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OPTIMIZATION METHODS IN MARITIME TRANSPORT AND LOGISTICS

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ABSTRACT

The purpose of the article is to present the goal of optimization of transport and logistics processes, followed by literature review in the field of optimization methods. The optimization methods were categorized and the most commonly used methods were listed. The tasks of static and dynamic optimization were formulated. An example of the single-criterion static and dynamic optimization and multi-criteria game optimization are given.

Keywords: maritime transport, optimization, ship control, computer simulation

DEVELOPMENT OF OPTIMIZATION METHODS - HISTORICAL OUTLINE

The primary goal of optimization is to implement the object control process in the best way. The process may be: physical phenomenon, technological process, technical object, economic system, production and transport planning, etc [1,4]. The mathematical description of the process formulated for the purpose of its optimization is by modelling. The optimization is as good as the mathematical model is adequate [3,9]. Formulating and solving the optimization task can be presented as in Fig. 1.

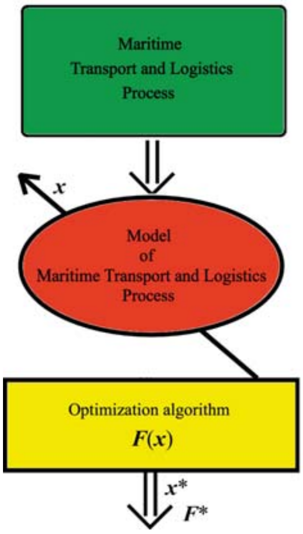

Fig. 1. Stages of formulating and solving the optimization task

The function $\boldsymbol{F}(\boldsymbol{x})$ means the evaluation of the quality of the object's operation or the course of the control process and assumes the name of the goal control function or control quality index, and $\boldsymbol{x}$ constitutes a set of the decision variables or state variables of the control process [7,10,13].

In many issues of transport and logistics there are many possible and acceptable solutions to the problem, of which only one is the optimal solution under the assumed criterion of the quality of the transport or logistics process [14,15,16].

Both static optimization and dynamic optimization methods are used to solve such issues. Increasingly, the practice of transport and logistics processes must meet both technical and economic criteria. Therefore, apart from the single-criterion optimization, multi-criteria optimization becomes more and more important.

And so, the beginning of the variational calculus is presented in the works of Lagrange (1736-1813), Hamilton (1805-1865), Weierstrass (1815-1897) and Pontriagin (1908-1988).

Modern methods of optimization dated beginning from 1939 are: logistics problems related to planning operations during World War II - linear programming (Dantzig (1914-2005)); integer programming and selection from among a finite number of decisions (Cabot (1922-1984), Balas (1922)); non-linear programming (Kuhn, Tucker and Georffrion).

The development of methodology of computer calculating has caused interest in numerical algorithms (Powell, Rossen, Fletcher) and dynamic programming (Bellman, Riccati).

Space research has focused on the optimization of rocket constructions and flight control in the stratosphere and space.

Optimization of economic processes includes: problems of production allocation, optimal composition of the investment portfolio, large scale problems and decomposition methods (Lasdon, Findeisen).

The development of methods for solving optimization tasks took place in the following stages:
- analytical classic methods, or methods of "mountain climbing": models developed by mathematicians of the 17th and 19th centuries, the "unpolluted" world of square functions of the target and ubiquitous derivatives,
- development of computer calculations: modifications of classic methods, algorithmization of calculations enabling application to practical problems of science and technology,
- soft computing, resistant methods: evolutionary, genetic algorithms, neural networks used to optimize complex process models [12].

In transport and logistics, the best possible control of an object is expressed in optimization dealing with how to describe and achieve the best, when we already know how to measure and change good and bad (Beightler, Phillips: *Foundations of Optimization*, 1979).

The general categorization of optimization methods considered the most representative, is shown in Fig. 2.

In practice, the following methods are most commonly used:
- non-gradient static optimization without constraints: golden division, bisection, Gauss-Seidel, division and constraints, division and isolation, Hooke-Jeeves, square interpolation, Nelder-Mead symplex, Rosenbrock, Davies-Swann-Campey;
- gradient static optimization without constraints: simple gradient, the fastest slope, Newton-Raphson, conjugate gradient Hestenes-Stiefel, Levenberg-Marquardt, Powell, Zangwill;
- non-gradient static optimization with constraints: Lagrange, linear programming, Kuhn-Tucker, Schmidt-Fox;
- gradient static optimization with constraints: Zoutendijk, Raster projected gradient;
- heuristics: grouping, Monte Carlo, simulated annealing, genetic algorithms, particle swarm;
- basic direct dynamic optimization: Euler's calculus, Bellman's principle of optimality, simple gradient in control space, conjugate gradient in control space, variable metrics, second variation;

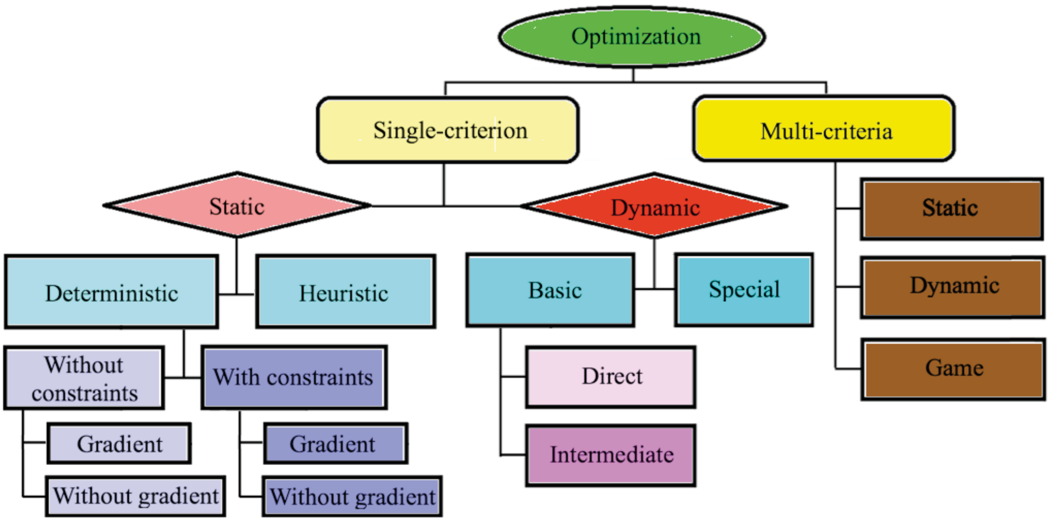

Fig. 2. Categorization of optimization methods

The optimization methods can be divided in respect of:
- object or process properties: into static and dynamic;
- constraints: into without constraints and with constraints;
- way of calculating the optimum: into gradient and non-gradient;
- type of object or process model: into deterministic and stochastic;
- type of calculations: into analytical and numerical;
- form of the goal function: into linear and non-linear;
- complexity of the goal function: into single-criterion and multi-criteria.

- basic intermediate dynamic optimization: maximum principle of Pontriagin, Newton in the state space, Newton-Rapson in the conjugate space;
- special dynamic optimization: time-optimal control of Neustadt, Gilbert, Barr, Balakrishnan punishment function, Findeisen's two-level optimization;
- static multi-criteria optimization: set of optimal Pareto points in the space of variants, Bentham's rule of utilitarianism, Rawls principle of justice, Salukvadze reference point, Benson weighted sum method, Haimes ε-constraints method, purposeful programming method;
- dynamic multi-criteria optimization: selection of weight coefficients;
- game multi-criteria optimization: multi-stage positional game, multi-step matrix game [9].

PROBLEMS AND METHODS OF OPTIMIZATION IN TRANSPORT AND LOGISTICS

STATIC OPTIMIZATION

The optimization task consists in determining such values of state variables $\boldsymbol{x}^*$ at which the function of the control goal $\boldsymbol{F}(\boldsymbol{x})$ reaches its minimum or maximum value.

The values of constituents of the state vector $\boldsymbol{x}$ cannot be arbitrary and are subject to various constraints. A distinction is made between inequalities:

$$\boldsymbol{g}_i(\boldsymbol{x}) \leq 0 \quad \text{for} \quad i=1, 2, \ldots, M \tag{1}$$

and equality constraints:

$$\boldsymbol{h}_p(\boldsymbol{x})=0 \quad \text{for} \quad p=1, 2, \ldots, P \tag{2}$$

The introduction of any equality constraint reduces the size of the optimization space by one and may be the reason for the lack of an optimal solution.

Single-criterion static optimization

The task of single-criterion static optimization is to search for a minimum or maximum of the objects output or its function:

$$\boldsymbol{F}(\boldsymbol{x})=\boldsymbol{f}(\boldsymbol{x}) \quad \text{for} \quad \boldsymbol{x}=x_1, x_2, \ldots, x_n \tag{3}$$

while meeting the constraints imposed on the variables $\boldsymbol{x}$.

Examples of single-criterion static optimization tasks in sea transport and logistics:

- optimization of the product range in the yard of commercial and fishing ships and yachts;
- optimization of rational cutting of sheets in the ship's construction process;
- optimization of transport process of containers, cars, citrus fruits and other loads;
- optimization of logistics of cargo transportation between the port and the recipients;
- optimization of the quantity and type of port equipment for handling ship transhipments;
- investment optimization of the construction or expansion of the port.

Multi-criteria static optimization

The task of multi-criteria static optimization is determining the optimal decision when there is more than one optimization criterion:

$$\boldsymbol{F}(\boldsymbol{x})= \text{for} \quad k=1, 2, \ldots, K \tag{4}$$

while meeting the constraints imposed on the variables $\boldsymbol{x}$.

Examples of multi-criteria static optimization tasks in sea transport and logistics:

- optimization of transport means works with minimal cost and delivery time;
- maximum use of cargo space with a minimum delivery time.

DYNAMIC OPTIMIZATION

Single-criterion dynamic optimization

The task of single-criterion dynamic optimization is to look for the minimum or maximum of a functional as the integral of a function:

$$\boldsymbol{F}(\boldsymbol{x}) = \int_{t_o}^{t_k} f_O(\boldsymbol{x}, \boldsymbol{u}, t)dt \tag{5}$$

where the dynamic properties of the control object are described by the equations of state and output:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) \tag{6}$$

$$\boldsymbol{y}=\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}, t) \tag{7}$$

and meeting constraints imposed on the state variables $\boldsymbol{x}$ and control variables $\boldsymbol{u}$.

The task of dynamic optimization can be solved analytically as a task of time-optimal control and minimization of the goal function in a square form, with linear state equations.

Examples of single-criterion dynamic optimization tasks in maritime transport:

- determination of the optimal route of the ship from the initial port to the port of destination, ensuring minimum fuel consumption, including navigational limits and hydro-meteorological forecasts;
- determination of the optimal anti-collision manoeuvre of own ship ensuring minimum risk of collision during passing the encountered ships;
- optimization of the main engine control of the ship, ensuring minimum fuel consumption;
- optimization of ship loading, ensuring maximum ship stability;
- optimization of power distribution between the ship propulsors, ensuring the maximum controllability of the ship;
- optimization of the ship's electrical system, ensuring maximum reliability of power supply for ship's equipment.

Multi-criteria dynamic optimization

The task of multi-criteria dynamic optimization is to look for the minimum or maximum of a functional as the integral of a function:

$$\boldsymbol{F}(\boldsymbol{x}) = \int_{t_o}^{t_k} \sum_{k=1}^{K} f_{Ok}(\boldsymbol{x}, \boldsymbol{u}, t)dt; \quad for \quad k=1, 2, \ldots, K \tag{8}$$

Examples of multi-criteria dynamic optimization tasks in maritime transport:

- optimization of sea transport process, ensuring maximum profit with a minimum risk;
- optimal control of the ship on a reference course, ensuring maximum control accuracy and minimum fuel consumption;
- safe traffic management ensuring minimum risk of collision and minimum road loss on passing objects.

GAME OPTIMIZATION

The game control of the marine object consists in minimizing goal function given in the form of an integral payment and final payment:

$$\boldsymbol{F}(\boldsymbol{x}) = \int_{t_o}^{t_k} f_O(\boldsymbol{x},\boldsymbol{u},t)dt + r_f(t_k) + d_f(t_k) \qquad (9)$$

The integral payment of the game determines loss of path of the own object on the passing of cooperating or non-operating objects that were met.

The final payment of the game determines the final risk of collision r_f and the final deviation of own object position d_f from its reference trajectory of movement [2,8,11].

A distinction is made between the following types of game control of a maritime transport object as follows:

- multi-stage positional, non-cooperative or cooperative game;
- multi-step matrix, non-cooperative or cooperative game.

EXAMPLES OF OPTIMIZATION TASKS IN MARITIME TRANSPORT AND LOGISTICS

OPTIMIZATION OF CONTAINER TRANSPORT BY MEANS OF LINEAR PROGRAMMING

The ship owner has five container ships: K1, K2, K3 and K4 with capacity of 2600, 4200, 2100, 1100 TEU containers, respectively. It is necessary to plan the transport of 10 000 containers from Asia to five European ports: P1, P2, P3, P4 and P5 in quantities of 1800, 2100, 3100, 1800, 1100 TEU, respectively, with the lowest total cost of transport from Asia to Europe. Tab. 1 shows the cost of transporting one TEU container.

Tab. 1. Data summary for the task of container transport optimization

Ships / Ports	 K1	 K2	 K3	 K4	Number of containers expected in port
P1 Lisbon	500 USD	450 USD	640 USD	620 USD	1800 TEU
P2 Le Havre	600 USD	540 USD	660 USD	690 USD	2100 TEU

Ships / Ports	 K1	 K2	 K3	 K4	Number of containers expected in port
P3 Bremerhaven	700 USD	610 USD	710 USD	730 USD	3100 TEU
P4 Gdańsk	740 USD	735 USD	870 USD	810 USD	1800 TEU
P5 Sankt Petersburg	900 USD	890 USD	960 USD	930 USD	1200 TEU
Load capacity of ship	2600 TEU	4200 TEU	2100 TEU	1100 TEU	

The standard and simultaneously canonical form of linear programming which it will take, is as follows:

$$F(\boldsymbol{x})=500x_{11}+450x_{12}+640x_{13}+620x_{14}+600x_{21}+540x_{22}+660x_{23}+690x_{24}+700x_{31}+610x_{32}+710x_{33}+730x_{34}+740x_{41}+735x_{42}+870x_{43}+810x_{44}+900x_{51}+890x_{52}+960x_{53}+930x_{54}\rightarrow \min \qquad (10)$$

$$\begin{aligned}
x_{11}+x_{12}+x_{13}+x_{14}&=1800\\
x_{21}+x_{22}+x_{23}+x_{24}&=2100\\
x_{31}+x_{32}+x_{33}+x_{34}&=3100\\
x_{41}+x_{42}+x_{43}+x_{44}&=1800\\
x_{51}+x_{52}+x_{53}+x_{54}&=1200\\
x_{11}+x_{21}+x_{31}+x_{41}+x_{51}&=2600\\
x_{12}+x_{22}+x_{32}+x_{42}+x_{52}&=4200\\
x_{13}+x_{23}+x_{33}+x_{43}+x_{53}&=2100\\
x_{14}+x_{24}+x_{34}+x_{44}+x_{54}&=1100
\end{aligned} \qquad (11)$$

$$(x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34}, x_{41}, x_{42}, x_{43}, x_{44}, x_{51}, x_{52}, x_{53}, x_{54}) \geq 0 \qquad (12)$$

The ***linprog*** function from MATLAB software has been used: [x, fval] = **linprog** (f, A, b, Aeq, beq, lb, ub), as a result of its operation there will be obtained a vector ***x*** with the solution and the containers transporting cost under the variable ***fval*** (Fig. 3).

The following results of container transport optimization in the MATLAB ***linprog*** software are obtained:

$$\begin{aligned}
&x_{11}^* = 0,\ x_{12}^* = 0,\ x_{13}^* = 1800,\ x_{14}^* = 0\\
&x_{21}^* = 0,\ x_{22}^* = 1000,\ x_{23}^* = 0,\ x_{24}^* = 1100\\
&x_{31}^* = 2600,\ x_{32}^* = 500,\ x_{33}^* = 0,\ x_{34}^* = 0\\
&x_{41}^* = 0,\ x_{42}^* = 1500,\ x_{43}^* = 300,\ x_{44}^* = 0\\
&x_{51}^* = 0,\ x_{52}^* = 1200,\ x_{53}^* = 0,\ x_{54}^* = 0\\
&F^*(x) = 7\ 007\ 500\ USD
\end{aligned} \qquad (13)$$

Tab. 2 shows the optimal loading of container ships.

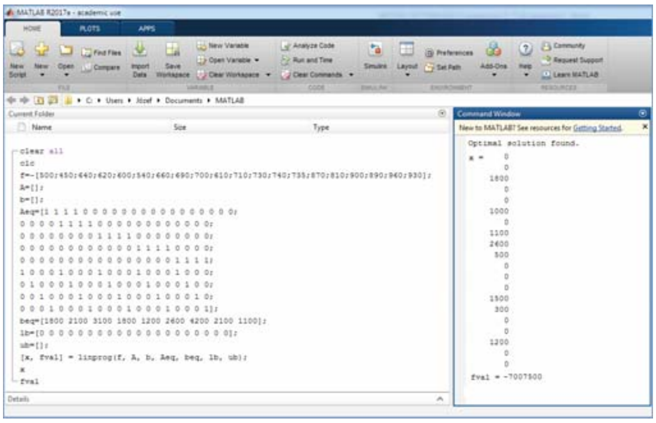

Fig. 3. Results of container transport optimization

Tab. 1. Optimal loading of container ships

Ships / Ports	 K1	 K2	 K3	 K4	Number of containers delivered in port
P1 Lisbon			1800 TEU		1800 TEU
P2 Le Havre		1000 TEU		1100 TEU	2100 TEU
P3 Bremerhaven	2600 TEU	500 TEU			3100 TEU
P4 Gdansk		1500 TEU	300 TEU		1800 TEU
P5 Sankt Petersburg		1200 TEU			1200 TEU
Loading the ship	2600 TEU	4200 TEU	2100 TEU	1100 TEU	

OPTIMIZATION OF SAFE SHIP CONTROL BY MEANS OF SWARM OF PARTICLES

A swarm of particles or a flock is a group of individuals of the same species, rarely different species of animals, insects, birds and fish living in a specific territory, related to each other due to more or less advanced fora of social organization. Mating of individuals into flocks is most often associated with breeding or searching for food.

The Particle Swarm Optimization – PSO was proposed in 1995 by Kennedy and Eberhart. The idea of the algorithm comes from imitating the behaviour of a population of living creatures - birds, ants, bees, fireflies, bats, cockroaches, krill, etc, in which a single individual has a very limited ability to make decisions and mutual communication (Fig. 4).

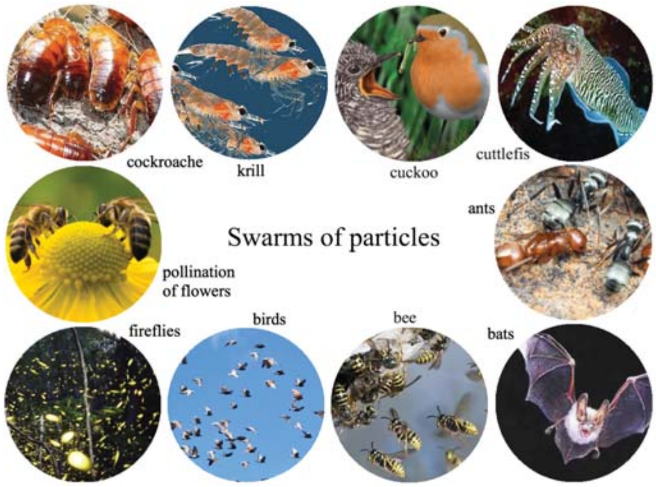

Fig. 4. Examples of individuals forming swarms of particles

The whole population, despite the lack of a central control system, demonstrates the features of having intelligence, that is, the responding to changes in the environment and the collective undertaking of related actions.

The numerical model of the behaviour of a group of objects treats the population as a swarm and each individual as a particle. During the next steps of the discretized time, the particles move to new positions, simulating the adoption of the swarm to the environment, i.e. they are looking for the optimum. The algorithm uses to search for the extreme value of the adaptive function as a function of the control target for a population of moving particles that can memorize the point of the best value of the objective function in the search space and transmit this information to whole population or its part.

Bird's algorithm – Particle Swarm Optimization PSO, presented in 1995 by R.C. Eberhart and J. Kennedy, imitates the gregarious behaviour of birds that communicate and observe each other exchanging information among themselves, improving the search of the area as a space for optimal solutions.

Base Bees algorithm – BBA mimics of feeding of honeybee swarms, developed by D.T. Pham in 2005.

Firefly algorithm – FA, developed in 2008 by Prof. Xin-She Yang, is inspired by the social behaviour of skylights, insects from the Lampyridae family, whose phenomenon is bioluminescent communication.

Cuckoo Search – CS algorithm uses cuckoo nesting habits, proposed in 2009 by Xin-She Yang and Suash Deb, mimicking the behaviour of some cuckoo species that use the nest of other birds to hatch eggs and raise their chicks.

Cockroach Swarm Optimization – CSO algorithm uses three cockroach behaviours as insects: swarming, dispersing and absolute behaviour, described by L. Cheng, Z.B. Wang, Y.H. Song and A.H. Guo in 2011.

Flower Pollination – FPA algorithm, inspired by the process of pollination of flowering plants, was developed by Xin-She Yang in 2012.

Cuttlefish algorithm* – *CFA algorithm, proposed by A.S. Eesa, Z. Orman and A.M.A. Brifcani in 2013, is inspired by the environmental change of cuttlefish skin colour.

Krill Herd* – *KH algorithm, described in 2012 by A.H. Gandami and A.H. Alavi, is based on the simulation of behaviour of herds of krill individuals.

Ant Colony Optimization* – *ACO algorithm, proposed in 1999 by Marco Dorigo, is a probabilistic technique for solving problems by looking for good roads in graphs, inspired by the behaviour of ants looking for food for their colony.

The use of ACO algorithm to determine the safe trajectory of a ship in a collision situation has been developed by A. Lazarowska [6].

Calculations of the safe trajectory of the own ship by using the form-based algorithm consist of three main stages which include:

- data initialization;
- constructing solutions;
- updating pheromone traces.

The ship route from the starting point wp_0 to the end point wp_e is divided into k stages. Ships are represented by hexagonal domains, which cannot be crossed by a respective own ship (Fig. 5).

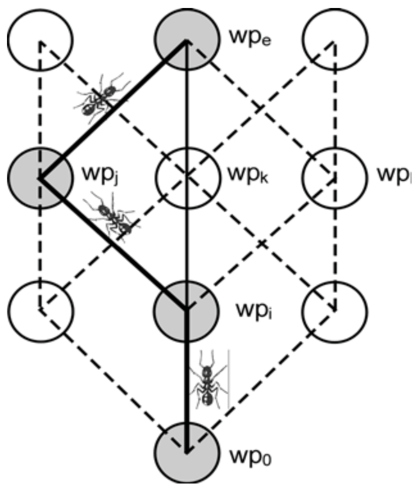

Fig. 5. An example of a route chosen by an ant [35]

The probability of choosing the next top by the ant is:

$$p_{wp_{ij}}(t)=\frac{\left[\tau_{wp_j}(t)\right]^{\alpha}\left(\eta_{wp_{ij}}\right)^{\beta}}{\sum_{l\in wp_i}\left[\tau_{wp_l}(t)\right]^{\alpha}\left(\eta_{wp_{il}}\right)^{\beta}} \qquad (14)$$

where:

t_{wpj} (t) – values of the pheromone trace at the apex j,
h_{wpij} – some heuristic information, called visibility, the inverse of the distance between the current vertex i and the neighbouring vertex j,
l – ant number,
α, β – coefficients of algorithm convergence.

Fig.6 shows an example of determining the safe trajectory of an own ship by using a ACO algorithm.

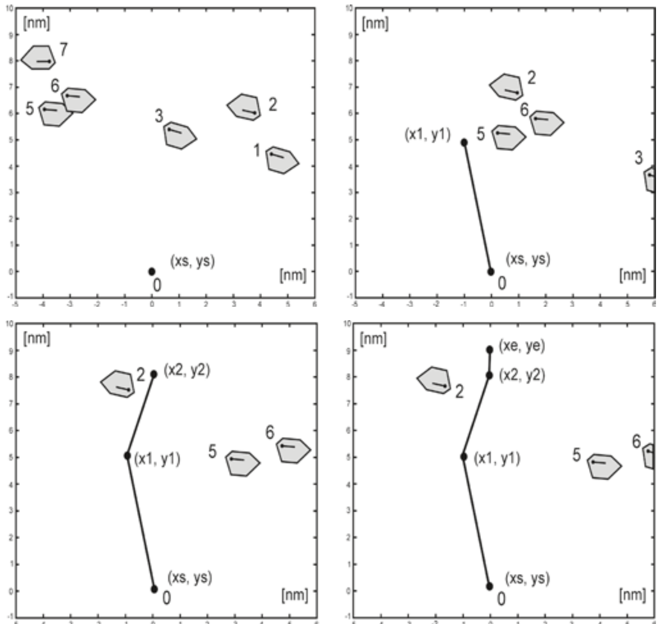

Fig. 6. Solution to the situation of meeting with 7 ships in the Kattegat Strait [35]

OPTIMIZATION OF SAFE SHIP CONTROL BY MEANS OF DYNAMIC PROGRAMMING

Determining the optimal control of the ship in the sense of a fixed control quality indicator can be made by using Bellman's principle of optimality. The principle defines the basic possession of an optimal strategy – regardless of the status and initial decisions, the remaining decisions must form strategies that are optimal from the point of view of the state resulting from the first decision. The principle of optimality is described by the Bellman functional equation:

$$\frac{\partial S}{\partial t}-\min_{u}\left[f_o(x,u,t)+f(x,u,t)\frac{\partial S}{\partial x}\right]=0 \qquad (15)$$

where: x – state of process, u – control, t – time, f_o – cost function, f – process state function.

Function S is:

$$S(x,t)=\min F=\min\int_0^{\infty} f_o(x,u,t)\,dt \qquad (16)$$

The optimal time for ship to travel k stages will be:

$$t_k^* = \min_{u_{1,k-2},u_{2,k-2}} \Big\{ t_{k-1}^*[x_{1,k}, x_{2,k}, x_{3,k-1}, x_{4,k-1}, x_{5,k-1}, x_{6,k-1}] + \Delta t_k [x_{1,k}, x_{2,k}, x_{1,k+1}(x_{1,k}, x_{3,k}(x_{3,k-1}, x_{4,k-1}(x_{4,k-2}, u_{1,k-2}, \Delta t_{k-2}), \Delta t_{k-1}), x_{5,k}(x_{5,k-1}, x_{6,k-1}, (x_{6,k-2}, u_{2,k-2}, \Delta t_{k-2})\Delta t_{k-1})), x_{2,k+1}(x_{2,k}, x_{3,k}(x_{3,k-1}, x_{4,k-1}(x_{4,k-2}, u_{1,k-2}, \Delta t_{k-2}, \Delta t_{k-1}) x_{5,k}(x_{5,k-1}, x_{6,k-1}(x_{6,k-2}, u_{2,k-2}, \Delta t_{k-2}), \Delta t_{k-1})), x_{5,k}(x_{5,k-1}, x_{6,k-1}(x_{6,k-2}, u_{2,k-2}, \Delta t_{k-2}), \Delta t_{k-1})] \Big\} \quad k = 3, 4, \ldots, K \tag{17}$$

where:

- coordinates of ship's position: $x_1=x$, $x_2=y$;
- ship's course: $x_3=\psi$;
- angular speed of the ship's return: $x_4=\dot{\psi}$;
- ship's speed: $x_5=V$;
- acceleration of the ship: $x_6=\dot{V}$;
- time: $x_7=t$;
- relative rudder deflection: $u_1=\alpha/a_m$;
- relative change in rotational speed of the main screw propeller: $u_2=n/n_m$ [5].

By moving from the first to the last stage, the Bellman function equation is obtained for the ship control process by changing the rudder angle and the rotational speed of the propeller.

Including constraints resulting from the safe approach, the right path recommendations consist in checking if the state variables did not exceed the limits in each considered node and reject the nodes in which a violation was detected.

Constraints of state variables and control variables form a separate calculation procedure in the algorithm for determining the dynamic safe trajectory of a ship.

Fig. 7 shows the division of the ship's path into k stages and n nodes.

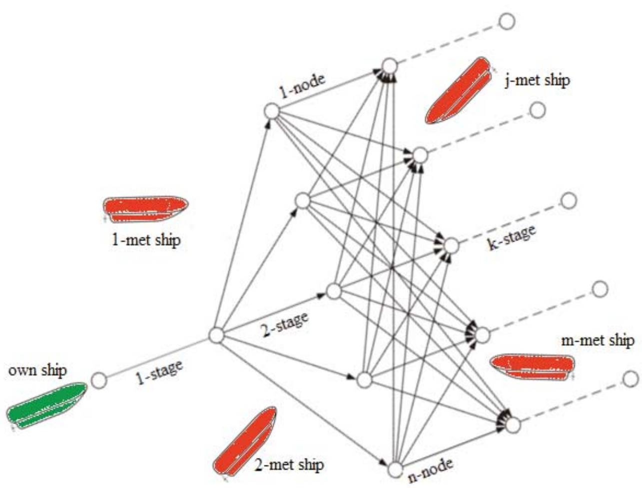

Fig. 7. Determination of the own ship safe and optimal trajectory by means of dynamic programming method

By going from the first stage to the last one the formula (17) determines the Bellman's functional equation for the process of the ship's control by the alteration of the rudder angle and the rotational speed of the propeller. The constraints for the state variables and the control values generate the *Neural Constraints (NC)* procedure in the computer algorithm Dynamic Programming Trajectory - DPT for the determination of the safe ship trajectory.

The consideration of the constraints resulting from maintaining safe approaching distance and the recommendations of the way priority law is performed by checking whether the state variables have not exceeded constraints in each of the considered intersections and by rejecting the intersections in which a violation has been discovered.

The safe trajectories of own ship in the situation of passing by J=10 ships met in conditions of good visibility at sea are shown in Fig. 8 , and in restricted visibility at sea – in Fig. 9, determined by the DPT algorithm.

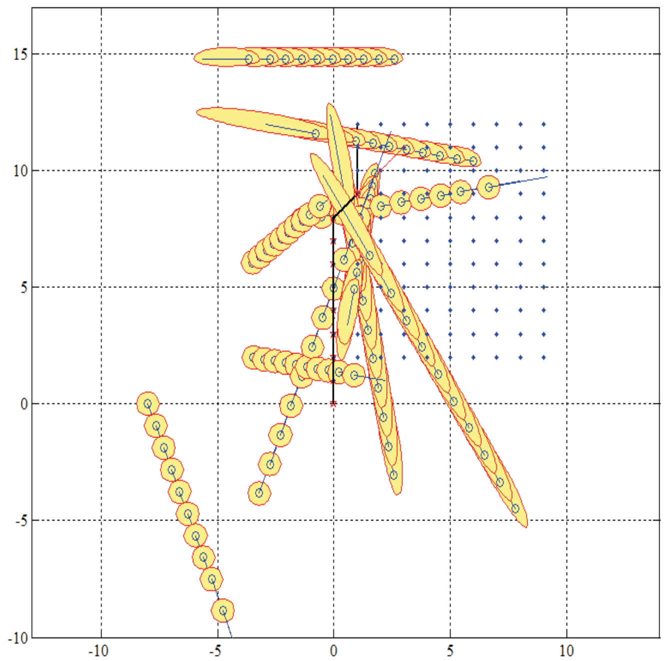

Fig. 8. Optimal and safe trajectory of own ship while passing by J=10 ships encountered in conditions of good visibility at sea, D_s=0.5 nm, t_k=49.66 min*

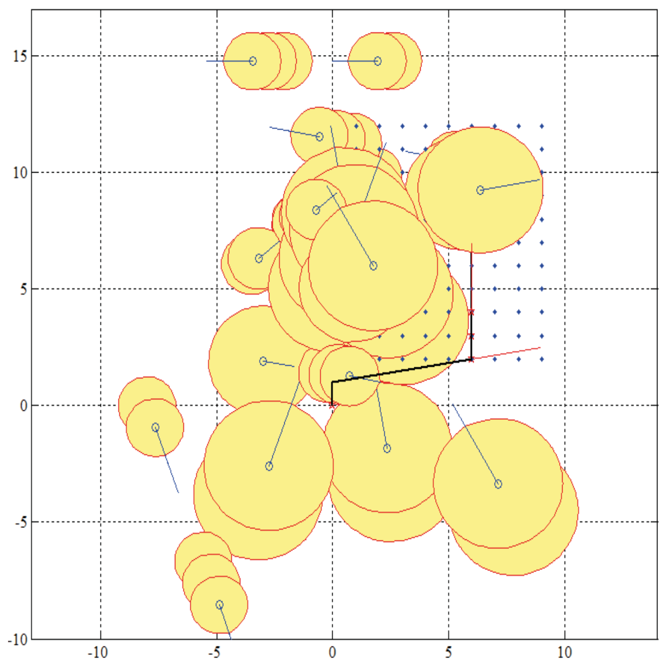

Fig. 9. Optimal and safe trajectory of own ship while passing by J=10 ships encountered in conditions of restricted visibility at sea, D_s=2.0 nm, t_k=68.33 min*

OPTIMIZATION OF SAFE SHIP CONTROL BY MEANS OF MULTI-CRITERIA GAME CONTROL

In order to ensure the safety of navigation, ships are obliged to respect the legal requirements in the form of COLREGs rules. However, the rules apply only to two ships in terms of good visibility. In the conditions of limited visibility, they only give general recommendations and are not able to take into account all the necessary conditions of the actual process. Thus, the actual process of passing ships occurs under indefinite conditions and conflict with inaccurate cooperation of ships in accordance with the principles of COLREGs.

Therefore, it is expedient to present the process of safe control of the ship as well as development of the appropriate control methods and testing their operation, by using the rules of game theory.

For practical synthesis of control algorithms, positional and matrix game models are used.

The essence of the positional game is the dependence of the own ship's strategy on the ship's position $p(t_k)$ at the current step k.

The optimal control of own ship for non-cooperative game is determined from the following criterion:

$$F(x)_{nc}^* = \min_{u_0^* \in U_0} \left\{ \max_{u_{j,0} \in U_{j,0}} \min_{u_{0,j} \in U_{0,j}} S[x_0(t_k)] \right\} \quad (18)$$

$$j = 1, 2, \ldots, J$$

and for cooperative game:

$$F(x)_{c}^* = \min_{u_0^* \in U_0} \left\{ \min_{u_{j,0} \in U_{j,0}} \min_{u_{0,j} \in U_{0,j}} S[x_0(t_k)] \right\} \quad (19)$$

$$j = 1, 2, \ldots, J$$

Trajectories of own ship in the situation of J=19 ships encountered in the Kattegat Strait in restricted visibility at sea with D_s=1.6 nm, determined according to the non-cooperative positional game algorithm are shown in Fig 10, and for cooperative positional game - in Fig. 11.

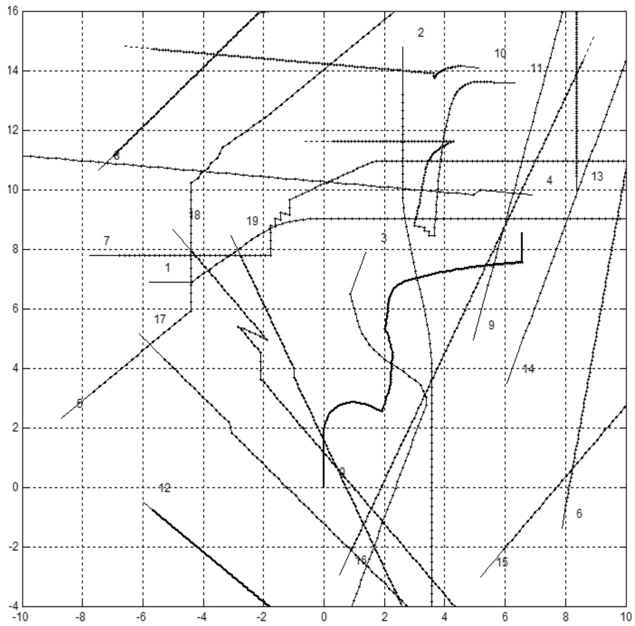

Fig. 10. Computer simulation of multi-stage non-cooperative positional game algorithm for safe own ship control in situation of passing J=19 encountered ships in restricted visibility at sea, D_s=1.4 nm, $d(t_k)$=6.56 nm

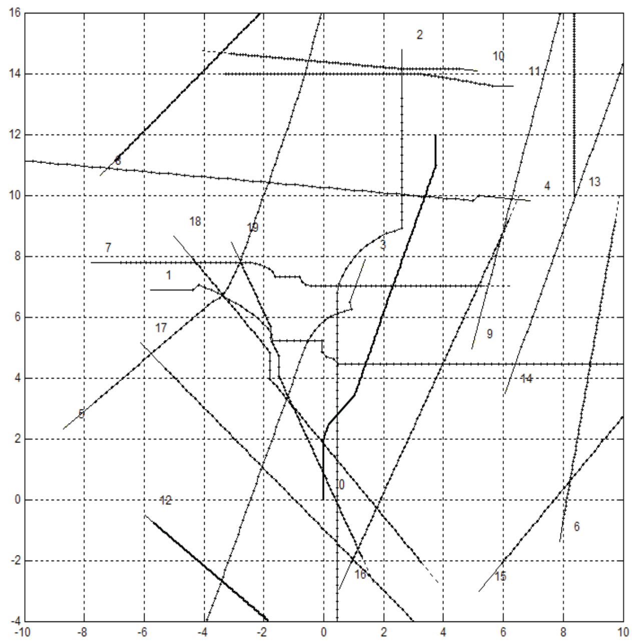

Fig. 11. Computer simulation of multi-stage cooperative positional game algorithm for safe own ship control in situation of passing J=19 encountered ships in restricted visibility at sea, D_s=1.4 nm, $d(t_k)$=3.75 nm

CONCLUSIONS

In synthesis of the controller or the optimal control algorithm for a given transport or logistic object, both static and dynamic analytical and numerical optimization methods can be used.

However, various optimization tasks in practical applications are most often solved by means of appropriate numerical methods of static and dynamic optimization.

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