Stiffness and Damping Coefficients of 8-lobe Journal Bearings

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Abstract

High-speed rotating machinery, such as steam turbines, turbo-compressors, pumps, turbo pumps, accelerating gearboxes contain multilobe journal bearings. The bearings with 8 lobes are applied in bearing system of grinding spindle. The reasons for the application of such bearings are not too high temperatures of bearing operation and specific static and dynamic characteristics. Stiffness and damping coefficients of 8-lobe journal bearings with cylindrical and multilobe profile of lobes were calculated by means of perturbation method. Different values of bearing length to diameter ratio, relative clearance and lobe relative clearance were assumed. Reynolds', energy and viscosity equations were solved by means of iterative procedure. Adiabatic oil film, laminar flow in the bearing gap and aligned orientation of journal in the bearing were considered.

Keywords: multilobe journal bearing, dynamic characteristic

1. Introduction

The bearings with 4, 6 or 8-lobes are applied in the turbo-compressors and the spindle bearings systems of grinding machines [1-4]. In case of 6 or 8-lobe bearings there are few information on their static and dynamic characteristics that include stiffness and damping coefficients of oil film. Such characteristics are required for determination of bearing stability. The acquaintance of both types of characteristics for these bearings allows choosing their correct design and operating parameters. The static characteristics are the input variables for obtaining the dynamic characteristics and they are the basis for the reliable and durable design of journal bearings [1-3].

In grinding machinery the journal bearings ensure the operation of spindle at proper oil film temperatures and at low level of vibrations what is very important in the design of grinders and grinding [5,6]. The stiffness, dynamic properties (damping of vibration), rotational accuracy and the heat [6,7] generated by spindle support of grinder affects the dimensional-and profile accuracy of grinded elements. An increase in efficiency of grinding process can be obtained by the increase in the rotational speed of grinding spindle, which is restricted by the design of bearings and the strength of grinding wheel. It means that the calculation of the journal bearings is of great importance in the design process of grinding machine.

The paper presents the results of the calculations of stiffness and damping coefficients of 8-lobe journal bearings operating at different length to diameter ratios, bearing and lobe relative clearances as well as different rotational speed of spindle. Numerical method by means of finite differences was applied for the simultaneous solution of geometry, Reynolds and energy, viscosity equations on the assumption of adiabatic oil film. The dynamic characteristics were calculated by means of perturbation method [9].

2. Stiffness and damping coefficients and application for rotor stability

On the assumption of the parallel axis of journal and bearing sleeve, the geometry of oil film gap of multilobe journal bearing (Fig. 1) is described by Eq. (1); the first term of this equation gives the geometry of multilobe bearing [7,8].

$$\overline{H}(\varphi) = \overline{H}_{Li}(\varphi) - \varepsilon \cdot \cos(\varphi - \alpha) \tag{1}$$

where: $\overline{H} = h/(R-r)$ - dimensionless oil film thickness, h - oil film thickness (m), R, r - sleeve and journal radius (m), α - attitude angle, (⁰), ε - relative eccentricity, φ - peripheral co-ordinate, (⁰)

$$\overline{H}_{Li}(\phi) = \psi_{si} + (\psi_{si} - 1) \cdot \cos(\phi - \gamma_i)$$
⁽²⁾

where: γ_i - angle of lobe centre point, (⁰), ψ_{si} - lobe relative clearance.



Figure 1. 8-lobe journal bearings; a) general view, b) cross-section of sleeve, c, d) geometry (load between lobes – LBL and load on lobe – LOL, Offset); F – bearing

load, O_b , O_j – bearing and journal centre, R_l – radius of lobe, ω - angular velocity,

The journal bearing performances for adiabatic model of oil film can be determined by the numerical solution of the oil film geometry, Reynolds, energy and viscosity equations on the assumption of static equilibrium position of the journal [6 -8]. The oil film pressure distribution was obtained from Eq. (3).

$$\frac{\partial}{\partial \varphi} \left(\frac{\overline{H}^{3}}{\overline{\eta}} \frac{\partial \overline{p}}{\partial \varphi} \right) + \left(\frac{D}{L} \right)^{2} \frac{\partial}{\partial \overline{z}} \left(\frac{\overline{H}^{3}}{\overline{\eta}} \frac{\partial \overline{p}}{\partial \overline{z}} \right) = 6 \frac{\partial \overline{H}}{\partial \varphi} + 12 \frac{\partial \overline{H}}{\partial \phi}$$
(3)

where: \overline{p} - dimensionless oil film pressure, $\overline{p} = p\psi^2/(\eta\omega)$, p - oil film pressure (MPa), (m), D, L - bearing diameter and length (m), t - time (sec), \overline{z} - axial co-ordinates, $\phi = \omega$ t - dimensionless time, $\overline{\eta}$ - dimensionless viscosity, ψ - bearing relative clearance, ψ = $\Delta R/R$ (‰), ΔR - bearing clearance, $\Delta R = R - r$ (m).

It has been assumed for the pressure region that on the bearing edges the oil film pressure $p(\varphi, z) \ge 0$ and in the regions of negative pressure, $p(\varphi, z) = 0$. The oil film pressure distribution computed from Eq. (3) was put in the transformed energy equation [6,7] to obtain the temperature and viscosity distributions. Temperature $T(\varphi, z)$ on the boundaries ($z = \pm L/2$) was determined by means of the parabolic approximation [7].

The equations of motion for the journal and the centre of elastic shaft [1, 9] are given in matrix form by Eqn. (4).

$$M \cdot \ddot{x} + B \cdot \dot{x} + C \cdot x = \hat{a} \cos \omega t + b \sin \omega t \tag{4}$$

where: *M*, *B*, *C* –matrices of mass, damping and stiffness, \hat{a}, \hat{b} coefficients of dynamic constraints.

After transformations of Eq. (4) the real and imaginary part was obtained [9]. The stability of elastic rotor-bearing system is analysed based on the following characteristic frequency equation of 6-th order with regard to (λ/ω) [9-11].

$$c_6 \lambda^6 + c_5 \lambda^5 + c_4 \lambda^4 c_3 \lambda^3 + c_2 \lambda^2 + c_1 \lambda + c_0 = 0$$
(5)

The assumption of the solution of Eq. (5) is $\lambda_j = -u_j + iv_j$ ($1 \le j \le 6$), with u as damping and v representing the self-vibrations. Stability of the linear vibrations of system occurs only when all real parts of eigenvalues λ_j are negative. The coefficients c_0 through c_6 in Eqn. (5) are the functions of a_0 , b_0 , g_{ik} , b_{ik} .

$$c_0, \ldots, c_6 = f(a_0, b_0, g_{ik}, b_{ik})$$
 (6)

where: a_0 - ratio of angular velocity ω to the angular self-frequency of stiff shaft, $a_0 = (\omega / \omega_c)^2$, ω_c - angular self-frequency of stiff rotor, $\omega_c = \sqrt{c^* / m}$, b_0 - ratio of Sommerfeld number to the relative elasticity of shaft, $b_0 = S_0/c_s$, c^* - shaft stiffness, (N m⁻¹), c_s - relative elasticity of shaft, $c_s = f/\Delta R = g/(\omega_c^2 \cdot \Delta R)$, f - static deflection of shaft, (m), F - resultant force of oil film (N), F_{stat} - static load of bearing, (N), g - acceleration of gravity, (m s⁻²), g_{ik} - dimensionless stiffness coefficients, $g_{ik} = S_0(\Delta R/F_{\text{stat}})$, g'_{ik} , - stiffness coefficients, (N/m), b_{ik} - dimensionless damping coefficients, $b_{ik} = So(\Delta R/F_{\text{stat}}) \omega b'_{ik}$, b'_{ik} - damping coefficients, (N s/ m), m - mass of the rotor, (kg), So - Sommerfeld number, $So = F \cdot \psi^2 / (L \cdot D \cdot \eta \cdot \omega)$.

The coefficients of the characteristic frequency equation of 6-th order [3] depend on the stiffness g_{ik} and damping b_{ik} coefficients, Sommerfeld number *So*, relative elasticity of shaft c_s and the ratio of angular velocity to the critical angular velocity of stiff rotor. As the result of transformations, the expression determining the ratio of boundary angular speed ω_b to the critical ω_c one, and the stability of rotor, has the form [1,9]:

$$\left(\frac{\omega_b}{\omega_c}\right) = \frac{1}{1 + b_0 \cdot \frac{A_3}{A_1}} \frac{A_2 \cdot A_3^2}{A_1^2 + A_1 \cdot A_3 \cdot A_4 + A_0 \cdot A_3^2}$$
(7)

where: A_0 , ..., A_4 are the combination of four stiffness g_{ik} and four damping b_{ik} coefficients.

3. Results of calculations

The calculations of static and dynamic characteristics were carried out on the assumption of static equilibrium position of journal [9-11]. The data applied for the calculations were as follows: bearing length to diameter ratios L/D=0.5 and L/D=0.8, relative clearances $\psi = 0.8\%$ and $\psi = 1.1\%$, lobe relative clearances $\psi_{si} = 1.0$ (case of cylindrical profile of lobe) and $\psi_s = 3.9$, rotational speed of journal 3000 rpm, 5000 rpm and 10000 rpm, shaft relative elasticity $c_s = 0.1$ and $c_s = 1.0$ and the temperature of supplied oil $T_0=40^{\circ}$ C. The heat number K_T [6-8] used in transformation of energy equation and for computation of oil film temperature distribution, depends on the applied speed, bearing clearance and supplied oil temperature $(K_T = \omega \cdot \eta_0 / (c_t \cdot \rho \cdot g \cdot T_0 \cdot \psi^2), c_t$ - specific heat of oil, (J/kgK), g - acceleration of gravity (m/s²), T₀ - temperature of supplied oil, (⁰C), η_0 - dynamic viscosity of supplied oil, Ns/m², ρ - oil density, (kg/m³).

Exemplary results of computation of journal displacements ε and static equilibrium position angles α_{eq} for different types of 8-lobe journal bearings (LOL) are shown in Fig. 2 (letters M, OF, C, mean multilobe, offset, cylindrical, respectively) and Fig. 3; 8 lobe bearing with larger lobe relative ratio and at assumed relative eccentricity shows the largest load capacity and the smallest values of static equilibrium position angles α_{eq} .



Figure 2. Relative eccentricity of 8 lobe Figure 3. Static equilibrium position angles journal bearings of 8 lobe journal bearings

The stiffness and damping coefficients versus Sommerfeld number *So* are presented in Fig. 4 through Fig. 11. The values of the coefficients of 8 lobe bearing with cylindrical profile of lobes (ψ_{si} =1.0), length to diameter ratio *L/D*=0.5, relative clearance $\psi = 0.8$ ‰ and at assumed rotational velocity of journal can be observed in Fig. 4 through Fig. 7; at lower rotational speed of journal all coefficients are larger than at higher speeds (e.g. Fig.4 and Fig. 6 or Fig. 5 and Fig. 7). At rotational speed 10000 rpm the largest values has the stiffness coefficient g_{22} and the smallest g_{12} (Fig. 8). Among the damping coefficients the largest values has the coefficient b_{22} and the smallest b_{21} (Fig. 9).

For the bearing with L/D=0.8, clearance $\psi = 0.8$ ‰, lobe relative clearance $\psi_{si}=3.9$ and Sommerfeld numbers, the largest values have the stiffness g22 and damping b22 coefficients but the smallest have the coefficients g_{12} (Fig. 10) and the coupled damping coefficients b₁₂ and b₁₂ (their values are equal in all cases, e.g. Fig. 5, Fig.11).



2

1,75

1,5

1,25

Figure 4. Stiffness coefficients of 8 lobe journal bearing (3000 rpm)

Figure 5. Damping coefficients of 8 lobe journal bearing (3000 rpm)

b_{ik} [-]

 b_{11}

 b_{12}

8C

L/D=0.5

LOL



Figure 6. Stiffness coefficients of 8 lobe journal bearing (ψ_{si} =1.0, 5000 rpm)



Figure 7. Damping coefficients of 8 lobe journal bearing (ψ_{si} =1.0, 5000 rpm)



Figure 8. Stiffness coefficients of 8 lobe Figure 9. Damping coefficients of 8 lobe journal bearing (10000 rpm) journal bearing (10000 rpm)



Figure 10. Stiffness coefficients of 8 lobe journal bearing (ψ_{si} =3.9, 5000 rpm)

Figure 11. Damping coefficients of 8 lobe journal bearing (ψ_{si} =3.9, 5000 rpm)

Stiffness and damping coefficients characterizing the bearing properties are applied for investigation into the stability and instability ranges, e.g. the simple, symmetric and stiff rotor; exemplary results are presented in Fig. 12 and Fig. 13 versus critical Sommerfeld number So_k (So_k - critical Sommerfeld number, $So_k = So \ \omega/\omega_c$). Fig. 12 presents the stability ranges for two values of shaft relative stiffness c_s rotating in the bearings with the width to diameter length L/D=0.8 at the speed of 5000 rpm; the angle τ determines the range of stability of considered bearing at assumed geometric and operational parameters.

The boundary lines of stability ranges for ideally stiff rotor ($c_s \rightarrow 0$) and for different bearing parameters and rotational speeds are given in Fig. 13. It can be seen, that for both assumed rotational speeds the best stability show 8 lobe bearings with cylindrical profile of lobes (Fig. 13 – the lines for bearing relative clearance $\psi = 0.8$ ‰ and lobe relative clearance $\psi_s = 1.0$).



Figure 12. Stability ranges of 8- lobe journal bearing (5000 rpm)

Figure 13. Boundary lines of the stability of 8-lobe journal bearings

4. Final remarks

The stiffness and damping coefficients of the oil film of 8-lobe journal bearings were obtained by means of author-developed code of numerical computation. It was stated that such parameters as bearing length to diameter ratio L/D, relative clearance ψ and lobe relative clearance ψ_s have significant effect on the computed coefficients as well as on the stability of simple symmetric rotor operating in considered bearing. It can be concluded that the variation of above-mentioned parameters can be applied for the design changes of considered bearing allowing at the same time for the operation at higher speeds

The coefficients computed for two types of 8-lobe bearings and stability ranges do not give clear answer which of both types of bearings is more convenient to apply. From the author's view it would be necessary to take into account the static characteristics and particularly the maximum oil film temperatures and the power losses. Developed code of calculations allows theoretical investigation into the static and dynamic characteristics comprising the basic parameters of bearing such as load capacity, static equilibrium position angles, power losses, stiffness and damping coefficients and the stability ranges.

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