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Interim fuel consumption estimation based on ship service parameters in real weather conditions with the use of ndCurveMaster curve fitting software

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Abstract

This paper deals with fuel consumption estimations relating to container ships on the basis of ship service and wave parameters. Data, on which to base estimations, was measured and recorded from a container ship during 96 months at sea. Approximating functions were calculated by the use of curve fitting techniques and regression methods, utilizing newly developed software named ndCurveMaster. The approximation function presented in this paper could have practical application for the estimation of container ship fuel consumption, while considering weather routing. In addition the study clearly shows the relationship between the fuel consumption of a container ship and the number of months since its last docking. These results may form the basis for further research in this direction.

Introduction

The existing methods for the prediction of fuel consumption, which have already been presented in many papers (Aligne, Papageorgiou & Ramos, 1997; Christiansen, 1999; Yiyo, 2010), are usually based on theoretical calculations. These methods only took into account ship resistance calculated theoretically in calm water and rarely in any other weather conditions. Based on a theoretically calculated ship resistance in addition to overall ship efficiency, theoretical fuel consumption can then be calculated according to (Cepowski, 2015).

Overall ship efficiency must take into account engine, propeller and hull efficiency as shown in Figure 1. There may be difficulties in finding a method that can simultaneously calculate propeller and hull efficiencies.

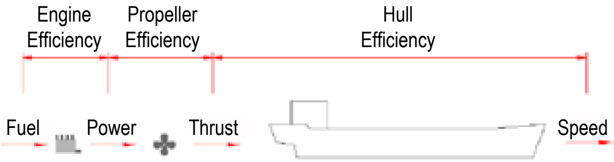

Figure 1. Overall ship efficiency (Cepowski, 2015)

In addition, any theoretical methods will:

- only estimate medium values but not assess interim fuel consumption;
- only take into account main engine parameters such as engine revolution, main engine load and theoretical engine efficiency;
- not consider ship service parameters such as wind, waves, drafts and trim, as well as propeller and hull fouling.

In summary, existing methods for the calculation of fuel consumption were developed primarily for the purpose of assisting in overcoming ship design issues mentioned earlier in (Szelangiewicz, Wiśniewski & Żelazny, 2014).

However, due to the fact that no better methods are available, theoretical methods are sometimes used in weather routing according to (Drozd, 2006; Wiśniewski, Medyna & Chomski, 2009, 2013).

Another key issue is the amount of fuel consumed; a big container vessel can consume up to 250 tons per day while travelling at sea. The main objective of weather routing calculations is to reduce the consumption of fuel which feeds directly into cost reduction. When making calculations, even a small error of around 1–2% can, in the long term, lead to huge economic losses resulting from the choice of a non-optimal route. Therefore, enhancing the accuracy of these calculations can bring clear financial benefit.

Due to this, ship owners generally assume that current calculation methods are not sufficiently accurate to utilize the large amounts of data, that nowadays are available from systems installed on-board to monitor the fuel consumption of their vessels. A typical example of such a simple method is usage of the equation (Wiśniewski, Medyna & Chomski, 2009, 2013):

$$\text{FOC} = k \cdot \text{RPM}^3 \qquad (1)$$

where:

FOC – fuel oil consumption (mt),

RPM – main engine revolution (1/min),

k – coefficient determined for specific vessel (or group of sister-vessels),

n – exponent determined for specific vessel (or group of sister-vessels),

This equation is commonly used for weather route optimisation when seeking the most fuel efficient route.

Contemporary systems installed on-board ships provide a large amount of raw data, allowing for post-voyage analysis. This data could potentially be used to reach much better estimations of ship's fuel consumptions.

This paper focusses on the presentation of approximations of container ship fuel consumption, based on the ship's recorded service and hydro-meteorological parameters. These estimations could be applied to enable optimal ship weather routing while at sea.

Estimation functions were calculated with ndCurveMaster software developed by the author. NdCurveMaster automatically determines optimal equations for the preparation of empirical data using a multiple linear regression method, and uses heuristic techniques for curve fitting (TCSOFTWARE, 2017). Data was obtained from one container vessel during 96 months of service – Figure 2 shows a graphical representation of its voyages during this period.

Figure 2. Sketch of ship's voyages – the source of recorded data used in an article

Research method

The aim of this research was to determine an estimation of function f as interim fuel consumption FOC, based on operational parameters $X_1, X_2, \ldots, X_n$ (Cepowski, 2015):

$$\text{FOC} = f(X_1, X_2, \ldots, X_n) \qquad (2)$$

where:

FOC – estimated interim fuel consumption;

$X_1, X_2, \ldots, X_n$ – operating parameters such as:

- loading conditions (draft and trim),
- ship propulsion system parameters (RPM, M/E load),
- environmental condition parameters (wind and wave parameters),

f – function for the estimation of interim fuel consumption FOC.

Function f can be determined according to the formula:

$$X_1, X_2, \ldots, X_n \xrightarrow{f} \text{FOC} \qquad (3)$$

- Navigational equipment – automatic record of vessel's positions, speed and course;
- Propulsion system – automatic record of RPM, M/E load, and fuel consumption;
- Ship's movement sensor – automatic record of ship's rolling and pitching;
- Ship's records – information about draft, trim, number of months since last docking.

Regression analysis to obtain relationship between recorded data and ship's fuel consumption

Historical hydro-meteorological data:

- Wind parameters – speed and direction;
- Primary wave height, direction, and period;
- Secondary wave height, direction, and period.

Figure 3. Outline of method of interim fuel consumption estimation, based on operational parameters

where: $X_1, X_2, \ldots, X_n$ – recorded operational parameter values on-board the ship; FOC – recorded interim fuel consumption values on-board the ship.

The study assumed that the function f in equation (3) is determined using a multiple regression method – a statistical process for estimating the relationships among variables. Multiple regressions include techniques for modelling and analysing several variables, when the focus is on the relationship between a dependent variable (ship's fuel consumption) and predictors $X_1, X_2, \ldots, X_n$. Regression analysis helps one to understand how the typical value of the dependent variable changes when any one of the independent variables is altered, while the other independent variables are held fixed. The results of the study are presented in the next part of this article.

Multiple linear regression of interim fuel consumption on the basis of operating parameters

The analysis took into account a set of 11 137 registered fuel consumption values and:

- loading condition parameters: mean draught and trim;
- ship propulsion system parameters, such as engine revolution and main engine load;
- ship motion parameters, such as roll and pitch amplitudes;
- environmental condition parameters, such as wind speed, wind direction, primary wave height, primary wave direction, primary wave period, secondary wave height, secondary wave direction and secondary wave period;
- sea-based operation parameters, such as the number of months since last docking.

Parameter range values are presented in Tables 1 and 2, which fully cover typical real operational conditions both with regards to RPM (M/E loads) sets, loading (draft and trim), as well as weather conditions. The maximum recorded roll (15°) and pitch (4.9°) angles indicate that the vessel was being operated within safety limits, with regard to her motions on the waves/swells.

The main ship parameters were as follows:

- length between perpendiculars $L = 304$ m;
- breadth moulded $B = 40$ m;
- number of TEU containers = 6500;
- deadweight DWT = 83 000 t;
- service speed $V = 25$ knots.

Table 1. Minimal and maximal values

	FOC [t/day]	*F* [deg]	*P* [deg]	*M*	*T* [m]	*t* [m]	RPM [rev/min]	*M*/*E* [%]
Min	1.2	0.2	0.2	0	10.6	–0.28	0	0
Max	163.2	15	4.9	96.2	12.9	1.44	91	58

where: FOC – fuel consumption, F – roll amplitude, P – pitch amplitude, M – number of months since last docking, T – mean draught, t – trim, RPM – engine revolution, M/E – main engine load.

Table 2. Minimal and maximal values

	VV [w]	VQ [deg]	PH [m]	PQ [deg]	PP [s]	SH [m]	SQ [deg]	SP [s]
Min	0	0	0	0	2.5	0	0	2.5
Max	43	180	7.1	180	22.1	3.9	180	20.1

where: VV – wind speed, VQ – wind direction (angle), PH – primary wave height, PQ – primary wave direction, PP – primary wave period, SH – secondary wave height, SQ – secondary wave direction, SP – secondary wave period.

Statistical analysis showed that container vessel interim fuel consumption FOC depends on all of the above parameters. Of all the investigated statistical relationships, the following proved to be the best:

$$\begin{aligned}
\mathrm{FOC} = {} & a0 + a1\cdot\phi^{-1} + a2\cdot\psi^{1/3} + a3\cdot(1/2)^{\mathrm{M}} + \\
& + a4\cdot[\exp(\mathrm{T})]^{5} + a5\cdot t^{16} + a6\cdot \mathrm{RPM}^{2} + a7\cdot \mathrm{L} + \\
& + a8\cdot \mathrm{VV}^{5} + a9\cdot \mathrm{VQ} + a10\cdot \mathrm{PH}^{14} + a11\cdot \ln^{8}(\mathrm{PP}) + \\
& + a12\cdot \mathrm{SH}^{1/5} + a13\cdot \ln^{7}(\mathrm{SP})
\end{aligned} \tag{4}$$

$$\begin{aligned}
\mathrm{FOC} = {} & a0 + a1\cdot(1/2)^{\psi} + a2\cdot \mathrm{T}^{-2} + a3\cdot t^{1/7} + \\
& + a4\cdot \mathrm{RPM}^{5} + a5\cdot \mathrm{L} + a6\cdot \mathrm{VV} + a7\cdot \mathrm{VQ}^{15} + a8\cdot \mathrm{PH} + \\
& + a9\cdot \mathrm{PQ}^{3} + a10\cdot \mathrm{PP} + a11\cdot \mathrm{SH} + a12\cdot \mathrm{SQ}^{1/4} + \\
& + a13\cdot \ln^{4}(\phi\cdot\psi) + a14\cdot\phi\cdot \mathrm{T}^{2} + a15\cdot\phi\cdot t^{1/5} + \\
& + a16\cdot\phi\cdot \mathrm{RPM} + a17\cdot\phi\cdot \mathrm{VV} + a18\cdot\phi\cdot \mathrm{PH}^{1/5} + \\
& + a19\cdot\phi\cdot \mathrm{PQ} + a20\cdot\phi\cdot \mathrm{PP}^{-1/2} + a21\cdot(1/2)^{\psi\cdot \mathrm{M}} + \\
& + a22\cdot\psi\cdot \mathrm{T} + a23\cdot\psi\cdot \mathrm{RPM} + a24\cdot\psi\cdot \mathrm{L} + a25\cdot\psi\cdot \mathrm{VV}^{2} + \\
& + a26\cdot(1/7)^{\psi\cdot \mathrm{VQ}} + a27\cdot\psi\cdot \mathrm{PQ} + a28\cdot\psi\cdot \mathrm{SP} + \\
& + a29\cdot \mathrm{M}\cdot \mathrm{T}^{12} + a30\cdot \mathrm{M}\cdot t + a31\cdot \mathrm{M}\cdot \mathrm{RPM}^{7} + \\
& + a32\cdot \mathrm{M}\cdot \mathrm{L}^{1/2} + a33\cdot \mathrm{M}\cdot \mathrm{VV}^{2} + a34\cdot(1/2)^{\mathrm{M}\cdot \mathrm{PH}} + \\
& + a35\cdot \mathrm{M}\cdot \mathrm{PQ}^{4} + a36\cdot \mathrm{M}\cdot \mathrm{PP}^{1/2} + a37\cdot \mathrm{M}\cdot \mathrm{SH} + \\
& + a38\cdot \mathrm{M}\cdot \mathrm{SQ}^{1/2} + a39\cdot \mathrm{M}\cdot \mathrm{SP}^{2} + a40\cdot \mathrm{T}\cdot t^{2} + \\
& + a41\cdot \mathrm{T}\cdot \mathrm{RPM}^{2} + a42\cdot \mathrm{T}\cdot \mathrm{VQ}^{4} + a43\cdot \mathrm{T}\cdot \mathrm{PH}^{16} + \\
& + a44\cdot \ln^{5}(\mathrm{T}\cdot \mathrm{PP}) + a45\cdot \mathrm{T}\cdot \mathrm{SH}^{2} + a46\cdot \mathrm{T}\cdot \mathrm{SQ} + \\
& + a47\cdot \mathrm{T}\cdot \mathrm{SP} + a48\cdot t\cdot \mathrm{RPM} + a49\cdot t\cdot \mathrm{L} + \\
& + a50\cdot t\cdot \mathrm{VV}^{1/11} + a51\cdot t\cdot \mathrm{VQ} + a52\cdot t\cdot \mathrm{PH}^{2} + \\
& + a53\cdot t\cdot \mathrm{PQ}^{2} + a54\cdot t\cdot \mathrm{PP}^{1/21} + a55\cdot t\cdot \mathrm{SP} + \\
& + a56\cdot \mathrm{RPM}\cdot \mathrm{L} + a57\cdot \mathrm{RPM}\cdot \mathrm{VV} + a58\cdot \mathrm{RPM}\cdot \mathrm{VQ}^{8} + \\
& + a59\cdot \mathrm{RPM}\cdot \mathrm{PH} + a60\cdot \mathrm{RPM}\cdot \mathrm{PQ} + a61\cdot \mathrm{RPM}\cdot \mathrm{PP} + \\
& + a62\cdot \mathrm{RPM}\cdot \mathrm{SH} + a63\cdot \mathrm{L}\cdot \mathrm{VV}^{3} + a64\cdot \mathrm{L}\cdot \mathrm{VQ} + \\
& + a65\cdot \mathrm{L}\cdot \mathrm{PH} + a66\cdot \mathrm{L}\cdot \mathrm{PQ} + a67\cdot \mathrm{L}\cdot \mathrm{PP}^{16} + \\
& + a68\cdot \mathrm{L}\cdot \mathrm{SH} + a69\cdot \mathrm{L}\cdot \mathrm{SP}^{11} + a70\cdot \mathrm{VV}\cdot \mathrm{SQ}^{2} + \\
& + a71\cdot \mathrm{VV}\cdot \mathrm{SP} + a72\cdot(1/2)^{\mathrm{VQ}\cdot \mathrm{PH}} + a73\cdot \mathrm{VQ}\cdot \mathrm{PP} + \\
& + a74\cdot \mathrm{VQ}\cdot \mathrm{SH}^{2} + a75\cdot \mathrm{VQ}\cdot \mathrm{SP}^{16} + a76\cdot \mathrm{PH}\cdot \mathrm{PP}^{1/21} + \\
& + a77\cdot(1/7)^{\mathrm{PH}\cdot \mathrm{SP}} + a78\cdot \mathrm{PQ}\cdot \mathrm{PP}^{14} + a79\cdot \mathrm{PQ}\cdot \mathrm{SH} + \\
& + a80\cdot \mathrm{PQ}\cdot \mathrm{SP}^{4} + a81\cdot \mathrm{PP}\cdot \mathrm{SH}^{4} + a82\cdot \mathrm{PP}\cdot \mathrm{SQ}^{2} + \\
& + a83\cdot(1/2)^{\mathrm{SH}\cdot \mathrm{SQ}} + a84\cdot \mathrm{SH}\cdot \mathrm{SP}
\end{aligned} \tag{5}$$

where:
FOC – fuel consumption [ton/day],
f – roll amplitude [deg],
ψ – pitch amplitude [deg],
M – number of months since last docking [–],
T – mean draught [m],
t – trim [m],
RPM – revolution of the engine [rev/min],
L – main engine load [%],
VQ – relative wind course [deg],
PH – primary wave height [m],
PQ – primary wave direction [deg],
PP – primary wave period [s],
SH – secondary wave height [m],
SQ – secondary wave direction [deg],
SP – secondary wave period [s],
a0 … a84 – coefficients – shown in Tables 3 and 4.

Equation (4) is characterized by:

- R-squared coefficient $R^2 = 0.96$;
- standard error SE = 4.37 ton/day;
- P-value (of F-test) = 5.55E–16;

while equation (5) is characterized by:

- R-squared coefficient $R^2 = 0.97$;
- standard error SE = 4.08 ton/day;
- P-value (of F-test) = 5.55E–16.

Analysis of variance, the overall P-values of F-test and the individual P-values presented in Tables 3 and 4 show that all of the predictors in equations (4) and (5) are statistically significant.

As clearly shown above, the accuracy of the obtained results (fuel estimation accuracy) improves with complicity of the equation.

Table 3. Predictor (P) and coefficients (a) values in equation (4)

a	Value of a	P-value	a	Value of a	P-value
a0	2.740801	3.90E–06	a7	2.112529	0.00
a1	–7.90E–01	2.19E–07	a8	1.39E–08	1.03E–03
a2	2.948419	4.34E–19	a9	0.00987	1.87E–30
a3	–1.40E+01	2.21E–299	a10	9.05E–12	1.10E–06
a4	–5.27E–29	4.69E–03	a11	–3.65E–04	3.40E–09
a5	0.001894	9.65E–03	a12	–1.12E+00	4.62E–03
a6	0.003652	4.24E–75	a13	7.56E–04	8.73E–08

Comparison of obtained results with the commonly used equation

The results obtained from equations (4) and (5) have been compared with the results of the commonly used equation (1). Based on the recorded data, coefficient k in equation (1) was calculated to be $k = 227.9\text{E–}6$.

Table 4. Predictor (P) and coefficients (a) values in equation (5)

a	Value of a	P–value	a	Value of a	P–value	a	Value of a	P–value	a	Value of a	P–value
a0	–2.87E+01	2.68E–08	a22	0.835032	1.83E–14	a44	–4.63E–03	3.04E–04	a66	–5.81E–04	1.14E–05
a1	–8.15E+00	6.95E–07	a23	–2.31E–01	2.00E–14	a45	–5.75E–03	3.53E–10	a67	9.27E–46	2.37E–02
a2	1938.957	1.03E–13	a24	0.198129	2.11E–10	a46	0.00228	2.86E–11	a68	0.293581	5.05E–08
a3	20.06238	1.17E–22	a25	–1.83E–04	9.17E–03	a47	0.012584	3.78E–04	a69	–1.22E–31	2.10E–06
a4	1.09E–08	2.85E–24	a26	–5.94E+00	7.50E–04	a48	–1.28E–01	8.05E–13	a70	–5.64E–08	4.90E–35
a5	3.764012	1.16E–69	a27	–1.01E–02	1.86E–04	a49	0.166426	1.38E–13	a71	–7.16E–03	9.27E–05
a6	0.401539	2.23E–08	a28	–1.77E–01	3.08E–07	a50	–2.47E+00	1.23E–03	a72	5.855514	5.74E–04
a7	–2.46E–34	5.33E–04	a29	–3.83E–37	7.38E–03	a51	–9.58E–03	1.10E–06	a73	–2.16E–03	1.21E–03
a8	–1.03E+01	3.41E–09	a30	–7.68E–02	2.12E–52	a52	0.05132	4.47E–04	a74	–1.83E–05	5.51E–07
a9	–1.86E–07	2.86E–02	a31	–2.81E–27	7.90E–09	a53	2.59E–05	1.20E–03	a75	1.77E–55	1.11E–03
a10	1.692808	3.80E–06	a32	0.31051	1.25E–50	a54	–7.56E+00	7.68E–10	a76	25.3368	3.03E–03
a11	16.71768	6.00E–09	a33	–3.95E–07	1.10E–06	a55	–1.33E–01	3.00E–06	a77	9.660327	3.89E–03
a12	–9.88E–01	4.96E–04	a34	8.805145	5.75E–16	a56	–3.24E–02	1.26E–22	a78	9.34E–49	5.47E–04
a13	0.054328	1.23E–05	a35	2.41E–17	1.32E–04	a57	–4.73E–03	3.20E–06	a79	0.008984	6.05E–15
a14	–3.12E–04	3.63E–03	a36	–2.27E–01	3.45E–10	a58	–2.28E–33	2.42E–13	a80	2.55E–14	1.60E–06
a15	–3.27E+00	2.60E–06	a37	–1.36E–02	2.01E–04	a59	0.209394	4.79E–08	a81	6.46E–07	2.25E–05
a16	0.012238	3.52E–04	a38	–1.96E–02	9.33E–04	a60	0.000383	3.04E–04	a82	–4.73E–07	3.25E–08
a17	0.009437	1.53E–02	a39	6.50E–07	2.80E–03	a61	–4.34E–03	3.53E–10	a83	–1.13	1.79E–08
a18	–7.52E+00	3.31E–05	a40	0.029596	2.03E–21	a62	–3.39E–01	2.86E–11	a84	0.171874	9.45E–03
a19	–3.23E–03	7.53E–05	a41	1.88E–05	1.30E–18	a63	9.63E–10	3.78E–04			
a20	–9.48E+00	4.25E–11	a42	1.76E–13	1.10E–16	a64	0.001198	8.05E–13			
a21	–1.16E+01	3.46E–44	a43	2.10E–31	2.82E–03	a65	–1.39E–01	1.38E–13			

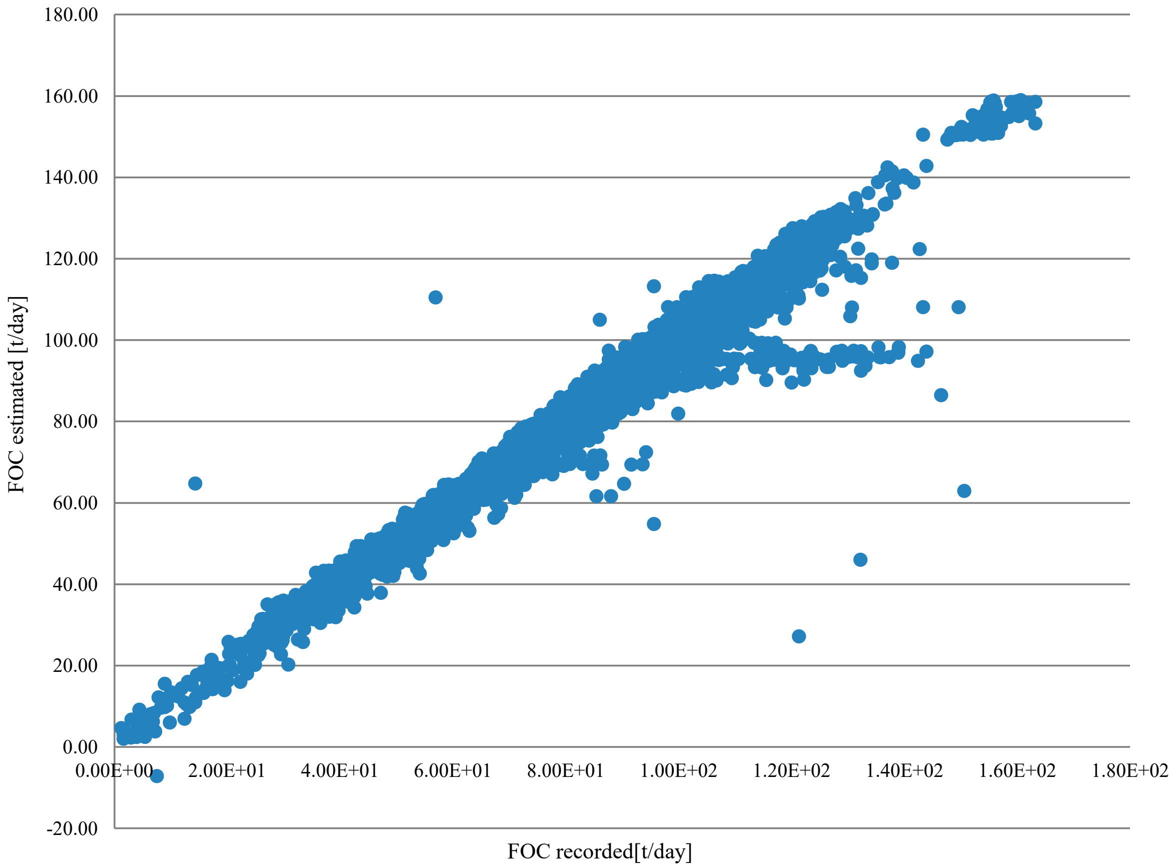

Figure 4. Approximations of interim fuel consumption depending on service parameters and its relationship (4) with calculated values in comparison to recorded data

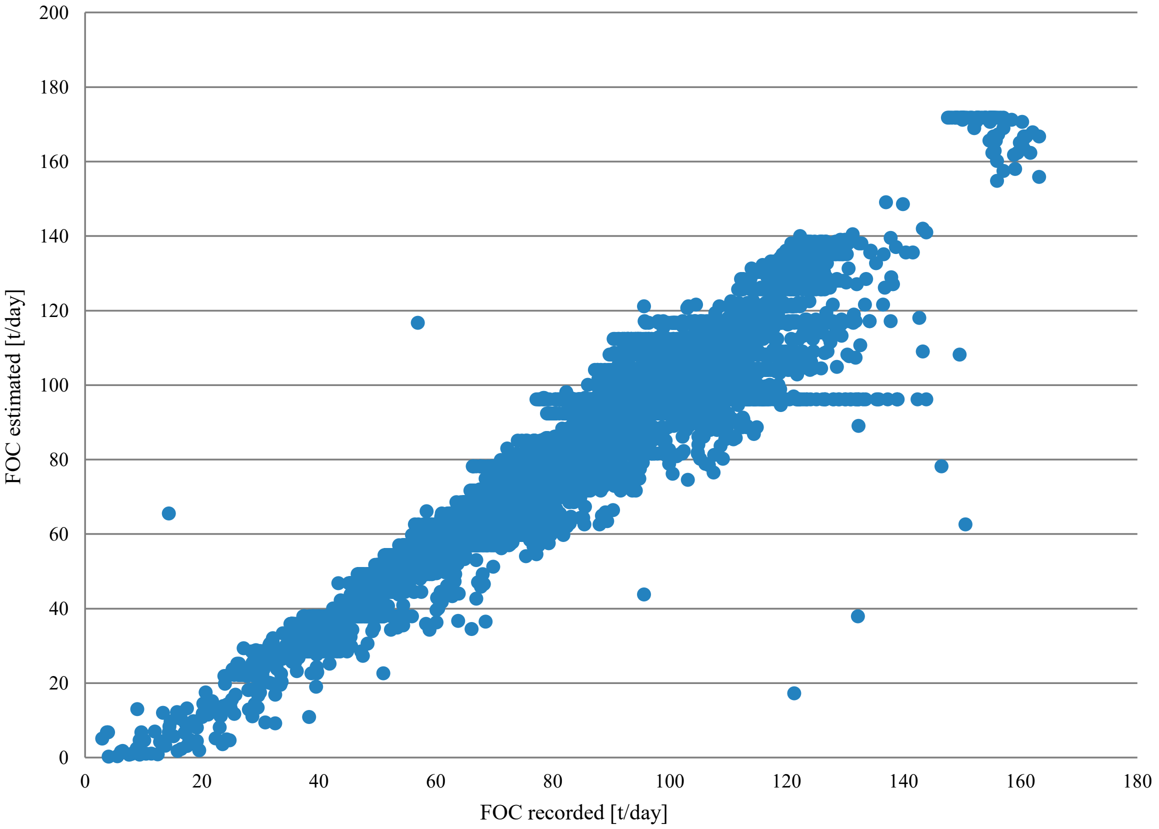

Figure 5. Approximations of interim fuel consumption depending on service parameters and its relationship (6) with calculated values in comparison to recorded data

Equation (1) and coefficient *k* lead to:

$$\text{FOC} = 227.9\text{E–}6 \cdot \text{RPM}^3 \qquad (6)$$

Equation (6) is characterized by:

- R-squared coefficient $R^2 = 0.91$;
- standard error SE = 7.69 ton/day.

Equation (6) is characterized by a lower correlation and almost twice as large a value of standard error as compared with equations (4) and (5).

Likewise, Figures 4 and 5 show the relation between equations (4) and (6) relative to the recorded data, demonstrating that equation (4) is more accurate than equation (6). It can be seen that there are many more points on the 1:1 line in Figure 4 than in Figure 5, though still a few points are far from the 1:1 line in both of these figures.

Conclusions

1. This research has shown that it is possible to develop approximate fuel consumption based on on-board ship recorded data.
2. The regression methods that were used enabled the creation of accurate estimations, characterized by low standard error.
3. The commonly used formula (6) is the least accurate and is characterized by almost twice as large a value of standard error as compared with equations (4) and (5).
4. Equations (4) and (5) take into account all relevant (significant/important) ship service parameters and weather parameters. This has never been seen before and we hope that it will advance the development of new ship theory.
5. Equation (5) is more accurate than equation (4), but equation (5) is far more complex.
6. Equation (5) takes into account all weather parameters, while equation (4) does not take into account primary and secondary wave direction.
7. This study clearly shows a relationship between fuel consumption and the number of months since last docking. This work could be said to be highly important as it makes it possible to take into account the relationship between hull and propeller fouling, and fuel consumption. These kinds of relationships have not been demonstrated before in ship theory. The results found here can be the basis for further research in this direction.
8. NdCurveMaster software is highly effective, meaning significant time savings were made in the search for the suitable equations. A curve fitting method has been efficiently implemented in ndCurveMaster.

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