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Cluster consensus of general fractional-order nonlinear multi agent systems via adaptive sliding mode controller

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In this paper cluster consensus is investigated for general fractional-order multi agent systems with nonlinear dynamics via adaptive sliding mode controller. First, cluster consensus for fractional-order nonlinear multi agent systems with general form is investigated. Then, cluster consensus for the fractional-order nonlinear multi agent systems with first-order and general form dynamics is investigated by using adaptive sliding mode controller. Sufficient conditions for achieving cluster consensus for general fractional-order nonlinear multi agent systems are proved based on algebraic graph theory, Lyapunov stability theorem and Mittag-Leffler function. Finally, simulation examples are presented for first-order and general form multi agent systems, i.e. a single-link flexible joint manipulator which demonstrates the efficiency of the proposed adaptive controller.

Key words: nonlinear multi agent systems, cluster consensus, fractional-order systems, adaptive sliding mode controller

1. Introduction

There are many applications of multi agent systems such as formation [3], swarming, flocking and synchronization in physical and chemical systems. Hence, multi agent systems attract more attention recently. Consensus is a cooperation control mission for multi agent systems in a sense that all the states/outputs converge to the same value [21]. There are two categories for consensus problem, consensus without leader [18] and consensus tracking with leader in which all agents follow the leader [20].

Cluster consensus is different from complete consensus which means that agents are divided into a few clusters and these clusters may change in different

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situations and environments [22]. All the agents in the same cluster converge to a same attitude which is different from other clusters [29].

Most of the literatures about consensus have studied about multi agent systems with integer-order dynamic. Consensus problem for multi agent systems is studied in [15] for first-order dynamics, in [11] for second-order dynamics, and in [5] for higher-order dynamics. Many behaviors of agents can be better described with fractional-order dynamics in complex environments, such as traveling aircraft in rainy and dusty environment [2, 9]. The fractional-order dynamics include the integer-order dynamics; however, there are a few studies about control of fractional-order multi agent systems [24]. In [2], the fractional-order linear multi agent systems are investigated to be controlled under the directed graph and the convergence speed to achieve consensus.

Consensus with leader and without leader is studied for fractional-order nonlinear multi agent systems under directed topologies in [6]. The consensus for fractional-order multi agent systems with nonlinear dynamics which satisfy Lipschitz condition, is studied in [7] with an unknown leader. In [28], the observer base consensus for fractional-order multi agent systems with the fractional-order smaller than two along with second-order leader is investigated. Therefore, cluster consensus for fractional-order multi agent systems with high-order nonlinear dynamics are a new study which is investigated in this paper.

The smallest non-zero eigenvalue of the Laplacian matrix is a feature which should be known to achieve consensus [13, 16]. Consensus via adaptive controller can overcome to this limitation [14]. Consensus for second-order nonlinear multi agent systems which satisfy the Lipschitz condition, for both with and without leader via adaptive controller under undirected topologies is studied in [27]. In [26], consensus for nonlinear multi agent systems in strict-feedback form with uncertainty is studied via adaptive controller. The distributed consensus is investigated in [17] for second-order multi agent systems which satisfy Lipschitz condition under directed topologies.

The main contribution of this paper is divided in twofold:

- The cluster consensus of fractional-order nonlinear system is presented for the first time. Note that the underlying dynamics are in general nonlinear form.
- The controller for multi agent system is designed by using adaptive sliding mode principles and the efficiency of which are compared with that of non-adaptive case.

The above discussion motivates that cluster consensus for fractional-order nonlinear multi agent systems can be investigated by using adaptive sliding mode controller. The Lyapunov stability theorem for fractional-order systems is used to prove the Mittag-Leffler stability and the convergence of consensus.

The rest of the paper is organized as follows. In Section 2, graph theory is discussed. A brief explanation of fractional calculus is given in Section 3. In Section 4, problem formulation for fractional-order nonlinear multi agent systems is discussed. In Section 5, simulation examples are given. Finally, the conclusion is presented in Section 6.

2. Graph theory

A fixed graph of multi agent systems can describe the information exchange between n agents which is shown by $G = (VEA)$, where $V = \{v_1, v_2, \ldots, v_n\}$ is the node set and $E = \left\{(v_j, v_i) : \ v_j, v_i \in V, \ v_j \neq v_i\right\} \in V \times V$ is the edge set of the graph. $\mathcal{N}_i = \left\{v_j \in V : (v_j, v_i) \in E\right\}$ is the neighbors set of the i-th agent which the i-th agent receives information from the j-th agent. The directed graph has a directed spanning tree when there is at least a directed path from one node to all other nodes. Fig. 1 shows the directed graph for multi agent system.

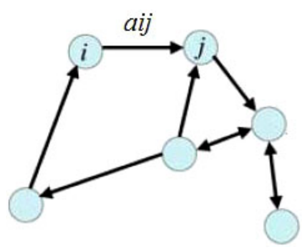

Figure 1: A directed graph

A and L are two matrices to describe the information exchange, where $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ is a matrix with $a_{ij} > 0$ if $\left(v_j, v_i\right) \in E$, otherwise $a_{ij} = 0$ and L is a positive semi-definite matrix which is defined as $L = D - A$ where $D = diag\left\{\sum_{j=1}^{n} a_{ij}\right\} \in \mathbb{R}^{n\times n}$. Lemma 1 defines some properties of the Kronecker product which is used for higher dimensional spaces.

Lemma 1 *There are some properties of the Kronecker product* $\otimes$ *which is defined as follows [4]:*

1. $(\xi A) \otimes B = A \otimes (\xi B)$,
2. $(A + B) \otimes C = A \otimes C + B \otimes C$,
3. $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$,
4. $(A \otimes B)^T = A^T \otimes B^T$,

where A, B, C *and* D *are matrices with appropriate dimensions and* $\xi \in \mathbb{R}$.

3. Fractional-order calculus

Definition 1 *Caputo fractional derivative used in this paper is defined below [19]:*

$$ {}_{0}^{c}\mathcal{D}_{t}^{\alpha}x(t) = \frac{1}{\Gamma(m-\alpha)}\int_{0}^{t}(t-\tau)^{m-\alpha-1}x^{(m)}(\tau)\,\mathrm{d}\tau, \tag{1}$$

where $m-1<\alpha<m$, $m\in\mathbb{N}_{+}$ *and* $\Gamma(.)$ *is the gamma function.*

The notation $x^{\alpha}(t)$ is used instead of ${}_{0}^{c}\mathcal{D}_{t}^{\alpha}x(t)$ for simplicity.

Definition 2 *The Mittag-Leffler function is used to solve fractional-order systems like the exponential function for integer-order systems. The two-parameter Mittag-Leffler function with* $\alpha,\ \beta>0$ *is defined as:*

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty}\frac{z^{k}}{\Gamma(k\alpha+\beta)}. \tag{2}$$

This function for $\beta=1$ *turns to the one-parameter Mittag-Leffler function.*

For investigating the stability of fractional-order nonlinear multi agent systems, the following Lemmas is are defined [1, 10, 30].

Lemma 2 *For any time* $t\geqslant t_0$*, the following inequality is defined:*

$$\frac{1}{2}{}_{t_0}^{c}\mathcal{D}_{t}^{\alpha}x^{2}(t)\leqslant x(t){}_{t_0}^{c}\mathcal{D}_{t}^{\alpha}x(t),\qquad \alpha\in(0,1), \tag{3}$$

where $x(t)\in\mathbb{R}$ *is a continuous and derivable function.*

Lemma 3 *The following inequality is defined:*

$${}_{t_0}^{c}\mathcal{D}_{t}^{\alpha}\left(x^{T}(t)Px(t)\right)\leqslant 2x^{T}(t)P{}_{t_0}^{c}\mathcal{D}_{t}^{\alpha}x(t),\qquad \alpha\in(0,1), \tag{4}$$

where $x(t)\in\mathbb{R}^{n}$ *is continuous and derivable and* $P\in\mathbb{R}^{n\times n}$ *is a positive definite symmetric matrix.*

The systems are proved to be asymptotically stable by using the Lyapunov direct method. The Lyapunov direct method for fractional order systems is derived from the Lyapunov direct method for integer-order systems and it leads to the Mittag-Leffler stability [12].

Lemma 4 *Mittag-Leffler stability for fractional-order system in* $t_0 = 0$ *at the equilibrium point* $x = 0$ *is proved if the Lyapunov function* $V(t, x(t))$ *exists with the following condition:*

$$\begin{aligned} &\alpha_1 \|x\|^c \leqslant V(t, x(t)) \leqslant \alpha_2 \|x\|^{cd}, \\ &{}_0^c\mathcal{D}_t^\alpha V(t, x(t)) \leqslant -\alpha_3 \|x\|^{cd}, \end{aligned} \tag{5}$$

where $V(t, x(t))\colon [0, +\infty) \times D \to \mathbb{R}$, $D \in \mathbb{R}^n$ *is a domain, that contains the origin;* $t \in \mathbb{R}^+$, $\alpha \in (0, 1]$, α_1, α_2, c *and* d *which are positive constants.*

Lemma 5 can be derived from Lemma 4.

Lemma 5 *Mittag-Leffler stability for the fractional-order system in* $t_0 = 0$ *at the equilibrium point* $x = 0$ *is proved if the Lyapunov function* $V(t, x(t))$ *exists with the following condition:*

$$\begin{aligned} &\alpha_1 \|x\|^c \leqslant V(t, x(t)) \leqslant \alpha_2 \|x\|^{cd}, \\ &{}_0^c\mathcal{D}_t^\alpha V(t, x(t)) \leqslant -\lambda V(t, x(t)), \end{aligned} \tag{6}$$

where $t \in \mathbb{R}^+$, $\alpha \in (0, 1]$, α_1, α_2, λ, c *and* d *are positive constants [25].*

4. Problem formulation

In this part, cluster consensus problem is explained, where the graph of agents is divided into two parts and each part reaches a different consistent value. Consider a graph with n agents divided like $l_1 = \{1, 2, \ldots, N_1\}$, $l_2 = \{N_1 + 1, \ldots, N_1 + N_2\}$ where $n = N_1 + N_2$.

Cluster consensus problem is achieved if

$$\lim_{t\to\infty} \left(x_i - x_j\right) = 0, \quad \forall i, j \in l_1,$$

$$\lim_{t\to\infty} \left(x_i - x_j\right) = 0, \quad \forall i, j \in l_2 .$$

Some assumptions for analyzing the cluster consensus problem are expressed in this section.

Assumption 1 *The Lipschitz condition is defined as follow for function* $f(x, t)$.

$$|f(x_2, t) - f(x_1, t)| \leqslant l\, |x_2 - x_1|, \quad \forall x_1, x_2 \in \mathbb{R}, \ \forall t \geqslant 0,$$

where l *is the Lipschitz constant.*

Assumption 2

(a) Each subnetwork or cluster has a directed spanning tree.

(b) The information exchange between two clusters should be balanced. In other words, two clusters graph, satisfy the following conditions: $\sum_{j=N_1+1}^{N_1+N_2} a_{ij} = 0$, *for all* $i \in l_1$, $\sum_{j=1}^{N_1} a_{ij} = 0$, *for all* $i \in l_2$.

4.1. Cluster consensus of fractional-order nonlinear multi agent systems

In this section, agents with general nonlinear model are studied. There are n agents which the i-th agent, $i = 1, 2, \ldots, n$ is modeled as follows [23]:

$$x_i^{\alpha} = Ax_i + Cf(x_i, t) + Bu_i(t), \tag{7}$$

where $x_i \in \mathbb{R}^p$ are the states of i-th agent for $i = 1, 2, \ldots, n$ with p dimensions, $f(x_i, t) = \left(f_1(x_i, t),\ f_2(x_i, t),\ \ldots,\ f_p(x_i, t)\right)^T$ is a nonlinear vector-valued function which satisfies Lipschitz condition, u_i is the control input, A, B and C are constant matrices.

Fig. 2 shows the block diagram of the cluster consensus algorithm. A cluster consensus algorithm for system (7) is given as follows:

$$u_i(t) = k\theta \sum_{j \in \mathcal{N}_i} a_{ij} \left[x_j(t) - x_i(t)\right], \quad i = 1, 2, \ldots, n, \tag{8}$$

where $k > 0$, $\theta \in \mathbb{R}^{1\times p}$ is the feedback gain matrix.

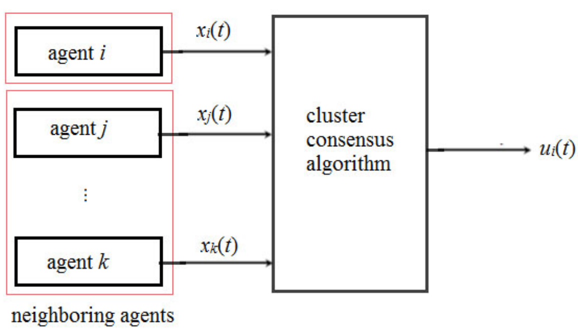

Figure 2: Block diagram of cluster consensus algorithm

By substituting the controller (8) into the system (7), we can write:

$$x_i^{\alpha} = Ax_i + Cf(x_i, t) + kB\theta \sum_{j \in \mathbb{N}_i} a_{ij} \left[x_j(t) - x_i(t)\right], \quad i = 1, 2, \ldots, n, \tag{9}$$

where
$X = [x_1, x_2, \ldots, x_{N_1}, x_{N_1+1}, \ldots, x_{N_1+N_2}]^T$ and
$F(X,t) = [f(x_1,t), \ldots, f(x_{N_1},t), f(x_{N_1+1},t), \ldots, f(x_{N_1+N_2},t)]^T$,
now system (9) can be written as:

$$X^\alpha = (I_n \otimes A)X + (I_n \otimes C)F(X,t) - k(H \otimes B\theta)X, \tag{10}$$

where H is the Laplacian matrix of G described for two clusters as follows:

$$H = \begin{bmatrix} L_1 & \Omega_{12} \\ \Omega_{21} & L_2 \end{bmatrix},$$

where L_i, $i = 1, 2$ are the Laplacian matrix of the subnetworks and Ω_{ij}, $i, j = 1, 2$ are the information exchange between the two subnetworks as:

$$\Omega_{12} = -\begin{bmatrix} a_{1(N_1+1)} & \cdots & a_{1(N_1+N_2)} \\ \vdots & \ddots & \vdots \\ a_{N_1(N_1+1)} & \cdots & a_{N_1(N_1+N_2)} \end{bmatrix},$$

$$\Omega_{21} = -\begin{bmatrix} a_{(N_1+1)1} & \cdots & a_{(N_1+1)N_1} \\ \vdots & \ddots & \vdots \\ a_{(N_1+N_2)1} & \cdots & a_{(N_1+N_2)N_1} \end{bmatrix}.$$

To investigate cluster consensus of system (10), we introduce error matrices as follows:

$$\begin{aligned} e_i &= x_i - x_{N_1}, \quad i = 1, \ldots, N_1 - 1, \\ e_j &= x_j - x_{N_1+N_2}, \quad j = N_1 + 1, \ldots, N_1 + N_2 - 1, \\ E &= \left[e_1, \ldots, e_{N_1-1}, x_{N_1}, e_{N_1+1}, \ldots, e_{N_1+N_2-1}, x_{N_1+N_2}\right]^T, \\ e &= [e_1, \ldots, e_{N_1-1}, e_{N_1+1}, \ldots, e_{N_1+N_2-1}]^T \\ &= [x_1 - x_{N_1}, \ldots, x_{N_1-1} - x_{N_1}, x_{N_1+1} - x_{N_1+N_2}, \ldots, x_{N_1+N_2-1} - x_{N_1+N_2}]^T. \end{aligned}$$

Arranging (10) in terms of e, the error system for two clusters is defined as follows:

$$e^\alpha = (I_{n-2} \otimes A)e + (I_{n-2} \otimes C)f_e(E,t) - k(H_G \otimes B\theta)e, \tag{11}$$

where $\otimes$ is the Kronecker product and

$$H_G = \begin{bmatrix} \widetilde{L}_1 & \widetilde{\Omega}_{12} \\ \widetilde{\Omega}_{21} & \widetilde{L}_2 \end{bmatrix},$$

$$\widetilde{L}_1 = \begin{bmatrix} l_{11} - l_{N_1 1} & \cdots & l_{1(N_1-1)} - l_{N_1(N_1-1)} \\ \vdots & \ddots & \vdots \\ l_{(N_1-1)1} - l_{N_1 1} & \cdots & l_{(N_1-1)(N_1-1)} - l_{N_1(N_1-1)} \end{bmatrix},$$

$$\widetilde{L}_2 = \begin{bmatrix} l_{(N_1+1)1} - l_{(N_1+N_2)(N_1+1)} & \cdots & l_{(N_1+1)(N_1+N_2-1)} - l_{(N_1+N_2)(N_1+N_2-1)} \\ \vdots & \ddots & \vdots \\ l_{(N_1+N_2-1)1} - l_{(N_1+N_2)(N_1+1)} & \cdots & l_{(N_1+N_2-1)(N_1+N_2-1)} - l_{(N_1+N_2)(N_1+N_2-1)} \end{bmatrix},$$

$$\widetilde{\Omega}_{12} = \begin{bmatrix} a_{N_1(N_1+1)} - a_{1(N_1+1)} & \cdots & a_{N_1(N_1+N_2-1)} - a_{1(N_1+N_2-1)} \\ \vdots & \ddots & \vdots \\ a_{N_1(N_1+1)} - a_{(N_1-1)(N_1+1)} & \cdots & a_{N_1(N_1+N_2-1)} - a_{(N_1-1)(N_1+N_2-1)} \end{bmatrix}$$

and

$$\widetilde{\Omega}_{21} = \begin{bmatrix} a_{(N_1+N_2)1} - a_{(N_1+1)1} & \cdots & a_{(N_1+N_2)(N_1-1)} - a_{(N_1+N_2)(N_1-1)} \\ \vdots & \ddots & \vdots \\ a_{(N_1+N_2)1} - a_{(N_1+N_2-1)1} & \cdots & a_{(N_1+N_2)(N_1-1)} - a_{(N_1+N_2-1)(N_1-1)} \end{bmatrix}.$$

Now, define:

$$\begin{aligned} f_e(E,t) &= \Big[f(x_1,t) - f(x_{N_1},t), \ldots, f(x_{N_1-1},t) - f(x_{N_1},t), f(x_{N_1+1},t) \\ &\quad - f(x_{N_1+N_2},t), \ldots, f(x_{N_1+N_2-1},t) - f(x_{N_1+N_2},t) \Big]^T \\ &= \Big[f(x_{N_1} + e_1,t) - f(x_{N_1},t), \ldots, f(x_{N_1} + e_{N_1-1},t) - f(x_{N_1},t), \\ &\quad \ldots, f(x_{N_1+N_2} + e_{N_1+N_2-1},t) - f(x_{N_1+N_2},t) \Big]^T. \end{aligned}$$

Theorem 1 *The cluster consensus problem for system (7) is achieved with control law (8) if P is a positive definite matrix satisfying:*

$$\begin{bmatrix} AP + PA^T + l^2CC^T - k\lambda_{\min}(H_G)BB^T + \beta P & P \\ P & -I_p \end{bmatrix} < 0, \tag{12}$$

where $\beta > 0$ and take $\theta = \frac{1}{2}B^T P^{-1}$.

Proof. First, select a Lyapunov function as follows to prove the cluster consensus and show that system (11) is asymptotically stable:

$$V(t) = e^T(t)\left(I_{n-2} \otimes P^{-1}\right)e(t). \tag{13}$$

Now, $V^{\alpha}(t)$ yields in by using Lemma 3:

$$\begin{aligned} V^{\alpha}(t) &\leqslant 2e^T(t)\left(I_{n-2}\otimes P^{-1}\right)e^{\alpha}(t) \\ &= 2e^T\left(I_{n-2}\otimes P^{-1}\right)\Big[(I_{n-2}\otimes A)\,e + (I_{n-2}\otimes C)f_e(E,t) \\ &\quad - k\,(H_G\otimes B\theta)\,e\Big] \\ &= e^T\left(I_{n-2}\otimes P^{-1}\right)(I_{n-2}\otimes A)\,e + e^T\left(I_{n-2}\otimes A^T\right)\left(I_{n-2}\otimes P^{-1}\right)e \\ &\quad + 2e^T\left(I_{n-2}\otimes P^{-1}C\right)f_e(E,t) - 2ke^T\left(I_{n-2}\otimes P^{-1}\right)(H_G\otimes B\theta)\,e \\ &= e^T\left[I_{n-2}\otimes\left(P^{-1}A + A^TP^{-1}\right)\right]e + 2e^T(I_{n-2}\otimes P^{-1}C)f_e(E,t) \\ &\quad - 2ke^T\left(H_G\otimes P^{-1}B\theta\right)e. \end{aligned} \tag{14}$$

Consider Lemma 6 to continue the proof of Lyapunov stability.

Lemma 6 *The following inequality with vectors x, y and matrices P, D and S is established:*

$$2x^TDSy \leqslant x^TDPD^Tx + y^TS^TP^{-1}Sy. \tag{15}$$

Using Lemma 6 and Assumption 1, is given (16) as:

$$\begin{aligned} V^{\alpha}(t)c &\leqslant e^T\left[I_{n-2}\otimes(P^{-1}A + A^TP^{-1})\right]e \\ &\quad + e^T\left[I_{n-2}\otimes(l^2P^{-1}CC^TP^{-1} + I_p)\right]\mathrm{e} - 2ke^T\left(H_G\otimes P^{-1}B\theta\right)e. \end{aligned} \tag{16}$$

Substituting $\theta = \dfrac{1}{2}B^TP^{-1}$, (16) is rewritten as follows:

$$\begin{aligned} V^{\alpha}(t) &\leqslant e^T\left[I_{n-2}\otimes(P^{-1}A + A^TP^{-1})\right]e \\ &\quad + e^T\left[I_{n-2}\otimes(l^2P^{-1}CC^TP^{-1} + I_p)\right]e \\ &\quad - ke^T\left(H_G\otimes P^{-1}BB^TP^{-1}\right)e. \end{aligned} \tag{17}$$

Now, (17) can be written as:

$$\begin{aligned} V^{\alpha}(t) &\leqslant \varepsilon^T\left[I_{n-2}\otimes(AP + PA^T + l^2CC^T + P^TP)\right]\varepsilon \\ &\quad - k\varepsilon^T\left(H_G\otimes BB^T\right)\varepsilon, \end{aligned} \tag{18}$$

where $\varepsilon = (I_{n-2}\otimes P^{-1})e$.

$$\begin{aligned} V^{\alpha}(t) &\leqslant \varepsilon^T\left[I_{n-2}\otimes(AP + PA^T + l^2CC^T + P^TP)\right]\varepsilon \\ &\quad - k\lambda_{\min}(H_G)\,\varepsilon^T\left(I_{n-2}\otimes BB^T\right)\varepsilon. \end{aligned} \tag{19}$$

Using (12) and Schur complement lemma [8], (19) is rewritten as follows:

$$V^{\alpha}(t) \leqslant -\beta\varepsilon^{T}\left(I_{n-2}\otimes P\right)\varepsilon = -\beta e^{T}\left(I_{n-2}\otimes P^{-1}\right)e = -\beta V(t). \tag{20}$$

Hence, $V^{\alpha} \leqslant -\beta V(t)$ which we can conclude from Lemma 5, Mittag-Leffler stability results in (11).

4.2. Cluster consensus of fractional-order nonlinear multi agent systems by adaptive law

4.2.1. First-order system

In this subsection, the consensus tracking problem for followers which are nonlinear is investigated. The followers with nonlinear dynamics are described as follows:

$$x_i^{\alpha}(t) = f_{\hat{i}}\left(t, x_i(t)\right) + u_i(t), \quad i = 1, 2, \ldots, n, \tag{21}$$

where $f_{\hat{i}}\left(t, x_i(t)\right)$ is a nonlinear function for cluster $\hat{i}$, which satisfies the Lipschitz condition like Assumption 1. $u_i(t)$ is the control input.

The model of i-th leader is described by:

$$x_{ri}^{\alpha}(t) = f_{\hat{i}}\left(t, x_{ri}(t)\right), \quad i = 1, 2, \ldots, n. \tag{22}$$

The cluster consensus with the following condition is achieved:

$$\begin{aligned} &\lim_{t\to\infty}\|x_i - x_{ri}\| = 0, \quad \forall i \in l_1, \\ &\lim_{t\to\infty}\|x_i - x_{ri}\| = 0, \quad \forall i \in l_2. \end{aligned} \tag{23}$$

The adaptive sliding mode controller and adaptive law are designed as follows:

$$\begin{aligned} u_i(t) = {} & -\theta_i\left(\sum_{j=1}^{n} a_{ij}\left[x_i - x_j\right] + d_i\left[x_i - x_{ri}\right]\right) \\ & - \omega sgn\left(\sum_{j=1}^{n} a_{ij}\left[x_i - x_j\right] + d_i\left[x_i - x_{ri}\right]\right), \\ & i, j = 1, 2, \ldots, n, \quad i \neq j, \end{aligned} \tag{24}$$

$$\begin{aligned} \theta_i^{\alpha} = {} & \beta_i\left(\sum_{j=1}^{n} a_{ij}\left[x_i - x_j\right] + d_i\left[x_i - x_{ri}\right]\right)^{T} \\ & \cdot\left(\sum_{j=1}^{n} a_{ij}\left[x_i - x_j\right] + d_i\left[x_i - x_{ri}\right]\right), \end{aligned} \tag{25}$$

where ω and β_i are any positive constants, $sgn(.)$ is the signum function, a_{ij}, $i, j = 1, 2, \ldots, n$ is the (i, j)-th element of the adjacency matrix A. θ_i is the adaptive gain for i-th agent. $D = diag(d_1 d_2 \ldots d_n)$ is the leader adjacency matrix. $d_i$is used for describing the leader information exchange to i-th agent. $d_i = 1$, if information is transmitted from leader to i-th agent and otherwise $d_i = 0$. For each cluster, $d_i = 1$ for only one follower.

Theorem 2 *The cluster consensus problem is achieved with control law (24) and adaptive law (25) for the system (21) if* $\theta_0 \geqslant \dfrac{l\lambda_{\max}(M)}{\lambda_{\min}(M)^2}$, $\lambda_{\min}$ *and* $\lambda_{\max}$ *are the smallest and largest eigenvalues with undirected graph G or directed graph with the symmetric positive definite matrix* M.

Proof. The system (21) can be rewritten as

$$\begin{aligned} x_i^{\alpha}(t) = f_{\hat{i}}(t, x_i(t)) + \theta_i \left(\sum_{j=1}^{n} a_{ij} \left[x_i - x_j \right] + d_i \left[x_i - x_{ri} \right] \right) \\ - \omega sgn \left(\sum_{j=1}^{n} a_{ij} \left[x_i - x_j \right] + d_i \left[x_i - x_{ri} \right] \right), \\ ij = 1, 2, \ldots, n, \quad i \neq j. \end{aligned} \tag{26}$$

Now system (26) can be written as:

$$X^{\alpha} = F(t, X) - \theta M X - \omega sgn(MX), \tag{27}$$

where

$$\begin{aligned} X &= [x_1, x_2, \ldots, x_{N_1}, x_{N_1+1}, \ldots, x_n]^T, \\ F(t, X) &= [f_{\hat{i}}(t, x_1(t)), f_{\hat{i}}(t, x_2(t)), \ldots, f_{\hat{i}}(t, x_{N_1}(t)), f_{\hat{i}}(t, x_{N_1+1}(t)), \\ &\quad \ldots, f_{\hat{i}}(t, x_n(t))]^T, \end{aligned}$$

and $\theta = diag(\theta_{1,}\theta_2, \ldots, \theta_n)$. $M = H + D$, where H is the Laplacian matrix of G described for two clusters which is described in (10) and M is a symmetric positive definite matrix. In the undirected graph G, M is always a symmetric positive definite matrix and in directed graph G, if information is sent and received between two nodes, M is a symmetric positive definite matrix.

Let $\widetilde{x}_i = x_i - x_{ri}$, so we can write as follows:

$$\widetilde{X}^{\alpha}(t) = F(t, X) - F(t, X_r) - \theta M \widetilde{X} - \omega sgn \left(M \widetilde{X} \right), \tag{28}$$

where $\widetilde{X} = [\widetilde{x}_1, \widetilde{x}_2, \ldots, \widetilde{x}_{N_1}, \widetilde{x}_{N_1+1}, \ldots, \widetilde{x}_n]^T$.

The following Lyapunov function is considered:

$$V = \frac{1}{2}\widetilde{X}^T M \widetilde{X} + \sum_{i=1}^{n} \frac{1}{2\beta_i}(\theta_i - \theta_0)^2, \tag{29}$$

where θ_0 is selected as:

$$\theta_0 \geqslant \frac{l\lambda_{\max}(M)}{\lambda_{\min}(M)^2}. \tag{30}$$

The fractional-order derivative of V is given as follows:

$$\begin{aligned} V^\alpha &\leqslant \widetilde{X}^T M \widetilde{X}^\alpha + \sum_{i=1}^{n} \frac{1}{\beta_i}(\theta_i - \theta_0)\,\theta_i^\alpha \\ &= \widetilde{X}^T M\left[F(t,X) - F(t,X_r)\right] - \left[M\widetilde{X}\right]^T \theta \left[M\widetilde{X}\right] \\ &\quad - \omega\left[M\widetilde{X}\right]^T sgn\left(M\widetilde{X}\right) + \left[M\widetilde{X}\right]^T \theta \left[M\widetilde{X}\right] - \theta_0\left[M\widetilde{X}\right]^T\left[M\widetilde{X}\right] \\ &\leqslant l\lambda_{\max}(M)\left\|\widetilde{X}\right\|^2 - \omega\left\|M\widetilde{X}\right\|_1 - \theta_0\lambda_{\min}(M)^2\left\|\widetilde{X}\right\|^2 \\ &\leqslant -\left(\theta_0\lambda_{\min}(M)^2 - l\lambda_{\max}(M)\right)\left\|\widetilde{X}\right\|^2. \end{aligned} \tag{31}$$

From (31), we can obtain that $V^\alpha \leqslant -\gamma\left\|\widetilde{X}\right\|^2$ where $\gamma = \theta_0\lambda_{\min}(M)^2 - l\lambda_{\max}(M) \geqslant 0$. So from Lemma 4, system is Mittag-Leffler stable which means $\|x_i(t) - x_{ri}(t)\| = 0$, $i = 1, 2, \ldots, n$ and the cluster consensus for the system (21) is achieved by using the controller (24) and adaptive law (25).

4.2.2. General form system

Consider a group of n agents which the model of i-th agent is described in general form as follows:

$$x_i^\alpha = Ax_i + Cf_{\hat{i}}(t, x_i) + Bu_i(t), \qquad i = 1, 2, \ldots, n, \tag{32}$$

where $x_i \in \mathbb{R}^p$ are the states of i-th agent for $i = 1, 2, \ldots, n$, $f_{\hat{i}}(t, x_i) = \left(f_1(t, x_i), f_2(t, x_i), \ldots, f_p(t, x_i)\right)^T$ is a nonlinear vector-valued function for each cluster which satisfies Lipschitz condition,B is constant matrix and u_i is the control input.

The adaptive sliding mode controller and adaptive law are designed as follows:

$$\begin{aligned} u_i(t) = -\theta_i k &\left(\sum_{j=1}^{n} a_{ij}\left[x_i - x_j\right] + d_i\left[x_i - x_{ri}\right]\right) \\ &- \omega sgn\left(\sum_{j=1}^{n} a_{ij}\left[x_i - x_j\right] + d_i\left[x_i - x_{ri}\right]\right), \\ &i, j = 1, 2, \ldots, n,, \quad i \neq j, \end{aligned} \tag{33}$$

$$\begin{aligned} \theta_i^{\alpha} = \beta_i &\left(k\sum_{j=1}^{n} a_{ij}\left[x_i - x_j\right] + d_i\left[x_i - x_{ri}\right]\right)^T \\ &\cdot\left(k\sum_{j=1}^{n} a_{ij}\left[x_i - x_j\right] + d_i\left[x_i - x_{ri}\right]\right). \end{aligned} \tag{34}$$

The model of i-th leader is described by:

$$x_{ri}^{\alpha} = Ax_{ri} + Cf_{\hat{i}}(t, x_{ri}) \quad i = 1, 2, \ldots, n. \tag{35}$$

Let $\widetilde{x}_i = x_i - x_{ri}$, so we can write as follows:

$$\begin{aligned} \widetilde{X}^{\alpha}(t) = (I_n \otimes A)\widetilde{X} + (I_n \otimes C)(F(t, X) - F(t, X_r)) - (\theta M \otimes Bk)\widetilde{X} \\ - (\omega I_n \otimes B)\, sgn\left((M \otimes k)\widetilde{X}\right), \end{aligned} \tag{36}$$

where $\widetilde{X} = [\widetilde{x}_1, \widetilde{x}_2, \ldots, \widetilde{x}_{N_1}, \widetilde{x}_{N_1+1}, \ldots, \widetilde{x}_n]^T$, $1_n = [11\ldots1]^T$, $M = H + D$, H is the Laplacian matrix of G described for two clusters which is described in (10) and $\otimes$ is Kronecker product.

Theorem 3 *The cluster consensus problem is achieved with control law (34) and adaptive law (35) for the systems (32) if P is a positive definite matrix satisfying:*

$$\begin{bmatrix} AP + PA^T + l^2CC^T - 2\theta_0\lambda_{\min}(M)BB^T + \beta P & P \\ P & -I_n \end{bmatrix} < 0, \tag{37}$$

where $\beta > 0$ and take $k = -B^T P^{-1}$.

Proof. The following Lyapunov function is considered:

$$V = \widetilde{X}^T(M \otimes P^{-1})\widetilde{X} + \sum_{i=1}^{n}\frac{1}{\beta_i}(\theta_i - \theta_0)^2. \tag{38}$$

The fractional-order derivative of V is given as follows:

$$\begin{aligned} V^{\alpha} \leqslant\ & \widetilde{X}^T(M\otimes P^{-1}A+A^TP^{-1})\widetilde{X}+2\widetilde{X}^T(M\otimes P^{-1}C)\left[F(t,X)-F(t,X_r)\right] \\ & +\widetilde{X}^T(M\otimes P^{-1})(\theta M\otimes Bk)\widetilde{X}+\widetilde{X}^T(\theta M\otimes k^TB^T)(M\otimes P^{-1})\widetilde{X} \\ & +2\widetilde{X}^T\omega(M\otimes P^{-1}B)sgn\left((M\otimes k)\widetilde{X}\right) \\ & +2\sum_{i=1}^{n}(\theta_i-\theta_0)\left(k\sum_{j=1}^{n}a_{ij}\left(\widetilde{x}_i-\widetilde{x}_j\right)+d_i\widetilde{x}_i\right)^T \\ & \cdot\left(k\sum_{j=1}^{n}a_{ij}\left(\widetilde{x}_i-\widetilde{x}_j\right)+d_i\widetilde{x}_i\right). \end{aligned} \tag{39}$$

Using Lemma 6 and Assumption and by getting $k=-B^TP^{-1}$ is given (40) as follows:

$$\begin{aligned} V^{\alpha} \leqslant\ & \widetilde{X}^T\left(M\otimes P^{-1}Av+vA^TP^{-1}\right)\widetilde{X}+\sum_{i=1}^{n}\left(\widetilde{x}_i^T\left(l^2P^{-1}CC^TP^{-1}+I\right)\widetilde{x}_i\right) \\ & +2\widetilde{X}^T(M\theta M\otimes P^{-1}Bk)\widetilde{X} \\ & +2\sum_{i=1}^{n}(\theta_i-\theta_0)\left(k\sum_{j=1}^{n}a_{ij}\left(\widetilde{x}_i-\widetilde{x}_j\right)+d_i\widetilde{x}_i\right)^T\left(k\sum_{j=1}^{n}a_{ij}\left(\widetilde{x}_i-\widetilde{x}_j\right)+d_i\widetilde{x}_i\right) \\ =\ & \widetilde{X}^T\left(M\otimes P^{-1}A+A^TP^{-1}+l^2P^{-1}CC^TP^{-1}+I\right)\widetilde{X} \\ & -2\sum_{i=1}^{n}\theta_i\left(\sum_{j=1}^{n}a_{ij}\left(\widetilde{x}_i-\widetilde{x}_j\right)+d_i\widetilde{x}_i\right)^TP^{-1}BB^TP^{-1} \\ & \cdot\left(\sum_{j=1}^{n}a_{ij}\left(\widetilde{x}_i-\widetilde{x}_j\right)+d_i\widetilde{x}_i\right)+2\sum_{i=1}^{n}(\theta_i-\theta_0)\left(\sum_{j=1}^{n}a_{ij}\left(\widetilde{x}_i-\widetilde{x}_j\right)+d_i\widetilde{x}_i\right)^T \\ & \cdot P^{-1}BB^TP^{-1}\left(\sum_{j=1}^{n}a_{ij}\left(\widetilde{x}_i-\widetilde{x}_j\right)+d_i\widetilde{x}_i\right) \\ =\ & \widetilde{X}^T\left(M\otimes P^{-1}A+A^TP^{-1}+l^2P^{-1}CC^TP^{-1}+I\right)\widetilde{X} \\ & -2\theta_0\widetilde{X}^T\left(M^2\otimes P^{-1}BB^TP^{-1}\right)\widetilde{X} \\ \leqslant\ & \widetilde{X}^T\left(M\otimes P^{-1}A+A^TP^{-1}+l^2P^{-1}CC^TP^{-1}+I\right. \\ & \left.-2\theta_0\lambda_{\min}(M)P^{-1}BB^TP^{-1}\right)\widetilde{X}. \end{aligned} \tag{40}$$

From (40), we can obtain $V^{\alpha} \leqslant -\beta V$ if Theorem 3 is established. So from Lemma 5, the system is Mittag-Leffler stable, which means

$$\lim_{t\to\infty} \|x_i(t) - x_{ri}(t)\| = 0, \quad i = 1, 2, \ldots, n$$

and the cluster consensus for the system (32) is achieved by using the controller (33) and the adaptive law (34).

5. Simulation results

In this section, three numerical simulation examples are given to evaluate the theoretical results.

Example 1. In this example, each agent is a single-link flexible-joint manipulator shown in Fig. 3. The revolving joints are actuated by a DC motor and a linear spring is used to model the elasticity of the joint. The state-space model which the states of the system are position and velocity of motor and link is as follows:

$$\begin{aligned}
\theta_m^{\alpha} &= \omega_m\,,\\
\omega_m^{\alpha} &= \frac{k}{J_m}(\theta_l - \theta_m) - \frac{k}{J_m}\omega_m + \frac{K_T}{J_m}u,\\
\theta_l^{\alpha} &= \omega_l\,,\\
\omega_l^{\alpha} &= -\frac{k}{J_l}(\theta_l - \theta_m) - \frac{mgh}{J_l}\sin(\theta_l),
\end{aligned}$$

where J_m and J_l are the inertia of the motor and link, θ_m and θ_l are the angular rotation of the motor and the angular position of the link, respectively; and ω_m and ω_l are the angular velocity of the motor and link [31].

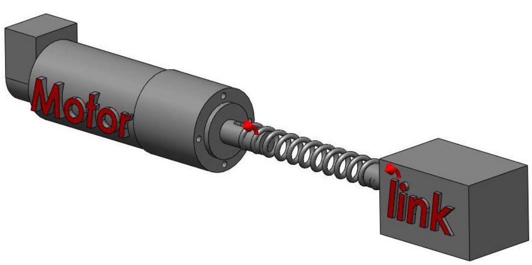

Figure 3: Single-link manipulator with a flexible joint

The manipulator is described by the form of (7) with $\alpha = 0.95$ and the numerical parameters are defined as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.26 & 48.6 & 0 \\ 0 & 0 & 0 & 10 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix}, \qquad C = I_4,$$

$$x_i(t) = [\theta_{m_i}, \ \omega_{m_i}, \ \theta_{l_i}, \ \omega_{l_i}]^T = [x_{i1}(t), \ x_{i2}(t), \ x_{i3}(t), \ x_{i4}(t)]^T,$$

$$f(x_i, t) = (0, \ 0, \ 0, \ 0.333 \sin(x_{i3}(t)))^T.$$

Cluster consensus for the manipulator is achieved for graph in Fig. 4 with $l = 0.333$, $\beta = 0.7$, $k = 28.75$ and P is achieved by LMI (12) as follows:

$$P = \begin{bmatrix} 0.5283 & -0.9689 & 0.4636 & -0.0826 \\ -0.9689 & 122.4712 & -0.0074 & 0.0009 \\ 0.4636 & -0.0074 & 0.5134 & -0.0482 \\ -0.0826 & 0.0009 & -0.0482 & 0.0195 \end{bmatrix}.$$

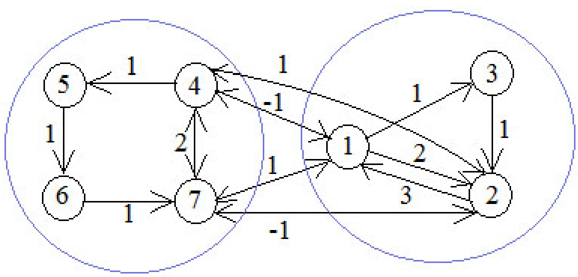

Figure 4: The topology interaction of the agents

Simulation results are shown in Fig. 5.

The state trajectories of agents are shown in Fig. 5. In Fig. 5(a), first state of agents x_{11} to x_{31} converge to one consensus value which is time varying and x_{41} to x_{71} converge to another consensus value at time 0.1 second. In Fig. 5(b), second state of agents x_{12} to x_{32} converge to one consensus value which is time-varying and x_{42} to x_{72} converge to another consensus value at time 0.1 second. In Fig. 5(c), third state of agents x_{13} to x_{33} converge to one consensus value which is time-varying and x_{43} to x_{73} converge to another consensus value at time 5 second. In Fig. 5(d), fourth state of agents x_{14} to x_{34} converge to one consensus value which is time varying and x_{44} to x_{74} converge to another consensus value at time 7 second. So cluster consensus is achieved for all states of agents.

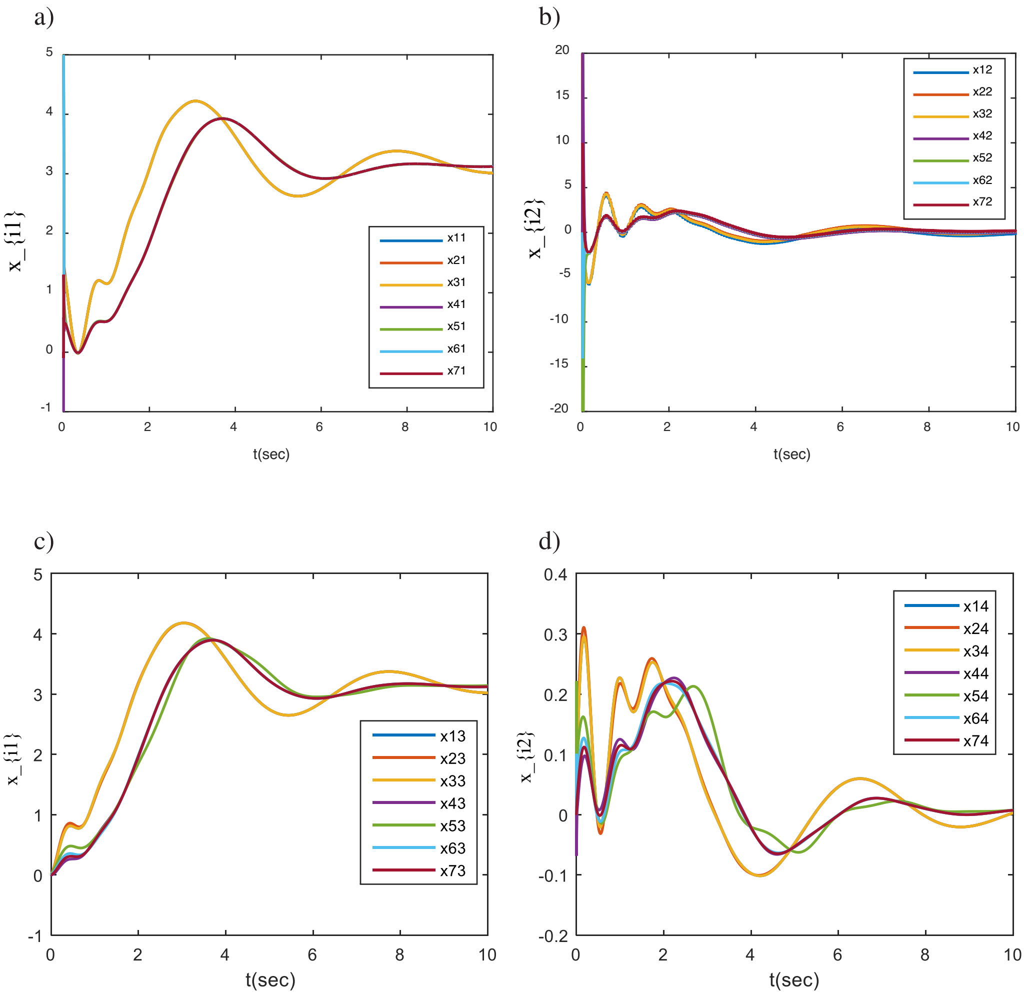

Figure 5: Cluster consensus for (a) $x_{i1}(t)$, (b) $x_{i2}(t)$, (c) $x_{i3}(t)$, (d) $x_{i4}(t)$, $i = 1, \ldots, 7$

Example 2. A directed graph with seven followers and two leaders which communicated with first and fourth agents is shown in Fig. 4. $\alpha = 0.95$, $\omega = 1$, $f_1(x_i(t)) = \sin\left(\dfrac{x_i(t)}{10}\right)$, $f_2(x_i(t)) = 2\sin\left(\dfrac{x_i(t)}{10}\right)$, $\beta_i = 1$, $i = 1, \ldots, n$ are chosen. The condition of Theorem 2 is satisfied and simulation results are shown in Figs 6, 7.

Fig. 6 shows the position trajectories of agents in clusters; the followers can track the leaders and cluster consensus achieves at time 1.5 second. The effectiveness of the controller (24) and adaptive law (25) is shown in Fig. 7 where both clusters are shown in same figure.

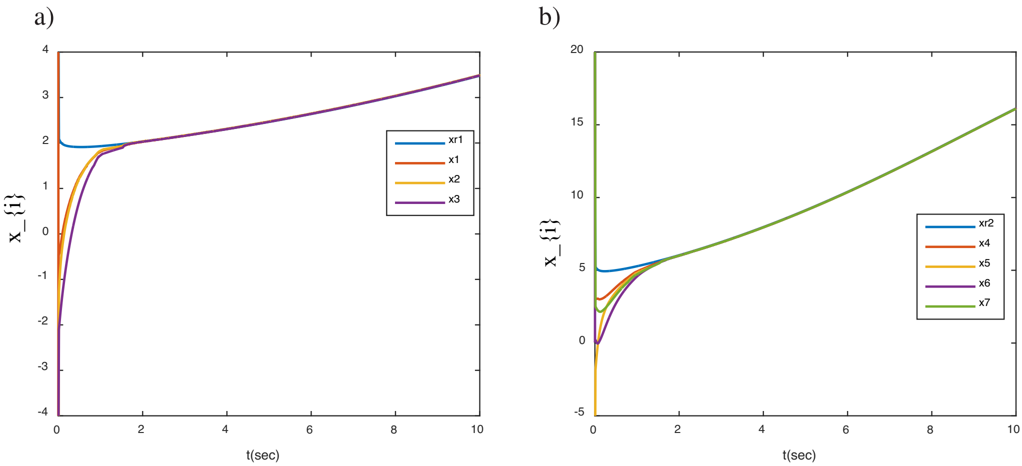

Figure 6: Cluster consensus for graph Fig. 2, (a) cluster 1, (b) cluster 2

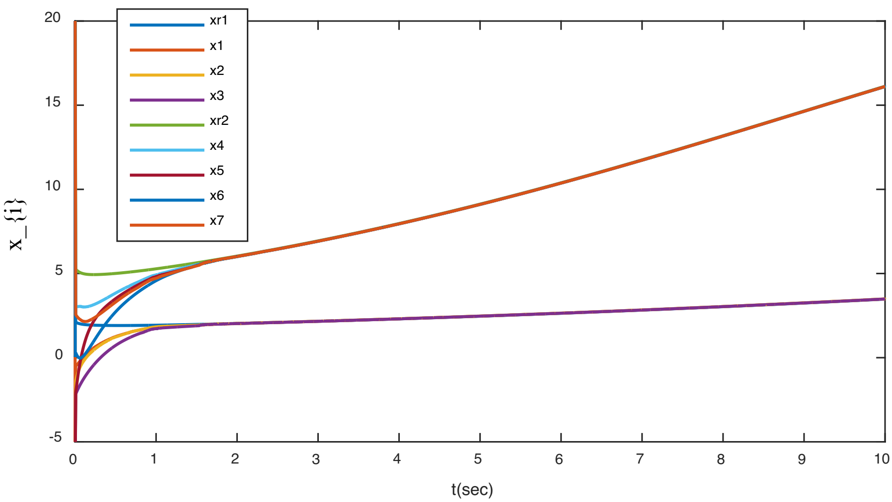

Figure 7: Cluster consensus for both clusters

Example 3. A directed graph with seven followers and two leaders which communicated with first and fourth agents is shown in Fig. 4. In this example, each agent is a single-link flexible-joint manipulator and this is described like Example 1 with $\alpha = 0.95$, $\omega = 1$, $f_1(t, x_i(t)) = \left(0,\ 0,\ 0,\ \sin\left(\frac{x_{i3}(t)}{10}\right)\right)^T$, $f_2(t, x_i(t)) = \left(0,\ 0,\ 0,\ 2\sin\left(\frac{x_{i3}(t)}{10}\right)\right)^T$, $l = 0.333$, $\beta = 0.7$, $\theta_0 = 105.77$ and P

is achieved by LMI (37) as follows:

$$P = \begin{bmatrix} 0.5349 & -0.9707 & 0.47 & -0.0828 \\ -0.9707 & 107.0333 & -0.0183 & 0.001 \\ 0.47 & -0.0183 & 0.5195 & -0.0484 \\ -0.0828 & 0.001 & -0.0484 & 0.0196 \end{bmatrix}.$$

Simulation results are shown in Fig. 8.

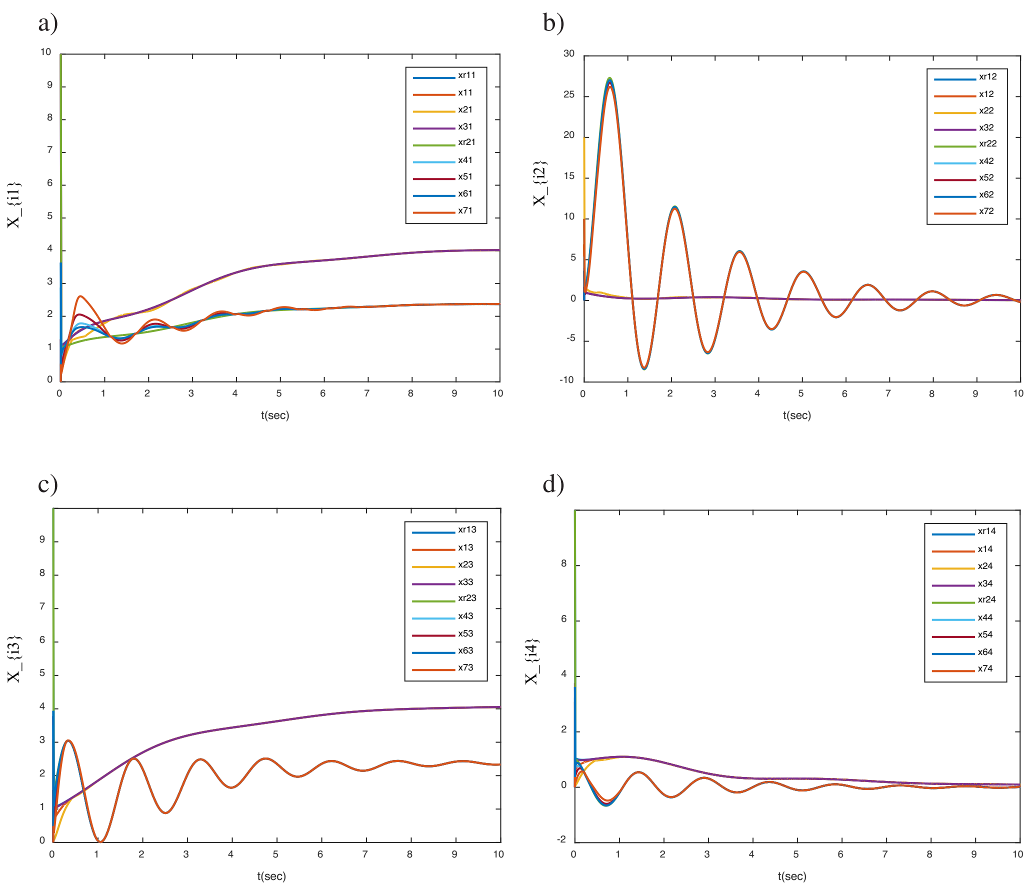

Figure 8: Cluster consensus for (a) $x_{i1}(t)$, (b) $x_{i2}(t)$, (c) $x_{i3}(t)$, (d) $x_{i4}(t)$, $i = 1, \ldots, 7$

Fig. 8 shows the position trajectories of agents for each state. In Fig. 8(a), first state of agents x_{11} to x_{31} follow the first leader and x_{41} to x_{71} follow the second leader after 3.5 second. In Fig. 8(b), second state of agents x_{12} to x_{32} follow the first leader and x_{42} to x_{72} follow the second leader after 0.5 second. In Fig. 8(c), third state of agents x_{13} to x_{33} follow the first leader and x_{43} to x_{73} follow the second leader after 0.5 second. In Fig. 8(d), fourth state of agents

x_{14} to x_{34} follow the first leader and x_{44} to x_{74} follow the second leader after 1 second. By comparison Fig. 3 and Fig. 8, the convergence speed increases by using adaptive sliding mode controller. Also, the comparison of these examples is shown in Table 1. So, it demonstrates the efficiency of the proposed adaptive controller.

Table 1: Comparison of examples

System	Graph	Adaptive sliding mode controller	Average consensus time of the states
General form system	7 agents		3 second
First-order system	7 agents	✓	1.5 second
General form system	7 agents	✓	1.4 second

6. Conclusions

In this paper, cluster consensus for the general fractional-order nonlinear multi agent systems has been studied. Adaptive sliding mode control has been proposed for multi agent systems. First-order nonlinear systems and general form nonlinear systems are both considered. Cluster consensus is achieved by an adaptive sliding mode controller and to judge cluster consensus problem, the sufficient conditions are given and the efficiency of the proposed adaptive controller is demonstrated. The effectiveness of the proposed results is shown by simulations. In this paper, fixed communication topologies are used, however, future work will concentrate on cluster consensus of the fractional-order nonlinear multi agent systems under switching communication topology.

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