the supply voltages. For constant loading torque, the speed of the motor has decreased by around $n = 100\, rev / min$, in comparison to of fault state.

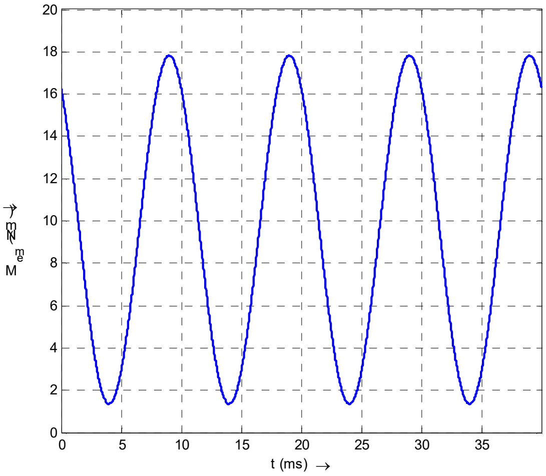

Fig. 10. Electromagnetic torque waveform

Fig.11 shows the stator and rotor current space vector trajectories for practically no loaded motor $(s = 0{,}001)$. From the picture it is clear that the power factor has worsened significantly.

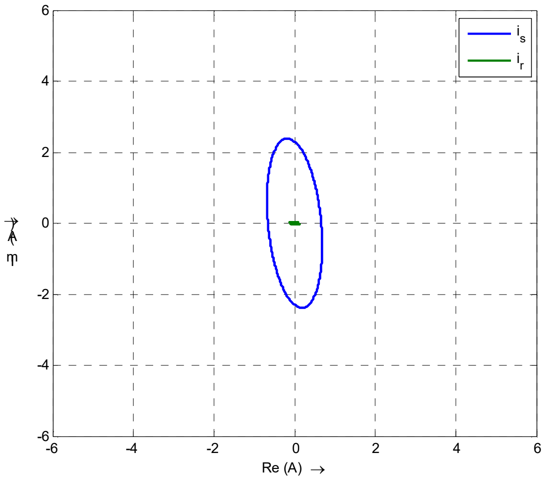

Fig. 11. Stator and rotor current space vector trajectories

In the Fig.12 is shown the electromagnetic torque waveform. The torque is very small, but again strongly rippled. The speed of the motor has increased to near synchronous speed $(n = 2997\, rev / min)$.

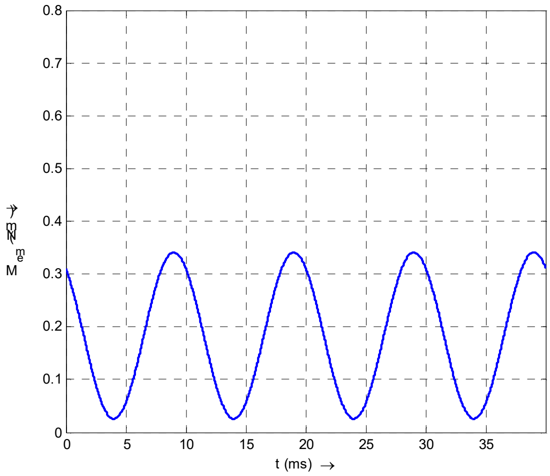

Fig.12. Electromagnetic torque waveform

5. Conclusion

The paper presents characteristics of a five phase induction motors with stator winding connected to the pentagram. Behavior of the operation of one supply phase is shown.

The motor electromagnetic torque is strongly rippled at a frequency equal to twice the supply frequency.

Disadvantage of the pentacle connection compare to the star one is that it is not possible to improve the torque ripple by a shifting of no failed phases to obtain again circular relative electromagnetic field.

Acknowledgment

The financial support of the Slovak Research and Development Agency under the contract N^0: APVV-16-0270 is acknowledged.

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