Application of Allan Variance in Gyroscopic Sensor Noise Analysis

Rafał Kotas, Marek Kamiński, Paweł Marciniak, Bartosz Sakowicz, Andrzej Napieralski, Anna Kurzych, and Zbigniew Krajewski

***Abstract*—The paper presents the investigation of gyroscopic sensor noise properties used in the construction of body position detection device for posturographic testing. The first part shows the sources of noise in gyro sensors and their measurement methods. In the second part of the paper, the authors describe their own research on the efficiency of the calculation of the Allan variance (one of the popular noise evaluation methods) and the wrong concept of calculations acceleration. The article concludes with an explanation of the reasons for the lack of theoretical research and the results of practical measurements of the sensors used in the project.**

***Index Terms*—Allan Variance, Gyroscope, Accelerometer, MEMS**

I. Introduction

The implementation of the "Innovative system for evaluation and rehabilitation of human imbalance" is very dependent on the construction of a suitable measuring system to accurately determine the position of the human body. The device placed on the patient should be able to monitor his movements and thus to diagnose his condition, evaluate his rehabilitation progress and partially prevent the patient from falling.

The essential element of the measuring system is an inertial system consisting of many MEMS sensors, all of which are thermally compensated and calibrated so that the motion readings are accurately measured in the orthogonal axis. The built-in gyroscope measures the rotation in three axis around a fixed point, and the three-axis accelerometer built into the system in turn gives information about the displacement (the acceleration in each axis). These data are sent to the microcontroller which then analyses them. It allows to determine, for example, the trajectory of the motion or the position in space (relative to the starting point).

Inertial measurements have following advantages:

- they are independent of any other external support systems;
- they can be used in tunnels and buildings;
- they are resistant to intentional or accidental interference.

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The most important disadvantage of inertial measuring process is the decrease in accuracy during its duration.

For this project the accuracy of the device was estimated at about 1 degree. This required a calibration exercise, and at the same time a look at the problem of errors in gyro sensors and their measurement. The authors took a closer look at the procedure for determining Allan's variance for gyro sensors.

The article presents the work done for this purpose and the interesting drawback of available scientific documentation leading to completely unsuccessful attempts to improve the calculation procedure.

II. Errors in Reading from Gyroscopes / Sources of Errors in Gyroscopes Measurements

The following inertial measurement errors can be observed [3, 4]:

- constant deviation from the real value, which is an individual characteristic of a given sensor;
- deviation that becomes different with temperature change;
- random error associated with each launch of the sensor;
- deviations of measured values from real values, fluctuating during measurement in an unpredictable manner;
- Random measurement noise – possible to reduce in the filtration process (during operation of the Kalman filter system) or at the analysis stage after the measurement.

The MEMS gyroscope measuring angular movement has several internal sources of error, which are manifested in the fact that zero reading in the gyroscope will drift in time as a result of system noise, changes in polarization voltage as well as imperfections in its design. Repeatability of this polarization can be achieved by calibrating the temperature range at which the IMU is designed to operate, but the instability of this polarization alone will result in errors in the angle measurement. These errors will accumulate over time and will cause a calculation error in the DSP.

In the case of accelerometers, the problem of drift and accumulation of measurement errors does not occur, however, these devices in turn are exposed to errors due to the vibration

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of the element or other accelerations beyond gravity. For this type of error, the axis of rotation about the vertical axis is most exposed, as the other two can be periodically calibrated in relation to the gravitational field, as measured by accelerometers. In the case of a vertical axis of rotation it cannot be calibrated, because the rotation around this axis does not affect the accelerated projection on the individual axis of the accelerometer, because the rotation is about the direction of the field.

One of the methods to eliminate the gyroscope measurement drift is a regular calibration of zero angular velocity for all gyro sensors in the system. Ideally, every time it is known that the axis does not rotate, the calibration process should zero its speed. How often this operation can be done very strongly depends on the specific application of the system, however, regardless of that, the system should use every 'stop' between man steps to zero the measurement drift.

III. Application of Allan Variance in Noise Analysis with Computational Complexity

Variance methods of data analysis are used for the analysis in the time domain of signal measurements from gyroscopes and accelerometers. The Allan variance analysis is a popular method for defining random error parameters for inertial sensors [3, 4]. The algorithm based on this method allows to analyse the following random errors of gyroscopes:

- bias instability;
- angle random walk;
- quantization noise;
- rate random walk;
- rate ramp.

Accelerometers are subjected to a similar analysis of random errors.

The magnitude of these errors is determined by the AV (tau) deviation graph recorded in the static conditions of the measured data.

In order to calculate Allan variance for gyroscopes it is possible to use two calculation procedures:

A) based on angular values (θ)

The algorithm consists of two steps:

Angle calculations based on angular velocity readings

$$\theta(t) = \int_0^t \Omega(t')dt' \tag{1}$$

Calculating Allan's variance based on angles

$$\theta^2(\tau) = \frac{1}{2\tau^2(N-2m)} \sum_{K=1}^{N-2m} (\theta_{K+2m} - 2\theta_{K+m} + \theta_K)^2 \tag{2}$$

N - number of samples

M - averaging factor ($\tau = m \cdot \tau0$)

T_0 - sampling period

The number of floating point operations for formula (2) can be estimated:

- for N addition operations for integration;
- for $\sum_{m=1}^{N/2} 3(m - 2\mathrm{N})$ addition operations for calculation of Allan variance;
- for $\sum_{m=1}^{N/2} N - 2m$ multiplication operations for calculation of Allan variance;

The main advantage of the formula (2) is its computational complexity at the level of n^2. In practice this means that this formula is readily used in practical calculations due to the short calculation time even for large N values.

B) Based on the average angular velocity (Ω)

$$\acute{\Omega}_K(\tau) = \frac{\theta_{K+m} - \theta_K}{\tau} \tag{3}$$

The equation (2) connecting the angular velocity to the angle means that the Allan variance can be calculated directly on the basis of the average angular velocity. This solution has similar computational complexity as the previous one but it has very unfavorable numerical properties. The use of the overlapping procedure (calculation of mean values over overlapping time periods (Figure 1)) allows to solve the problem of results instability.

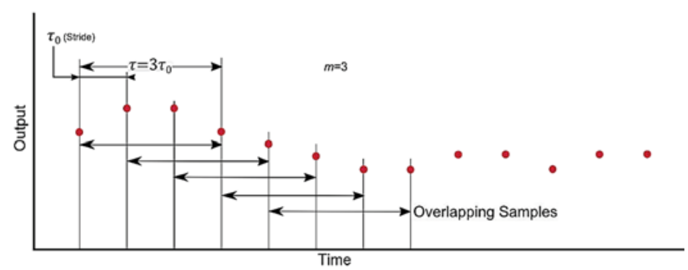

Fig. 1. Overlapping procedure [6]

The formula for calculating the variance of Allan, which includes overlapping, has the form [5 (page 15, equation 9)] [6 (page 4, equation 8)]:

$$(\tau) = \frac{1}{2m^2(N-2m)} \sum_{j=1}^{N-2m} \left\{ \sum_{K=j}^{j+m-1} (\acute{\Omega}_{K+m}(\tau) - \acute{\Omega}_K(\tau))^2 \right\} \tag{4}$$

The number of floating point operations depending on N can be estimated at:

- for $\sum_{m=1}^{N/2} 2(Nm - 2m^2)$ addition operations for calculation of Allan variance;
- for $\sum_{m=1}^{N/2} Nm - 2m^2$ multiplication operations for calculation of Allan variance;

The computational complexity of this formula is n^2. This means a much worse time parameters in comparison with the formula (2). For larger values of N (in the described tests N = 500k) the use of formula (4) becomes impossible.

An additional very adverse element of the formula (from the implementation point of view) is the gradual increase in the number of operations (computation time) for the subsequent segment sizes (m). This causes great difficulties in parallel calculations (despite the form of the formula suggesting easy balancing) due to the proper division of tasks into individual machines cores.

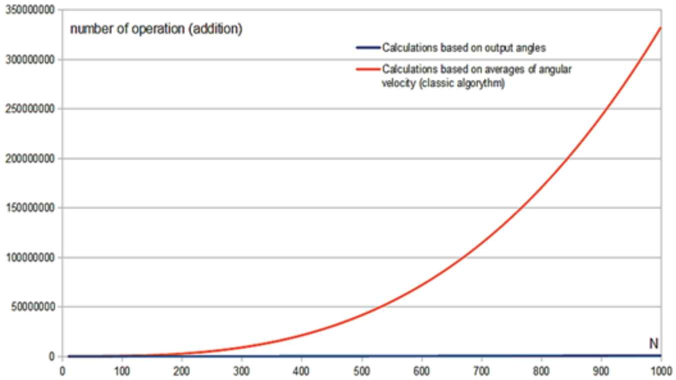

Fig. 2. Comparison of the number of addition actions from N for both algorithms.

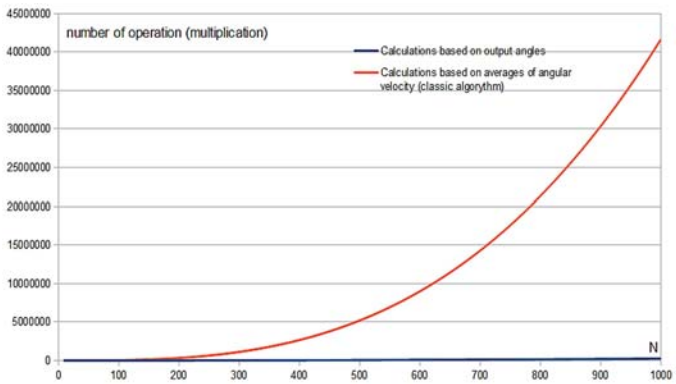

Fig. 3. Comparison of the number of multiplication operations from N for both algorithms

In addition to the calculation of Allan variance, the authors have developed a simple numerical method (the way of arranging the algorithm) which significantly reduces the number of actions performed for the formula. This led to shortening of the calculation time to the level which is similar to that of the procedure A. The algorithm does not affect the results obtained in any way.

IV. Methodology of Calculation Acceleration

Precise analysis of formula 4 shows that many operations are repeatedly performed. The internal sum in formula 4 for the next j differs only by two extreme values. Therefore, it is relatively easy to convert an algorithm into one that eliminates unnecessary operations without changing the results.

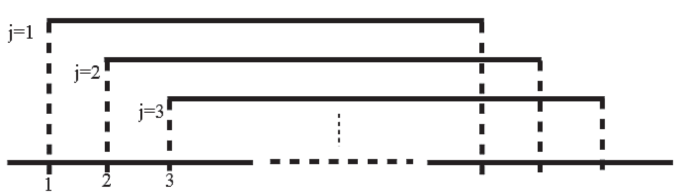

Fig. 4. Scheme for calculating further sums in formula 4.

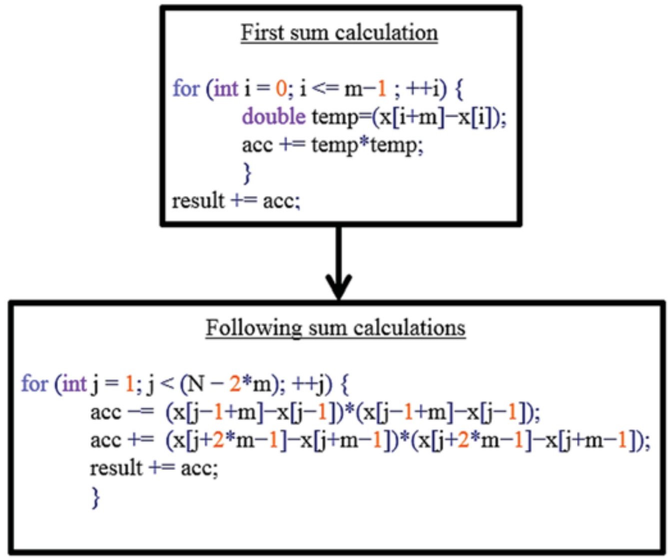

Fig. 5. Algorithm of the modified method

In a similar way (although much less important) the segmentation of samples can be accelerated. The new version of the algorithm is characterized by the following number of actions performed:

For calculation of Allan variance:

addition:

$$\sum_{m=1}^{N/2} 2\left(Nm-2m^2\right)$$

multiplication:

$$\sum_{m=1}^{N/2} Nm-2m^2$$

It can be estimated that the time needed to complete the calculation is proportional to the n^2 (number of samples).

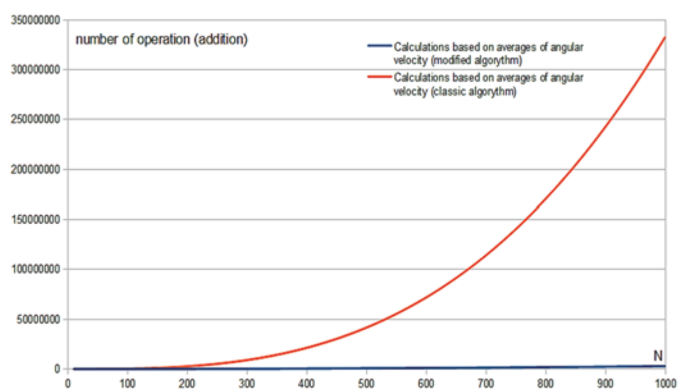

Fig. 6. Comparison of the number of addition operations from N for accelerated and classical algorithms

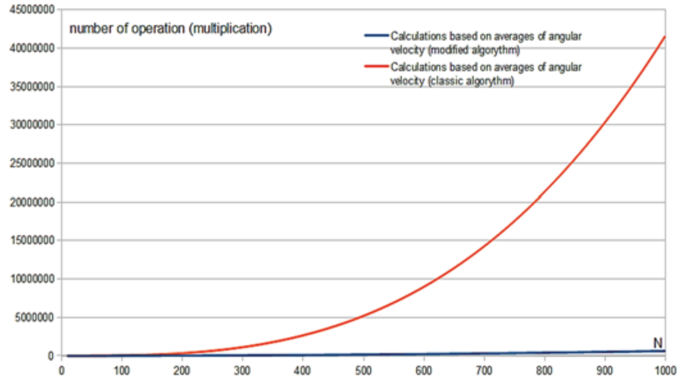

Fig. 7. Comparison of the number of multiplication operations from N for accelerated and classical algorithms

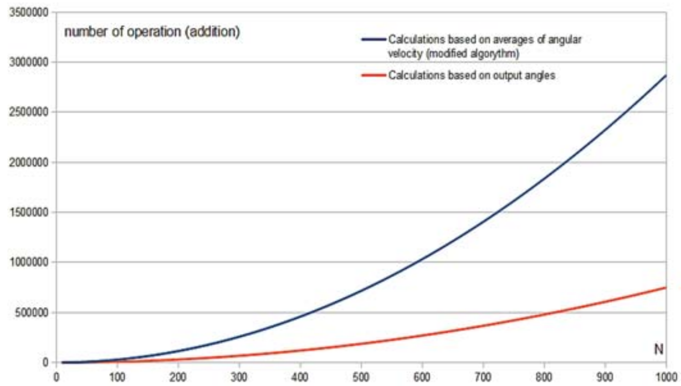

Fig. 8. Comparison of the number of addition operations from N for formulas 2 and 4

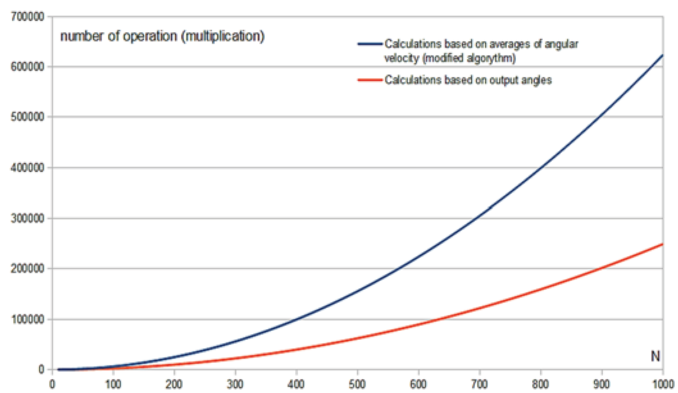

Fig. 9. Comparison of the number of multiplication operations from N for formulas 2 and 4

Calculation time is a bit bigger than for formula 2, but the difference is relatively small. The computational complexity of both algorithms can be estimated as identical.

V. Results

In order to verify the described concept, Allan variance calculations (based on LSM9DS1 sensor readings) were performed. The authors used a typical three-axis MEMS gyroscope.

In order to test the concept of acceleration calculations about 514 thousand samples were collected. Calculations were made on a laptop with an Intel Core i7-3610QM 2.3GHz and 16GB of RAM. Table 1 shows the comparison of the calculation times.

TABLE I.
Calculation times

Algorithm	Calculation time [s]
Calculations based on the angles (2)	283
Calculations using angular velocity without overlapping	641
Calculations using angular velocity with overlapping (4) accelerated version	2007
Calculations using angular velocity with overlapping (4) standard version	~ 600000

During the work it turned out that the whole concept was based on erroneous assumptions.

Comparison of both formulas: The formula (2) calculated from the angles (previously calculated on the basis of angular velocity integration) and formula (4) calculated directly on the basis of angular velocity gave completely different results.

There are two errors in formula (4). First, at angular velocities there should be no dashes indicating the averaging (the internal sum is used for averaging purposes) and the second is in the wrong place. Instead of the present sum of squares there should be a square of sum (of course the inner). Then the inner sum without the squares is exactly the overlapping averaging, and the outer (squared) is the traditional formula for the Allan variance. In the second article there is also an error with incorrect placement of the squares.

After correcting the formula (without special effort), it is possible to transform the equation (2) into equation (4) and both have the same result. Nobody suspected the mistake in the formula (4) because this formula was not used due to very long calculation time.

The author's idea of speeding up the calculations makes sense only with the sum of squares. In the case of a squared sum, the only sensible approach is the prior conversion of sums (in angles) and the use of formula (2).

Therefore, the author's proposal ceased to have any value in terms of substance - formulas (2) and (4) are identical, give the same result, and (2) will always be faster (number of addition and multiplication operations are smaller.

Figures 10-12 show the results of the Allan variance calculations using angles (2) and angular velocities without overlapping. Measurement time was limited to 50,000 samples. Calculations were performed for all three axes of the system. Formulas 2 and 4 (angular velocities with overlapping) give the same results.

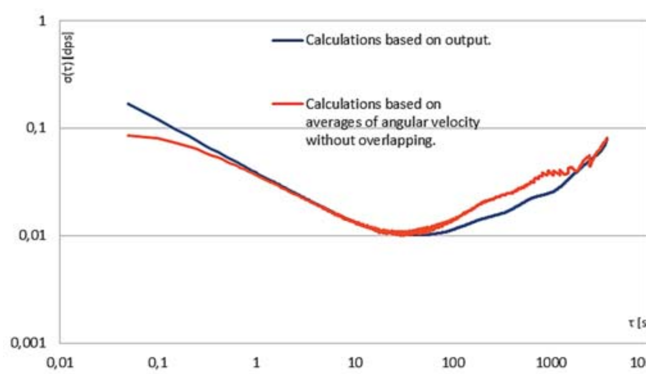

Fig. 10. Results of the Allan variance g_x (x axis)

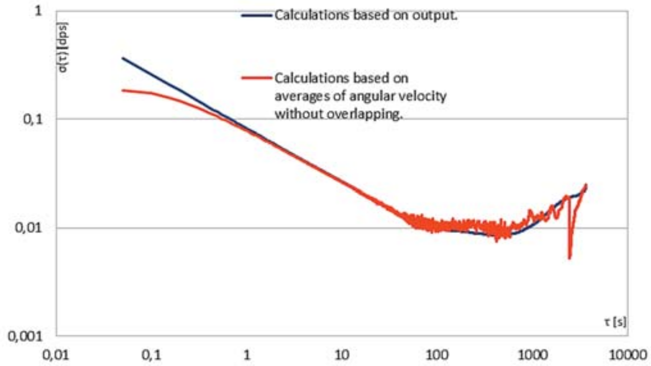

Fig. 11. Results of the Allan variance g_y (y axis)

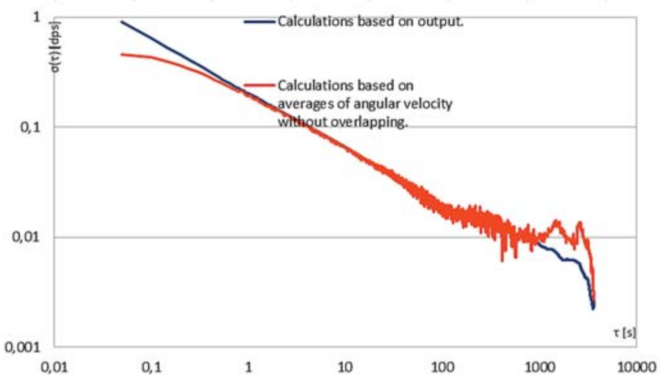

Fig. 12. Results of the Allan variance g_z (z axis)

VI. Conclusions

Works presented in the article were focused on two issues. On the one hand, they concerned the evaluation of the position detection sensor noise characteristic. The issue of noise is particularly relevant for gyroscope sensors. The authors plan to develop their own MEMS sensor. This means that the authors need to carry out research to assess their own and commercial solutions. At this stage of the work, noise characteristics of commercial sensors were collected. They will be used to compare it with the sensor developed by the authors. The slope of the graphs means that the primary source of noise in a wide frequency range is white noise.

TABLE II.
Slope chart for each axis

Sensor axis	Slope chart
x	-0.48
y	-0.5
z	-0.5

Table 2 presents slope charts for one of five sensor units. Obtained results show that the noise characteristic is practically identical for all tested sensors. The second part of the research was an observation that the methods proposed in bibliographic sources could be optimized for computational efficiency. One of the formulas (described as never used) has been significantly accelerate. Unfortunately, the comparison of the results obtained from different equations and a more accurate analysis of the problem revealed errors in the documentation that undermined the proposed solutions. The authors decided to describe all the unsuccessful work motivated by a strong emotional connection with the subject. The issue of course will no longer be taken.

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