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Port oil terminal operation cost optimization

Keywords

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Abstract

The operation model of a complex system changing its functional structure and its instantaneous operation costs during the variable at time operation states and linear programming are proposed to optimize the system operation process in order to get the system total operation cost minimal. The optimization method allowing to find the optimal values of the transient probabilities of the complex system operation process at the particular operation state that minimize the system total operation cost mean value under the assumption that the system conditional operation costs mean values at the particular operation states are fixed or in the safety state subset not worse than the critical safety state are presented. The procedure of finding the optimal mean value of system total operation cost for the fixed operation time or in the safety state subset not worse than the critical safety state are applied to the port oil terminal operation cost minimization.

1. Introduction

In today's industrial landscape, the optimization of operation costs plays a vital role in achieving efficiency and maximizing profitability. Complex multistate ageing technical systems (Kołowrocki, 2014, 2022; Xue, 1995; Xue & Yang, 1985), characterized by their dynamic operation and changing safety states (Ferreira & Pacheco, 2007; Glynn & Haas, 2006; Grabski, 2002, 2015; Limnios & Oprisan, 2005; Mercier, 2008; Tang et al., 2007), present unique challenges in cost management. This article delves into the realm of cost optimization for such systems, aiming to identify strategies that minimize operation expenses while ensuring the highest levels of safety and performance.

The operation costs of multistate ageing technical systems have a direct impact on the overall financial health of industries such as manufacturing, transportation, and critical infrastructure management. With the continuous evolution of these systems and their components, cost optimization becomes a critical factor in maintaining competitiveness and sustainability.

To minimize the system total operation cost for the fixed operation time or in the safety state subset not worse than the critical safety state, the linear programming (Klabjan & Adelman, 2006) is used through finding the optimal values of the system operation process limit transient probabilities at the particular operation states under the fixed the system total conditional operation cost in these operation states (Kołowrocki, Magryta-Mut, 2020a; Magryta-Mut, 2020). In both considered cases of the system operation cost optimization, the procedures of changing the system operation process for minimizing the mean value of the system total operation cost are proposed.

The chapter is organized into 6 parts, this Introduction as Section 1, Sections 2–5 and Conclusion as Section 6. Section 2 introduces the modeling approach for system operation cost. It presents two models: cost model 1, which focuses on the total operation cost for a fixed operation time, and cost model 2, which analyzes the total operation

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cost in the safety state subsets. Section 3 is dedicated to the optimization of system operation costs. By considering the characteristics of the multistate ageing system's operation process and the conditional instantaneous operation costs at different operation states, linear programming techniques are applied. This allows for the identification of the optimal mean value of the system's total operation cost for a fixed operation time using cost model 1, as proposed in Section 2. Additionally, for safety state subsets not worse than the critical safety state, cost model 2 is employed. In Section 4, the practical aspect of the study is explored. The focus is on conducting cost examinations of real and complex system, specifically the port oil terminal critical infrastructure. In Section 5, the proposed procedure for system operation cost optimization is applied to real system analyzed in the Chapter. The obtained optimal values are compared to the values prior to the optimization process. The chapter concludes with a Summary section, which evaluates the obtained results and proposes future research directions within the subject matter. Lastly, a Bibliography section is provided, containing relevant references related to the chapter topic.

2. Operation cost

2.1. System operation cost model for fixed operation time

Similarly to safety analysis of the system impacted by its operation process, we may investigate the system operation total cost for fixed operation time. Namely, we firstly define the instantaneous system operation cost in the form of the vector

$$\boldsymbol{C}(t) = [[C(t)]^{(1)}, [C(t)]^{(2)}, \ldots, [C(t)]^{(\nu)}], \quad (1)$$

$t \in \langle 0, \infty)$,

with the coordinates

$$[C(t)]^{(b)}, t \in \langle 0, \infty), b = 1,2,\ldots,\nu, \quad (2)$$

that are the system conditional instantaneous operation costs at the system operation states z_b, $b = 1,2,\ldots,\nu$.

Further, it is natural to assume that the system operation total cost during the fixed operation time depends significantly on the system operation total costs at the operation states. This dependency is clearly expressed in mean value of the system operation total cost during the system operation time θ, given by

$$\hat{\boldsymbol{C}}(\theta) = \sum_{b=1}^{\nu} p_b [\hat{\boldsymbol{C}}(\theta)]^{(b)}, \theta > 0, \quad (3)$$

where $p_b, b = 1,2,\ldots,\nu$, are limit transient probabilities at operation states defined by:

$$p_b = \lim_{t\to\infty} p_b(t) = b = 1,2,\ldots,\nu, \quad (4)$$

of transient probabilities

$$p_b(t) = P(Z(t) = z_b), \quad (5)$$

$t \in \langle 0, \infty), b = 1,2,\ldots,\nu$,

of the system operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,\nu$, and $[C(\theta)]^{(b)}$, $b = 1,2,\ldots,\nu$, are the mean values of the system conditional operation total costs at the particular system operation states z_b, $b = 1,2,\ldots,\nu$, given by

$$[\hat{\boldsymbol{C}}(\theta)]^{(b)} = \int_0^{\hat{M}_b} [\boldsymbol{C}(t)]^{(b)} dt, \theta > 0, \quad (6)$$

$b = 1,2,\ldots,\nu$,

where $\hat{M}_b$, $b = 1,2,\ldots,\nu$, are the mean values of the system operation process total sojourn times at the operation states during the fixed system operation time θ, (Kołowrocki, Magryta-Mut, 2022a, 2022b; Kołowrocki & Soszyńska-Budny, 2011/2015; Magryta-Mut, 2023) given by

$$\hat{M}_b = E[\hat{\theta}_b] = p_b\theta, b = 1,2,\ldots,\nu, \quad (7)$$

and $[C(t)]^{(b)}$, $t \in \langle 0, \infty)$, $b = 1,2,\ldots,\nu$, are the system conditional instantaneous operation costs at the system particular operation states defined by (3).

2.2. System operation cost model in safety state subsets

Similarly to safety analysis of the system impacted by its operation process, we may investigate the system operation total costs in the safety state subsets. Namely, we define the instantaneous system operation cost in the form of the vector

$\boldsymbol{C}(t,\cdot) = [\boldsymbol{C}(t,1),\ldots,\boldsymbol{C}(t,z)],\ t \in \langle 0,\infty),$ (8)

with the coordinates given by

$\boldsymbol{C}(t,u) \cong \sum_{b=1}^{\nu} p_b[\boldsymbol{C}(t,u)]^{(b)},$ (9)

$t \in \langle 0,\infty), u = 1,2,\ldots,z,$

where $[\boldsymbol{C}(t,u)]^{(b)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,\nu$, are the coordinates of the vector

$[\boldsymbol{C}(t,\cdot)]^{(b)} = [[\boldsymbol{C}(t,1)]^{(b)},\ldots,[\boldsymbol{C}(t,z)]^{(b)}],$

$t \in \langle 0,\infty), b = 1,2,\ldots,\nu,$

representing the system conditional instantaneous operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the system operation states z_b, $b = 1,2,\ldots,\nu$, and p_b, $b = 1,2,\ldots,\nu$, are the system operation process limit transient probabilities in the particular operation states (Grabski, 2015). Thus, it is naturally to assume that the system instantaneous operation cost depends significantly on the system operation state and the system operation cost at the operation state as well. This dependency is also clearly expressed in mean value of the system total operation cost

$\boldsymbol{C}(\cdot) = [\boldsymbol{C}(1), \boldsymbol{C}(2),\ldots,\boldsymbol{C}(z)],$ (10)

with coordinates given by the linear equations

$\boldsymbol{C}(u) \cong \sum_{b=1}^{\nu} p_b[\boldsymbol{C}(u)]^{(b)}, u = 1,2,\ldots,z,$ (11)

for the mean values of the system total unconditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, where $\boldsymbol{C}(t,u)]^{(b)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,\nu$, are the mean values of the system total conditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the particular system operation states z_b, $b = 1,2,\ldots,\nu$, determined by

$[\boldsymbol{C}(u)]^{(b)} \cong \int_0^{[\boldsymbol{\mu}(u)]^{(b)}} [\boldsymbol{C}(t,u)]^{(b)}\, dt,$ (12)

$u = 1,2,\ldots,z, b = 1,2,\ldots,\nu,$

where

$[\boldsymbol{\mu}(u)]^{(b)} = E\big[[T(u)]^{(b)}\big],$ (13)

$u = 1,2,\ldots,z,$

are the mean values of the system conditional lifetimes $[T(u)]^{(b)}$ in the safety state subset $\{u,u+1,\ldots,z\}$ at the operation state z_b, $b = 1,2,\ldots,\nu$, given by (Kołowrocki & Magryta, 2020c; Kołowrocki & Soszyńska-Budny, 2011/2015):

$[\boldsymbol{\mu}(u)]^{(b)} = \int_0^{\infty} [\boldsymbol{S}(t,u)]^{(b)}\, dt, u = 1,2,\ldots,z,$

and $[\boldsymbol{S}(t,u)]^{(b)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,\nu$, are the system safety function defined above and p_b are limit transient probabilities defined in (Kołowrocki & Soszyńska-Budny, 2011/2015).

3. System operation cost optimization

3.1. System operation cost optimization model for fixed operation time

From the linear equation (3) of the system operation cost model introduced in Section 2.1, we can see that the mean value of the system total unconditional operation cost $\boldsymbol{C}(\theta), \theta > 0$, is determined by the limit values of transient probabilities p_b, $b = 1,2,\ldots,\nu$, of the system operation process at the operation states z_b, $b = 1,2,\ldots,\nu$, defined by

$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3)],\ t \in \langle 0,\infty),$ (14)

coordinate given by (Kołowrocki & Soszyńska-Budny, 2011/2015)

$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{\nu} p_b[\boldsymbol{S}(t,u)]^{(b)},$ (15)

$t \in \langle 0,\infty), u = 1,2,\ldots,z,$

where $p_b, b = 1,2,\ldots,\nu$, are the limit transient probabilities of the system operation process at the operation states z_b, $b = 1,2,\ldots,\nu$, and by the mean values $[C(\theta)]^{(b)}$, $\theta > 0$, $b = 1,2,\ldots,\nu$, of the system conditional operation total costs at the particular system operation states z_b, $b = 1,2,\ldots,\nu$, determined by (6). Therefore, the system operation cost optimization based on the linear programming (Klabjan & Adelman, 2006), can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,\ldots,\nu$, of the limit transient probabilities p_b, $b = 1,2,\ldots,\nu$, of the system operation process at the operation states to minimize the mean value $\boldsymbol{C}(\theta)$ of the system unconditional operation total cost under the assumption that the mean values $[C(\theta)]^{(b)}$, $b = 1,2,\ldots,\nu$, of the system conditional operation total costs at the particular system operation states z_b, $b = 1,2,\ldots,\nu$, are fixed.

Thus, we may formulate the optimization problem as a linear programming model with the objective function of the form given by (3) with the bound constraints

$$\check{p}_b \le p_b \le \hat{p}_b,\ b = 1,2,\ldots,\nu,$$

$$\sum_{b=1}^{\nu} p_b = 1, \quad (16)$$

where

$$[C(\theta)]^{(b)}, [C(\theta)]^{(b)} \ge 0, \quad (17)$$

$$b = 1,2,\ldots,\nu,$$

are fixed mean values of the system conditional operation total costs at the operation states z_b, $b = 1,2,\ldots,\nu$, determined according to (6) and

$$\check{p}_b,\ 0 \le \check{p}_b \le 1 \text{ and } \hat{p}_b,\ 0 \le \hat{p}_b \le 1,\ \check{p}_b \le \hat{p}_b,$$

$$b = 1,2,\ldots,\nu, \quad (18)$$

are lower and upper bounds of the unknown transient probabilities $p_b, b = 1,2,\ldots,\nu$, respectively.

Now, we can find the optimal solution of the formulated by (3), (6), (16)–(18) the optimization problem, i.e. we can determine the optimal values $\dot{p}_b$, of the transient probabilities p_b, $b = 1,2,\ldots,\nu$, that minimize the objective function given by (3). The minimizing procedure is described in (Magryta-Mut, 2023).

Finally, after applying this procedure, we can get the minimum value of the system unconditional operation total cost, defined by the linear equation (3), in the following form

$$\dot{\boldsymbol{C}}(\theta) = \sum_{b=1}^{\nu} \dot{p}_b [C(\theta)]^{(b)}. \quad (19)$$

3.2. System operation cost optimization model in safety state subsets

From the linear equations (8), we can see that the mean value of the system total unconditional operation cost $\boldsymbol{C}(u)$, $u = 1,2,\ldots,z$, is determined by the limit values of transient probabilities p_b, $b = 1,2,\ldots,\nu$, of the system operation process at the operation states z_b, $b = 1,2,\ldots,\nu$, and by the mean values $[\boldsymbol{C}(u)]^{(b)}$. Therefore, the system operation cost optimization based on the linear programming (Klabjan & Adelman, 2006), can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,\ldots,\nu$, of the transient probabilities p_b, $b = 1,2,\ldots,\nu$, to minimize the mean value $\boldsymbol{C}(u)$ under the assumption that the mean values $[\boldsymbol{C}(u)]^{(b)}$, $b = 1,2,\ldots,\nu$, $u = 1,2,\ldots,z$, of the system total conditional operation costs in the safety state subsets $\{u, u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the particular system operation states z_b, $b = 1,2,\ldots,\nu$, are fixed. As a special and practically important case of the above formulated system operation cost optimization problem for $u = r$, where if r, $r = 1,2,\ldots,z$, is a system critical safety state, we may look for the optimal values $\dot{p}_b$, $b = 1,2,\ldots,\nu$, of the transient probabilities p_b, $b = 1,2,\ldots,\nu$, of the system operation process at the system operation states to minimize the mean value $\boldsymbol{C}(r)$, $r = 1,2,\ldots,z$, under the assumption that the mean values $[\boldsymbol{C}(r)]^{(b)}$, $b = 1,2,\ldots,\nu$, $r = 1,2,\ldots,z$, are fixed. More exactly, we may formulate the optimization problem as a linear programming model (Klabjan & Adelman, 2006) with the objective function of the following form

$$\boldsymbol{C}(r) \cong \sum_{b=1}^{\nu} p_b [\boldsymbol{C}(r)]^{(b)}, \quad (20)$$

for a fixed $r \in \{1,2,\ldots,z\}$ and with the following bound constraints

$$\check{p}_b \le p_b \le \hat{p}_b,\ b = 1,2,\ldots,\nu,$$

$$\sum_{b=1}^{\nu} p_b = 1, \quad (21)$$

where

$$[\boldsymbol{C}(r)]^{(b)}, [\boldsymbol{C}(r)]^{(b)} \ge 0,\ b = 1,2,\ldots,\nu,$$

and

$$\check{p}_b,\ 0 \le \check{p}_b \le 1 \text{ and } \hat{p}_b,\ 0 \le \hat{p}_b \le 1,\ \check{p}_b \le \hat{p}_b,$$

$b = 1,2,\ldots,\nu,$ (22)

are lower and upper bounds of the unknown transient probabilities p_b, $b = 1,2,\ldots,\nu$, respectively.
Now, we can obtain the optimal solution of the linear programming problem given by (20)–(21), i.e. we can find the optimal values $\dot{p}_b$, of the transient probabilities p_b, $b = 1,2,\ldots,\nu$, that minimize the objective function given by (20).
We arrange the mean values of the system total conditional operation costs $[\boldsymbol{C}(r)]^{(b)}$, $b = 1,2,\ldots,\nu$, in non-decreasing order

$$[C(r)]^{(b_1)} \leq [C(r)]^{(b_2)} \leq \ldots \leq [C(r)]^{(b_i)},$$

where $b_i \in \{1,2,\ldots,\nu\}$ for $i = 1,2,\ldots,\nu$.
Next, we substitute

$$x_i = p_{b_i}, \check{x}_i = \check{p}_{b_i}, \hat{x}_i = \hat{p}_{b_i} \quad (23)$$

for $i = 1,2,\ldots,\nu$,

and we minimize with respect to x_i, $i = 1, 2,\ldots,\nu$, the linear form (20) that after this transformation takes the form

$$\boldsymbol{C}(r) \cong \sum_{b=1}^{\nu} x_i [\boldsymbol{C}(r)]^{(b_i)}, \quad (24)$$

for a fixed $r \in \{1,2,\ldots,z\}$ with

$$\check{x}_i \leq x_i \leq \hat{x}_i, i = 1,2,\ldots,\nu, \sum_{b=1}^{\nu} x_i = 1, \quad (25)$$

where

$$[C(r)]^{(b_1)}, [C(r)]^{(b_1)} \geq 0, i = 1,2,\ldots,\nu,$$

are arranged in non-decreasing order and

$$\check{x}_i, 0 \leq \check{x}_i \leq 1, \text{ and } \hat{x}_i, 0 \leq \hat{x}_i \leq 1, \check{x}_i \leq \hat{x}_i,$$

$$i = 1,2,\ldots,\nu, \quad (26)$$

are lower and upper bounds of the unknown probabilities x_i, $i = 1,2,\ldots,\nu$ respectively.
To find the optimal values of x_i, $i = 1,2,\ldots,\nu$, we define

$$\check{x} = \sum_{b=1}^{\nu} \check{x}_i, \hat{y} = 1 - \check{x}, \quad (27)$$

and

$$\check{x}^0 = 0, \hat{x}^0 = 0 \text{ and } \check{x}^I = \sum_{i=1}^{I} \check{x}_i, \hat{x}^I = \sum_{i=1}^{I} \hat{x}_i,$$

for $I = 1,2,\ldots,\nu$. (28)

Next, we find the largest value $I \in \{0,1,\ldots,n\}$ such that

$$\hat{x}^I - \check{x}^I < \hat{y} \quad (29)$$

and we fix the optimal solution that minimize (24) in the following way:
i) if $I = 0$, the optimal solution is

$$\dot{x}_1 = \hat{y} + \check{x}_1 \text{ and } \dot{x}_i = \hat{x}_i \text{ for } i = 2,3,\ldots,\nu, \quad (30)$$

ii) if $0 < I < \nu$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1, 2,\ldots, I,$$

$$\dot{x}_{I+1} = \hat{y} - \hat{x}^I + \check{x}^I + \check{x}_{I+1}$$

$$\text{and } \dot{x}_i = \check{x}_i \text{ for } i = I + 2, I + 3, \ldots,\nu, \quad (31)$$

iii) if $I = \nu$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,\ldots,\nu \quad (32)$$

Finally, after making the inverse to (23) substitution, we get the optimal limit transient probabilities

$$\dot{p}_{b_i} = \dot{x}_i \text{ for } i = 1,2,\ldots,\nu \quad (33)$$

that minimize the mean value of the system total unconditional operation costs in the safety state subset $\{r, r+1,\ldots,z\}$, defined by the linear form (27), giving its minimum value in the following form

$$\dot{\boldsymbol{C}}(r) \cong \sum_{b=1}^{\nu} \dot{p}_b [\boldsymbol{C}(r)]^{(b)}, \quad (34)$$

for a fixed $r \in \{1,2,\ldots,z\}$.
From the expression (34) for the minimum mean value $\dot{\boldsymbol{C}}(r)$ of the system unconditional operation cost in the safety state subset $\{r, r+1,\ldots,z\}$, replacing in it the critical safety state r by the safety state u, $u = 1,2,\ldots,z$, we obtain the corresponding optimal solutions for the mean values of the system unconditional operation costs in the safety state subsets $\{r, r+1,\ldots,z\}$ of the form

$$\dot{\boldsymbol{C}}(u) \cong \sum_{b=1}^{\nu} \dot{p}_b [\boldsymbol{C}(u)]^{(b)}, \; u = 1,2,\ldots,z. \qquad (35)$$

According to (10)–(11), the mean value of the system optimal total operation cost can by expressed by

$$\dot{\boldsymbol{C}}(\cdot) = [\dot{\boldsymbol{C}}(1), \ldots, \dot{\boldsymbol{C}}(z)], \qquad (36)$$

with coordinates given by the linear equations (34)

$$\dot{\boldsymbol{C}}(u) \cong \sum_{b=1}^{\nu} \dot{p}_b [\boldsymbol{C}(u)]^{(b)}, \; u = 1,2,\ldots,z, \qquad (37)$$

for the mean values of the system optimal total unconditional operation costs in the safety state subsets $\{u, u+1, \ldots, z\}$, $u = 1,2,\ldots,z$, where$[\boldsymbol{C}(u)]^{(b)}$, $b = 1,2,\ldots,\nu$, $u = 1,2,\ldots,z$, are the mean values of the system total conditional operation costs in the safety state subsets $\{u, u+1, \ldots, z\}$, $u = 1,2,\ldots,z$, at the particular system operation states z_b, $b = 1,2,\ldots,\nu$, and $\dot{p}_b$, $b = 1,2,\ldots,\nu$, are the system operation process optimal limit transient probabilities at these operation states given by (33).

The expressions for the optimal mean values of the system total operation costs in the particular safety states are

$$\dot{\bar{\boldsymbol{C}}}(u) = \dot{\boldsymbol{C}}(u) - \dot{\boldsymbol{C}}(u+1), \; u = 1,2,\ldots,z-1,$$

$$\dot{\bar{\boldsymbol{C}}}(z) = \dot{\boldsymbol{C}}(z), \qquad (38)$$

where $\dot{\boldsymbol{C}}(u)$, $u = 1,2,\ldots,z$, are the optimal mean values of the system total unconditional operation costs in the safety state subsets $\{u, u+1, \ldots, z\}$, $u = 1,2,\ldots,z$, given by (35).

4. Port oil terminal operation cost

4.1. Terminal description

The port oil terminal placed at the Baltic seaside is designated for receiving oil products from ships, storage and sending them by carriages or trucks to inland. Additionally, the terminal is capable of functioning in the opposite direction. A comprehensive description of the terminal can be found in (Kołowrocki & Soszyńska-Budny, 2019a, 2019b).

The terminal under consideration consists of three interconnected parts, namely A, B and C, linked by the piping transportation system with the pier. The estimated length of the oil transportation system within the port, consisting of pipelines, amounts to approximately 25 km.

The main technical assets (components) of the port oil terminal critical infrastructure are:

A_1 – port oil piping transportation system,
A_2 – internal pipeline technological system,
A_3 – supporting pump station,
A_4 – internal pump system,
A_5 – port oil tanker shipment terminal,
A_6 – loading railway carriage station,
A_7 – loading road carriage station,
A_8 – unloading railway carriage station,
A_9 – oil storage reservoir system.

The asset A_1, the port oil piping transportation system operating at the port oil terminal critical infrastructure consists of three subsystems:

- the subsystem S_1 composed of two pipelines, each composed of 176 pipe segments and 2 valves,
- the subsystem S_2 composed of two pipelines, each composed of 717 pipe segments and 2 valves,
- the subsystem S_3 composed of three pipelines, each composed of 360 pipe segments and 2 valves.

Its operation is the main activity of the port oil terminal involving the remaining assets $A_2 – A_9$.

The port oil transportation system is a series system composed of two series-parallel subsystems S_1, S_2, each containing two pipelines (assets) and one series – “2 out of 3” subsystem S_3 containing 3 pipelines (assets).

The subsystems S_1, S_2 and S_3 are forming a general series port oil transportation system safety structure presented in Figure 1.

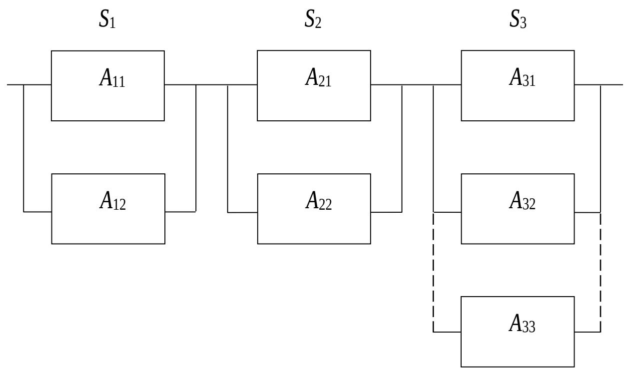

Figure 1. The port oil piping transportation system safety structure.

4.2. Operation process

We consider the port oil terminal critical infrastructure impacted by its operation process.

On the basis of the statistical data and expert opinions, it is possible to fix and to evaluate the following unknown basic parameters of the oil terminal critical infrastructure operation process.

The number of operation process states $\nu = 7$. We distinguish the following operation process states:

- the operation state z_1 – transport of one kind of medium from the terminal part B to part C using two out of three pipelines of the subsystem S_3 of the asset A_1 and assets A_2, A_4, A_6, A_7, A_9,
- the operation state z_2 – transport of one kind of medium from the terminal part C to part B using one out of three pipelines of the subsystem S_3 of the asset A_1 and assets A_2, A_4, A_8, A_9,
- the operation state z_3 – transport of one kind of medium from the terminal part B through part A to pier using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 of the asset A_1 and assets A_2, A_4, A_5, A_9,
- the operation state z_4 – transport of one kind of medium from the pier through parts A and B to part C using one out of two pipelines of the subsystem S_1, one out of two pipelines in subsystem S_2 and two out of three pipelines of the subsystem S_3 of the asset A_1 and assets A_2, A_3, A_4, A_5, A_6, A_7, A_9,
- the operation state z_5 – transport of one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 of the asset A_1 and assets A_2, A_3, A_4, A_5, A_9,
- the operation state z_6 – transport of one kind of medium from the terminal part B to C using two out of three pipelines of the subsystem S_3, and simultaneously transport one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 of the asset A_1 and assets A_2, A_3, A_4, A_5, A_6, A_7, A_9,
- the operation state z_7 – transport of one kind of medium from the terminal part B to C using one out of three pipelines of the subsystem S_3, and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines of the subsystem S_3 of the asset A_1 and assets A_2, A_4, A_6, A_7, A_8, A_9.

To identify the unknown parameters of the port oil piping transportation system operation process the suitable statistical data coming from its real realizations should be collected. On the basic of this data (GMU, 2018), it is possible to estimate these parameters and to fix the port oil terminal characteristics (Kołowrocki & Soszyńska-Budny, 2011/2015):

- the limit values of transient probabilities of the operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\dots,7$:

$$p_1 = 0.395,\ p_2 = 0.060,\ p_3 = 0.003,\ p_4 = 0.002,\ p_5 = 0.20,\ p_6 = 0.058,\ p_7 = 0.282, \quad (39)$$

- the expected values of the total sojourn times $\hat{\theta}_b$, $b = 1,2,\dots,7$, of the system operation process at the particular operation states z_b, $b = 1,2,\dots,7$, during the fixed operation time $\theta = 1$ year = 365 days:

$$\begin{aligned}
M_1 &= E[\hat{\theta}_1] = 0.395 \text{ year} = 144.175 \text{ days},\\
M_2 &= E[\hat{\theta}_2] = 0.060 \text{ year} = 21.9 \text{ days},\\
M_3 &= E[\hat{\theta}_3] = 0.003 \text{ year} = 1.095 \text{ days},\\
M_4 &= E[\hat{\theta}_4] = 0.002 \text{ year} = 0.73 \text{ days},\\
M_5 &= E[\hat{\theta}_5] = 0.20 \text{ year} = 73 \text{ days},\\
M_6 &= E[\hat{\theta}_6] = 0.058 \text{ year} = 21.17 \text{ days},\\
M_7 &= E[\hat{\theta}_7] = 0.282 \text{ year} = 102.93 \text{ days}.
\end{aligned} \quad (40)$$

4.3. Operation cost for fixed operation time

The port oil terminal critical infrastructure operation process $Z(t)$ main characteristics are the limit values of transient probabilities of the operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\dots,7$, (Kołowrocki & Soszyńska-Budny, 2011/2015; Magryta, 2020).

The asset A_1, the port oil terminal system is composed of 2880 components and the number of the system components operating at the various operation states, are different. Namely, there are operating 1086 system components at the operation states z_1, z_2 and z_7, 1794 system components at the operation states z_3 and z_5, 2880 system components at the operation states z_4 and z_6.

According to the information coming from experts, the approximate value of the instantaneous operation cost of the single basic component of the asset A_1 used during the operation time interval of $\theta = 1$ year at the operation state z_b,

$b = 1,2,\ldots,7$, is constant and amounts 9.6 PLN, $t \in \langle 0, 1\rangle$, $b = 1,2,\ldots,7$, whereas, the cost of each its singular basic component that is not used is equal to 0 PLN.
Hence, the number of components in a subsystems S_1, S_2, S_3 and their use at particularly operation states imply that the asset A_1 conditional instantaneous operation costs $[C_1(t)]^{(b)}$, $t \in \langle 0, \theta\rangle$, $b = 1,2,\ldots,7$, introduced by (2), are:

$[\boldsymbol{C}_1(t)]^{(1)} = 1086 \cdot 9.6 = 10425.6,$
$[\boldsymbol{C}_1(t)]^{(2)} = 1086 \cdot 9.6 = 10425.6,$
$[\boldsymbol{C}_1(t)]^{(3)} = 1794 \cdot 9.6 = 17222.4,$
$[\boldsymbol{C}_1(t)]^{(4)} = 2880 \cdot 9.6 = 27648,$
$[\boldsymbol{C}_1(t)]^{(5)} = 1794 \cdot 9.6 = 17222.4,$
$[\boldsymbol{C}_1(t)]^{(6)} = 2880 \cdot 9.6 = 27648,$
$[\boldsymbol{C}_1(t)]^{(7)} = 1086 \cdot 9.6 = 10425.6.$ (41)

Applying the formula (40) and (41), we get the approximate mean values $[C_1(\theta)]^{(b)}$, $b = 1,2,\ldots,7$, of the asset A_1 conditional operation total costs at the operation state z_b, $b = 1,2,\ldots,7$, during the operation time $\theta = 1$ year:

$[\widehat{\boldsymbol{C}}_1(\theta)]^{(1)} = 144.175 \cdot 10425.6 = 1503110.88,$
$[\widehat{\boldsymbol{C}}_1(\theta)]^{(2)} = 21.9 \cdot 10425.6 = 228320.64,$
$[\widehat{\boldsymbol{C}}_1(\theta)]^{(3)} = 1.095 \cdot 17222.4 = 18858.528,$
$[\widehat{\boldsymbol{C}}_1(\theta)]^{(4)} = 0.73 \cdot 27648 = 20183.04,$
$[\widehat{\boldsymbol{C}}_1(\theta)]^{(5)} = 73 \cdot 17222.4 = 1257235.2,$
$[\widehat{\boldsymbol{C}}_1(\theta)]^{(6)} = 21.17 \cdot 27648 = 585308.16,$
$[\widehat{\boldsymbol{C}}_1(\theta)]^{(7)} = 102.93 \cdot 10425.6 = 1073107.008.$
(42)

The corresponding mean values of the total conditional operation costs for the remaining assets $A_2 - A_9$, during the operation time $\theta = 1$ year, assumed arbitrarily (we do not have data at the moment) equal to 10000 PLN, in all operation states if they are used and equal to 0 PLN if they are not used. Under this assumption, considering the procedure of using assets $A_2 - A_9$ at particular operation states and the total operation costs of asset A_1 given in (42), we fix the total costs of the entire port oil terminal at the particular operation states $z_b, b = 1,2,\ldots,7$, (Magryta-Mut, 2023), given by:

$[\widehat{\boldsymbol{C}}(\theta)]^{(1)} = 3805344 + 50000 = 3855344,$
$[\widehat{\boldsymbol{C}}(\theta)]^{(2)} = 3805344 + 40000 = 3845344,$
$[\widehat{\boldsymbol{C}}(\theta)]^{(3)} = 6286176 + 40000 = 6326176,$
$[\widehat{\boldsymbol{C}}(\theta)]^{(4)} = 10091520 + 70000 = 10161520,$
$[\widehat{\boldsymbol{C}}(\theta)]^{(5)} = 6286176 + 50000 = 6336176,$
$[\widehat{\boldsymbol{C}}(\theta)]^{(6)} = 10091520 + 70000 = 10161520,$
$[\widehat{\boldsymbol{C}}(\theta)]^{(7)} = 3805344 + 60000 = 3865344.$ (43)

Considering the values of the total costs $[\widehat{\boldsymbol{C}}_1(\theta)]^{(b)}$, $b = 1,2,\ldots,7$, from (113) and the values of transient probabilities p_b, $b = 1,2,\ldots,7$, given by (39), the port oil terminal total operation mean cost during the operation time $\theta = 1$ year is given by

$$[\widehat{\boldsymbol{C}}(\theta)] \cong p_1[\widehat{\boldsymbol{C}}(\theta)]^{(1)} + p_2[\widehat{\boldsymbol{C}}(\theta)]^{(2)} + p_3[\widehat{\boldsymbol{C}}(\theta)]^{(3)} + p_4[\widehat{\boldsymbol{C}}(\theta)]^{(4)} + p_5[\widehat{\boldsymbol{C}}(\theta)]^{(5)} + p_6[\widehat{\boldsymbol{C}}(\theta)]^{(6)} + p_7[\widehat{\boldsymbol{C}}(\theta)]^{(7)} \cong 4739513.456 \text{ PLN}. \quad (44)$$

4.4. Operation cost in safety state subsets

The number of components in a subsystems S_1, S_2, S_3 and their use at particularly operation states, the asset A_1 conditional instantaneous operation costs $[\boldsymbol{C}(t,u)]^{(b)}$, $u = 1, 2$, $b = 1,2,\ldots,7$, in the safety state subsets $\{1,2\}$, $\{2\}$ for $t \in \langle 0, \infty)$, $b = 1,2,\ldots,7$, expressed in PLN, are:

$[\boldsymbol{C}_1(t,1)]^{(1)} = [\boldsymbol{C}_1(t,2)]^{(1)} = 1086 \cdot 9.6 = 10425.6,$
$[\boldsymbol{C}_1(t,1)]^{(2)} = [\boldsymbol{C}_1(t,2)]^{(2)} = 1086 \cdot 9.6 = 10425.6,$
$[\boldsymbol{C}_1(t,1)]^{(3)} = [\boldsymbol{C}_1(t,2)]^{(3)} = 1794 \cdot 9.6 = 17222.4,$
$[\boldsymbol{C}_1(t,1)]^{(4)} = [\boldsymbol{C}_1(t,2)]^{(4)} = 2880 \cdot 9.6 = 27648,$
$[\boldsymbol{C}_1(t,1)]^{(5)} = [\boldsymbol{C}_1(t,2)]^{(5)} = 1794 \cdot 9.6 = 17222.4,$
$[\boldsymbol{C}_1(t,1)]^{(6)} = [\boldsymbol{C}_1(t,2)]^{(6)} = 2880 \cdot 9.6 = 27648,$
$[\boldsymbol{C}_1(t,1)]^{(7)} = [\boldsymbol{C}_1(t,2)]^{(7)} = 1086 \cdot 9.6 = 10425.6.$
(45)

The mean values $[\boldsymbol{\mu}(u)]^{(b)}$, $u = 1,2$, of the terminal conditional lifetimes $[T(u)]^{(b)}$, $u = 1,2$, in the safety state subset $\{1,2\}$, $\{2\}$ at the operation state $z_b, b = 1,2,\ldots,7$, determined in Section 3, respectively (expressed in years) are:

$[\boldsymbol{\mu}(1)]^{(1)} \cong 8.08342,$
$[\boldsymbol{\mu}(1)]^{(2)} \cong 8.16593,$
$[\boldsymbol{\mu}(1)]^{(3)} = [\boldsymbol{\mu}(1)]^{(5)} \cong 7.60179,$
$[\boldsymbol{\mu}(1)]^{(4)} = [\boldsymbol{\mu}(1)]^{(6)} \cong 6.80805,$
$[\boldsymbol{\mu}(1)]^{(7)} \cong 8.00256,$
$[\boldsymbol{\mu}(2)]^{(1)} = 5.15695,$
$[\boldsymbol{\mu}(2)]^{(2)} = 5.21069,$
$[\boldsymbol{\mu}(2)]^{(3)} = [\boldsymbol{\mu}(2)]^{(5)} \cong 4.85232,$

$[\boldsymbol{\mu}(2)]^{(4)} \cong 4.34292,$
$[\boldsymbol{\mu}(2)]^{(6)} \cong 4.3429,$
$[\boldsymbol{\mu}(2)]^{(7)} \cong 5.10431.$ (46)

Applying the formula (45) and (46) we get the approximate mean values $[\boldsymbol{C}_1(1)]^{(b)}$, $b = 1,2,\ldots,7$, of the total costs of the entire port oil terminal at the particular operation states given by:

$[\boldsymbol{C}_1(1)]^{(1)} = 8.08342 \cdot 10425.6 \cong 84274.50355,$
$[\boldsymbol{C}_1(1)]^{(2)} = 8.16593 \cdot 10425.6 \cong 85134.71981,$
$[\boldsymbol{C}_1(1)]^{(3)} = 7.60179 \cdot 17222.4 \cong 130921.06810,$
$[\boldsymbol{C}_1(1)]^{(4)} = 6.80805 \cdot 27648 \cong 188228.9664,$
$[\boldsymbol{C}_1(1)]^{(5)} = 7.60179 \cdot 17222.4 \cong 130921.06810,$
$[\boldsymbol{C}_1(1)]^{(6)} = 6.80805 \cdot 27648 \cong 188228.96640,$
$[\boldsymbol{C}_1(1)]^{(7)} = 8.00256 \cdot 10425.6 \cong 83431.48954,$ (47)

in the safety state subset {1,2} and

$[\boldsymbol{C}_1(2)]^{(1)} = 5.15695 \cdot 10425.6 \cong 53764.29792,$
$[\boldsymbol{C}_1(2)]^{(2)} = 5.21069 \cdot 10425.6 \cong 54324.56966,$
$[\boldsymbol{C}_1(2)]^{(3)} = 4.85232 \cdot 17222.4 \cong 83568.59597,$
$[\boldsymbol{C}_1(2)]^{(4)} = 4.34292 \cdot 27648 \cong 120073.05216,$
$[\boldsymbol{C}_1(2)]^{(5)} = 4.85232 \cdot 17222.4 \cong 83568.59597,$
$[\boldsymbol{C}_1(2)]^{(6)} = 4.3429 \cdot 27648 \cong 120072.49920,$
$[\boldsymbol{C}_1(2)]^{(7)} = 5.10431 \cdot 10425.6 \cong 53215.49434,$ (48)

in the safety state subset {2}.

The corresponding mean values of the total conditional operation costs for the remaining assets $A_2 - A_9$, we assume arbitrarily (we do not data at the moment) equal to 10000 PLN, in all operation states if they are used and equal to 0 PLN if they are not used. Under this assumption, considering the procedure of using asset $A_2 - A_9$ at particular operation state and the total cost of asset A_1 given in (47) and (48), we fix the total costs of the entire port oil terminal at the particular operation states given by:

$[\boldsymbol{C}_1(1)]^{(1)} = 84274.50355 + 50000$
$= 134274.50355,$
$[\boldsymbol{C}_1(1)]^{(2)} = 85134.71981 + 40000$
$= 125134.71981,$
$[\boldsymbol{C}_1(1)]^{(3)} = 130921.06810 + 40000$
$= 170921.06810,$
$[\boldsymbol{C}_1(1)]^{(4)} = 188228.9664 + 70000$
$= 258228.9664,$
$[\boldsymbol{C}_1(1)]^{(5)} = 130921.06810 + 50000$
$= 180921.06810,$
$[\boldsymbol{C}_1(1)]^{(6)} = 188228.96640 + 70000$
$= 258228.96640,$
$[\boldsymbol{C}_1(1)]^{(7)} = 83431.48954 + 60000$
$= 143431.48954,$ (49)

in the safety state subset {1,2} and

$[\boldsymbol{C}_1(2)]^{(1)} = 53764.29792 + 50000$
$= 103764.29792,$
$[\boldsymbol{C}_1(2)]^{(2)} = 54324.56966 + 40000$
$= 94324.56966,$
$[\boldsymbol{C}_1(2)]^{(3)} = 83568.59597 + 40000$
$= 123568.59597,$
$[\boldsymbol{C}_1(2)]^{(4)} = 120073.05216 + 70000$
$= 190073.05216,$
$[\boldsymbol{C}_1(2)]^{(5)} = 83568.59597 + 50000$
$= 133568.59597,$
$[\boldsymbol{C}_1(2)]^{(6)} = 120072.49920 + 70000$
$= 190072.49920,$
$[\boldsymbol{C}_1(2)]^{(7)} = 53215.49434 + 60000$
$= 113215.49434,$ (50)

in the safety state subset {2}.

Considering the values of $[\boldsymbol{C}(u)]^{(b)}$, $u = 1,2$, $b = 1,2,\ldots,7$, from (49)–(50) and the values of transient probabilities $p_b, b = 1,2,\ldots,7$, the port oil terminal total unconditional operation cost is given by

$$\begin{aligned}\boldsymbol{C}(1) &\cong p_1[\boldsymbol{C}(1)]^{(1)} + p_2[\boldsymbol{C}(1)]^{(2)} + p_3[\boldsymbol{C}(1)]^{(3)} \\ &+ p_4[\boldsymbol{C}(1)]^{(4)} + p_5[\boldsymbol{C}(1)]^{(5)} + p_6[\boldsymbol{C}(1)]^{(6)} \\ &+ p_7[\boldsymbol{C}(1)]^{(7)} \\ &\cong 0.395 \cdot 134274.50355 \\ &+ 0.06 \cdot 125134.71981 \\ &+ 0.003 \cdot 170921.06810 \\ &+ 0.02 \cdot 258228.9664 \\ &+ 0.2 \cdot 180921.06810 \\ &+ 0.058 \cdot 258228.96640 \\ &+ 0.282 \cdot 143431.48954 \\ &\cong 153184.90695 \text{ PLN}, \end{aligned} \quad (51)$$

in the safety state subset {1,2} and

$$\begin{aligned}\boldsymbol{C}(2) &\cong p_1[\boldsymbol{C}(2)]^{(1)} + p_2[\boldsymbol{C}(2)]^{(2)} + p_3[\boldsymbol{C}(2)]^{(3)} \\ &+ p_4[\boldsymbol{C}(2)]^{(4)} + p_5[\boldsymbol{C}(2)]^{(5)} + p_6[\boldsymbol{C}(2)]^{(6)} \\ &+ p_7[\boldsymbol{C}(2)]^{(7)} \\ &\cong 0.395 \cdot 103764.29792 \\ &+ 0.06 \cdot 94324.56966 \\ &+ 0.003 \cdot 123568.59597 \\ &+ 0.002 \cdot 190073.05216 \end{aligned}$$

$+ 0.2 \cdot 133568.59597$
$+ 0.058 \cdot 190072.49920$
$+ 0.282 \cdot 113215.49434$
$\cong 117061.91730$ PLN, (52)

in the safety state subset {2}.

5. Port oil terminal operation cost optimization

5.1. Operation cost optimization model for fixed operation time

Considering (39) to find the minimum value of the port oil terminal mean cost, we define the objective function given by (3), in the following form

$$\boldsymbol{C}(\theta) = p_1 \cdot 3855344 + p_2 \cdot 3845344 + p_3 \cdot 6326176 + p_4 \cdot 10161520 + p_5 \cdot 6336176 + p_6 \cdot 10161520 + p_7 \cdot 3865344. \quad (53)$$

The lower $\check{p}_b$ and upper $\hat{p}_b$ bounds of the unknown optimal values of transient probabilities $p_b, b = 1,2,\ldots,7,$ respectively are (Magryta, 2020):

$\check{p}_1 = 0.31$, $\check{p}_2 = 0.04$, $\check{p}_3 = 0.002$, $\check{p}_4 = 0.001$,
$\check{p}_5 = 0.15$, $\check{p}_6 = 0.04$, $\check{p}_7 = 0.25$,

$\hat{p}_1 = 0.46$, $\hat{p}_2 = 0.08$, $\hat{p}_3 = 0.006$, $\hat{p}_4 = 0.004$,
$\hat{p}_5 = 0.26$, $\hat{p}_6 = 0.08$, $\hat{p}_7 = 0.40$. (54)

Therefore, according to (16)–(17), we assume the following bound constraints

$0.31 \le p_1 \le 0.46$, $0.04 \le p_2 \le 0.08$,
$0.002 \le p_3 \le 0.006$, $0.001 \le p_4 \le 0.004$,
$0.15 \le p_5 \le 0.26$, $0.04 \le p_6 \le 0.08$,
$0.25 \le p_7 \le 0.40$, $\sum_{i=1}^{7} p_b = 1$. (55)

Now, before we find optimal values $\dot{p}_b$ of the transient probabilities $p_b, b = 1,2,\ldots,7,$ that minimize the objective function (53), we arrange the mean values of the port oil terminal conditional operation costs $[\boldsymbol{C}(\theta)]^{(b)}$, $b = 1,2,\ldots,7,$ in non-decreasing order

$$3845344 \le 3845344 \le 3845344 \le 6326176 \le 6336176 \le 10161520 \le 10161520,$$

i.e.

$$[\widehat{\boldsymbol{C}}(\theta)]^{(2)} \le [\widehat{\boldsymbol{C}}(\theta)]^{(1)} \le [\widehat{\boldsymbol{C}}(\theta)]^{(7)} \le [\widehat{\boldsymbol{C}}(\theta)]^{(3)} \le [\widehat{\boldsymbol{C}}(\theta)]^{(5)} \le [\widehat{\boldsymbol{C}}(\theta)]^{(4)} \le [\widehat{\boldsymbol{C}}(\theta)]^{(6)}. \quad (56)$$

Further, we substitute

$$x_1 = p_2,\ x_2 = p_1,\ x_3 = p_7,\ x_4 = p_3,\ x_5 = p_5,\ x_6 = p_4,\ x_7 = p_6, \quad (57)$$

and

$\check{x}_1 = \check{p}_2 = 0{,}04, \check{x}_2 = \check{p}_1 = 0{,}31,$
$\check{x}_3 = \check{p}_7 = 0{,}25, \check{x}_4 = \check{p}_3 = 0{,}002,$
$\check{x}_5 = \check{p}_5 = 0{,}15, \check{x}_6 = \check{p}_4 = 0{,}001,$
$\check{x}_7 = \check{p}_6 = 0{,}04,$

$\hat{x}_1 = \hat{p}_2 = 0{,}08, \hat{x}_2 = \hat{p}_1 = 0{,}46,$
$\hat{x}_3 = \hat{p}_7 = 0{,}40, \hat{x}_4 = \hat{p}_3 = 0{,}006,$
$\hat{x}_5 = \hat{p}_5 = 0{,}26, \hat{x}_6 = \hat{p}_4 = 0{,}004,$
$\hat{x}_7 = \hat{p}_6 = 0{,}08,$ (58)

and we minimize with respect to x_i, $i = 1,2,\ldots,7,$ the linear form (53) which takes the form

$$\widehat{\boldsymbol{C}}(\theta) = x_1 \cdot 3845344 + x_2 \cdot 3855344 + x_3 \cdot 3865344 + x_4 \cdot 6326176 + x_5 \cdot 6336176 + x_6 \cdot 10161520 + x_7 \cdot 10161520 \quad (59)$$

with the following bound constraints

$0.04 \le x_1 \le 0.08$, $0.31 \le x_2 \le 0.46$,
$0.25 \le x_3 \le 0.40$, $0.002 \le x_4 \le 0.006$,
$0.15 \le x_5 \le 0.26$, $0.001 \le x_6 \le 0.004$,
$0.04 \le x_7 \le 0.08$, $\sum_{i=1}^{7} x_i = 1$. (60)

We calculate

$$\check{x} = \sum_{i=1}^{7} \check{x}_i = 0.793,$$
$$\hat{y} = 1 - \check{x} = 1 - 0.793 = 0.207 \quad (61)$$

and we find

$\check{x}^0 = 0, \hat{x}^0 = 0, \hat{x}^0 - \check{x}^0 = 0,$
$\check{x}^1 = 0.04, \hat{x}^1 = 0.08, \hat{x}^1 - \check{x}^1 = 0.04,$
$\check{x}^2 = 0.35, \hat{x}^2 = 0.54, \hat{x}^2 - \check{x}^2 = 0.19,$
$\check{x}^3 = 0.06, \hat{x}^3 = 0.94, \hat{x}^3 - \check{x}^3 = 0.34,$
$\check{x}^4 = 0.602, \hat{x}^4 = 0.946, \hat{x}^4 - \check{x}^4 = 0.344,$
$\check{x}^5 = 0.752, \hat{x}^5 = 1.206, \hat{x}^5 - \check{x}^5 = 0.454,$
$\check{x}^6 = 0.753, \hat{x}^6 = 1.21, \hat{x}^6 - \check{x}^6 = 0.457,$
$\check{x}^7 = 0.793, \hat{x}^7 = 1.29, \hat{x}^7 - \check{x}^7 = 0.497.$ (62)

From the above, since the expression takes the form

$$\hat{x}^I - \check{x}^I < 0.207, \quad (63)$$

then it follows that the largest value $I \in \{0,1,\ldots,7\}$ such that this inequality holds is $I = 5$. Therefore, we fix the optimal solution that minimize linear function (53). Namely, we get

$$\dot{x}_1 = \hat{x}_1 = 0.08, \dot{x}_2 = \hat{x}_2 = 0.46,$$
$$\dot{x}_3 = \hat{y} - \hat{x}^2 + \check{x}^2 + \check{x}_3$$
$$= 0.207 - 0.54 + 0.35 + 0.25 = 0.267,$$
$$\dot{x}_4 = \check{x}_4 = 0.002, \dot{x}_5 = \check{x}_5 = 0.15,$$
$$\dot{x}_6 = \check{x}_6 = 0.001, \dot{x}_7 = \check{x}_7 = 0.04. \quad (64)$$

Finally, after making the substitution inverse to (57), we get the optimal transient probabilities

$$\dot{p}_2 = \dot{x}_1 = 0.08, \dot{p}_1 = \dot{x}_2 = 0.46,$$
$$\dot{p}_7 = \dot{x}_3 = 0.267, \dot{p}_3 = \dot{x}_4 = 0.002,$$
$$\dot{p}_5 = \dot{x}_5 = 0.15, \dot{p}_4 = \dot{x}_6 = 0.001,$$
$$\dot{p}_6 = \dot{x}_7 = 0.04. \quad (65)$$

that minimize the mean value of the port oil terminal operation total cost $\boldsymbol{C}(\theta)$ during the operation time $\theta = 1$ year, expressed by the linear form (115) and considering (44), its minimal value is

$$\dot{\boldsymbol{C}}(\theta) \cong 0.46 \cdot 3855344 + 0.08 \cdot 3845344$$
$$+ 0.002 \cdot 6326176 + 0.001 \cdot 10161520$$
$$+ 0.15 \cdot 6336176 + 0.04 \cdot 10161520$$
$$+ 0.267 \cdot 3865344 \cong 4\,492\,833.68 \text{ PLN}. \quad (66)$$

5.2. Cost optimization model in safety state subsets

Assuming the critical safety state $r = 1$ and considering (49) to find the minimum value of this cost, we define the objective function given by (20), in the following form

$$\boldsymbol{C}(1) \cong p_1[\boldsymbol{C}(1)]^{(1)} + p_2[\boldsymbol{C}(1)]^{(2)} + p_3[\boldsymbol{C}(1)]^{(3)}$$
$$+ p_4[\boldsymbol{C}(1)]^{(4)} + p_5[\boldsymbol{C}(1)]^{(5)} + p_6[\boldsymbol{C}(1)]^{(6)}$$
$$+ p_7[\boldsymbol{C}(1)]^{(7)}$$
$$\cong p_1 \cdot 134274.50355 + p_2 \cdot 125134.71981$$
$$+ p_3 \cdot 170921.06810 + p_4 \cdot 258228.9664$$
$$+ p_5 \cdot 180921.06810 + p_6 \cdot 258228.96640$$
$$+ p_7 \cdot 143431.48954. \quad (67)$$

The lower $\check{p}_b$ and upper $\hat{p}_b$ bounds of the unknown optimal values of transient probabilities p_b, $b = 1,2,\ldots,7$, respectively are (Kołowrocki & Soszyńska-Budny, 2011/2015):

$$\check{p}_1 = 0.31, \check{p}_2 = 0.04, \check{p}_3 = 0.002, \check{p}_4 = 0.001,$$
$$\check{p}_5 = 0.15, \check{p}_6 = 0.04, \check{p}_7 = 0.25,$$

$$\hat{p}_1 = 0.46, \hat{p}_2 = 0.08, \hat{p}_3 = 0.006, \hat{p}_4 = 0.004,$$
$$\hat{p}_5 = 0.26, \hat{p}_6 = 0.08, \hat{p}_7 = 0.40. \quad (68)$$

Therefore, according to (21)–(22), we assume the following bound constraints

$$0.31 \le p_1 \le 0.46,\ 0.04 \le p_2 \le 0.08,$$
$$0.002 \le p_3 \le 0.006,\ 0.001 \le p_4 \le 0.004,$$
$$0.15 \le p_5 \le 0.26,\ 0.04 \le p_6 \le 0.08,$$
$$0.25 \le p_7 \le 0.40,\ \sum_{i=1}^{7} p_b = 1. \quad (69)$$

Now, before we find optimal values $\dot{p}_b$ of the transient probabilities $p_b, b = 1,2,\ldots,7$, that minimize the objective function (56), we arrange the mean values of the port oil terminal conditional operation costs $[C(1)]^{(b)}$, $b = 1,2,\ldots,7$, determined by (49), in non-decreasing order

$$125134.71981 \le 134274.50355$$
$$\le 143431.48954 \le 170921.06810$$
$$\le 180921.06810 \le 258228.96640$$
$$\le 2588228.9664,$$

i.e.

$$[\boldsymbol{C}(1)]^{(2)} \le [\boldsymbol{C}(1)]^{(1)} \le [\boldsymbol{C}(1)]^{(7)} \le$$
$$[\boldsymbol{C}(1)]^{(3)} \le [\boldsymbol{C}(1)]^{(5)} \le [\boldsymbol{C}(1)]^{(4)} \le [\boldsymbol{C}(1)]^{(6)}, \quad (70)$$

Further, we substitute

$$x_1 = p_2,\ x_2 = p_1,\ x_3 = p_7,\ x_4 = p_3,$$
$$x_5 = p_5,\ x_6 = p_4,\ x_7 = p_6, \quad (71)$$

and

$$\check{x}_1 = \check{p}_2 = 0{,}04, \check{x}_2 = \check{p}_1 = 0{,}31,$$
$$\check{x}_3 = \check{p}_7 = 0{,}25, \check{x}_4 = \check{p}_3 = 0{,}002,$$
$$\check{x}_5 = \check{p}_5 = 0{,}15, \check{x}_6 = \check{p}_4 = 0{,}001,$$
$$\check{x}_7 = \check{p}_6 = 0{,}04,$$

$$\hat{x}_1 = \hat{p}_2 = 0{,}08, \hat{x}_2 = \hat{p}_1 = 0{,}46,$$
$$\hat{x}_3 = \hat{p}_7 = 0{,}40, \hat{x}_4 = \hat{p}_3 = 0{,}006,$$
$$\hat{x}_5 = \hat{p}_5 = 0{,}26, \hat{x}_6 = \hat{p}_4 = 0{,}004,$$
$$\hat{x}_7 = \hat{p}_6 = 0{,}08, \quad (72)$$

and we minimize with respect to x_i, $i = 1,2,\ldots,7$, the linear form (67) that according to (23)–(24) and (70)–(71) takes the form

$$\boldsymbol{C}(1) = x_1 \cdot 125134.71981 + x_2 \cdot 134274.50355 + x_3 \cdot 143431.48954 + x_4 \cdot 170921.06810 + x_5 \cdot 180921.06810 + x_6 \cdot 258228.9664 + x_7 \cdot 258228.9664, \quad (73)$$

with the following bound constraints

$$0.04 \le x_1 \le 0.08,\ 0.31 \le x_2 \le 0.46,\ 0.25 \le x_3 \le 0.40,\ 0.002 \le x_4 \le 0.006,\ 0.15 \le x_5 \le 0.26,\ 0.001 \le x_6 \le 0.004,\ 0.04 \le x_7 \le 0.08,\ \sum_{i=1}^{7} x_i = 1. \quad (74)$$

We calculate

$$\breve{x} = \sum_{i=1}^{7} \breve{x}_i = 0.793,\quad \hat{y} = 1 - \breve{x} = 1 - 0.793 = 0.207 \quad (75)$$

and we find

$$\breve{x}^0 = 0, \hat{x}^0 = 0, \hat{x}^0 - \breve{x}^0 = 0,$$
$$\breve{x}^1 = 0.04, \hat{x}^1 = 0.08, \hat{x}^1 - \breve{x}^1 = 0.04,$$
$$\breve{x}^2 = 0.35, \hat{x}^2 = 0.54, \hat{x}^2 - \breve{x}^2 = 0.19,$$
$$\breve{x}^3 = 0.06, \hat{x}^3 = 0.94, \hat{x}^3 - \breve{x}^3 = 0.34,$$
$$\breve{x}^4 = 0.602, \hat{x}^4 = 0.946, \hat{x}^4 - \breve{x}^4 = 0.344,$$
$$\breve{x}^5 = 0.752, \hat{x}^5 = 1.206, \hat{x}^5 - \breve{x}^5 = 0.454,$$
$$\breve{x}^6 = 0.753, \hat{x}^6 = 1.21, \hat{x}^6 - \breve{x}^6 = 0.457,$$
$$\breve{x}^7 = 0.793, \hat{x}^7 = 1.29, \hat{x}^7 - \breve{x}^7 = 0.497. \quad (76)$$

From the above, since the expression (29) takes the form

$$\hat{x}^I - \breve{x}^I < 0.207, \quad (77)$$

then it follows that the largest value $I \in \{0,1,\ldots,7\}$ such that this inequality holds is $I = 2$. Therefore, we fix the optimal solution that minimize linear function (67). Namely, we get

$$\dot{x}_1 = \hat{x}_1 = 0.08, \dot{x}_2 = \hat{x}_2 = 0.46,$$
$$\dot{x}_3 = \hat{y} - \hat{x}^2 + \breve{x}^2 + \breve{x}_3$$
$$= 0.207 - 0.54 + 0.35 + 0.25 = 0.267,$$
$$\dot{x}_4 = \breve{x}_4 = 0.002, \dot{x}_5 = \breve{x}_5 = 0.15,$$
$$\dot{x}_6 = \breve{x}_6 = 0.001, \dot{x}_7 = \breve{x}_7 = 0.04. \quad (78)$$

Finally, after making the substitution inverse to (23), we get the optimal transient probabilities

$$\dot{p}_2 = \dot{x}_1 = 0.08, \dot{p}_1 = \dot{x}_2 = 0.46,$$
$$\dot{p}_7 = \dot{x}_3 = 0.267, \dot{p}_3 = \dot{x}_4 = 0.002,$$
$$\dot{p}_5 = \dot{x}_5 = 0.15, \dot{p}_4 = \dot{x}_6 = 0.001,$$
$$\dot{p}_6 = \dot{x}_7 = 0.04. \quad (79)$$

that minimize the mean value of the port oil terminal total operation cost $\boldsymbol{C}(1)$ expressed by the linear form (67), its optimal value in the safety state subset {1,2} is

$$\dot{\boldsymbol{C}}(1) \cong 0.46 \cdot 134274.50355 + 0.08 \cdot 125134.71981 + 0.002 \cdot 170921.06810 + 0.001 \cdot 258228.9664 + 0.15 \cdot 180921.06810 + 0.04 \cdot 258228.96640 + 0.267 \cdot 143431.48954 \cong 148140.64690 \text{ PLN}, \quad (80)$$

and further, the optimal mean value of the port oil terminal operation total cost in the safety state subset {2} is

$$\dot{\boldsymbol{C}}(2) \cong 0.46 \cdot 103764.29792 + 0.08 \cdot 94324.56966 + 0.002 \cdot 123568.59597 + 0.001 \cdot 190073.05216 + 0.15 \cdot 133568.59597 + 0.04 \cdot 190072.49920 + 0.267 \cdot 113215.49434 \cong 113581.47921 \text{ PLN}. \quad (81)$$

Hence, and according to (38), the optimal values of the port oil terminal total operation costs in the particular safety states 1 and 2, respectively are:

$$\dot{\bar{C}}(1) \cong 148140.64690 - 113581.47921 = 34559.16769,$$
$$\dot{\bar{C}}(2) \cong 113581.47921. \quad (82)$$

The analyzed costs after optimization are lower than before it, what is respectively expressed by comparison of the results before the optimization given in (51)–(52) and the results after the optimization given in (80)–(81).

6. Conclusion

To embark on this scientific topic, it is essential to understand the intricate nature of complex multistate ageing technical systems. These systems are characterized by their gradual degradation over time, varying operation states, and changing safety requirements. As components age, their performance may decline, necessitating maintenance, repairs, or replacement to ensure continued functionality and safety.

In the realm of cost optimization, the challenges associated with multistate ageing technical systems are twofold. Firstly, there is a need to minimize operation costs while considering the dynamic nature of the systems. Unlike static systems, multistate ageing technical systems demand adaptive strategies that account for the transitions between operation states and their associated costs. Secondly, safety considerations cannot be compromised during cost optimization. Ensuring the integrity and reliability of these systems is paramount to prevent catastrophic failures, accidents, and potential environmental hazards (Bogalecka, 2020; Dąbrowska & Kołowrocki, 2019a, 2019b, 2020a, 2020b).

The integration of system operation cost models and multistate system safety models offers a comprehensive approach to address these challenges. By incorporating the impact of operation process on both cost and safety, a holistic understanding of the system's performance and its optimization potential is achieved. This joint model enables decision makers to strike a balance between cost reduction and safety enhancement, ultimately driving sustainable and efficient operations.

The application of the proposed cost optimization procedure (Gouldby et al., 2010; Habibullah et al., 2009; Kołowrocki & Magryta, 2020a, 2020c; Kołowrocki et al., 2016; Lauge et al., 2015; Magryta-Mut, 2022) extends beyond theoretical realms. Real-world example, such as port oil terminals exemplify the practical significance of this research. These complex technical systems, subject to varying operation and safety states, demand sophisticated strategies for cost reduction without compromising safety standards. The findings of this study provide valuable in-sights and practical guidelines for industries operating within such critical infrastructures. By fostering further research and development, this study contributes to the advancement of efficient and economically sustainable industrial practices in the face of complex multistate ageing technical systems.

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