Power Electronics and Drives
Volume 5(40), 2020 DOI: 10.2478/pead-2020-0003

sciendo

Power Electronics and Drives

Analysis of Transients in a 4-Level Flying Capacitor Converter: Time Domain Approach. Part 2: Small Normalised Voltage Command

Research Article

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Received: October 03, 2019; Accepted: January 05, 2020

Abstract: A 4-level flying capacitor converter (FCC) operation is considered on a base of discrete state-space model. A transition matrix is obtained for a pulse width modulation (PWM) period for small normalised voltage command values [0, 1/3]. The transition matrix elements are expanded into power series by small parameters. The matrix eigenvalues are presented in the form of power series as well. Six separate transients are constructed for six possible initial FCC states on a PWM period. Inductor current and capacitors' voltage transients are found for the voltage source power-up as the arithmetic average of the six separate transients. Finally, the discrete solutions are replaced by equivalent continuous ones. Simple and accurate formulas for inductor current and capacitors' voltage transients demonstrate good agreement with simulation results.

Keywords: *4-level DC–DC converter • flying capacitor • natural balancing • transients*

1. Introduction

The first part of analytical study of transients in a 4-level DC–DC flying capacitor converter (FCC) for large reference voltages D is presented in Reznikov et al. (2019 submitted to). In that study, the approach described in details is used for transient matrix calculation with elements as series by powers of small parameter, calculation of eigenvalues and eigenvectors of this matrix and, finally, calculation of transients of the converter. As a result, simple and accurate formulas were obtained for $1/3 \le D \le 1$. In this paper, the analytical study is continued for a range of small D ($0 \le D \le 1/3$), thus completing the analytical description of 4-level DC–DC FCC. As in Reznikov et al. (2019 submitted to), the time-domain approach will be taken as a basis of analytical research. The theoretical essence of this approach was described in detail in Reznikov et al. (2019 submitted to). Therefore, here some explanations will be omitted with references to the previous papers provided instead. The goal of this paper as in Reznikov et al. (2019 submitted to) is to obtain simple and accurate formulas for average natural balancing transients in case of small D values as functions of FCC circuit and load parameters and also pulse width modulation (PWM) period.

2. Discrete Model Construction

Figure 1a presents a 4-level DC–DC FCC circuit. All the following analysis is based on the same assumptions accepted for a 4-level FCC in case of large D (Reznikov et al., 2019 submitted to), namely, the load is considered

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as a series connection of inductor and resistor, the bi-directional switches have zero resistance in the conductive state (ON) and infinite in the open-circuit one (OFF), and switching rise and fall times equal zero.

Just as in the case of large D (Reznikov et al., 2019 submitted to), the FCC operation will be analysed only for positive D values due to the symmetry considerations. 4-level FCC balancing dynamics depend on modulation strategy. The classic realisation of carrier-based PWM is described in detail in Reznikov and Ruderman (2009).

The modulation strategy is determined by the switching algorithm of the complementary switch pairs $S1-\overline{S}1, S2-\overline{S}2$ and $S3-\overline{S}3$. This algorithm is the same for both ranges of large and small D. A PWM period T_{PWM} is divided into six successive time intervals $\Delta t_1, \Delta t_2, \Delta t_3, \Delta t_4, \Delta t_5, \Delta t_6$. The intervals with even (odd) indices have the same duration, that is $\Delta t_2 = \Delta t_4 = \Delta t_6$ ($\Delta t_1 = \Delta t_3 = \Delta t_5$). Each interval index corresponds to its own equivalent circuit (topology) with the number equal to the interval index. The topologies with odd numbers are presented in Figure 1b and those with even ones in Figure 1c. The difference from the case of large D is in the fact that, instead of one topology for all even indices, three different topologies participate in FCC operation in the case of small D.

The topologies are defined by the following switch states. In Topology 1, switches S2 and S3 are ON, and switch S1 is OFF. In Topology 3, switches S1 and S3 are ON, and switch S2 is OFF. In Topology 5, switches S1 and S2 are ON, and switch S3 is OFF. In Topology 2, switch S3 is ON, and switches S1 and S2 are OFF. In Topology 4, switch S1 is ON, and switches S2 and S3 are OFF. In Topology 6, switch S2 is ON, and switches S1 and S3 are OFF.

Similar to the large D case, space vector $X(t) = \left(i_L(t)\ v_{C_1}(t)\ v_{C_2}(t)\right)^T$ is introduced and the matrix relation for arbitrary time interval Δt_k is written as

$$X\left(t+\Delta t_k\right) = A_k\left(\Delta t_k\right)X\left(t\right) + B_k\left(\Delta t_k\right)V/2 \tag{1}$$

where matrices A_k and vectors B_k relate to their corresponding topologies and time interval durations.

Associating subsequently equation (1) with each of six PWM period T_{PWM} intervals and using the continuation of vector X coordinates at the switching moments lead to

$$X^{(1)}\left(t+T_{PWM}\right) = A^{(1)}X^{(1)}\left(t\right) + B^{(1)}V/2 \tag{2}$$

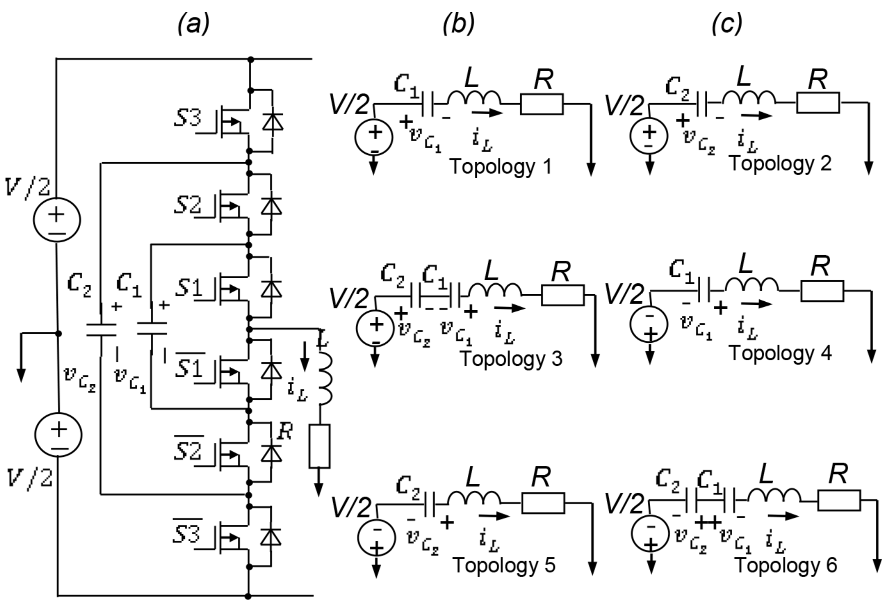

Fig. 1. 4-level FCC circuit (a) and its switched topologies for the small D case (b and c).

where transient matrix $A^{(1)}$ and vector $B^{(1)}$ are defined as follows:

$$A^{(1)} = A_6 A_5 A_4 A_3 A_2 A_1 \tag{3}$$

$$B^{(1)} = A_6\left(A_5\left(A_4(A_3\left(A_2 B_1 + B_2\right) + B_3\right) + B_4\right) + B_5) + B_6 \tag{4}$$

Equations (3) and (4) are written down assuming that the PWM period starts with the time interval 1 as pointed by the superscript in parentheses.

Similar to the large D range, the proposed analysis ignores the FCC oscillating behaviour inside the PWM period and is aimed at describing the average behaviour in "long" discrete time $t_k = kT_{PWM.}$ In this way, equation (2) may be rewritten as

$$X^{(1)}\left(t_{k+1}\right) = A^{(1)} X^{(1)}\left(t_k\right) + B^{(1)} V/2, \;\; t_{k+1} = t_k + T_{PWM} \tag{5}$$

As shown in Reznikov and Ruderman (2009), the time interval durations for small D become

$$\Delta t_1 = \Delta t_3 = \Delta t_5 = \frac{1/3 + \left|D\right|}{2} T_{\text{PWM}} \tag{6}$$

$$\Delta t_2 = \Delta t_4 = \Delta t_6 = \frac{1/3 - \left|D\right|}{2} T_{\text{PWM}} \tag{7}$$

Let $\omega_1 = \sqrt{(1/LC_1) - \alpha^2}$, $\omega_2 = \sqrt{(1/LC_2) - \alpha^2}$ and $\omega_3 = \sqrt{((C_2 + C_1)/LC_2C_1) - \alpha^2}$ with $\alpha = R/(2L)$ be the oscillation frequencies for topologies 1 and 4, 5 and 2 and 3 and 6, correspondingly. Then, using the reasoning similar to that for large D (Reznikov et al., 2019 submitted to), in order to simplify the calculations, let us introduce the small parameter $\beta = (\omega_1(1/3 + |D|)/2)T_{\text{PWM}}$. As, due to the symmetry considerations, only positive D is considered, D is used instead of its absolute value in the following. Just as in the case of large D, let us use the following designations: $s\_(\omega) = \cos\omega\tau - (\alpha/\omega)\sin\omega\tau$, $s_+(\omega) = \cos\omega\tau + (\alpha/\omega)\sin\omega\tau$, $r_c = C_1/C_2$. Simple transformations lead to $\omega_2 = k_1\omega_1$ and $\omega_3 = k_2\omega_1$, where $k_1 = \sqrt{r_c(1+r^2) - r^2}$ and $k_2 = \sqrt{r_c(1+r^2) + 1}$.

Next, denote $w\_(k,\tau) = \cos k\tau\beta - (r/k)\sin k\tau\beta$ and $w_+(k,\tau) = \cos k\tau\beta + (r/k)\sin k\tau\beta$. Then, the above designations allow to compactly represent matrices A_k and vectors A_k as functions of β:

$$A_1 = e^{-r\beta}\begin{pmatrix} w\_(1,1) & -\dfrac{\sin\beta}{\omega_1 L} & 0 \\ \omega_1 L(1+r^2)\sin\beta & w_+(1,1) & 0 \\ 0 & 0 & e^{r\beta} \end{pmatrix}$$

$$A_3 = e^{-r\beta}\begin{pmatrix} w\_(k_2,1) & \dfrac{\sin k_2\beta}{k_1\omega_1 L} & -\dfrac{\sin k_2\beta}{k_1\omega_1 L} \\ -\dfrac{\omega_1}{k_1}L(1+r^2)\sin k_2\beta & \dfrac{w_+(k_2,1) + r_c e^{r\beta}}{1+r_c} & \dfrac{-w_+(k_2,1) + e^{r\beta}}{1+r_c} \\ \dfrac{\omega_1 r_c}{k_1}L(1+r^2)\sin k_2\beta & r_c\dfrac{-w_+(k_2,1) + e^{r\beta}}{1+r_c} & \dfrac{r_c w_+(k_2,1) + e^{r\beta}}{1+r_c} \end{pmatrix}$$

$$A_5 = e^{-r\beta}\begin{pmatrix} w_{-}(k_1,1) & 0 & \dfrac{\sin k_1\beta}{k_1\omega_1 L} \\ 0 & e^{r\beta} & 0 \\ \dfrac{-\omega_1 r_c}{k_1}L(1+r^2)\sin k_1\beta & 0 & w_{+}(k_1,1) \end{pmatrix}$$

$$A_2 = e^{-r\beta k_t}\begin{pmatrix} w_{-}(k_1,k_t) & 0 & -\dfrac{\sin k_1 k_t\beta}{\omega_1 L} \\ 0 & \exp(r\beta k_t) & 0 \\ \dfrac{\omega_1 L r_c(1+r^2)\sin k_1 k_t\beta}{k_1} & 0 & w_{+}(k_1,k_t) \end{pmatrix}$$

$$A_4 = e^{-r\beta k_t}\begin{pmatrix} w_{-}(1,k_t) & 0 & -\dfrac{\sin k_t\beta}{\omega_1 L} \\ \omega_1 L(1+r^2)\sin k_t\beta & w_{+}(1,k_t) & 0 \\ 0 & 0 & e^{r\beta k_t} \end{pmatrix}$$

$$A_6 = e^{-r\beta}\begin{pmatrix} w_{-}(k_2,k_t) & -\dfrac{\sin k_2 k_t\beta}{k_2\omega_1 L} & -\dfrac{\sin k_2 k_t\beta}{k_2\omega_1 L} \\ \dfrac{\omega_1}{k_2}L(1+r^2)\sin k_2 k_t\beta & \dfrac{w_{+}(k_2,k_t)+r_c e^{r\beta}}{1+r_c} & \dfrac{-w_{+}(k_2,k_t)+e^{r\beta}}{1+r_c} \\ \dfrac{-\omega_1 r_c}{k_2}L(1+r^2)\sin k_2 k_t\beta & r_c\dfrac{-w_{+}(k_2,k_t)+e^{r\beta}}{1+r_c} & \dfrac{r_c w_{+}(k_2,k_t)+e^{r\beta}}{1+r_c} \end{pmatrix}$$

$$B_1 = \begin{pmatrix} \dfrac{\exp(-r\beta)\sin\beta}{\omega L} \\ 1-\exp(-r\beta)\,w_{+}(1,1) \\ 0 \end{pmatrix}$$

$$B_3 = \begin{pmatrix} \dfrac{\exp(-r\beta)\sin(\beta k_2)}{\omega L k_2} \\ \dfrac{\exp(-r\beta)\,w_{+}(k_2,1)-1}{1+r_c} \\ \dfrac{1-\exp(-r\beta)\,w_{+}(k_2,1)}{1+r_c}r_c \end{pmatrix}$$

$$B_5 = \begin{pmatrix} -\dfrac{\exp(-r\beta)\sin(\beta k_1)}{\omega L k_1} \\ 0 \\ 1-\exp(-r\beta)\,w_{+}(k_1,1) \end{pmatrix}$$

$$B_2 = \begin{pmatrix} \dfrac{\exp(-r\beta k_t)\sin(\beta k_1 k_t)}{\omega L k_1} \\ 0 \\ 1-\exp(-r\beta)\, w_+(k_1, k_t) \end{pmatrix}$$

$$B_4 = \begin{pmatrix} -\dfrac{\exp(-r\beta k_t)\sin(\beta k_t)}{\omega L} \\ 1-\exp(-r\beta k_t)\, w_+(1, k_t) \\ 0 \end{pmatrix}$$

$$B_6 = \begin{pmatrix} -\dfrac{\exp(-r\beta k_t)\sin(\beta k_2 k_t)}{\omega L k_2} \\ \dfrac{\exp(-r\beta k_t)\, w_+(k_2, k_t)-1}{1+r_c} \\ \dfrac{1-\exp(-r\beta k_t)\, w_+(k_2, k_t)}{1+r_c} r_c \end{pmatrix}$$

3. Eigenvalues

Expansion of the matrix A_i elements into a series of β followed by their multiplication according to equation (3) and expansion of elements of the matrix product (3) in the form of series of β yield

$$A^{(1)} = \left\{ a_{ij}^{(1)} \right\}_{\substack{i=1 \\ j=1}}^{3} \tag{8}$$

$$a_{11}^{(1)} = 1 - \frac{12r\beta}{1+3D} + \frac{72r^2\beta^2}{(1+3D)^2} + \frac{4r\left((1+r^2)\left(27D^2 - 2r_c(9D-7)\right) - 211r^2 + 5\right)\beta^3}{3(1+3D)^3} + \ldots \tag{9}$$

$$a_{12}^{(1)} = \frac{24rD\beta^2}{\omega L(1+3D)^2} - \frac{3\left(r_c(1+r^2)(D-1)(3D^2-1) - 8r^2(3D^2-6D-1)\right)\beta^3}{\omega L(1+3D)^3} + \ldots \tag{10}$$

$$a_{13}^{(1)} = \frac{12r(1-D)\beta^2}{\omega L(1+3D)^2} - \frac{6\left(r^2(3D^3+6D^2-17D+10) + D(3D^2-1)\right)\beta^3}{\omega L(1+3D)^3} + \ldots \tag{11}$$

$$a_{21}^{(1)} = \frac{24\omega Lr(1+r^2)\beta^2}{(1+3D)^2} + \frac{3\omega L(1+r^2)\left(r_c(1+r^2)(3D^2-1)(D-1) - 8r^2(3D^2+6D-1)\right)\beta^3}{(1+3D)^3} + \ldots \tag{12}$$

$$a_{22}^{(1)} = 1 - \frac{4r(1+r^2)(27D^2+5)\beta^3}{3(1+3D)^3} + \ldots \tag{13}$$

$$a_{23}^{(1)}=-\frac{3\left(1+r^2\right)\left(3D^2-1\right)\beta^2}{(1+3D)^2}+\frac{2r\left(1+r^2\right)\left(27D^2-36D+5\right)\beta^3}{3(1+3D)^3}+\ldots \tag{14}$$

$$a_{31}^{(1)}=-\frac{12\omega Lr_c r\left(1+r^2\right)\left(D-1\right)\beta^2}{(1+3D)^2}+\frac{6\omega Lr_c\left(1+r^2\right)\left(r^2\left(3D^3+6D^2+7D-14\right)+3D\left(D^2-1\right)\right)\beta^3}{(1+3D)^3}+\ldots \tag{15}$$

$$a_{32}^{(1)}=\frac{3r_c\left(1+r^2\right)\left(3D^2-1\right)\beta^2}{(1+3D)^2}+\frac{2r\left(1+r^2\right)r_c\left(27D^2-36D+5\right)\beta^3}{3(1+3D)^3}+\ldots \tag{16}$$

$$a_{33}^{(1)}=1+\frac{8r\left(1+r^2\right)r_c\left(9D-7\right)\beta^3}{3(1+3D)^3}+\ldots \tag{17}$$

Matrix $A^{(1)}$ characteristic polynomial is found as $p(\lambda)=\det\left(A^{(1)}-\lambda E\right)$, where E is a 3 × 3 unity matrix. The polynomial roots are matrix $A^{(1)}$ eigenvalues. As noted in Reznikov et al. (2019 submitted to), all other matrices $A^{(i)}$, $i = 1, 2, 3, 4, 5$, have the same eigenvalues because they are obtained by a cyclic permutation of $A^{(1)}$ factor matrices. The characteristic polynomial becomes

$$p\left(\lambda\right)=\lambda^3+g_1\lambda^2+g_2\lambda+g_3 \tag{18}$$

where the coefficient series expansions in β amount to

$$g_1=-3+\frac{12r\beta}{1+3D}-\frac{72r^2\beta^2}{\left(1+3D\right)^2}+\frac{288r^3\beta^3}{3\left(1+3D\right)^3}+\frac{(F_1r^4+F_2r^2+9r_c\left(3D^2-1\right)^2)\beta^4}{\left(1+3D\right)^4}+\ldots \tag{19}$$

$$g_2=3-\frac{24r\beta}{1+3D}+\frac{144r^2\beta^2}{\left(1+3D\right)^2}-\frac{576r^3\beta^3}{\left(1+3D\right)^3}-\frac{((F_1+864)r^4+F_2r^2+9r_c(3D^2-1)^2)\beta^4}{\left(1+3D\right)^4}+\ldots \tag{20}$$

$$g_3=-1+\frac{12r\beta}{1+3D}-\frac{72r^2\beta^2}{\left(1+3D\right)^2}+\frac{288r^3\beta^3}{\left(1+3D\right)^3}-\frac{864r^4\beta^4}{\left(1+3D\right)^4}+\ldots \tag{21}$$

with

$$F_1=81r_cD^4-198r_cD^2-144D^2+89r_c-784$$

$$F_2=162r_cD^4-252r_cD^2-144D^2+98r_c+80$$

Following the method described in details in Reznikov et al. (2019 submitted to), we present the polynomial roots as series expansions in β:

$$\lambda_i=u_0^{(i)}+u_1^{(i)}\beta+u_2^{(i)}\beta^2+u_3^{(i)}\beta^3+\ldots \tag{22}$$

with the superscript i equal to the root number. The convergence of the series (22) is justified by the same reasons as in the large D case (Reznikov et al., 2019 submitted to).

Now, substitute equation (22) into equation (18) and equate the series to zero:

$$p(\lambda_i) = c_0^{(i)} + c_1^{(i)}\beta + c_2^{(i)}\beta^2 + c_3^{(i)}\beta^3 + c_4^{(i)}\beta^4 + \ldots = 0 \tag{23}$$

By carrying out the same operations as for large D (Reznikov et al., 2019 submitted to) that is successively equating the coefficients $c_k^{(i)}$ $(k = 0, 1, 2,\ldots)$ to zero, one can obtain the values of $u_j^{(i)}$ $(j = 0,1,2,\ldots)$ for each of the three roots of characteristic polynomial (18). This calculation yields the following representations of the roots (or matrix $A^{(1)}$ eigenvalues):

$$\lambda_1 = 1 - \frac{12r\beta}{1+3D} + \frac{72r^2\beta^2}{(1+3D)^2} + \frac{4r((1+r^2)(9D^2-5)(1+r_c)+216r^2)\beta^3}{3(1+3D)^3} + O(\beta^4) \tag{24}$$

$$\lambda_{2,3} = 1 \mp j\frac{3(1+r^2)(1-3D^2)r_c\beta^2}{(1+3D)^2} + \frac{2r(1+r^2)(9D^2-5)(1+r_c)\beta^3}{3(1+3D)^3} + O(\beta^4) \tag{25}$$

The complex conjugate roots λ_2 and λ_3 can be presented in the exponential form:

$$\lambda_2 = M\exp(j\varphi), \;\; \lambda_3 = M\exp(-j\varphi) \tag{26}$$

where the module M and argument φ series expansions are

$$M = \sqrt{\mathrm{Re}^2(\lambda_2) + \mathrm{Im}^2(\lambda_2)} = 1 - \frac{2r(1+r^2)(1+r_c)(5-9D^2)}{3(1+3D)^3}\beta^3 + O(\beta^4) \tag{27}$$

$$\varphi = \left|\mathrm{arc}tg\left(\frac{\mathrm{Im}(\lambda_2)}{\mathrm{Re}(\lambda_2)}\right)\right| = \frac{3(1+r^2)\sqrt{r_c}(1-3D^2)}{(1+3D)^2}\beta^2 + O(\beta^4) \tag{28}$$

As follows from (24) and (27), the modules of all three roots are less than unity that means that the system is stable and its transients converge.

4. Partial Transients of Natural Balancing Dynamics

The three different eigenvalues λ_1, λ_2 and λ_3 define three eigenfunctions λ_1^k, λ_2^k and λ_3^k of the difference equations system (5) and accordingly three eigenvectors $\Gamma_1 = \lambda_1^k(\gamma_1^{(1)} \;\; \gamma_2^{(1)} \;\; \gamma_3^{(1)})^T$, $\Gamma_2 = \lambda_2^k(\gamma_1^{(2)} \;\; \gamma_2^{(2)} \;\; \gamma_3^{(2)})^T$ and $\Gamma_3 = \lambda_3^k(\gamma_1^{(3)} \;\; \gamma_2^{(3)} \;\; \gamma_3^{(3)})^T$. As the eigenvectors are defined up to a constant multiplier, one of their components can be chosen arbitrarily, for example, equal to 1. Select $\gamma_1^{(1)} = 1$, $\gamma_1^{(2)} = 1$ and $\gamma_1^{(3)} = 1$. Then, the second and third eigenvector components are found from the system of linear equations in the matrix form:

$$\begin{pmatrix} a_{11}^{(1)} - \lambda_i & a_{12}^{(1)} & a_{13}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} - \lambda_i & a_{23}^{(1)} \\ a_{31}^{(1)} & a_{32}^{(1)} & a_{33}^{(1)} - \lambda_i \end{pmatrix} \begin{pmatrix} 1 \\ \gamma_2^{(i)} \\ \gamma_3^{(i)} \end{pmatrix} = 0 \tag{29}$$

As the system (29) is linear dependent, any two equations can be used to find $\gamma_2^{(i)}$ and $\gamma_3^{(i)}$. Substituting λ_1, λ_2 and λ_3 in (29) and expanding the result into a series yields

$$\gamma_2^{(1)} = -\frac{2\omega LD}{1+3D}(1+r^2)\beta + \frac{2r(1+r^2)(3D^2-1)\omega L}{(1+3D)^2}\beta^2 + \ldots \tag{30}$$

$$\gamma_3^{(1)} = \frac{r_c\left(1+r^2\right)\omega L\left(D-1\right)}{1+3D}\beta - \frac{r_c r\left(1+r^2\right)\left(D-1\right)\omega L}{1+3D}\beta^2 + \ldots \tag{31}$$

$$\gamma_2^{(2)} = \frac{\omega L\left(1+3D\right)\left(2D - j\sqrt{r_c}\left(D-1\right)\right)}{r_c\left(1-D\right)^2 + 4D^2}\beta^{-1} + \frac{r\omega L\left(F1r + j\sqrt{r_c}F1i\right)}{9\left(3D^2-1\right)\left(r_c\left(1-D\right)^2 + 4D^2\right)^2} + \ldots \tag{32}$$

$$\gamma_3^{(2)} = \frac{-\omega L\left(1+3D\right)\left(r_c\left(D-1\right) + 2Dj\sqrt{r_c}\right)}{r_c\left(1-D\right)^2 + 4D^2}\beta^{-1} + \frac{r\omega L\left(F2r + 2j\sqrt{r_c}F2i\right)}{9\left(3D^2-1\right)\left(r_c\left(1-D\right)^2 + 4D^2\right)^2} + \ldots \tag{33}$$

$$\gamma_2^{(3)} = \frac{\omega L\left(1+3D\right)\left(2D + j\sqrt{r_c}\left(D-1\right)\right)}{r_c\left(1-D\right)^2 + 4D^2}\beta^{-1} - \frac{r\omega L\left(F1r + j\sqrt{r_c}F1i\right)}{9\left(3D^2-1\right)\left(r_c\left(1-D\right)^2 + 4D^2\right)^2} + \ldots \tag{34}$$

$$\gamma_3^{(3)} = \frac{\omega L\left(1+3D\right)\left(r_c\left(1-D\right) + 2Dj\sqrt{r_c}\right)}{r_c\left(1-D\right)^2 + 4D^2}\beta^{-1} + \frac{r\omega L\left(F2r - 2j\sqrt{r_c}F2i\right)}{9\left(3D^2-1\right)\left(r_c\left(1-D\right)^2 + 4D^2\right)^2} + \ldots \tag{35}$$

where

$$F1r = -2((1-D)^3(9D^2-5)r_c^2 - fr_1(D)r_c + 4D^2 fr_2(D))$$

$$F1i = fr_3(D)r_c + fr_4(D)$$

$$fr_1(x) = (1-x)(81x^5 - 18x^4 - 36x^3 - 14x^2 - x + 4)$$

$$fr_2\left(x\right) = 4x^2\left(81x^4 - 9x^3 - 45x^2 + 5x + 4\right)$$

$$fr_3\left(x\right) = \left(3x+1\right)\left(x-1\right)\left(9x^2 - 6x - 1\right)$$

$$fr_4\left(x\right) = 4x\left(27x^3 - 18x^2 - 15x + 8\right)$$

$$F2r = fr_5\left(D\right)r_c^2 + fr_6\left(D\right)r_c + 16D^3\left(9D^2 - 5\right)$$

$$F2i = fr_7\left(D\right)r_c + fr_8\left(D\right)$$

$$fr_5\left(x\right) = \left(x-1\right)^2\left(81x^4 - 18x^3 - 54x^2 - 2x + 9\right)$$

$$fr_6\left(x\right) = 4x\left(81x^5 - 117x^4 - 72x^3 + 86x^2 + 19x - 13\right)$$

$$fr_7(x) = (x-1)\left(81x^5 + 27x^4 - 36x^3 - 48x^2 - x + 9\right)$$

$$fr_8(x) = 4x^2\left(81x^4 - 36x^2 - 1\right)$$

The general solution of (5) is a linear combination of eigenvectors and can be written as

$$X^{(1)}(k) = Q_1\lambda_1^k\begin{pmatrix}1\\ \gamma_2^{(1)}\\ \gamma_3^{(1)}\end{pmatrix} + Q_2\lambda_2^k\begin{pmatrix}1\\ \gamma_2^{(2)}\\ \gamma_3^{(2)}\end{pmatrix} + Q_3\lambda_3^k\begin{pmatrix}1\\ \gamma_2^{(3)}\\ \gamma_3^{(3)}\end{pmatrix} \tag{36}$$

where $Q_{1,}$ Q_2 and Q_3 are arbitrary constants.

Now a particular solution of the homogeneous system (a solution for zero supply voltage) may be found. Suppose that for the initial time instant $k = 0$, the inductor current equals i_0 and the capacitors voltages $v1_0$ and $v2_0$. Then (36) can be represented as

$$\begin{pmatrix}i_0\\ v1_0\\ v2_0\end{pmatrix} = Q_1\begin{pmatrix}1\\ \gamma_2^{(1)}\\ \gamma_3^{(1)}\end{pmatrix} + Q_2\begin{pmatrix}1\\ \gamma_2^{(2)}\\ \gamma_3^{(2)}\end{pmatrix} + Q_3\begin{pmatrix}1\\ \gamma_2^{(3)}\\ \gamma_3^{(3)}\end{pmatrix} \tag{37}$$

Resolving equation (37) with respect to Q_1, Q_2 and Q_3 yields

$$Q_1 = i_0 - \frac{2Dv1_0 + (1-D)v2_0}{(1+3D)\omega L}\beta + O\left(\beta^2\right) \tag{38}$$

$$Q_2 = \left(\frac{D}{(1+3D)\omega L} - j\frac{(1-D)\sqrt{r_c}}{2(1+3D)\omega L}\right)v1_0\beta + \left(\frac{1-D}{2\omega L(1+3D)} + j\frac{D}{(1+3D)\omega L\sqrt{r_c}}\right)v2_0\beta + O\left(\beta^2\right) \tag{39}$$

$$Q_3 = \left(\frac{D}{(1+3D)\omega L} + j\frac{(1-D)\sqrt{r_c}}{2(1+3D)\omega L}\right)v1_0\beta + \left(\frac{1-D}{2\omega L(1+3D)} - j\frac{D}{(1+3D)\omega L\sqrt{r_c}}\right)v2_0\beta + O\left(\beta^2\right) \tag{40}$$

Let us represent the eigenfunctions in a trigonometric form using equation (26) for complex conjugate roots:

$$\lambda_{2,3}^k = M^k\left(\cos(k\varphi) \mp j\bullet\sin(k\varphi)\right) \tag{41}$$

Substituting equations (38), (39) and (40) into equation (36) and accounting for equation (41) yields

$$i_L(k) = F_{11}\lambda_1^k + F_{12}M^k\cos(k\varphi) + F_{13}M^k\sin(k\varphi) \tag{42}$$

$$v_{C_1}(k) = F_{21}\lambda_1^k + F_{22}M^k\cos(k\varphi) + F_{23}M^k\sin(k\varphi) \tag{43}$$

$$v_{C_2}(k) = F_{31}\lambda_1^k + F_{32}M^k\cos(k\varphi) + F_{33}M^k\sin(k\varphi) \tag{44}$$

where

$$F_{11}=i_0-\frac{2Dv1_0+\left(1-D\right)v2_0}{\left(1+3D\right)\omega L}\beta+O\left(\beta^2\right)$$

$$F_{12}=\frac{2D\beta}{\left(1+3D\right)\omega L}v1_0+\frac{\left(1-D\right)\beta}{\left(1+3D\right)\omega L}v2_0+O\left(\beta^2\right)$$

$$F_{13}=\frac{\left(D-1\right)\sqrt{r_c}}{\left(1+3D\right)\omega L}\beta v1_0+\frac{2D}{\left(1+3D\right)\omega L\sqrt{r_c}}\beta v2_0+O\left(\beta^2\right)$$

$$F_{21}=-\frac{2D\omega L}{1+3D}\left(1+r^2\right)\beta i_0+O\left(\beta^2\right)$$

$$F_{22}=\frac{2D\omega L}{1+3D}\left(1+r^2\right)\beta i_0+v_{C_1}\left(0\right)+O\left(\beta^2\right)$$

$$F_{23}=\frac{\left(1-D\right)\omega L\sqrt{r_c}}{1+3D}\left(1+r^2\right)\beta i_0+\frac{1}{\sqrt{r_c}}v2_0-\frac{2r\left(9D^2-5\right)\beta}{9\left(1+3D\right)\left(3D^2-1\right)\sqrt{r_c}}\left(\left(r_c-1\right)v1_0+v2_0\right)+O\left(\beta^2\right),$$

$$F_{31}=\frac{\left(D-1\right)\omega Lr_c}{1+3D}\left(1+r^2\right)\beta i_0+O\left(\beta^2\right)$$

$$F_{32}=\frac{\left(1-D\right)\omega Lr_c}{1+3D}\left(1+r^2\right)\beta i_0+v2_0+O\left(\beta^2\right)$$

$$F_{33}==-\frac{2D\omega L\sqrt{r_c}\left(1+r^2\right)\beta}{1+3D}i_0-\sqrt{r_c}v1_0+\frac{2r\left(3D+1\right)\left(9D^2-5\right)\beta}{9\left(1+3D\right)\left(3D^2-1\right)\sqrt{r_c}}\left(\left(1-r_c\right)v2_0+r_cv1_0\right)+O\left(\beta^2\right)$$

A particular solution of the non-homogeneous system of equations or, in other words, a partial transient process may be obtained if the steady-state values of the inductor current and capacitor voltages are known. These values exist because, as noted above, the system is stable.

To obtain the steady-state values, first, find vector $B^{(1)}$ according to equation (4). By accomplishing the operations in equation (4) and making a series expansion, vector $B^{(1)}$ is found as

$$B^{(1)}=\begin{pmatrix}\frac{(-1+3D)}{\omega L(1+3D)}\beta-\frac{r(-1+3D)^2}{3\omega L(1+3D)^2}\beta^2+O(\beta^3)\\-\frac{(-1+3D)^2(1+r^2)}{2(1+3D)^2}\beta^2-\frac{r(1+r^2)(-1+3D)^3}{3(1+3D)^3}\beta^3+O(\beta^4)\\\frac{r_c(1+r^2)(-1+3D)^2}{2(1+3D)^2}\beta^2+\frac{r_cr(1+r^2)(-1+3D)^3}{3(1+3D)^3}\beta^3+O(\beta^4)\end{pmatrix}\qquad(45)$$

The steady-state values of the vector $X^{(1)}$ coordinates can be obtained from equation (36) for k striving to ∞. Let us denote the vector to be found as $X^{(1)}(\infty)$. As the partial transient tends to zero, for sufficiently large k, the vector does not vary and equals to $X^{(2)}(\infty)$.

$$X^{(1)}(\infty) = A^{(1)}X^{(1)}(\infty) + B^{(1)}V/2 \tag{46}$$

From equation (46), $X^{(1)}(\infty)$ is represented as

$$X^{(1)}(\infty) = \begin{pmatrix} i_L(\infty) \\ v_{C_1}(\infty) \\ v_{C_2}(\infty) \end{pmatrix} = \left(E - A^{(1)}\right)^{-1} B^{(1)}V/2 \tag{47}$$

Calculation of vector $X^{(1)}(\infty)$ from equation (47) followed by the power series expansion yields

$$X^{(1)}(\infty) = \begin{pmatrix} \frac{D}{R} + r\frac{3D-1}{3R}\beta + O(\beta^2) \\ \frac{2}{3} - \frac{(1+r^2)D^2}{(1+3d)r}\beta + O(\beta^2) \\ \frac{4}{3} + \frac{r_c(1+r^2)D(D-1)}{2(1+3d)r}\beta + O(\beta^2) \end{pmatrix} V/2 \tag{48}$$

The non-homogeneous system particular solution is obtained by the replacement of initial conditions – i_0 by $i_0 - i_L(\infty)$, $v1_0$ by $v1_0 - v_{C_1}(\infty)$ and $v2_0$ by $v2_0 - v_{C_2}(\infty)$ – in Q_1, Q_2 and Q_3 (formula (36)) and the addition of the term $X^{(1)}(\infty)$ on the right side.

Just as for large D, it is interesting to consider the FCC power-up for zero initial conditions. Corresponding calculations lead to the following expressions:

$$i_L(k) = i_L(\infty) + I_1\lambda_1^k + I_2M^k\cos(k\varphi) + I_3M^k\sin(k\varphi) \tag{49}$$

$$v_{C_1}(k) = v_{C_1}(\infty) + U_1\lambda_1^k + U_2M^k\cos(k\varphi) + U_3M^k\sin(k\varphi) \tag{50}$$

$$v_{C_2}(k) = v_{C_2}(\infty) + U_3\lambda_1^k + U_4M^k\cos(k\varphi) + U_5M^k\sin(k\varphi) \tag{51}$$

where up to small values of the first order

$$I_1 = \left(-\frac{D}{R} + \frac{3r(1-D^2)}{R(1+3D)}\beta\right)\frac{V}{2}$$

$$I_2 = -\frac{8r}{3R(1+3D)}\beta\frac{V}{2}$$

$$I_3 = \frac{4r\left(4D - r_c\left(1-D\right)\right)}{3R\sqrt{r_c}\left(1+3D\right)}\beta\frac{V}{2}$$

$$U_1 = \frac{\left(1+r^2\right)D^2}{r\left(1+3D\right)}\beta\frac{V}{2}$$

$$U_2 = -\frac{V}{3}$$

$$U_3 = \left(-\frac{4}{3\sqrt{r_c}} - \frac{4\left(9D^2-5\right)\left(r_c+1\right)r}{27\sqrt{r_c}\left(1-3D^2\right)\left(1+3D\right)}\beta\right)\frac{V}{2}$$

$$U_4 = r_c\frac{\left(1+r^2\right)D\left(1-D\right)}{2r\left(1+3D\right)}\beta\frac{V}{2}$$

$$U_5 = -\frac{2V}{3}$$

$$U_6 = \left(\frac{2\sqrt{r_c}}{3} - \frac{4\left(9D^2-5\right)r\left(r_c-2\right)}{27\sqrt{r_c}\left(1+3D\right)\left(1-3D^2\right)}\beta\right)\frac{V}{2}$$

and $i_L(\infty)$, $v_{C_1}(\infty)$ and $v_{C_2}(\infty)$ are taken from equation (48).

The obtained partial transient corresponds to the time interval 1 as the first one of the PWM period. Other five possible partial transients are calculated in a similar way using the cyclic permutations of the factor indices in formulas (3) and (4). The results of other partial transient calculations are represented in the following. Let us first define the auxiliary functions:

$$s_1\left(m\right) = \begin{cases} 1, & m = 1,\ 2,6 \\ -1, & m = 3,4,5 \end{cases}$$

$$s_2\left(m\right) = \begin{cases} m+4, & m = 1,\ 2 \\ m-2, & m = 3,4,5,6 \end{cases}$$

$$\mathrm{offs}_1\left(m\right) = \begin{cases} \dfrac{3\left(1-D^2\right)}{1+3D}, & m = 1,4 \\ \dfrac{\left(1-3D\right)\left(1-D\right)}{1+3D}, & m = 2,3 \\ 1+D, & m = 5,6 \end{cases}$$

$$\mathrm{offs}_2\left(m\right) = \begin{cases} \dfrac{2}{1+3D}, & m = 1,4 \\ \dfrac{1-3D}{1+3D}, & m = 2,3 \\ 1, & m = 5,6 \end{cases}$$

$$\mathrm{offs}_3(m)=\begin{cases}-4D+(-1)^{m+1}\left(1+(-1)^{m}D\right), & m=1,4\\ 2(-1)^{m+1}\left(1+(-1)^{m+1}D\right)r_c, & m=2,5\\ 2(-1)^{m+1}\left(1+(-1)^{m+1}D\right)+2rD, & m=3,6\end{cases}$$

$$\mathrm{offs}_4(m)=\begin{cases}D^2, & m=1,4\\ \dfrac{-D(1+D)}{2}, & m=2,3\\ \dfrac{D(1-D)}{2}, & m=5,6\end{cases}$$

Then

$$i_L^{(n)}(k)=i_L^{(n)}(\infty)+I_1^{(n)}\lambda_1^k+M^k(I_2^{(n)}\cos(k\varphi)+I_3^{(n)}\sin(k\varphi)) \tag{52}$$

$$v_{C_1}^{(n)}(k)=v_{C_1}^{(n)}(\infty)+U_1^{(n)}\lambda_1^k+M^k(U_2^{(n)}\cos(k\varphi)+U_3^{(n)}\sin(k\varphi)) \tag{53}$$

$$v_{C_2}^{(n)}(k)=v_{C_2}^{(n)}(\infty)+U_4^{(n)}\lambda_1^k+M^k(U_5^{(n)}\cos(k\varphi)+U_6^{(n)}\sin(k\varphi)) \tag{54}$$

where up to small values of the first order

$$I_1^{(n)}=\left(-\frac{D}{R}+r\frac{s_1(n)\mathrm{offs}_1(n)}{R}\beta\right)\frac{V}{2}$$

$$I_2^{(n)}=-4r\frac{\mathrm{offs}_2(n)s_1(n)}{3R}\beta\frac{V}{2}$$

$$I_3^{(n)}=4r\frac{\mathrm{offs}_3(n)}{3R\sqrt{r_c}(1+3D)}\beta\frac{V}{2}$$

$$U_1^{(n)}=\frac{(1+r^2)\mathrm{offs}_4(m)}{r(1+3D)}\beta\frac{V}{2}$$

$$U_2^{(n)}=-\frac{V}{3}$$

$$U_3^{(n)}=\left(-\frac{4}{3\sqrt{r_c}}-\frac{4(9D^2-5)(r_c+1)r}{27\sqrt{r_c}(1-3D^2)(1+3D)}\beta\right)\frac{V}{2}$$

$$U_4^{(n)}=\frac{r_c(1+r^2)\mathrm{offs}_4(s_2(n))}{r(1+3D)}\beta\frac{V}{2}$$

$$U_5^{(n)} = -\frac{2V}{3}$$

$$U_6^{(n)} = \left(\frac{2\sqrt{r_c}}{3} - \frac{4\left(9D^2-5\right)\left(r_c-2\right)r}{27\sqrt{r_c}\left(1-3D^2\right)\left(1+3D\right)}\beta \right)\frac{V}{2}$$

$$i_L^{(n)}(\infty) = \left(\frac{D}{R} + \frac{(-1)^{n-1} r(1-3D)}{3R}\beta \right)\frac{V}{2}$$

$$v_{C_1}^{(n)}(\infty) = \left(\frac{2}{3} - \frac{\left(1+r^2\right)\mathrm{off}s_4(n)}{r(1+3D)}\beta \right)\frac{V}{2}$$

$$v_{C_2}^{(n)}(\infty) = \left(\frac{4}{3} - \frac{r_c\left(1+r^2\right)\mathrm{off}s_4\left(s_2(n)\right)}{r(1+3D)}\beta \right)\frac{V}{2}$$

The process of partial transients continuation from the discrete points of time to the entire time axis is the same as for large D. It involves the replacement of functions λ^k by exponential terms with decay factors $\sigma = \ln(\lambda)/T_{\mathrm{PWM}}$ and trigonometric functions argument $k\varphi$ by Ωt, where $\Omega = \varphi/T_{\mathrm{PWM}}$. Substitution of λ_1 from equation (24) and M from equation (27) followed by the power series expansion yields the expressions (55) and (56):

$$\sigma_1 = \frac{1}{T_{\mathrm{PWM}}}\left(-\frac{12r}{1+3D}\beta - \frac{4r\left(1+r^2\right)\left(9D^2-5\right)\left(r_c+1\right)}{3\left(1+3D\right)^3}\beta^3 + O\left(\beta^4\right) \right) \tag{55}$$

$$\sigma_2 = \frac{1}{T_{\mathrm{PWM}}}\left(\frac{2r\left(1+r^2\right)\left(9D^2-5\right)\left(r_c+1\right)}{3\left(1+3D\right)^2}\beta^3 + O\left(\beta^4\right) \right). \tag{56}$$

5. Natural Balancing Dynamics

Now let us find the arithmetic mean of all six partial transients. In accordance with the definition of the FCC transient given in Reznikov et al. (2019 submitted to), this is the FCC transient. Carrying out these simple calculations and neglecting the small values of orders higher than first leads to the following expressions:

$$\bar{i}_L(t) = \bar{i}_L(\infty) + \bar{I}_1\exp\left(\sigma_1 t\right) + \exp\left(\sigma_2 t\right)\left(\bar{I}_2\cos\Omega t + \bar{I}_3\sin\Omega t\right) \tag{57}$$

$$\bar{v}_{C_1}(t) = \bar{v}_{C_1}(\infty) + \bar{U}_1\exp\left(\sigma_1 t\right) + \exp\left(\sigma_2 t\right)\left(\bar{U}_2\cos\Omega t + \bar{U}_3\sin\Omega t\right) \tag{58}$$

$$\bar{v}_{C_2}(t) = \bar{v}_{C_2}(\infty) + \bar{U}_4\exp\left(\sigma_1 t\right) + \exp\left(\sigma_2 t\right)\left(\bar{U}_5\cos\Omega t + \bar{U}_6\sin\Omega t\right) \tag{59}$$

where

$$\bar{i}_L(\infty) = \frac{D}{2R}V$$

$$\bar{I}_1 = -\frac{D}{2R}V$$

$$\bar{I}_2 = 0$$

$$\bar{I}_3 = 0$$

$$\bar{v}_{C_1}(\infty) = \frac{V}{3}$$

$$\bar{U}_1 = 0$$

$$\bar{U}_2 = -\frac{V}{3}$$

$$\bar{U}_3 = \left(-\frac{2}{3\sqrt{r_c}} - \frac{2r(r_c+1)(9D^2-5)\beta}{27\sqrt{r_c}(1+3D)(3D^2-1)}\right)V$$

$$\bar{v}_{C_2}(\infty) = \frac{2V}{3}$$

$$\bar{U}_4 = 0$$

$$\bar{U}_5 = -\frac{2V}{3}$$

$$\bar{U}_6 = \left(\frac{\sqrt{r_c}}{3} + \frac{2r(r_c-2)(9D^2-5)\beta}{27\sqrt{r_c}(1+3D)(3D^2-1)}\right)V$$

After substituting β, r_c and r values

$$\bar{i}_L(t) = \frac{D}{2R}V\left(1-\exp(\sigma_1 t)\right) \tag{60}$$

$$\bar{v}_{C_1}(t) = \frac{V}{3}\left(1-\exp(\sigma_2 t)\left(\cos\Omega t + \sqrt{\frac{C_2}{C_1}}\left(2+\frac{(C_1+C_2)(9D^2-5)RT_{\mathrm{PWM}}}{54LC_2(3D^2-1)}\right)\sin\Omega t\right)\right) \tag{61}$$

$$\bar{v}_{C_2}(t) = \frac{2V}{3}\left(1-\exp(\sigma_2 t)\left(\cos\Omega t - \sqrt{\frac{C_1}{C_2}}\left(\frac{1}{2}+\frac{(C_1-2C_2)(9D^2-5)RT_{\mathrm{PWM}}}{108LC_1(3D^2-1)}\right)\sin\Omega t\right)\right) \tag{62}$$

where

$$\sigma_1 = -\frac{R}{L}$$

$$\sigma_2 = \frac{RT_{\text{PWM}}^2 (C_1 + C_2)}{648 L^2 C_1 C_2} (9D^2 - 5)$$

$$\Omega = \frac{T_{\text{PWM}} (1 - 3D^2)}{12 L \sqrt{C_1 C_2}}$$

The values of σ_1 and σ_2 correspond to the time constants found in Reznikov and Ruderman (2009) (see formulas (20) and (24)).

6. Discussion

In Figure 2, the simulation results are compared with theoretical ones for set 1 of parameters: $L = 0.0004\,\text{H}$, $C_1 = 0.0002\,\text{F}$, $C_2 = 0.0001\,\text{F}$, $R = 1\Omega$, $T_{PWM} = 0.0001$ s, $D = 0.5$ and $V = 100$ V. In Figure 3, similar comparison is being made for set 2 of parameters: $L = 0.0001\,\text{H}$, $C_1 = 0.0006\,\text{F}$, $C_2 = 0.0004\,\text{F}$, $R = 0.6\Omega$, $T_{PWM} = 0.0005$ s, $D = 0.15$ and $V = 100$ V.

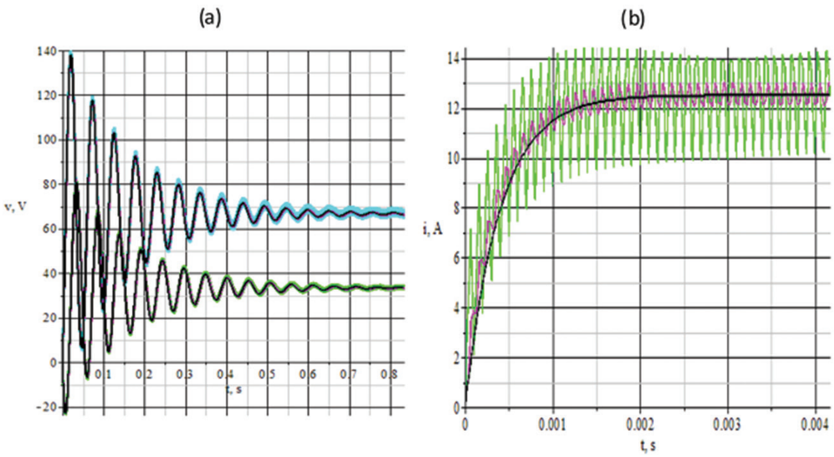

Fig. 2. The capacitor voltage transients (a) and the inductor current transients (b) for set 1.

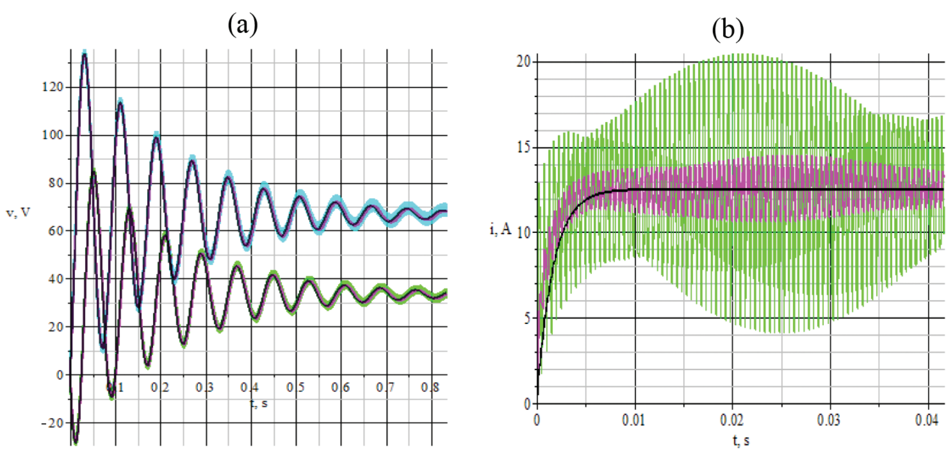

Fig. 3. The capacitor voltage transients (a) and the inductor current transients (b) for set 2.

Figures 2a and 3a show the graphs of the capacitor voltage simulation (the green curve for v_{C_1} and the light blue curve for v_{C_2}) and the transition process calculated by formulas (61) and (62) (the black curves). Figures 2b and 3b show similar curves for the inductor current. The black curve is calculated by formula (60). Magenta curves, which serve by references for comparison with theoretic results, on all the pictures correspond to the simulation curves filtered by a first-order low-pass filter with a time constant T_f. For the voltage curves, the time constant was selected as $T_f = T_{PWM}$ for the current curve – $T_f = T_{PWM}/2$. All the figures show good agreement between the simulation and calculation results.

Despite the overall third order of the system, the inductor current transient practically does not differ much from the pure exponential curve and the capacitor voltage transients – from the damped oscillating curves typical for second-order system transients. The capacitor voltage transients decay much slower in comparison with the inductor current one. For instance, for the set 1 of parameters, $\sigma_1 = -2500\text{s}^{-1}$ and $\sigma_2 = -6.42\text{s}^{-1}$. For the set 2 of parameters, $\sigma_1 = -600\text{s}^{-1}$ and $\sigma_2 = -4.627\text{s}^{-1}$. The average current steady-state value amounts to $\bar{i}_L(\infty) = VD/2R = 25A^{-1}$ for the set 1 and 12.5 A for the set 2. The same for the capacitor voltages: $\bar{v}_{C_1}(\infty) = V/3 = 33.3\,\text{V}$ and $\bar{v}_{C_2}(\infty) = 2V/3 = 66.7\,\text{V}$ for both sets. By inspection, the transients for large D values interval $1/3 \le D \le 1$ (Reznikov et al., 2019 submitted to) and for the small values one $0 \le D \le 1/3$ coincide for $D = 1/3$, which separates the two ranges. So, the sanity check for the continuity is an additional evidence of the correctness of theoretical calculations.

7. Conclusion

The article presents the time domain analysis of the four-level DC–DC FCC for small values of reference voltage D, $0 \le D \le 1/3$. Theoretical average balancing dynamics curves obtained by formulas (60)–(62) practically coincide with the accurate simulation ones with the fast “short” time oscillations averaged out by a low-pass filter. Simple theoretical formulas (60)–(62) accurately describe the averaged “long” time processes in the 4-level FCC for small D values and coincide with corresponding formulas for large D values at the separating point $D = 1/3$.

References

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