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INFLUENCE OF THE NUMBER OF MEASURED PARCEL BOUNDARY POINTS ON THE ACCURACY OF LAND PARCEL AREA CALCULATION*

ANALIZA WPŁYWU LICZBY MIERZONYCH PUNKTÓW GRANICZNYCH DZIAŁKI NA DOKŁADNOŚĆ OKREŚLANIA JEJ POLA POWIERZCHNI

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SUMMARY: The question which the research attempted to answer was to what degree the measurement of parcel boundary points influences the error in the calculation of the area of each land parcel. The field measurements relied on continuous GPS measurements of land parcels with various recording intervals. The accuracy of the simulated area as calculated from Gauss' formula increases with an increase in the number of boundary points. The experiment of using continuous measurement by GPS did not confirm the accuracy of the simulated area in all the point number ranges. The largest difference was observed where there was a small percentage of measurement points, where the prognosis error was much higher (almost two times) than the error obtained during measurement. The relationship between the number of points and accuracy of the area measurement was also compared with the literature. The results of our research partly confirmed results published in the literature with the main discrepancy being observed for a lower number of boundary points. In our research, area error increases with decreasing number of points; in the literature area error decreases with a decreasing number of boundary points. The explanation of this phenomenon requires further research especially as the field measurements from this research do not fully correlate with the simulations found in the literature.

1. INTRODUCTION

Global Positioning System (GPS) is applied in control procedures conducted in Integrated Administration and Control System (IACS). IACS is an information system used for decision support in EU agricultural financial subsidies. Each year farmers declare areas of crops. Some of declarations are checked in situ or using remote sensing data. Cultivated parcels are measured and their area is compared to the area declared by the farmer. There are some discrepancies allowed between the two values. It should not be greater than 5% according to the technical specification for parcel area measurements. Also, other tolerances are defined, depending on the measurements' technology. In this case tolerance is defined as a width of buffer around the parcel perimeter.

The permitted discrepancy between area from the cadastre and from the field measurements is usually also defined. In Poland technical specification G-5 is now obligatory (5). According to the regulation (G-5) discrepancy defined above can be calculated from following equation:

(1) $$\Delta P = 0.001P + 0.2\sqrt{P}$$

where: P – parcel area.

There are also other approaches, basing on empirical equations:

(2) $$\Delta P = 0.4\sqrt{2P} + \sqrt{\frac{1+K^2}{2K}}$$

(3) $$\Delta P = 2(0.002P + 0.2\sqrt{P})$$

(4) $$\Delta P = 0.001P + 0.0002M\sqrt{P}$$

where:
P – parcel area
K – parcel elongation coefficient (ratio of the length of long to short side of the parcel)
M – denominator of map scale.

There is however a problem – the equation used for cadastre purposes (1) is very strict and could be used in the case where parcel vertexes are marked in the field by stones. Otherwise obtaining such a high accuracy is not possible because of difficulties in parcel border definition in the field. Equations (1-4) have empirical not analytical backgrounds. Nowadays parcel areas are still calculated in an analytic way, usually from the Gauss' formula on the bases of vertex coordinates. In this case the question is raised: why not to calculate the area error strictly from analytical equations? Based on our research (Hejmanowska, 2003, eq. 5) and some approach from literature (Bogaert *et. al.*, 2005, eq. 6) parcel area error (m_P, σ_S) can be calculated from an analytical equation as follows:

(5) $$m_P = m_{pkt}\sqrt{\sum_{i=1}^{n}\frac{(y_{i+1}-y_i)^2+(x_{i-1}-x_{i+1})^2}{8}}$$

where:
m_{pkt} – point position error,
x_i, x_j, y_i, y_j – vertex coordinates in Cartesian coordinate system,

$$\sigma_S^2 = \frac{1}{2}\sigma_\varepsilon^2\left(\sum_{i=1}^{n}\left(r_i^2 - r_i r_{i+2}\cos\left(\alpha_{i+2} - \alpha_i\right)\right)\right) \tag{6}$$

where:
σ_S – standard deviation of parcel area,
σ_ε – standard deviation of point position,
r_i, α_i – vertex coordinate in polar coordinate system (radius, angle).

The above two equations were tested on the basis of GPS measurements and using parcels obtained from ortho-photomaps (Hejmanowska *et al.,* 2005) and have given the same results. Some inconvenience appears occasionally in eq. 5 for small angles, as the results are unexpectedly high.

Three components: parcel area, point position error and number of vertexes influence the parcel area error. The first two components are evident but the influence of the third one on area error can be discussed. Each vertex is measured during normal surveying, for example by Total Station. Vertex is understood as a point where the change appears in parcel border direction. Any points where parcel border direction does not change are not included, so parcel border is defined by the minimum amount of measurements points (vertexes). However from standalone GPS measurements parcel area is defined by much more points on the border than normally obtained from tachymeter surveying. During GPS measurements operator walks or moves using some vehicle along the parcel border. GPS measures point position with assumed frequency. Some tests were performed with different speeds (v) of the operator moving along the parcel border (Bogaert *et al.*, 2005; Fig. 1; Fig. 2). As the speed of the operator increases the amount of points on the parcel border decreases (Fig. 1). There are many points on the border obtained with a speed of 0.25 m/s (Fig. 1, upper left) compared with a speed of 10 m/s (Fig. 1, bottom right). In the first case, variation of point position along the parcel border is high but the random errors do not influence the area because of the averaging process. In the last case, the averaging does not exist and some systematic error (shifts) can be noticed. In two cases area error is less than in the other, intermediate ones. If we have many points (v tends to 0) even that the variance of point position is high, area variance will drop to 0. On the other hand if speed of the operator is very high (v tends to ∞) only four corners area are measured. We can observe correlation of point position measurements and systematic error but it does not influence parcel area, variance of parcel area will be small. Based on the simulation parcel area variances tend to zero if the speed of operator tends to 0 or to infinity (Fig. 2).

Both formulas for parcel area error calculation are derived assuming lack of the cross-correlation between X and Y coordinates. Point position from GPS is influenced by ephemeris models, satellite clock drifts and ionospheric modelling, that may cause systematic errors. However it changes in time, if measurements are made quickly, there is a systematic point position error that does not influence parcel area. If the measurement time is long enough we observe lack of X, Y cross-correlation (Bogaert *et al.*, 2005).

Results presented in the paper (Bogaert *et al.*, 2005, Fig. 1, Fig. 2) are derived based on computer simulations. It encouraged us to test them in the field, so we performed a series of experiments on the football field (Woźniak, 2009).

We defined the following two questions:

– How does the difference in number of points influence the parcel area error?
– Is it possible to predict parcel area error in an analytical way (eq. 5 and 6)?

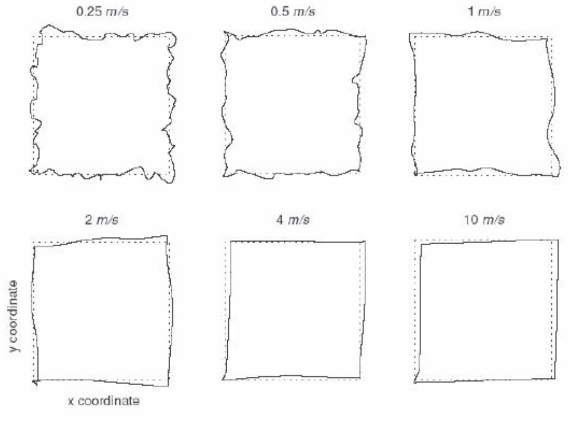

Fig. 1. Simulation of sequentially measured coordinates along the borders of a 100x 100 m parcel for an operator moving at a constant speed specified on the figures (in m*s-1) with a frequency of 1 measurement/s (Bogaert *et al.*, 2005).

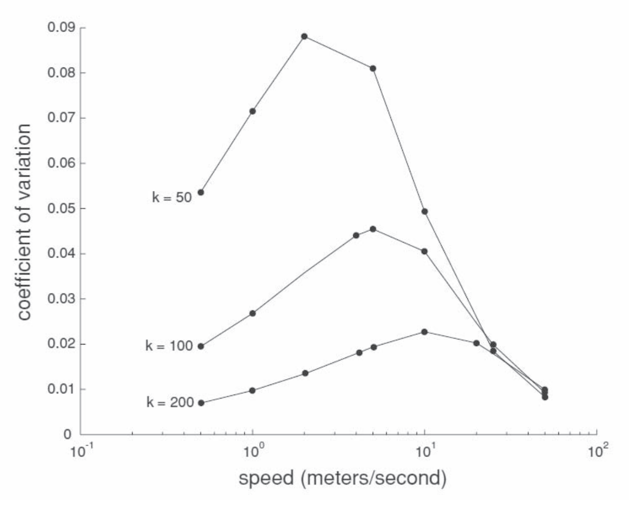

Fig. 2. Evolution of the coefficient of variation CVs for the area estimation of a k x k m parcel as function of the speed of an operator moving along the borders (in m*s-1); the operator is assumed to move at constant speed, with frequency of 1 measurement/s (Bogaert *et. al.*, 2005).

2. METHODOLOGY

A football pitch, as a test field (Fig. 3), was measured using total station and stand alone GPS with different point registration speed (distance between neighbouring points on the border: respectively 1, 2, 3, 4, 5 m, Fig. 4). Also, area of the football pitch was calculated from GPS measurements performed on only 4 corners of the polygon. Different registration speeds lead to different quantities of points, each measurement gives slightly different amount of points; average values were 200, 110, 90, 70, 50 for intervals 1, 2, 3, 4 and 5 s, respectively.

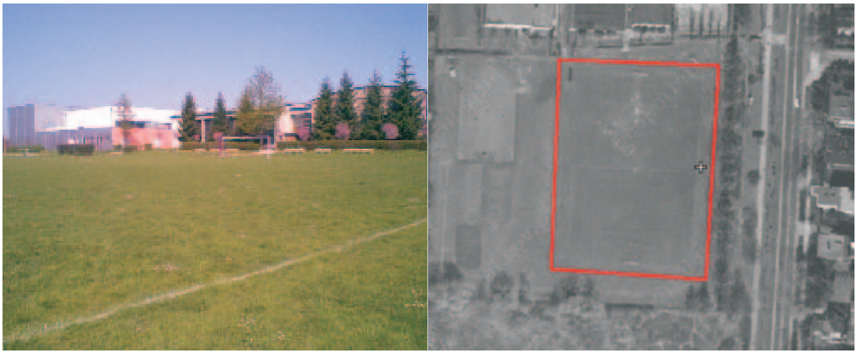

Fig. 3. Football pitch as a test field.

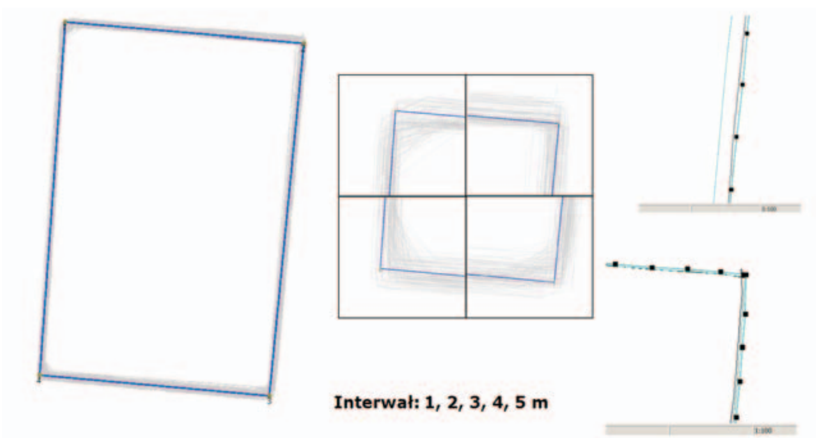

Fig. 4. GPS measurements of football pitch with different intervals of point registration, in the middle - zoom of the corners (see cut corners), on the right - example of point's position on the parcel border and also on the corner.

Accuracy of parcel area is the object of our research. As a reference area, the area calculated from total station measurement was assumed. Our test polygon was measured in 6 modes, with different registration speeds. Measurements with one mode were made 3 times. A set of 18 measurements (6 modes x 3 times) was made during one day. Since the experiment was performed over 3 days, we collected a total of 54 measurements (6 modes x 3 times x 3 days). Accuracy analysis were based on two kinds of relative discrepancies (errors - V_i) between areas calculated from GPS measurements (P_i) and:

– mean values of area measured in the experiment for each mode (P_r), variant 1,
– reference area (P_r), variant 2,

(7)
$$V_i = (P_i - P_r) / P_r$$

For the analysis, relative area error was taken into account. The following values of the area error, characterising this discrepancies, were calculated:

– mean error (ME)
– standard deviation (SD)
– root mean square (RMS)

(8)
$$ME = \frac{\sum_{i=1}^{n} V_i}{n}$$

(9)
$$SD = \sqrt{\frac{\sum (V_i - ME)^2}{n-1}}$$

(10)
$$RMS = \sqrt{\frac{\sum_{i}^{n} V_i V_i}{n-1}}$$

where: n – number of measurements in one mode.

Influence of the number of measured points on the area error was analysed on the charts. Finally, courses of the curves from our research were compared to the curves from the literature on Fig. 2.

3. RESULTS

On the charts below relationships between area error and numbers of points on the parcel border are presented. Influence of number of points on the area error is presented on Fig. 5, assuming as a reference mean value of the area (variant 1). Mean error (ME) is almost equal to 0 for all numbers of points, and standard deviation (SD) decreases from 0.7% to slightly more than 0.2%. In the case when we assumed measurements by total station as reference (variant 2), the relationship between error and number of points is something else because of consideration of mean error (Fig. 6). Mean error (ME), understood as a bias or systematic error varies from about -0.4 to 0.2%. Standard deviation (SD) in this case decreases from 0.8 to 0.2% and the course of the curve is more flat in comparison to the one on Fig. 5.

Mean error (ME) and standard deviation (SD) could be expressed together as root mean square (RMS). On the chart (Fig. 7) two lines are also added for comparison of our results with allowed discrepancy according to technical specifications. One of them shows the limit of area error (for parcel with the area equal to the area of the football pitch) calculated on the basis of now obligatory regulation G-5 (eq. 1). Only the measurements with registration of 1 second fulfil the requirements of regulation G-5. The second one shows value of area error limit obtained from less rigorous regulation, (eq. 3), obligatory in the past (published in 1998). In this case measurements of all modes would be allowed.

Root mean square (RMS) of the measurements is shown on Fig. 8. Additionally parcel area error, calculated from the eq. 5, assuming point position error of 1m was also presented on the diagram. Runs of the two curves (modelled and measured) vary.

Area error modelling using eq. 5 overestimates the error for a number of points, less than 50 points. Slightly underestimation could be observed for more than 50 points with registration more quickly than each 5 s.

Speed of GPS signal registration influences the accuracy of calculated area. The relationship is not recognised well enough. On the chart Fig. 8 vertical line shows numbers of points corresponding to the speed marked on Fig. 9. Notice that on Fig. 8 amount of points increases to the right (x-axis), and corresponding speed of GPS signal registration decreases, and on Fig. 9 the opposite. Firstly, compare curve courses on the two charts starting from the vertical line – on the right at Fig. 8 and on the left at Fig. 9. The error decreases in both cases and the curve courses look more or less similar. However the phenomenon

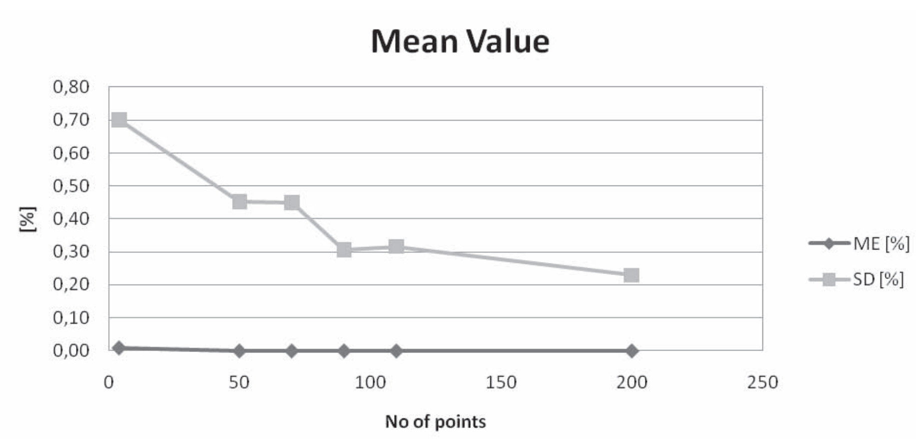

Fig. 5. Relationship between mean error (ME), standard deviation (SD) and number of points on the parcel area (variant 1).

changes on the opposite side of the vertical line. On the Fig. 8 (left from the vertical line), error increases continuously with decreasing of number of points. On the figure Fig. 9 (right from the vertical line), there is an inflection point on the error curve where the curve courses are changing, increasing speed (decreasing number of points) causes decreasing of the error.

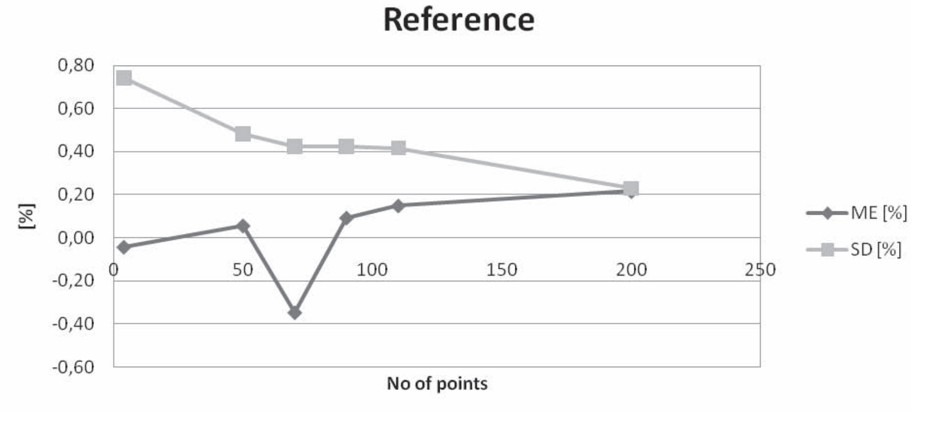

Fig. 6. Relationship between mean error (ME), standard deviation (SD) and number of points on the parcel area (variant 2).

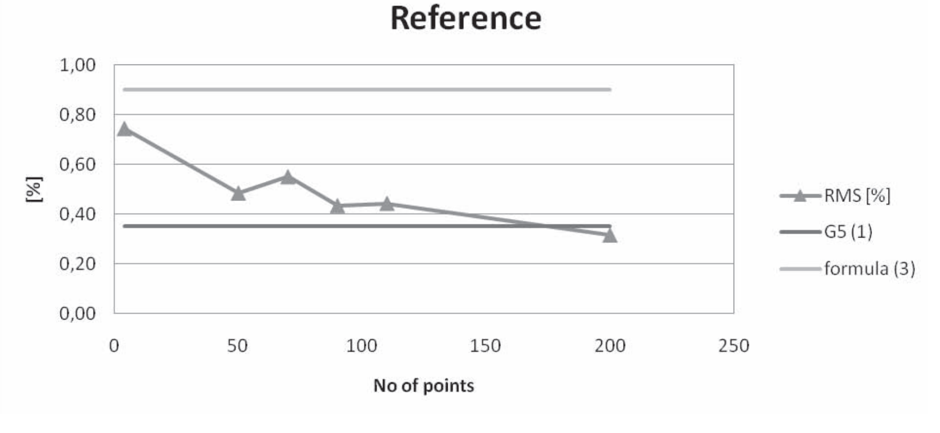

Fig. 7. Relationship between root mean square (RMS) and number of points on the parcel area; line presented allowed errors according technical specification G-5 (eq. 1) and eq. 3.

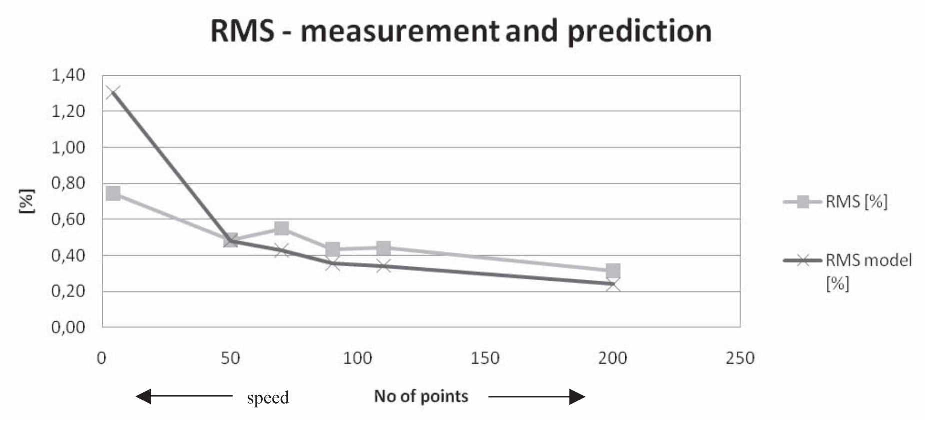

Fig. 8. Relationship between root mean square (RMS) from GPS measurements, RMS predicted from the model (eq. 4) and number of points on the parcel area; vertical line shows speed on Fig. 9.

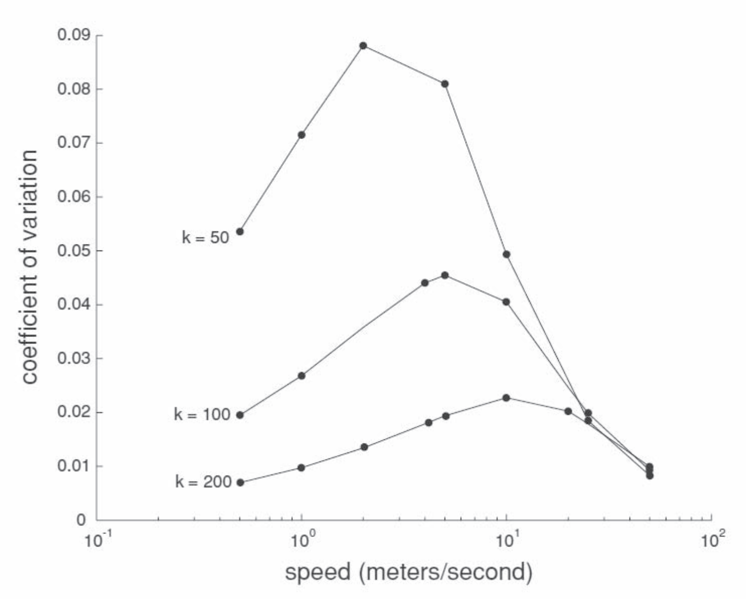

Fig. 9. Relationship between coefficient of variation and speed of point registration; vertical line shows respectively number of points on Fig. 8 (modified from Bogaert *et al*., 2005).

4. CONCLUSIONS

In our research we have tested the influence of numbers of points defining the parcel border on the parcel area error. Different numbers of points can be caused by the decreasing of operator speed (with constant speed of GPS registration) or by different GPS speed registration with constant operator speed. The two cases are not identical but similar; the experiments however can give also the similar amount of points. Generally the more points on the parcel border the better, and parcel error decreasing. We have not obtained a clear answer about reducing the amount of registered on the parcel border points.

The explanation stated in the introduction is logic that in the case of only 4 corners measured we can expect correlation between measurements causing the small error of parcel area, but we have not confirmed it in the field. To determine the inconsistency new measurements are needed possibly in the same conditions as in the two above experiments.

Some discrepancies between errors obtained based on the field measurements and modelled from formula (5) should be noticed (especially for a small number of points). So in our opinion this registration conditions: speed of operator more than 10 m/s and registration frequencies less than 1 point/5 s should be taken into account in the future. Increasing speed of operator more than 10 m/s and register with frequencies less than 1point/5 s is not so realistic like corner GPS measurement, which should be treated as an especially case. However experiments should cover all ranges of the values of operator speed or point registration frequency to recognition of the discussed phenomenon.

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SŁOWA KLUCZOWE: ciągły pomiar GPS, błąd powierzchni działki, IACS

STRESZCZENIE: Przeprowadzone badania miały odpowiedzieć na następujące pytanie: jaki jest wpływ liczby punktów pomierzonych na granicy działki na jej błąd powierzchni? Pomiary polowe metodą ciągłej rejestracji z wykorzystaniem ręcznego odbiornika GPS zostały przeprowadzone z różną częstotliwością rejestracji. Symulowany ze wzoru Gaussa błąd powierzchni maleje wraz ze wzrostem liczby punktów. Jednakże przeprowadzony eksperyment nie potwierdził modelowanej dokładności w całym zakresie liczby punktów. Największa różnica wystąpiła dla małej liczby punktów, gdzie prognozowana wartość błędu znacznie, niemal dwukrotnie, przekroczyła otrzymaną eksperymentalnie wartość błędu. Zależność pomiędzy liczbą zarejestrowanych punktów a błędem powierzchni została porównana z wynikami z literatury. Główna rozbieżność pojawiła się również w zakresie małej liczby punktów. Zgodnie z naszymi badaniami zmniejszanie liczby punktów powoduje zwiększanie się błędu powierzchni, podczas gdy na podstawie wyników zamieszczonych w literaturze można wyciągnąć wniosek, że powoduje to zmniejszanie się błędu powierzchni. Wyjaśnienie tego zjawiska wymaga przeprowadzenia dalszych badań, ponieważ nasze pomiary polowe nie w pełni odpowiadały warunkom symulacji z literatury.

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