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Optimization of critical infrastructure operation and safety with considering climate-weather change influence – maximizing safety lifetime

Keywords

critical infrastructure, safety, operation process, climate-weather change, optimization, safety lifetime

Abstract

The method based on the results of the joint model linking a semi-Markov modelling of the critical infrastructure operation process with a multistate approach to critical infrastructure safety and linear programming are proposed to the critical infrastructure operation and safety optimization. This method determining the optimal values of limit transient probabilities at the critical infrastructure operation states that maximize the critical infrastructure safety lifetime in the safety state subsets is proposed.

1. Introduction

The critical infrastructure operating at a fixed area may be vulnerable to damage caused by external threats and on the other hand, it may cause threats to other critical infrastructures [Lauge et al., 2015]. This fact should be considered to construct a global network of interconnected and interdependent critical infrastructure networks existing at this operating area what is highly reasonable as usually the critical infrastructures are not isolated and they create a system of interconnected and interdependent critical infrastructures. The proposed approach, taking into account threats associated with critical infrastructure and its components / assets operation and with natural climatic hazards [Caldwel et al., 2002], [HELCOM, 2009] can help to indicate which of critical infrastructures can be affected by and which ones can affect other critical infrastructures in their operating area. In this context, the safety analysis and prediction of a single critical infrastructure composed of a number of assets impacted by its operation process and climate-weather change is very important [Kołowrocki, 2014], [Kołowrocki, Soszyńska-Budny, 2011], [Kołowrocki, Soszyńska-Budny, 2012a-b]. Therefore, in the report, the method of a single critical infrastructure related to its operation process safety optimization with considering climate-weather change process influence is proposed.

2. Critical infrastructure operation process related to climate-weather change process

We consider the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in < 0, \infty)$, in a various way at this process states zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$. We assume that the changes of the states of operation process related to the climate-weather change process $ZC(t)$, $t \in < 0, \infty)$, at the critical infrastructure operating area have an influence on the critical infrastructure safety structure and on the safety of the critical infrastructure assets A_i, $i = 1,2,...,n$, as well [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017].
We assume, as in [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017], that the critical infrastructure during its operation process is taking $\nu, \nu \in N$, different operation states $z_1, z_2, \ldots, z_\nu$. We define the critical infrastructure operation process $Z(t)$, $t \in < 0, +\infty)$, with discrete operation states

from the set $\{z_1, z_2, \ldots, z_\nu\}$. Further, we assume that we have either calculated analytically or evaluated approximately by experts the vector

$$[p_b]_{1\times\nu} = [p_1, p_2, ..., p_\nu] \quad (1)$$

of limit values of transient probabilities (OPC1)

$$p_b(t) = P(Z(t) = z_b), \; t \in <0,+\infty), \; b = 1,2,...,\nu,$$

of the critical infrastructure operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,...,\nu$. Moreover, as in [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017], we assume that the climate-weather change process $C(t)$, $t \in <0,+\infty)$, at the critical infrastructure operating area is taking w, $w \in N$, different climate-weather states $c_1, c_2, \ldots, c_w$. We assume that we have either calculated analytically or evaluated approximately by experts the vector

$$[q_l]_{1xl} = [q_1, q_2, ..., q_w] \quad (2)$$

of limit values of transient probabilities (C-WCPC1)

$$q_l(t) = P(C(t) = c_l), \; t \in <0,+\infty), \; l = 1,2,...,w,$$

of the climate-weather change process $C(t)$ at the particular climate-weather states c_l, $l = 1,2,...,w$. Under the above assumptions about the critical infrastructure operation process $Z(t)$, $t \in <0,+\infty)$, and the climate-weather change process $C(t)$, we introduce the joint process of critical infrastructure operation process and climate-weather change process called the critical infrastructure operation process related to climate-weather change marked by

$$ZC(t), \; t \in <0,+\infty),$$

and we assume that it can take $\nu w, \nu, w \in N$, different operation states related to the climate-weather change

$$zc_{11}, zc_{12}, \ldots, zc_{\nu w},$$

We assume that the critical infrastructure operation process related to climate-weather change $ZC(t)$, at the moment $t \in <0,+\infty)$, is at the state zc_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, if and only if at that moment, the operation process $Z(t)$ is at the operation states z_b, $b = 1,2,...,\nu$, and the climate-weather change process $C(t)$ is at the climate-weather state c_l, $l = 1,2,...,w$, what we express as follows:

$$(ZC(t) = zc_{bl}) \Leftrightarrow (Z(t) = z_b \cap C(t) = c_l),$$
$$t \in <0,+\infty), \; b = 1,2,...,\nu, \; l = 1,2,...,w.$$

Further, the transient probabilities of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states zc_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$, can be defined by

$$pq_{bl}(t) = P(ZC(t) = zc_{bl}), \; t \in <0,+\infty), \; b = 1,2,...,\nu,$$
$$l = 1,2,...,w.$$

In the case when the processess $Z(t)$ and $C(t)$ are independent the expession for the transient probabilities can be expressed in the following way

$$pq_{bl}(t) = P(ZC(t) = zc_{bl}) = P(Z(t) = z_b \cap C(t) = c_l)$$
$$= P(Z(t) = z_b) \cdot P(C(t) = c_l) = p_b(t) \cdot q_l(t),$$
$$t \in <0,+\infty), \; b = 1,2,...,\nu, \; l = 1,2,...,w,$$

where $p_b(t)$, $b = 1,2,...,\nu$, are the transient probabilities of the operation process $Z(t)$ at the particular operation startes z_b, $b = 1,2,...,\nu$, and $q_l(t)$, $l = 1,2,...,w$, are the trsansient probabilities of the climate-weather change process $C(t)$ at the particular clmate-weather startes c_l, $b = 1,2,...,w$. Hence the limit values of the transient probabilities of the critical infrastructure operation process related to climate-weather change $ZC(t)$ at the operation states zc_{bl}, $b = 1,2,...,\nu$, $l = 1,2,...,w$,

$$pq_{bl} = \lim_{t\to\infty} pq_{bl}(t), \; b = 1,2,...,\nu, \; l = 1,2,...,w, \quad (3)$$

can be found from [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017]

$$pq_{bl} = p_b q_l, \; b = 1,2,...,\nu, \; l = 1,2,...,w, \quad (4)$$

where p_b, $b = 1,2,...,\nu$, are the limit trsansient probabilities of the operation process $Z(t)$ at the particular operation startes z_b, $b = 1,2,...,\nu$, and q_l, $l = 1,2,...,w$, are the limit trsansient probabilities of the climate-weather change process $C(t)$ at the particular clmate-weather startes c_l, $b = 1,2,...,w$.

Other interesting characteristics of the critical infrastructure operation process $ZC_{bl}(t)$ are its total sojourn times $\hat{\theta}\hat{C}_{bl}$, $b=1,2,...,\nu$, $l=1,2,...,w$, at the particular operation states zc_{bl}, $b=1,2,...,\nu$, $l=1,2,...,w$, during the fixed sufficiently large critical infrastructure operation time θ. They have approximately normal distributions with the expected values given by

$$\hat{M}\hat{N}_{bl} = E[\hat{\theta}\hat{C}_{bl}] = pq_{bl}\theta, \ b=1,2,...,\nu,$$
$$l=1,2,...,w, \qquad (5)$$

where pq_{bl}, $b=1,2,...,\nu$, $l=1,2,...,w$, are defined by (3) and given by (4) in the case the processes $Z(t)$ and $C(t)$ are independent.

3. Optimization of operation and safety of critical infrastructure

3.1. Optimal transient probabilities of critical infrastructure operation process at operation states related to climate-weather change process

Considering that the coordinates of the unconditional safety function of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$,

$$\boldsymbol{S}^4(t,\cdot) = [1, \ \boldsymbol{S}^4(t,1), \ ..., \ \boldsymbol{S}^4(t,z)], \ t \in <0,\infty),$$

are given by

$$\boldsymbol{S}^4(t,u) \cong \sum_{b=1}^{\nu}\sum_{l=1}^{w} pq_{bl}[\boldsymbol{S}^4(t,u)]^{(bl)} \ \text{for} \ t \in <0,\infty),$$
$$u=1,2,...,z, \qquad (6)$$

where

$$[\boldsymbol{S}^4(t,u)]^{(bl)}, \ u=1,2,...,z, \ b=1,2,...,\nu,$$
$$l=1,2,...,w,$$

are the coordinates of the critical infrastructure impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, conditional safety functions [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017]

$$[\boldsymbol{S}^4(t,\cdot)]^{(bl)} = [1, \ [\boldsymbol{S}^4(t,1)]^{(bl)}, \ ..., \ [\boldsymbol{S}^4(t,z)]^{(bl)}],$$
$$t \in <0,\infty), \ b=1,2,...,\nu, \ l=1,2,...,w,$$

and pq_{bl}, $b=1,2,...,\nu$, $l=1,2,...,w$, are the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, at the critical infrastructure operating area limit transient probabilities at the states zc_{bl}, $b=1,2,...,\nu$, $l=1,2,...,w$, defined by (3), it is natural to assume that the critical infrastructure operation process has a significant influence on the critical infrastructure safety.

This influence is also clearly expressed in the equation for the mean lifetime of the critical infrastructure in the safety state subset $\{u,u+1,...,z\}$, $u = 1,2,...,z$, given by

$$\boldsymbol{\mu}^4(u) = \int_0^{\infty}[\boldsymbol{S}^4(t,u)]dt \ \cong \sum_{b=1}^{\nu}\sum_{l=1}^{w} pq_{bl}[\mu^4(u)]^{(bl)},$$
$$u=1,2,...,z, \qquad (7)$$

where $[\boldsymbol{\mu}^4(r)]^{(bl)}$, $u=1,2,...,z$, $b=1,2,...,\nu$, $l=1,2,...,w$, are the mean values of the critical infrastructure conditional lifetimes $[T^4(u)]^{(bl)}$, $u=1,2,...,z$, $b=1,2,...,\nu$, $l=1,2,...,w$, in the safety state subset $\{u,u+1,...,z\}$ at the critical infrastructure operating process related to the climate-weather change state zc_{bl}, $b=1,2,...,\nu$, $l=1,2,...,w$, given by

$$[\mu^4(u)]^{(bl)} = \int_0^{\infty}[\boldsymbol{S}^4(t,u)]^{(bl)}dt, \ u=1,2,...,z,$$
$$b=1,2,...,\nu, \ l=1,2,...,w. \qquad (8)$$

From the linear equation

$$\boldsymbol{\mu}^4(u) \ \cong \sum_{b=1}^{\nu}\sum_{l=1}^{w} pq_{bl}[\mu^4(u)]^{(bl)}, \ u=1,2,...,z, \qquad (9)$$

we can see that the mean value of the critical infrastructure unconditional lifetime $\boldsymbol{\mu}^4(u)$, $u=1,2,...,z$, is determined by the limit values of transient probabilities pq_{bl}, $b=1,2,...,\nu$, $l=1,2,...,w$, of the critical infrastructure operation process at the operation states given by (3) and the mean values $[\boldsymbol{\mu}^4(r)]^{(bl)}$, $u=1,2,...,z$, $b=1,2,...,\nu$, $l=1,2,...,w$, are the mean values of the critical infrastructure conditional lifetimes in the safety state subset $\{u,u+1,...,z\}$ at the critical infrastructure operating process related to the climate-weather change process zc_{bl}, $b=1,2,...,\nu$, $l=1,2,...,w$, given by (8).

Therefore, the critical infrastructure lifetime optimization approach based on the linear programming [Klabjan, Adelman, 2006], [Kołowrocki, Soszyńska-Budny, 2011] can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}q_{bl}$, $b=1,2,...,\nu$, $l=1,2,...,w,$ of the limit transient probabilities pq_{bl}, $b=1,2,...,\nu$, $l=1,2,...,w,$ of the critical infrastructure operation process at the operation states zc_{bl}, $b=1,2,...,\nu$, $l=1,2,...,w,$ to maximize the mean value $\boldsymbol{\mu}^4(u)$, $u=1,2,...,z,$ of the unconditional critical infrastructure lifetimes in the safety state subsets $\{u,u+1,...,z\}$, $u=1,2,...,z,$ under the assumption that the mean values $[\boldsymbol{\mu}^4(u)]^{(bl)}$, $u=1,2,...,z,$ $b=1,2,...,\nu$, $l=1,2,...,w,$ of the critical infrastructure conditional lifetimes in the safety state subsets $\{u,u+1,...,z\}$, $u=1,2,...,z,$ are fixed. As a special case of the above formulated critical infrastructure lifetime optimization problem, if r, $r=1,2,...,z,$ is a critical infrastructure critical safety state, we want to find the optimal values $\dot{p}q_{bl}$, $b=1,2,...,\nu$, $l=1,2,...,w,$ of the critical infrastructure operation process limit transient probabilities $\dot{p}q_{bl}$, $b=1,2,...,\nu$, $l=1,2,...,w,$ at the operation states zc_{bl}, $b=1,2,...,\nu$, $l=1,2,...,w,$ to maximize the mean value $\boldsymbol{\mu}^4(r)$ of the unconditional critical infrastructure lifetimes in the safety state subset $\{r,r+1,...,z\}$, under the assumption that the mean values $[\boldsymbol{\mu}^4(r)]^{(bl)}$, $b=1,2,...,\nu$, $l=1,2,...,w,$ of the critical infrastructure conditional lifetimes in the safety state subset $\{r,r+1,...,z\}$ are fixed. More exactly, we formulate the optimization problem as a linear programming model with the objective function of the following form

$$\boldsymbol{\mu}^4(r) \cong \sum_{b=1}^{\nu}\sum_{l=1}^{w} pq_{bl}[\mu^4(r)]^{(bl)}, \tag{10}$$

for a fixed $r\in\{1,2,...,z\}$ and with the following bound constraints

$$\breve{p}q_{bl} \le pq_{bl} \le \hat{p}q_{bl},\ b=1,2,...,\nu,\ l=1,2,...,w, \tag{11}$$

$$\sum_{b=1}^{\nu}\sum_{l=1}^{w} pq_{bl} = 1, \tag{12}$$

where

$$[\boldsymbol{\mu}^4(r)]^{(bl)},\ [\boldsymbol{\mu}^4(r)]^{(bl)} \ge 0,\ b=1,2,...,\nu,\ l=1,2,...,w, \tag{13}$$

are fixed mean values of the critical infrastructure conditional lifetimes in the safety state subset $\{r,r+1,...,z\}$ and

$$\breve{p}q_{bl},\ 0\le \breve{p}q_{bl}\le 1 \text{ and } \hat{p}q_{bl},\ 0\le \hat{p}q_{bl}\le 1,\ \breve{p}q_{bl}\le \hat{p}q_{bl},\ b=1,2,...,\nu,\ l=1,2,...,w, \tag{14}$$

are lower and upper bounds of the transient probabilities pq_{bl}, $b=1,2,...,\nu$, $l=1,2,...,w,$ respectively.

Now, we can obtain the optimal solution of the formulated by (10)-(14) the linear programming problem, i.e. we can find the optimal values $\dot{p}q_{bl}$, $b=1,2,...,\nu$, $l=1,2,...,w,$ of the transient probabilities pq_{bl}, $b=1,2,...,\nu$, $l=1,2,...,w,$ that maximize the objective function given by (10).

First, we arrange the critical infrastructure conditional lifetime mean values $[\boldsymbol{\mu}^4(r)]^{(bl)}$, $b=1,2,...,\nu$, $l=1,2,...,w,$ in non-increasing order

$$[\boldsymbol{\mu}^4(r)]^{(bl_1)} \ge [\boldsymbol{\mu}^4(r)]^{(bl_2)} \ge \ldots \ge [\boldsymbol{\mu}^4(r)]^{(bl_{\nu w})}$$

where $bl_i \in\{1,2,...,\nu w\}$ for $i=1,2,...,\nu w.$

Next, we substitute

$$x_i = p_{bl_i},\ \breve{x}_i = \breve{p}_{bl_i},\ \hat{x}_i = \hat{p}_{bl_i} \text{ for } i=1,2,...,\nu w \tag{15}$$

and we maximize with respect to x_i, $i=1,2,...,\nu w,$ the linear form (10) that after this transformation takes the form

$$\boldsymbol{\mu}^4(r) = \sum_{i=1}^{\nu w} x_i[\boldsymbol{\mu}^4(r)]^{bl_i} \tag{16}$$

for a fixed $r\in\{1,2,...,z\}$ with the following bound constraints

$$\breve{x}_i \le x_i \le \hat{x}_i,\ i=1,2,...,\nu w, \tag{17}$$

$$\sum_{i=1}^{\nu w} x_i = 1, \tag{18}$$

where

$$\boldsymbol{\mu}_{bl_i}(r),\ \boldsymbol{\mu}_{bl_i}(r)\ge 0,\ i=1,2,...,\nu w,$$

are fixed mean values of the critical infrastructure conditional lifetimes in the safety state subset $\{r,r+1,...,z\}$ arranged in non-increasing order and

$$\breve{x}_i,\ 0 \le \breve{x}_i \le 1 \text{ and } \hat{x}_i,\ 0 \le \hat{x}_i \le 1,\ \breve{x}_i \le \hat{x}_i,\quad i = 1,2,\ldots,vw, \tag{19}$$

are lower and upper bounds of the unknown probabilities x_i, $i = 1,2,\ldots,vw,$ respectively.

To find the optimal values of x_i, $i = 1,2,\ldots,vw,$ we define

$$\breve{x} = \sum_{i=1}^{vw} \breve{x}_i,\ \ \hat{y} = 1 - \breve{x} \tag{20}$$

and

$$\breve{x}^0 = 0,\ \hat{x}^0 = 0 \text{ and } \breve{x}^I = \sum_{i=1}^{I} \breve{x}_i,\ \hat{x}^I = \sum_{i=1}^{I} \hat{x}_i \quad \text{for } I = 1,2,\ldots,vw. \tag{21}$$

Next, we find the largest value $I \in \{0,1,\ldots,vw\}$ such that

$$\hat{x}^I - \breve{x}^I < \hat{y} \tag{22}$$

and we fix the optimal solution that maximize (16) in the following way:

i) if $I = 0,$ the optimal solution is

$$\dot{x}_1 = \hat{y} + \breve{x}_1 \text{ and } \dot{x}_i = \breve{x}_i \text{ for } i = 1,2,\ldots,vw; \tag{23}$$

ii) if $0 < I < vw,$ the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,\ldots,I,\ \dot{x}_{I+1} = \hat{y} - \hat{x}^I + \breve{x}^I + \breve{x}_{I+1} \text{ and } \dot{x}_i = \breve{x}_i \text{ for } i = I+2, I+3,\ldots,vw; \tag{24}$$

iii) if $I = vw,$ the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,\ldots,vw. \tag{25}$$

Finally, after making the inverse to (15) substitution, we get the optimal limit transient probabilities

$$\dot{p}q_{bl_i} = \dot{x}_i \text{ for } i = 1,2,\ldots,vw, \tag{26}$$

that maximize the critical infrastructure mean lifetime in the safety state subset $\{r, r+1,\ldots,z\},$ defined by the linear form (10), giving its maximum value in the following form

$$\dot{\boldsymbol{\mu}}^4(r) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl}[\boldsymbol{\mu}^4(r)]^{(bl)} \tag{27}$$

for a fixed $r \in \{1,2,\ldots,z\}$.

3.2. Critical infrastructure optimal safety and resilience indicators

From the expression (27) for the maximum mean value $\dot{\boldsymbol{\mu}}^4(r)$ of the critical infrastructure unconditional lifetime in the safety state subset $\{r, r+1,\ldots,z\},$ replacing in it the critical safety state r by the safety state $u,$ $u = 1,2,\ldots,z,$ we obtain the corresponding optimal solutions for the mean values of the critical infrastructure unconditional lifetimes in the safety state subsets $\{u, u+1,\ldots,z\}$ of the form

$$\dot{\boldsymbol{\mu}}^4(u) = \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_b[\boldsymbol{\mu}^4(u)]^{(bl)} \text{ for } u = 1,2,\ldots,z. \tag{28}$$

Further, according to (6), the corresponding optimal unconditional multistate safety function of the critical infrastructure is the vector

$$\dot{\boldsymbol{S}}(t,\cdot) = [1,\ \dot{\boldsymbol{S}}(t,1),\ldots,\ \dot{\boldsymbol{S}}(t,z)\ ], \tag{29}$$

with the coordinates given by

$$\dot{\boldsymbol{S}}^4(t,u) \cong \sum_{b=1}^{v}\sum_{l=1}^{w} \dot{p}q_{bl}[\boldsymbol{S}^4(t,u)]^{(bl)} \text{ for } t \in <0,\infty),\quad u = 1,2,\ldots,z. \tag{30}$$

By applying (7.23) from [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017], the corresponding optimal values of the variances of the critical infrastructure unconditional lifetimes in the critical infrastructure safety state subsets are

$$\dot{\boldsymbol{\sigma}}^{4^2}(u) = 2\int_0^\infty t\ \dot{\boldsymbol{S}}^4(t,u)dt - [\dot{\boldsymbol{\mu}}^4(u)]^2,\quad u = 1,2,\ldots,z, \tag{31}$$

where $\dot{\boldsymbol{\mu}}^4(u)$ is given by (28) and $\dot{\boldsymbol{S}}^4(t,u)$ is given by (30).

And, by (7.25) from [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017], the optimal solutions for the mean values of the critical infrastructure unconditional lifetimes in the particular safety states are

$$\dot{\bar{\boldsymbol{\mu}}}^4(u) = \dot{\boldsymbol{\mu}}^4(u) - \dot{\boldsymbol{\mu}}^4(u+1),\ u = 1,\ldots,z-1,\quad \dot{\bar{\boldsymbol{\mu}}}^4(z) = \dot{\boldsymbol{\mu}}^4(z). \tag{32}$$

Moreover, considering (7.7) and (7.12) from [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017], the corresponding optimal critical infrastructure risk function and the optimal moment when the risk exceeds a permitted level δ, respectively are given by

$$\dot{\boldsymbol{r}}^4(t) = 1 - \dot{\boldsymbol{S}}^4(t,r),\ t \in <0,\infty), \quad (33)$$

and

$$\dot{\boldsymbol{\tau}}^4 = \dot{\boldsymbol{r}}^{4^{-1}}(\delta), \quad (34)$$

where $\dot{\boldsymbol{S}}^4(t,r)$ is given by (30) for $u = r$ and $\dot{\boldsymbol{r}}^{4^{-1}}(t)$, if it exists, is the inverse function of the optimal risk function $\dot{\boldsymbol{r}}^4(t)$.

The optimal intensities of degradation of the critical infrastructure / the optimal intensities of critical infrastructure departure from the safety state subset $\{u,u+1,...,z\}$, u = 1,2,...,z, impacted by the operation process related to the climate-weather change (SafI9), i.e. the coordinates of the vector

$$\dot{\boldsymbol{\lambda}}^4(t,\cdot) = [0,\ \dot{\boldsymbol{\lambda}}^4(t,1),\ \dots,\ \dot{\boldsymbol{\lambda}}^4(t,z)\ ],$$
$$t \in <0,+\infty), \quad (35)$$

are given by

$$\dot{\boldsymbol{\lambda}}^4(t,u) = \frac{-\dfrac{d\dot{\boldsymbol{S}}^4(t,u)}{dt}}{\dot{\boldsymbol{S}}^4(t,u)},\ t \in <0,+\infty),$$
$$u = 1,2,...,z. \quad (36)$$

The optimal coefficients of the operation process related to the climate-weather change impact on the critical infrastructure intensities of degradation / the coefficients of the operation process related to the climate-weather change impact on critical infrastructure intensities of departure from the safety state subset $\{u,u+1,...,z\}$ (SI10), i.e. the coordinates of the vector are given by

$$\dot{\boldsymbol{\rho}}^4(t,\cdot) = [0,\ \dot{\boldsymbol{\rho}}^4(t,1),\ \dots,\ \dot{\boldsymbol{\rho}}^4(t,z)\,],$$
$$t \in <0,+\infty), \quad (37)$$

where

$$\dot{\boldsymbol{\lambda}}^4(t,u) = \dot{\boldsymbol{\rho}}^4(t,u) \cdot \boldsymbol{\lambda}^0(t,u),\ t \in <0,+\infty),$$
$$u = 1,2,...,z, \quad (38)$$

i.e.

$$\dot{\boldsymbol{\rho}}^4(t,u) = \frac{\dot{\boldsymbol{\lambda}}^4(t,u)}{\boldsymbol{\lambda}^0(t,u)},\ t \in <0,+\infty),\ u = 1,2,...,z, \quad (39)$$

and $\boldsymbol{\lambda}^0(t,u)$, $t \in <0,+\infty)$, $u = 1,2,...,z$, are the intensities of degradation of the critical infrastructure without of the operation process related to the climate-weather change impact, i.e. the coordinate of the vector [Kołowrocki et al., EU-CIRCLE Report D3.3-Part3, 2017]

$$\boldsymbol{\lambda}^0(t,\cdot) = [0,\ \boldsymbol{\lambda}^0(t,1),,\ \dots,\boldsymbol{\lambda}^0(t,z)\,],\ t \in <0,+\infty), \quad (40)$$

and $\dot{\boldsymbol{\lambda}}^4(t,u)$, $t \in <0,+\infty)$, $u = 1,2,...,z$, are the optimal intensities of degradation of the critical infrastructure impacted by the operation process related to the climate-weather change, i.e. the coordinate of the vector

$$\dot{\boldsymbol{\lambda}}^4(t,\cdot) = [0,\ \dot{\boldsymbol{\lambda}}^4(t,1),\dots,\ \dot{\boldsymbol{\lambda}}^4(t,z)\,],\ t \in <0,+\infty). \quad (41)$$

The optimal indicator of critical infrastructure resilience to operation process related to climate-weather change impact (ResI4) is given by

$$\dot{\boldsymbol{RI}}^4(t,r) = \frac{1}{\dot{\rho}^4(t,r)},\ t \in <0,+\infty), \quad (42)$$

where $\dot{\boldsymbol{\rho}}^4(t,r)$, $t \in <0,+\infty)$, is the optimal coefficients of operation process related to climate-weather change impact on the critical infrastructure intensities of degradation given by (39) for $u = r$.

3.3. Optimal sojourn times of critical infrastructure operation process at operation states related to climate-weather change process and operation strategy

Assuming that the critical infrastructure operation process and the climate-weather change process are independent, i.e.

$$\dot{p}q_{bl} = \dot{p}_b \cdot q_l,\ b = 1,2,...,v,\ l = 1,2,...,w,$$

and replacing in (2.22) from [Kołowrocki, Soszyńska-Budny, 2011], the limit transient probabilities p_b of the critical infrastructure operation process at the operation states by

$$\dot{p}_b = \frac{\dot{p}b_{bl}}{b_l} = \frac{\dot{p}_b \cdot b_l}{b_l} = \dot{p}_b,\ l = 1,2,...,w,$$

$$b = 1,2,\ldots,v, \tag{43}$$

where

$$\dot{p}q_{bl} = \dot{p}_b \cdot q_l, \ b = 1,2,\ldots,v, \ l = 1,2,\ldots,w,$$

are the optimal values of transient probabilities pq_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, found by the equations (26) in Section 2.1 and the mean values M_b of the unconditional sojourn times at the operation states z_b, $b = 1,2,\ldots,v$, defined by (2.21) in [Kołowrocki, Soszyńska-Budny, 2011] by their corresponding unknown optimal values $\dot{M}_b$ maximizing the mean value of the critical infrastructure lifetime in the safety states subset $\{r, r+1,\ldots,z\}$ defined by (10), we get the critical infrastructure of equations

$$\dot{p}_b = \frac{\pi_b \dot{M}_b}{\sum_{l=1}^{v} \pi_l \dot{M}_l}, \ b = 1,2,\ldots,v. \tag{44}$$

Another very useful and much easier to be applied in practice tool that can help in planning more reliable and safe operation process of the critical infrastructures are the optimal mean values of the total critical infrastructure operation process sojourn times $\hat{\theta}_b$ at the particular operation states z_b, $b = 1,2,\ldots,v$, during the fixed critical infrastructure operation time θ, that can be obtained by the replacing in the formula (2.24) from [Kołowrocki, Soszyńska-Budny, 2011], the transient probabilities p_b at the operation states z_b by their optimal values $\dot{p}_b$ and resulting in the following expression

$$\dot{\hat{M}}_b = \dot{E}[\hat{\theta}_b] = \dot{p}_b \theta, \ b = 1,2,\ldots,v. \tag{45}$$

Similarly, replacing in the formula (5) the transient probabilities pq_{bl} at the operation states z_{bl} by their optimal values $\dot{p}q_{bl}$, we get the optimal mean values of the total critical infrastructure operation process sojourn times $\hat{\theta}\hat{C}_{bl}$ at the particular operation states zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, during the fixed critical infrastructure operation time θ, given by

$$\dot{\hat{M}}\hat{N}_{bl} = \dot{E}[\hat{\theta}\hat{C}_{bl}] = \dot{p}q_{bl}\theta, \ b = 1,2,\ldots,v,$$
$$l = 1,2,\ldots,w. \tag{46}$$

The knowledge of the optimal values $\dot{M}_b$ of the mean values of the unconditional sojourn times and the optimal values $\dot{M}_{bl}$ of the mean values of the conditional sojourn times at the operation states and the optimal mean values $\dot{\hat{M}}_b$ of the total sojourn times at the particular operation states z_b, $b = 1,2,\ldots,v$, and the optimal mean values $\dot{\hat{M}}\hat{N}_{bl}$ of the total sojourn times at the particular operation states related to climate-weather change zc_{bl}, $b = 1,2,\ldots,v$, $l = 1,2,\ldots,w$, during the fixed critical infrastructure operation time may by the basis for changing the critical infrastructures operation processes in order to ensure these critical infrastructures operation more safe. Their knowledge may also be useful in these critical infrastructures operation cost analysis.

4. Conclusions

The proposed optimization method presented in this paper can be used in critical infrastructure operation and safety optimization with considering climate-weather change impact. The optimization method application can be the basis for the elaboration of practical procedures of critical infrastructure safety improvement. The optimization model will be applied in Case Study 2 supported by suitable computer software that is placed at the GMU Safety Interactive Platform http://gmu.safety.am.gdynia.pl/. The GMU platform can be linked with CIRP and SimICI platforms of EU-CIRCLE project.

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